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ORIGINAL ARTICLE

Consistencies of the capability indices based on the normal probability distribution

Consistências dos índices de capacidade baseados na distribuição normal de probabilidade

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Abstract: Capability analysis seeks to estimate the probability that a process will produce compliant products. The capability indices are dimensionless parameters that measure how well the process can meet specifications. In the literature, eight capability indices are listed, among others, considering a stable process under statistical control and based on the normal probability distribution, defined by: C_p, P_p, C_{pk}, P_{pk}, C_{pm}, P_{pm}, C_{pmk}, and P_{pmk}. Basically, the index formulas differ in the calculations of the variability within and total, and of the shifts of the mean in relation to the nominal value and the nearest specification limit. The objective of this article was to compare these capacity indexes, and for that, it was chosen the most consistent estimator, that is, the one that improved the accuracy and efficiency as the number of observations increased. Thus, a simulation of 30,000 values of a normal random variable with a mean equal to zero and a standard deviation equal to one was performed. This made it possible to sample this process 1,000 times using 5, 10, 15, 20, 25, and 30 rational subgroups with individual observations or sample elements. Subsequently, 20 mean shifts were provoked, with values ranging from 0.1 to 2 and varying by 0.1 unit. According to the results, it was concluded that the indexes C_{pk} and P_{pk} were the most consistent in presenting higher accuracy and efficiency for at least 15 rational subgroups or sample elements, regardless of the magnitude of the mean displacement in relation to the nominal value.

Keywords: Capability index; Estimator; Quality control.

Resumo: A análise de capacidade busca estimar a probabilidade de um processo produzir produtos em conformidade. Os índices de capacidade são parâmetros adimensionais que medem o quanto o processo consegue atender às especificações. Na literatura são listados, além de outros, oito índices da capacidade, considerando um processo estável sob controle estatístico e baseado na distribuição normal de probabilidades, definidos por: C_p, P_p, C_{pk}, P_{pk}, C_{pm}, P_{pm}, C_{pmk}, e P_{pmk}. Basicamente, as fórmulas dos índices se diferenciam nos cálculos da variabilidade dentro e total, e dos deslocamentos da média em relação ao valor nominal e ao limite de especificação mais próximo. O objetivo deste artigo foi comparar estes índices de capacidade, e para isso, buscou-se escolher o estimador mais consistente, ou seja, que melhora a acurácia e a eficiência à medida que se aumenta o número de observações. Desse modo, foi realizada uma simulação de 30.000 valores de uma variável aleatória

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normal com média igual a zero e desvio-padrão igual a um. Isso possibilitou amostrar este processo em 1.000 vezes utilizando-se, para isso, 5, 10, 15, 20, 25 e 30 subgrupos racionais com observações individuais ou elementos amostrais. Posteriormente, foram provocados 20 deslocamentos da média, com valores de 0,1 a 2 e variando 0,1 unidade. De acordo com os resultados, concluiu-se que os índices C_{pk} e P_{pk} foram os mais consistentes, por apresentarem maiores acurácias e eficiências para pelo menos 15 subgrupos racionais ou elementos amostrais, independentemente da magnitude do deslocamento da média em relação ao valor nominal.

Palavras-chave: Índice de capacidade; Estimador; Controle de qualidade.

1 Introduction

For a product to be considered of quality, it is necessary that it meet the customer's needs and expectations; that is, the specifications. For this, it needs to be produced by a process that is stable or replicable and capable of producing products with pre-defined nominal values and little variability. In this context, Statistical Process Control (SPC) is widely used to obtain process stability and to improve capacity by reducing variability (Montgomery, 2019).

Control charts are the main statistical methods of SPC for analyzing data from sampling, replacing the mere detection and correction or exchange of defective products by the study and prevention of problems related to quality, aiming to prevent defective products from being produced (Souza et al., 2014). A process is under statistical control or stable when the variability is associated only with random causes.

Once the process is under statistical control, one can evaluate how well the process is able to generate products that meet the specifications. For this, the capability indices seek to detect whether the process meets, on average, the specification nominal value and, in relation to variability, whether it presents dispersion that meets the specifications that are established in the process (Gonçalez & Werner, 2009).

The variability caused by the process can be estimated by forming rational subgroups with individual observations or with repetitions, as proposed by Shewhart. In this case, the capability indices are referred to by the letter C. On the other hand, with or without the use of rational subgroups, when the estimate of the standard deviation is obtained by means of all sampled values, from the total variation, the capability indices are referred to by the letter P.

From this, it is possible to estimate process capability through the following indexes: C_p or P_p that consider the mean centered on the nominal value; C_{pk} or P_{pk} that consider where the mean is located in relation to the specification limits; C_{pm} or P_{pm} that include the expected quadratic deviation from the nominal value; and C_{pmk} or P_{pmk} that include the restrictions of indexes C_{pk} and P_{pk} with those of indexes C_{pm} and P_{pm}. For all of them, process stability and normality of the random variable are required. Souza et al. (2014) reported that, depending on the index used, the conclusions about the process capability can be differentiated, demonstrating the importance of choosing appropriate capability indices according to the behavior of each process.

Given the importance of capability indices for CEP, several authors have compared capability indices, described guidelines for their use, analyzed theory and practice, among other studies. Some important references are: Pearn et al. (1992), Kushler & Hurley (1992), Kotz et al. (1993), Vannman (1995), Pearn et al. (1998), Stoumbos (2002), Parchami & Mashinchi (2007), Wu et al. (2009), Miao et al. (2011), Souza et al. (2014), Álvarez et al. (2015) and Wang et al. (2021).

In this context, confirms the importance to contribute in the deepening of the theme and the importance of choosing an index that enables the closest possible estimate of its true capacity, since there are eight capability indices that provide different estimates for the capability of the same process. So, the objective of this work was to analyze the consistency of the eight indices of process capacity under stability and normality conditions. Consistency is a property that allows us to evaluate if the estimates of the capability indexes get closer to the true value of the capability (parameter) as the sample size increases.

2 Theoretical reference

2.1 Process capability

Any production process will always contain natural or inherent variability, that is, variation that happens due to random or common causes, which are not amenable to control (Montgomery, 2019). However, there are special causes that are targeted by CEP, i.e., major disturbances that increase the variability of the process and can be identified and eliminated.

Control charts, initially idealized by Shewhart, are the main statistical methods of CEP used for monitoring the mean and variability of one or more characteristics evaluated in products or services that respond to the quality of the process (Ribeiro, 2013). In Figure 1 is shown a scheme with the steps of CEP, in which control charts are used to monitor the quality of a process and determine whether it is in a state of statistical control (stable), which would indicate that its production has a variation due only to random causes (Álvarez et al., 2015).

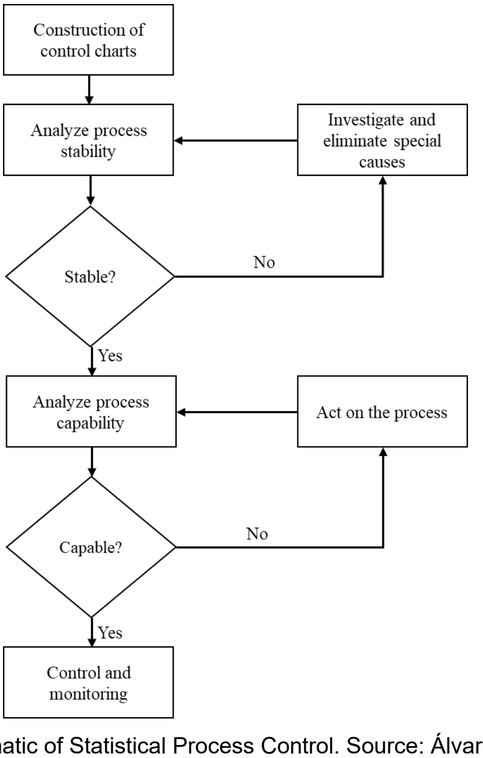

Figure 1. Schematic of Statistical Process Control. Source: Álvarez et al. (2015).

A process under statistical control or stable is one that has variability associated only with random causes; that is, it follows a predictable pattern over time. However, this stable process pattern may or may not be able to produce products that meet customer or project specifications. Once the special causes are eliminated, one can then evaluate the real capability of the process by comparing its variability (associated only with random causes) with the specifications (Ribeiro & Caten, 2012).

When the process is out of statistical control or is unstable, that is, when there are, besides random causes, special causes, the evaluation of its capacity is irrelevant, because it reflects only a certain moment, since the process does not present a predictable behavior. Therefore, the evaluation of the capability of a stable process is used to verify whether or not it meets the specifications established for its products. Therefore, this evaluation will represent its ability to produce them in accordance with the specification, i.e., the ability of the process to produce quality products or services (Ribeiro, 2013).

If the variability due to random causes is excessive, that is, greater than the specification range comprised by the lower specification limits (LEL) and upper specification limits (OSL), the process is said to be not capable, and management must act on it. If the inherent variability of the process is smaller than the specification range, the process is said to be capable. In this case, you can measure the capability of the process by means of indexes.

2.2 Capacity key figures

The objective of a capability analysis is to evaluate how well a process produces products within the specification range, comprised of the LEL and LSE (Equation 1). Therefore, the probability of products not conforming to specification with respect to a random variable Y that follows a normal distribution, given by $Y \sim N(\mu; \sigma^2)$, is obtained by:

$$P(nconf) = P(Y < LIE) + P(Y > LSE) \tag{1}$$

Otherwise, as a function of the parameter (θ) of the capacity index, we have:

$$P(nconf) = 2\Phi(-3\theta) \tag{2}$$

As can be seen in Equation 2, the capability index parameter is expressed in terms of a process that seeks to bring 6σ of variation within the specification range. When the capability index parameter equals one, you have a 3σ process in which 99.73% of the products will conform. When it is 1.33, you have a 4σ process with 99.994% of the products conforming. In this context, it is very common to consider a process capable when the parameter of the capability index is greater than or equal to 1.33.

The capability indices are dimensionless parameters that allow evaluating how much a process produces products that meet the specification (Ribeiro, 2013). To enable the estimation, it is necessary, as already mentioned, that the variable of interest has independent values and a normal distribution with mean (μ) and standard deviation (σ) (Werkema, 1995; Rodrigues, 2001).

When the estimate of σ is obtained by calculations related to the construction of Shewhart control charts for monitoring variability due to the formation of rational subgroups with individual observations or with repetitions, the capability indices are referred to by the letter C: C_p, C_{pk}, C_{pm}, and C_{pmk}. These consider the short-term variation or variation within, which is estimated based on the average variation occurring within the rational subgroups. When the estimate of σ is obtained directly by means of all values of the variable, with or without the formation of rational subgroups, from the total variation or long-term variation, the capability indexes are referred to by the letter P: P_p, P_{pk}, P_{pm}, and P_{pmk} (Ribeiro, 2013).

When only one observation per rational subgroup is considered, the first estimate of, σ, called within standard deviation (s_D), is obtained by means of Equation 3, where $\overline{am}$ is the estimate of the mean of the moving amplitudes as presented in Equation 4, where m is the number of rational subgroups and d_2 is the tabulated constant, which is 1.128. In this case, we have:

$$s_D = \frac{\overline{am}}{d_2} \tag{3}$$

$$\overline{am} = \frac{\sum_{i=2}^{m} |y_i - y_{i-1}|}{m-1} \tag{4}$$

The second estimate of, σ, called total standard deviation (s_T), is obtained from all values of the random variable (Y) of a process, as shown in Equation 5, where n is the number of values of the sample and $\overline{y}$ is the estimate of the mean μ given by $\overline{y} = \sum_{i=1}^{n} y_i / n$. In this case, one has:

$$s_T = \sqrt{\frac{\sum_{i=1}^{n} (y_i - \overline{y})^2}{n-1}} \tag{5}$$

To evaluate the process capability, at least three criteria are considered: (i) the process variability; (ii) the distance of the process average in relation to the nominal value (VN); and (iii) the distance of the average to the nearest specification limit (LIE or LSE). Therefore, the estimates of the capability indices can be obtained through the equations presented in Square 1.

Square 1. Estimates of the capacity indices.

Standard deviation within	Total standard deviation
$\hat{C}_p = \frac{LSE - LIE}{6 s_D}$	$\hat{P}_p = \frac{LSE - LIE}{6 s_T}$
$\hat{C}_{pk} = minimum\left(\frac{LSE - \overline{y}}{6 s_D}, \frac{\overline{y} - LIE}{6 s_D}\right)$	$\hat{P}_{pk} = minimum\left(\frac{LSE - \overline{y}}{6 s_T}, \frac{\overline{y} - LIE}{6 s_T}\right)$
$\hat{C}_{pm} = \frac{LSE - LIE}{6\sqrt{{s_D}^2 + (\overline{y} - VN)^2}}$	$\hat{P}_{pm} = \frac{LSE - LIE}{6\sqrt{{s_T}^2 + (\overline{y} - VN)^2}}$
$\hat{C}_{pmk} = minimum\left(\frac{LSE - \overline{y}}{3\sqrt{{s_D}^2 + (\overline{y} - VN)^2}}, \frac{\overline{y} - LIE}{3\sqrt{{s_D}^2 + (\overline{y} - VN)^2}}\right)$	$\hat{P}_{pmk} = minimum\left(\frac{LSE - \overline{y}}{3\sqrt{{s_T}^2 + (\overline{y} - VN)^2}}, \frac{\overline{y} - LIE}{3\sqrt{{s_T}^2 + (\overline{y} - VN)^2}}\right)$

The estimates of the capability indices Cp and Pp, given by $\hat{C}_p$ and $\hat{P}_p$, respectively, consider that the process is centered on the nominal value of the specification, i.e., μ = VN. These indices relate only the variability allowed to the process to the natural variability provided by the process (Barreto et al., 2016).

The estimates of the capability indices C_{pk} ($\hat{C}_{pk}$) and P_{pk} ($\hat{P}_{pk}$) on the other hand, take into account the distance from the process mean μ to the nearest specification limit. When the process is centered on the specification nominal value, one has: C_p C_{pk} and P_p P_{pk}. Otherwise, if C_p C_{pk} or P_p P_{pk}, the process is off-center and the mean μ does not coincide with the nominal specification value (Kane, 1986). The capability indices C_{pm} and P_{pm} include the expected squared deviation from the nominal value as a way of considering the distance of the mean μ from it.

And finally, the estimates of the capability indices C_{pmk} and P_{pmk}, that is, $\hat{c}_{pmk}$ and $\hat{p}_{pmk}$, respectively, further restrict the evaluations, since they consider the smallest distance between the mean μ of the process from the specification limits and the expected quadratic deviation from the nominal value (Gonçalez & Werner, 2009). For Chen & Ding (2001), this shows that the capability indices C_{pmk} and P_{pmk} are the most sensitive in detecting the possible violations that may be occurring in the process and, therefore, will provide lower estimates.

This means that there are several ways to estimate the capacity of a process. In the search for articles that compare the performances of process capability indices, the following were identified: Kotz et al. (1993), Mittag & Germany (1997), Tang & Than (1999), Gonçalez & Werner (2009), Álvarez et al. (2015), Dianda et al. (2016) and Riaz & Hamid (2016). The existence of other studies confirms the relevance of the topic, since there are eight capacity indices with different formulas, which therefore provide different estimates for the capacity of a process. In this context, given that there are eight estimators to estimate the same parameter, it is important that these estimators are accurate, efficient and consistent, which are desirable properties of estimators. Among the identified works, in addition to others listed by Yum & Kim (2011), no articles were identified that analyzed and compared the consistency of process capability indices with normal distribution.

According to Devore (2006), starting with the definition of a parameter of interest, the goal of estimation is to use a sample to calculate a number that provides, in a sense, a good prediction of the parameter. In other words, according to Montgomery & Runger (2018), a point estimate of some parameter of a population is a numerical value that can be considered a sensible value for the parameter. To obtain a point estimate, one must select a suitable formula (estimator) and from it calculate its value using the sample data. Thus, the basic estimation problem is to determine the formula (estimator) that best estimates the parameter.

An estimator $\hat{\theta}$ is said to be an accurate estimator, that is, non-biased, non-trending, or unbiased estimator of the population parameter θ if the mathematical hope of $\hat{\theta}$ is equal to θ, that is, if $E(\hat{\theta}) = \theta$. In other words, an accurate estimator is one in which the mean is exactly on the "target".

Although accuracy is a desirable quality for estimators, it is not the only property for selecting an estimator. Another desirable property is that the estimator is efficient—that is, that it has a minimum variance. If $\hat{\theta}_1$ and $\hat{\theta}_2$ are two accurate estimators of the same parameter, and if $\hat{\theta}_1$ is more efficient than $\hat{\theta}_2$, then it follows that $V(\hat{\theta}_1) < V(\hat{\theta}_2)$, that is, that the variance of $\hat{\theta}_1$ is smaller than the variance of $\hat{\theta}_2$.

However, accuracy and efficiency may depend on sampling. Therefore, a consistent estimator is one that focuses completely on its "target" as the sample size (n) increases indefinitely. If $\{\hat{\theta}_n\}$ is a sequence of estimators of θ, if $\lim_{n\to\infty} E(\hat{\theta}n) = \theta$ and if $\lim_{n\to\infty} V(\hat{\theta}n) = 0$, then $\hat{\theta}$ is a consistent estimator of θ. Consequently, there will be a smaller n-value associated with sampling that is appropriate for the technical objectives.

3 Methodology

3.1 Setting the parameters

To evaluate the process capability, the following specification interval was defined: LIE = -4, VN = 0 and LSE = 4. On the other hand, 21 stable processes were established, with 21 different means (μ) and σ = 1 for a random variable Y that follows normal distribution. In Table 1 are presented in parametric terms, the means, the probabilities of non-compliance and the capacities of the respective processes. And in Figure 2, their normal distributions are visualized.

Table 1. Averages, probabilities of non-compliance and process capability (θ).

μ	P(nconf)	θ	μ	P(nconf)	θ
0	0.000063	1.3333	1.1	0.001866	1.0369
0.1	0.000069	1.3268	1.2	0.002555	1.0056
0.2	0.000086	1.3093	1.3	0.003467	0.9743
0.3	0.000116	1.2846	1.4	0.004661	0.9432
0.4	0.000165	1.2560	1.5	0.006210	0.9122
0.5	0.000236	1.2257	1.6	0.008198	0.8813
0.6	0.000339	1.1945	1.7	0.010724	0.8505
0.7	0.000485	1.1630	1.8	0.013903	0.8199
0.8	0.000688	1.1314	1.9	0.017864	0.7895
0.9	0.000968	1.0999	2.0	0.022750	0.7592
1.0	0.001350	1.0684			

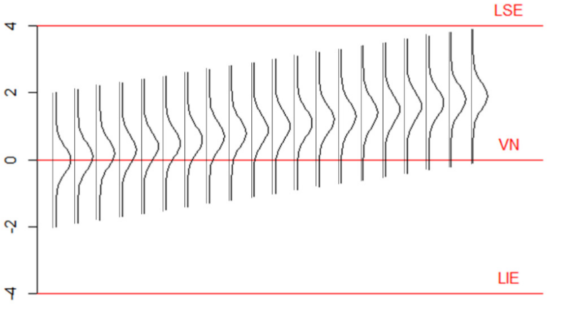

Figure 2. Normal distributions with means (μ) equal to 0; 0.1; 0.2; 0.3; 0.4; 0.5; 0.6; 0.7; 0.8; 0.9; 1.0; 1.1; 1.2; 1.3; 1.4; 1.5; 1.6; 1.7; 1.8; 1.9; 2.0 and standard deviation (σ) equal to 1.

To obtain the within standard deviation (s_D) by means of Shewhart control charts, we considered 5, 10, 15, 20, 25, and 30 rational subgroups with individual observations (m) in each rational subgroup. As presented by Souza et al. (2008), the estimate of, σ, both for rational subgroups with individual observations or with repetitions, approximates the true parameter σ in the absence of special causes. On the other hand, the total standard deviation (s_T) was obtained considering sample sizes (n), without the formation of rational subgroups, equal to 5, 10, 15, 20, 25, and 30, respectively.

3.2. Simulation data

The study was conducted by simulating 30,000 values with a mean (μ) equal to zero and a standard deviation (σ) equal to one, that is, Y ~ N (0; 1), using the Microsoft Excel, 2013 version. These values were organized in a spreadsheet with 1,000 rows and 30 columns as shown in Table 2. The rows (i) represent the quantities of analyses performed for the process with μ = 0 and the columns, the rational subgroups (m), or sample sizes (n), for m, n = 5, 10, 15, 20, 25, and 30.

Table 2. Algebraic representation of the simulation data.

i	m, n														
	1	**2**	**3**	**4**	**5**	**...**	**10**	**...**	**15**	**...**	**20**	**...**	**25**	**...**	**30**
1	$y_{1.1}$	$y_{1.2}$	$y_{1.3}$	$y_{1.4}$	$y_{1.5}$	…	$y_{1.10}$	…	$y_{1.15}$	…	$y_{1.20}$	…	$y_{1.25}$	…	$y_{1.30}$
2	$y_{2.1}$	$y_{2.2}$	$y_{2.3}$	$y_{2.4}$	$y_{2.5}$	…	$y_{2.10}$	…	$y_{2.15}$	…	$y_{2.20}$	…	$y_{2.25}$	…	$y_{2.30}$
3	$y_{3.1}$	$y_{3.2}$	$y_{3.3}$	$y_{3.4}$	$y_{3.5}$	…	$y_{3.10}$	…	$y_{3.15}$	…	$y_{3.20}$	…	$y_{3.25}$	…	$y_{3.30}$
4	$y_{4.1}$	$y_{4.2}$	$y_{4.3}$	$y_{4.4}$	$y_{4.5}$	…	$y_{4.10}$	…	$y_{4.15}$	…	$y_{4.20}$	…	$y_{4.25}$	…	$y_{4.30}$
…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…
1000	$y_{1000.1}$	$y_{1000.2}$	$y_{1000.3}$	$y_{1000.4}$	$y_{1000.5}$	…	$y_{1000.10}$	…	$y_{1000.15}$	…	$y_{1000.20}$	…	$y_{1000.25}$	…	$y_{1000.30}$

The study was conducted by simulating 30,000 values with a mean (μ) equal to zero and a standard deviation (σ) equal to one, that is, Y ~ N (0; 1). These values were organized in a spreadsheet with 1,000 rows and 30 columns as shown in Table 2. The rows (i) represent the quantities of analyses performed for the process with μ = 0 and the columns, the rational subgroups (m), or sample sizes (n), for m, n = 5, 10, 15, 20, 25, and 30.

Equation 6 shows the estimate of the mean for each process sampled with 5, 10, 15, 20, 25, and 30 rational subgroups with individual observations (m) or sampled elements (n), separately, in the analysis of order i (i = 1, 2, ..., 1000). Thus, we have:

$$\overline{y}_i = \frac{\sum_{j=1}^{m} y_{ij}}{m} \text{ or } \overline{y}_i = \frac{\sum_{j=1}^{n} y_{ij}}{n} \tag{6}$$

Equation 7 and Equation 8 show the within (s_D) and total (s_T) standard deviations of each process considering m, n = 5, 10, 15, 20, 25, and 30, separately, in the analysis of order i (i = 1, 2,..., 1000). Thus, we have:

$$s_{Di} = \frac{\overline{am_i}}{1{,}128}, \text{ for } \overline{am_i} = \frac{\sum_{j=2}^{m} |y_{ij} - y_{i(j-1)}|}{m-1} \tag{7}$$

$$s_{T_i} = \sqrt{\frac{\sum_{j=1}^{n} (y_{ij} - \overline{y_i})^2}{n-1}} \tag{8}$$

3.3 Capacity key figures

The estimates of the eight capability indices were obtained separately, for the 21 stable processes in each combination between the i-order analysis (i = 1, 2, ..., 1000), with the number of rational subgroups or sampled elements (m, n = 5, 10, 15, 20, 25, and 30) as presented in Square 2.

Square 2. Estimates of the capability indices, for i = 1, 2, ... 1000.

Standard deviation within	Total standard deviation
$\hat{C}_{pi} = \frac{4-(-4)}{6s_{Di}}$	$\hat{P}_{pi} = \frac{4-(-4)}{6s_{Ti}}$
$\hat{C}_{pki} = minimum\left(\frac{4-\overline{y}_i}{6s_{Di}}, \frac{\overline{y}_i-(-4)}{6s_{Di}}\right)$	$\hat{P}_{pki} = minimum\left(\frac{4-\overline{y}_i}{6s_{Ti}}, \frac{\overline{y}_i-(-4)}{6s_{Ti}}\right)$
$\hat{C}_{pmi} = \frac{4-(-4)}{6\sqrt{s_{Di}^2 + (\overline{y}_i - 0)^2}}$	$\hat{P}_{pmi} = \frac{4-(-4)}{6\sqrt{s_{Ti}^2 + (\overline{y}_i - 0)^2}}$
$\hat{C}_{pmki} = minimum\left(\frac{4-\overline{y}_i}{3\sqrt{s_{Di}^2 + (\overline{y}_i - 0)^2}}, \frac{\overline{y}_i-(-4)}{3\sqrt{s_{Di}^2 + (\overline{y}_i - 0)^2}}\right)$	$\hat{P}_{pmki} = minimum\left(\frac{4-\overline{y}}{3\sqrt{s_{Ti}^2 + (\overline{y}_i - 0)^2}}, \frac{\overline{y}-(-4)}{3\sqrt{s_{Ti}^2 + (\overline{y}_i - 0)^2}}\right)$

After the estimates were obtained, dispersion diagrams were constructed with Microsoft Excel so that it was possible to visualize the behavior of the eight process capability indices as a function of the 20 displacements of the averages in relation to the nominal value (VN = 0) for each number of rational subgroups or sample sizes. The dispersion diagrams were made based on the averages of 1000 analyses of the referred scenario.

For each capacity index, we estimated the bias obtained by the difference of the average of 1000 estimates in relation to the true parameter, i.e., $bias = \hat{\theta} - \theta$. In addition, the coefficient of variation (CV) was estimated, obtained by the standard deviation divided by the average of 1000 estimates for each capability index. And finally, the consistency of each was estimated in each of the 21 processes as a function of the increase from 5 to 30 rational subgroups or sample elements. For this, the reduction of biases and CVs were observed in terms of magnitudes. In this work, we chose to use the CV instead of the variance, to exclude the influence of the different magnitudes of the estimates of the eight capability indices.

4 Results

4.1 Capability indices

Figure 3 shows the estimates of the eight capability indices as a function of the shifts of the means relative to the nominal value (VN = 0) for each number of rational subgroups (m) or sample elements (n). In it, the dotted red line represents the true and respective parameters.

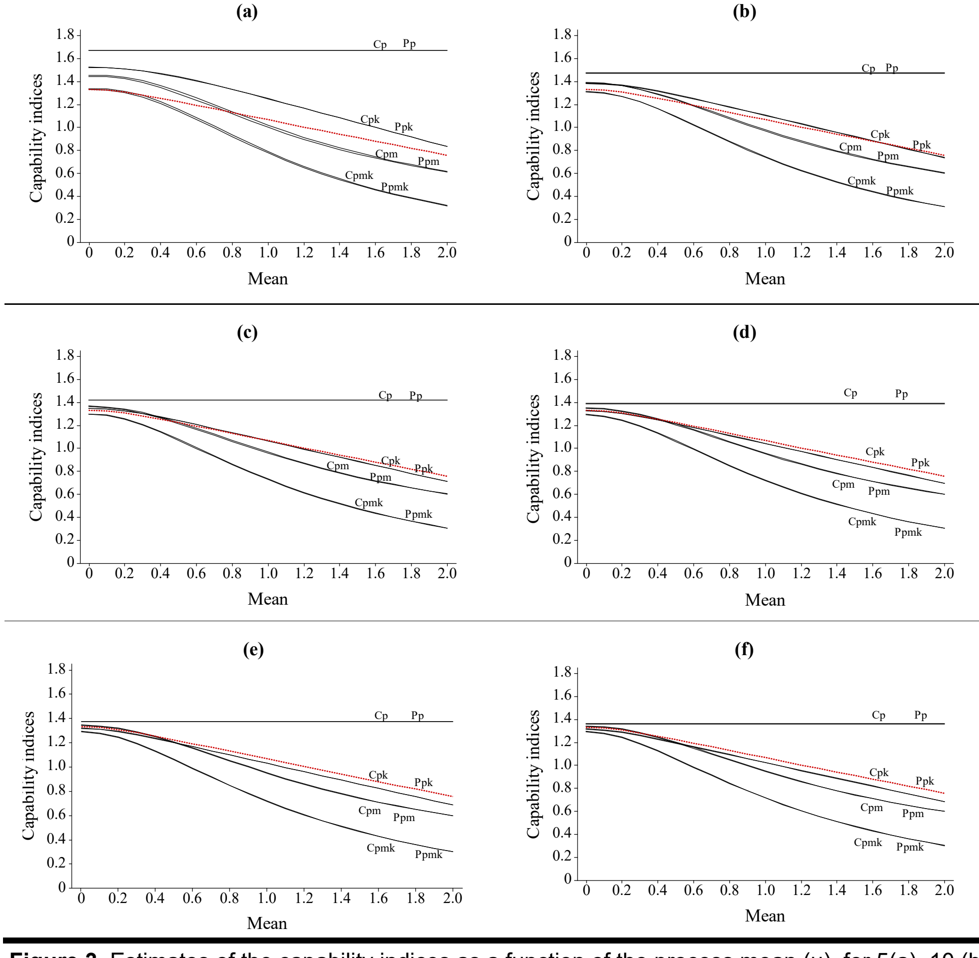

Figure 3. Estimates of the capability indices as a function of the process mean (μ), for 5(a), 10 (b), 15 (c), 20 (d), 25 (e) and 30 (f) rational subgroups (m) or sample elements (n).

It is possible to see that the within (s_D) and total (s_T) standard deviations did not interfere in the estimates of the capability indices. This can be observed since, in all scatterplots, the index pairs C_p and P_p, C_{pk} and P_{pk}, C_{pm} and P_{pm}, and C_{pmk} and P_{pmk} showed overlapping results (Figure 3).

The estimates of the capability indices C_p and P_p do not change with the shifts of the process averages. This is because, by definition, they assume that the meaning of the process is centered on the nominal value. Therefore, since they will provide wrong estimates that are larger than their respective parameters, the C_p and P_p indexes are considered theoretical because they measure the potential of each process.

On the other hand, the capability indices C_{pmk} and P_{pmk}, although they, too, provided wrong estimates, were lower than the respective parameters. In this case, they can be interpreted as the worst that each process can behave.

However, the estimates of the capability indices C_{pk} and P_{pk}, followed by C_{pm} and P_{pm}, were the closest to the respective parameters.

4.2 Bias

Figure 4 shows the biases of the estimates of the eight capability indices as a function of the shifts of the means relative to the nominal value (VN = 0) for each number of rational subgroups (m) or sample elements (n).

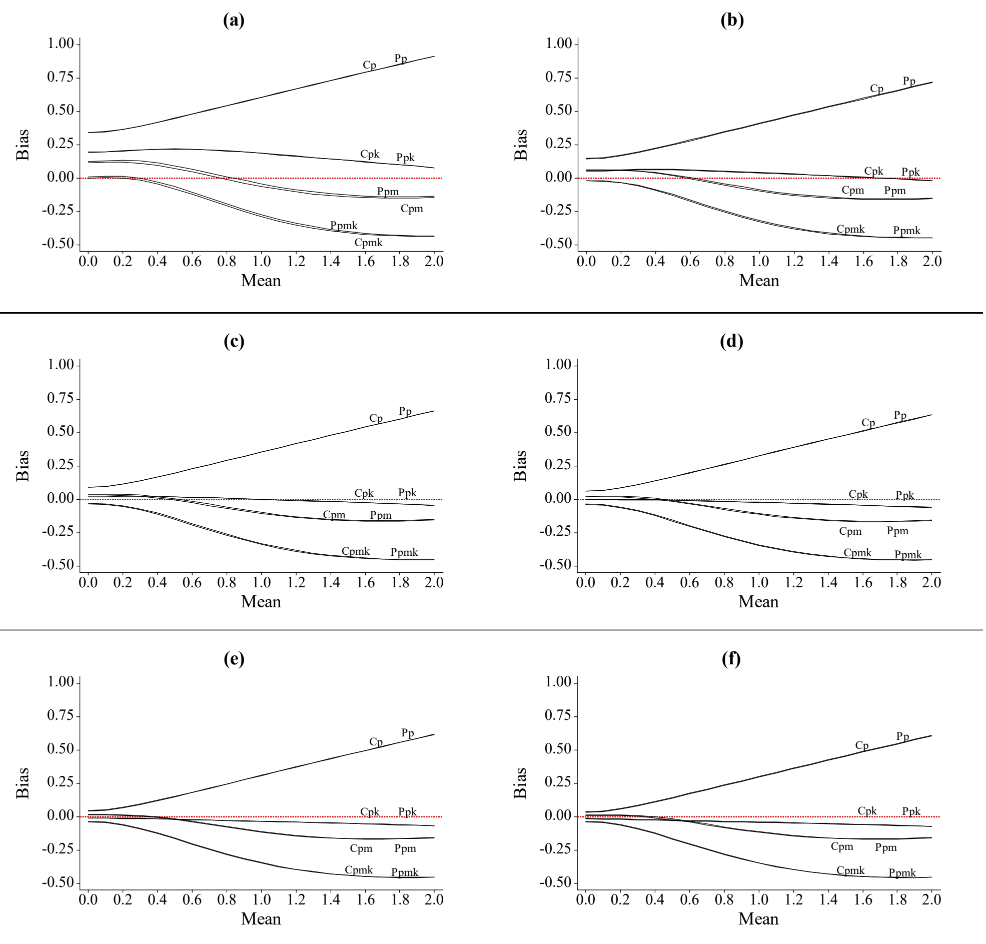

Figure 4. Biases of the capability indices as a function of the process mean (μ), for 5 (a), 10 (b), 15 (c), 20 (d), 25 (e) and 30 (f) rational subgroups (m) or sample elements (n).

As already mentioned, the indices C_p and P_p overestimate the process capability, regardless of the number of rational subgroups (m) or sample elements (n). On the other hand, the capability indices C_{pmk} and P_{pmk} underestimate the process capability. Furthermore, from 15 rational subgroups or sample elements, the estimates of all capability indices are nearly the same.

The capability indices that presented the smallest biases for all shifts of the averages were the C_{pk} and P_{pk} indices. The capability indices, C_{pm} and P_{pm}, were slightly lower than the previous two.

4.3 Coefficient of variation

Figure 5 shows the coefficients of variation (CV) of the eight capability indices as a function of the displacements of the means in relation to the nominal value (VN = 0), for 5 and 10 rational subgroups (m) or sample elements (n). In Figure 6, for 15, 20, 25, and 30.

Since the indices C_p, P_p, C_{pmk}, and P_{pmk} have already been identified as the least accurate, they were not selected as good estimators of capacity, regardless of their respective CVs.

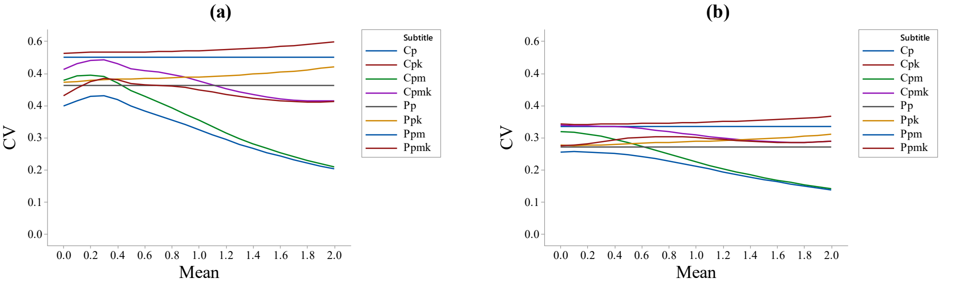

Figure 5. Coefficients of variation of the capability indices as a function of the process mean (μ), for 5 (a) and 10 (b) rational subgroups (m) or sample elements (n).

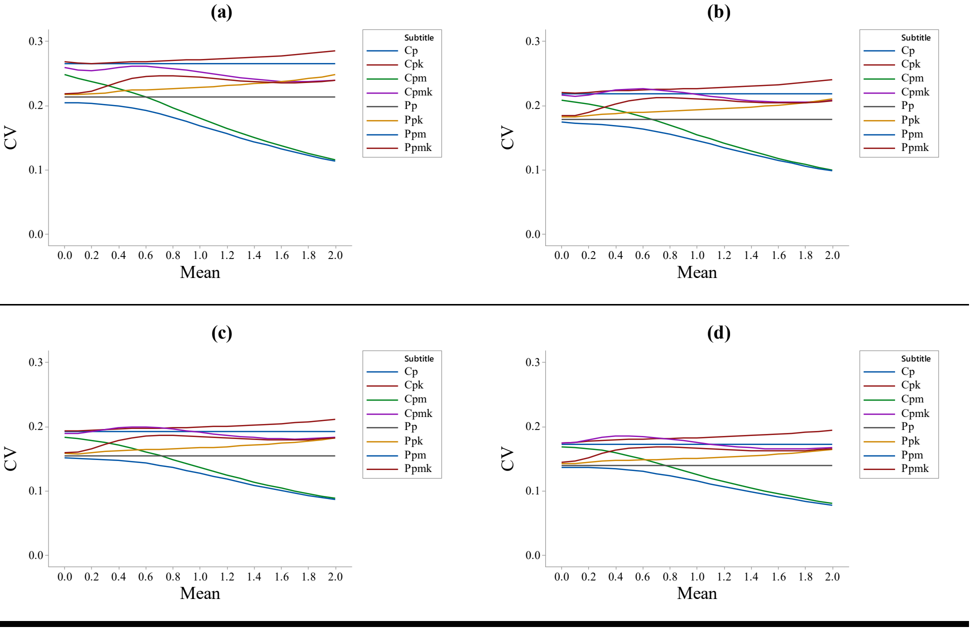

Figure 6. Coefficients of variation of the capability indices as a function of the process mean (μ), for 15 (a), 20 (b), 25 (c), 30 (d) rational subgroups (m) or sample elements (n).

For 5 and 10 rational subgroups or sample elements, the indices P_p, P_{pk}, P_{pm}, and P_{pmk}, were the ones that provided the lowest CV when compared to the indices C_p, C_{pk}, C_{pm}, and C_{pmk}, respectively (Figure 5). And as the number of rational subgroups or sample elements increases, the CV decreases, i.e., the capability indices become more efficient as the standard deviation decreases. Importantly, the C_{pm} and P_{pm} indices became more efficient as the mean shift increased (Figure 5 and Figure 6). Again, the CVs of all eight capability indices were lower when 15 or more rational subgroups or sample elements were added (Figure 6).

4.4 Consistency

The consistency of the estimator is analyzed if, as the number of observations increases, the estimates approach the “target”. In this work, this could be analyzed by increasing the number of rational subgroups or sample elements from 5 to 30. This would require that all estimates approach the parameter, that is, if the bias and standard deviation of the estimates decrease.

Analyzing the bias, it was possible to observe that the biases of the capability indexes C_p, P_p, C_{pmk}, and P_{pmk} did not reduce or reduced little. The indices C_{pk}, P_{pk}, C_{pm}, and P_{pm} showed reductions as the number of rational subgroups (m) or sample elements (n) increased.

Regarding the CV, it could be observed that its decrease is a function of the increase of m or n, for all capability indices. Consequently, there was a reduction in the standard deviation.

Thus, analyzing the bias and the efficiency of the capacity indexes, it can be concluded that the indexes C_{pk}, P_{pk}, C_{pm}, and Ppm were the most consistent. Among them, the indexes C_{pk} and P_{pk} were the most accurate, and the indexes C_{pm} and P_{pm}, the most efficient.

5 Final considerations

According to the results obtained, it was possible to observe that, with the displacement of the average in relation to the nominal value, there are indexes that estimate the process capability better than others. When an index underestimates the process capability, which occurred with the indexes C_{pmk} and P_{pmk}, it provides a worse quality estimate than the one that actually exists. However, this will be less harmful than when the index overestimates the process capability, which occurred with the C_p and P_p indexes, providing a higher estimate of the true process capability. According to Costa et al. (2018), the C_p and P_p indices are insensitive to changes in the process mean and therefore should only be used when the process mean remains centered on the target.

In order to reduce the problem of overestimation or underestimation of capacity, the most accurate were the indexes C_{pk} and P_{pk}, and the most efficient were the indexes C_{pm} and P_p. Therefore, the indexes C_{pk}, P_{pk}, C_{pm}, and P_{pm} were more consistent.

As presented by Álvarez et al. (2015), the results showed that creating 5 rational subgroups or collecting 5 sample elements was not enough to estimate the parameter. In this study, it is recommended to use at least 15, and beyond this value, the estimates did not show substantial improvement.

The way of estimating the standard deviation () of the process, considering rational subgroups (indices with C) or considering all the values of the sample (P indexes), did not interfere with the accuracy of the capacity indexes. However, the P indexes were more efficient than the C indexes.

The displacements of the averages were important to analyze the behavior of the capability indices since the estimate of the average will rarely be the nominal value of the process. As the mean shifted from the nominal value, the biases of the indices C_p, P_p, C_{pmk}, and P_{pmk} increased, showing that they are not good for estimating process capability.

Thus, since the capability indices C_{pk} and P_{pk} were the most accurate and equally efficient to the indices C_{pm} and P_{pm} for 15 or more rational subgroups or sample elements, the former two are recommended for these sample conditions. This means that the indices C_{pk} and P_{pk} were the most consistent in estimating process capability. It is also ratified that these indices, C_{pk} and P_{pk}, were designed to monitor process capability under stable and normal conditions and are not recommended for non-normal distributions.

As another work opportunity, it is suggested to impose the displacement of the mean and a gradual asymmetry in the predefined normal distribution, in order to verify how much the C_{pk} and P_{pk} indices can withstand the changes. This verification can be performed through the properties of accuracy, efficiency and consistency of the estimators. Furthermore, another opportunity is to evaluate the properties of new process capability indices, such as those proposed by Chen & Ding (2001), Abdolshah et al. (2009) and Pan & Lee (2010).

References

Abdolshah, M., Yusuff, R. M., Hong, T. S., & Yusof, M. Y. H. (2009). New process capability index using Taguchi loss functions. *Journal of Applied Sciences*, 9(20), 3775-3779. http://dx.doi.org/10.3923/jas.2009.3775.3779.

Álvarez, E., Moya-Férnandez, P. J., Blanco-Encomienda, F. J., & Muñoz, J. F. (2015). Methodological insights for industrial quality control management: the impact of various estimators of the standard deviaton on the process capability index. *Journal of King Saud University. Science*, 27(3), 271-277. http://dx.doi.org/10.1016/j.jksus.2015.02.002.

Barreto, R. R., Rocha, H. D., & Borges, C. A., Jr. (2016). Análise da capacidade de um processo de revestimento de bobinas de aço. In *Anais do XIII Simpósio de Excelência em Gestão e Tecnologia*. Rezende: SEGET.

Chen, J. P., & Ding, C. G. (2001). A new process capability index for non-normal distributions. *International Journal of Quality & Reliability Management*, 18(6-7), 762-770. http://dx.doi.org/10.1108/02656710110396076.

Costa, A. F. B., Epprecht, E. K., & Carpinetti, L. C. R. (2018). *Controle estatístico de qualidade* (2. ed.). São Paulo: Atlas.

Devore, J. L. (2006). *Probabilidade e estatísticas: para engenharia e ciências*. São Paulo: Cengage Learning.

Dianda, D. F., Quaglino, M. B., & Pagura, J. A. (2016). Performance of multivariate process capability indices under normal and non-normal distributions. *Quality and Reliability Engineering International*, 32(7), 2345-2366. http://dx.doi.org/10.1002/qre.1939.

Gonçalez, P. U., & Werner, L. (2009). Comparação dos índices de capacidade do processo para distribuições não-normais. *Gestão & Produção*, 16(1), 121-132. http://dx.doi.org/10.1590/S0104-530X2009000100012.

Kane, V. E. (1986). Process capability indices. *Journal of Quality Technology*, 1(1), 41-52. http://dx.doi.org/10.1080/00224065.1986.11978984.

Kotz, S., Pearn, W. L., & Johnson, N. L. (1993). Some process capability índices are more reliable than one might think. *Journal of the Royal Statistical Society. Series A, (Statistics in Society)*, 42, 55-62. http://dx.doi.org/10.2307/2347409.

Kushler, R., & Hurley, P. (1992). Confidence bounds for capability indices. *Journal of Quality Technology*, 24(4), 188-195. http://dx.doi.org/10.1080/00224065.1992.11979400.

Miao, R., Zhang, X., Yang, D., Zhao, Y., & Jiang, Z. (2011). A conjugate bayesian approach for calculating process capability índices. *Expert Systems with Applications*, 38(7), 8099-8104. http://dx.doi.org/10.1016/j.eswa.2010.12.151.

Mittag, H. J., & Germany, H. (1997). Measurement error effects on the performance of process capability índices. In H. J. Lenz & P. T. Wilrih (Eds.), *Frontiers in statistical quality control 5* (pp. 195-206). Berlin: Springer-Verlag Berlin Heidelberg. http://dx.doi.org/10.1007/978-3-642-59239-3_15.

Montgomery, D. C. (2019). *Introdução ao Controle Estatístico da Qualidade* (7. ed.). Rio de Janeiro: Livros Técnicos e Científicos S.A.

Montgomery, D. C., & Runger, G. C. (2018). *Estatística aplicada e probabilidade para engenheiros* (6. ed.). Rio de Janeiro: LTC.

Pan, J., & Lee, C. (2010). New capability indices for evaluating the performance of multivariate manufacturing processes. *Quality and Reliability Engineering International*, 26(1), 3-15. http://dx.doi.org/10.1002/qre.1024.

Parchami, A., & Mashinchi, M. (2007). Fuzzy estimation for process capability índices. *Information Sciences*, 177(6), 1452-1462. http://dx.doi.org/10.1016/j.ins.2006.08.016.

Pearn, W. L., Kotz, S., & Johnson, N. L. (1992). Distributional and inferential properties of process capability indices. *Journal of Quality Technology*, 24(4), 216-231. http://dx.doi.org/10.1080/00224065.1992.11979403.

Pearn, W. L., Lin, G. H., & Chen, K. S. (1998). Distributional and inferential properties of process accuracy and process precision indices. *Communications in Statistics. Theory and Methods*, 27(4), 985-1000. http://dx.doi.org/10.1080/03610929808832139.

Riaz, M., & Hamid, T. (2016). On the performance of different capability indices under normal and non-normal distributions. *Zhongguo Gongcheng Xuekan*, 39(8), 889-899. http://dx.doi.org/10.1080/02533839.2016.1220265.

Ribeiro, J. I., Jr. (2013). *Métodos estatísticos aplicados ao controle da qualidade* (1. ed.). Viçosa: Editora UFV.

Ribeiro, J. L., & Caten, C. S. (2012). *Série monográfica Qualidade – Projeto de experimentos*. Porto Alegre: Universidade Federal do Rio Grande do Sul. Retrieved in 2022, February 16, from http://www.producao.ufrgs.br/arquivos/disciplinas/117_apostila_pe_2011.pdf.

Rodrigues, L. A. (2001). *Índices de avaliação de processos: abordagem univariada e multivariada* (Dissertação de mestrado). Universidade Federal do Rio Grande do Sul, Porto Alegre.

Souza, F. S., Pedrini, D. C., & Caten, C. S. (2014). Proposta de fluxograma orientativo para aplicação de índices de capacidade. *Gestão & Produção*, 21(4), 882-894. http://dx.doi.org/10.1590/0104-530x496-13.

Souza, L. M., Ribeiro, J. I., Jr., Reis, G. M., & Ide, M. S. (2008). Eficiência dos gráficos de controle Xbarra, EWMA e CUSUM. *Revista Produção e Engenharia*, 1, 81-94. http://dx.doi.org/10.18407/issn.1983-9952.2008.v1.n1.p81-94.

Stoumbos, Z. G. (2002). Process capability indices: overview and extensions. *Nonlinear Analysis Real World Applications*, 3(2), 191-210. http://dx.doi.org/10.1016/S1468-1218(01)00022-0.

Tang, L. C., & Than, S. E. (1999). Computing process capability indices for non-normal data: a review and comparative study. *Quality and Reliability Engineering International*, 15(5), 339-353. http://dx.doi.org/10.1002/(SICI)1099-1638(199909/10)15:5<339::AID-QRE259>3.0.CO;2-A.

Vannman, K. (1995). A unified approach to capability índices. *Statistica Sinica*, 5, 805-820. Retrieved in 2022, February 16, from https://www.jstor.org/stable/24305072

Wang, S., Chiang, J. Y., Tsai, T. R., & Qin, Y. (2021). Robust process capability indices and statistical inference based on model selection. *Computers & Industrial Engineering*, 156, 107265. http://dx.doi.org/10.1016/j.cie.2021.107265.

Werkema, M. C. (1995). *Ferramentas estatísticas básicas para o gerenciamento de processos* (Vol. 2). Belo Horizonte: Fundação Christiano Ottoni.

Wu, C., Pearn, W. L., & Kotz, S. (2009). An overview of theory and practice on process capability índices for quality assurance. *International Journal of Production Economics*, 117(2), 338-359. http://dx.doi.org/10.1016/j.ijpe.2008.11.008.

Yum, B. J., & Kim, K. W. (2011). A bibliography of the literature on process capability indices: 2000–2009. *Quality and Reliability Engineering International*, 27(3), 251-268. http://dx.doi.org/10.1002/qre.1115.

Authors contribution

Jaqueline Akemi Suzuki Sediyama and José Ivo Ribeiro Júnior worked on the conceptualization and theoretical-methodological approach. Jaqueline Akemi Suzuki Sediyama was responsible for theoretical review and data collection. Jaqueline Akemi Suzuki Sediyama, Daibou Alassane and Raphael Henrique Teixeira da Silva participated in the data analysis. All authors participated in the writing and final revision of the manuscript.