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Ocean energy, fluxes and an anti-anti-turbulence conjecture

WK Dewar[1,2*] and B Deremble[1]

[1]Institut des Géosciences de l'Environnement, Centre National de la Recherche Scientifique, France,
[2]Department of Earth, Ocean and Atmospheric Science, Florida State University, USA
*Corresponding author: wdewar@fsu.edu

Abstract

The energy sources for convection and the general circulation are revisited through an analysis of the compressible equations of motion, rather than the Boussinesq equations. We are motivated in this endeavor by a more straightforward connection in the compressible equations between thermodynamics and dynamics, and the continuing debate in the field regarding the suggestion, made first in the form of Sandström's theorem, that surface buoyancy fluxes can not drive the overturning circulation. While ultimately supporting the Sandström position, the analysis leads to some new insights into ocean energetics and surface energy fluxes. We argue the ultimate role of buoyancy fluxes are to damp the circulation and that ocean energy cycles between internal and kinetic energy. Ocean heating due to the general circulation, geothermal heat flux and the biosphere are evaluated for their roles and we suggest the latter two provide energy to the overturning much more effectively than surface forcing. All three also contribute significantly to net ocean surface energy flux, an effect that influences the interpretation of ocean heat content imbalances.

Keywords: Energetics, Turbulence

INTRODUCTION

Turbulent diapycnal buoyancy flux is centrally involved in the overturning circulation (Cessi 2019). By definition, such a flux involves modifying fluid buoyancy through mixing, which in turn requires energy. A long-standing question in physical oceanography is the identification of the energy sources behind abyssal mixing. Sandström's theorem (Sandström 1908) addresses this question, stating that horizontal convection does not drive large-scale circulation, and has a history that dates nearly to the inception of physical oceanography. The conclusion was based on laboratory experiments showing that the maintenance of a sustained circulation by buoyancy forcing required that the heat source be positioned at a level beneath the cooling source. Equivalently, oceanic surface buoyancy fluxes, perhaps most properly termed as 'horizontal convection', where heating and cooling are almost at the same geopotential level, could not be responsible for the observed large-scale overturning circulation. That the theorem has been subject to several reinterpretations and restatements since it was first proposed cannot be denied. In spite of its continued debate in the community and investigation with advanced theoretical, numerical and laboratory techniques, the field remains divided in how to view Sandström's theorem and the role of buoyancy fluxes in general in driving a component of the overturning circulation. Assuming that ocean mixing proceeds with a gross 'efficiency' of about 20%, something like 2 TW (1 TW = 10^{12}W) must be provided to the ocean

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interior to maintain the observed stratification (Munk & Wunsch 1998, Wunsch & Ferrari 2004). Existing estimates have assumed buoyancy forcing plays a negligible role in supplying this energy, and this is the position most hotly debated in the literature. It is the objective of this paper to examine the question of buoyancy fluxes and global fluid energetics from as fundamental a perspective as possible, with a view towards addressing these uncertainties.

Several classical publications followed the original, notably Sandström (1916), Sandström (1922), Jeffreys (1925), and Defant (1961), where the conclusion and its interpretation were vigorously debated. In the last decade, interest in the ocean energy budget rekindled the discussion, with important contributions too numerous to effectively, or even usefully, cite. A recent overview is given in Coman et al. (2006), where attempts at reproduction of Sandström's results failed, and it was concluded that the original experiments were flawed. A related, useful discussion is also found in Kuhlbrot (2008).

A powerful and elegant reexamination of the fundamental question raised by Sandström was provided by Paparella & Young (2002), who supported the basic Sandström result by arguing the flows driven by horizontal convection were non-turbulent, failing the 'zeroth law' of turbulence, which defines turbulent flow by its ability to sustain finite dissipation in the limit of vanishing viscosity. Thus, Paparella & Young (2002) imply diapycnal mixing in an ocean forced solely by horizontally placed buoyancy sources must be governed by molecular processes, in which case all stratification will be confined to a surface trapped boundary layer whose width is set by a diffusive length scale. This does not resemble the observed abyssal stratification, leading to the conclusion that wind forcing is the primary source of interior ocean turbulence. Other notable publications endorsing or adopting this perspective include Huang (1998), Huang (1999), Wunsch & Ferrari (2004) and Ferrari & Wunsch (2009).

A vigorous counterpoint to this theme has been mounted by several, with a thorough discussion to be found in Hughes et al. (2009). More recently, Gayen et al. (2013) and Vreugdenhil et al. (2016) have argued that buoyancy effectively drives mixing through pathways that emphasize the generation of available potential energy. Clear evidence of large-scale flows arising from purely horizontally driven convection have been found numerically and in the laboratory, although often confined to regions of weak stratification. Vreugdenhil et al. (2016), in a paper discussing turbulence and convection resolving DNS simulations, find buoyancy forcing can maintain deep stratification by means of an extraordinarily high mixing efficiency.

In our view, at least part of the disparity in viewpoints comes from the use of the Boussinesq equations in examining this problem, because Boussinesq thermodynamics remain somewhat foggy, to say the least, particularly with regards to the role of internal energy. This general topic received a significant boost by Young (2010) who put Boussinesq thermodynamics on a much firmer foundation by identifying dynamic enthalpy as the effective potential energy of a Boussinesq fluid with a realistic equation of state. With this identification, the resulting equations could be shown to have an energy conservation principle, up to diabatic processes. While applauding this effort, we note that full inclusion of non-conservative processes was not considered. Divorcing dynamics from thermodynamics, i.e. the primary advantage of the Boussinesq equations, is very useful, but internal energy is generally several orders of magnitude greater than either kinetic or potential energy. Although the Boussinesq equations are a relatively accurate approximation of the full compressible equations, even tiny errors in estimating the internal energy budget can cause large errors in the kinetic and potential energy budgets.

We address this point in the present paper by examining the full compressible equations in order to extract exact statements about fluid energetics subject to external inputs of mechanical and thermal energy. As such, these statements are not subject to uncertainties in their interpretations, and are more easily assessed when approximations are introduced. They also allow us to identify pathways between dynamics, thermodynamics and dissipation.

Accordingly, we find that the role of buoyant forcing in the modern ocean is to damp the general circulation, and probably to have done so throughout geological history. In the theoretical setting of no wind-forcing, addressed by Sandström's theorem, net buoyant flux must vanish. The analysis also argues that mixing, whose presence is inferred from dissipation, builds internal energy rather than potential energy. Related to this, correlations between sea surface height and buoyancy flux act as sources and sinks to internal energy and contribute to the global energy budget at rates not constrained by molecular diffusivity. Modern observations of these variables suggest that buoyancy fluxes can work counter to the overturning, although the uncertainties on this estimate are too large for a definitive result. Speculations about the sea surface shape in the absence of wind forcing also argue buoyancy fluxes do not elevate mixing levels. In any case, this analysis supports Sandström's conjecture about the necessity of wind for the maintenance of deep stratification.

Estimates of the role played by geothermal heat flux in driving a 'bottom-up' convection, and the marine biosphere acting as a distributed internal heat source, are larger sources for dissipation than the contributions from surface fluxes by a factor of ten to sixty, although they remain small compared to modern energy inputs due to the wind. Finally, an evaluation of the impact of circulation dissipation, geothermal heat flux and total metabolic rate of the biosphere on surface fluxes suggests that they measurably impact the observed changes in ocean heat content.

The next section reviews the derivation of the energy equations for a fully compressible fluid. Integral constraints then demonstrate the ultimately damping role of buoyancy fluxes on the modern circulation, such that when restricted to horizontal convection, net buoyancy fluxes must vanish. We argue the dynamical role played by buoyancy fluxes is not in their direct contribution to potential energy, but rather in their indirect interactions with internal energy. This connection is contained in the Boussinesq equations, although its appearance is masked somewhat so that it resembles a contribution to potential energy. We end with a discussion of the results.

BACKGROUND

The stratified Navier-Stokes momentum and mass equations for a compressible binary fluid are

$$\begin{gathered}\rho(\frac{d}{dt}\boldsymbol{u}+\boldsymbol{f}\times\boldsymbol{u}) = \\ -\nabla P-\rho\nabla G+\mu\nabla^2\boldsymbol{u}+(\lambda+\mu)\nabla\nabla\cdot\boldsymbol{u} \\ \rho_t+\nabla\cdot\boldsymbol{u}\rho=0\end{gathered} \tag{1}$$

where $\frac{d}{dt}$ denotes a material derivative, $\boldsymbol{u}$ velocity, $\boldsymbol{f}=2\boldsymbol{\Omega}$ where Ω is rotation rate of the Earth, P total pressure, $\rho=1/\alpha$ fluid density, G the geopotential, μ molecular viscosity and λ bulk viscosity (Batchelor 1967, Landau & Lifshitz 1959). The kinetic energy equation is obtained from (1) by vector multiplying by $\boldsymbol{u}$ and using mass conservation

$$\begin{gathered}\frac{d}{dt}K=(\rho K)_t+\nabla\cdot\rho\boldsymbol{u}K= \\ -\boldsymbol{u}\cdot\nabla P-\rho wg-\rho\epsilon+\frac{\partial}{\partial x_j}(u_i\tau_{ij})\end{gathered} \tag{2}$$

where $K=\boldsymbol{u}\cdot\boldsymbol{u}/2$ is kinetic energy, $g=|\nabla G|$ gravity and w vertical velocity (with the vertical direction defined by ∇G). The stress tensor τ_{ij} is given by

$$\underline{\underline{\tau}}=\tau_{ij}=\mu(\frac{\partial}{\partial x_j}u_i+\frac{\partial}{\partial x_i}u_j)+\lambda\delta_{ij}\frac{\partial}{\partial l}u_l \tag{3}$$

where Einstein summation is implied and δ_{ij} is the Kronecker delta function. Dissipation, ϵ, is given by

$$\epsilon=\tau_{ij}\frac{\partial}{\partial x_j}u_i \tag{4}$$

which can be shown to be positive definite. We have neglected spatial variations of the viscosities.

The full internal energy equation of a binary compressible fluid is

$$\frac{d}{dt}e=-P\frac{d}{dt}\alpha+T\frac{d}{dt}\eta+\gamma\frac{d}{dt}S \tag{5}$$

where e is internal energy, T in-situ temperature, η entropy, γ the relative chemical potential of seawater and S salinity. Energy conservation is insured by the fundamental thermodynamic relation (Landau & Lifshitz 1959, IOC, SCOR and IAPSO 2010)

$$T\frac{d}{dt}\eta+\gamma\frac{d}{dt}S=-\frac{1}{\rho}\nabla\cdot\boldsymbol{F}_Q+\epsilon+Q \tag{6}$$

where $\boldsymbol{F}_Q$ is the generalized heat flux of a two component fluid and Q represents sources of internal heating aside from dissipation, such as the total metabolic rate of the biosphere (Dewar et al. 2006).

Thus

$$(\rho e)_t + \nabla\cdot(\rho \boldsymbol{u} e) = -P\nabla\cdot\boldsymbol{u} - \nabla\cdot\boldsymbol{F}_Q + \rho\epsilon + \rho Q \quad (7)$$

where mass conservation has again been used. A curious fact is that (2) and (7), comprising a full statement of fluid energetics, make no explicit mention of salinity, although its presence is implicit in $\boldsymbol{F}_Q$ and internal energy, e.

We now integrate (2) and (7) over a closed domain. The nature of the boundaries will depend on the application. When discussing the ocean, we classify the bottom as rigid, no slip and either nonconductive or conductive at a known rate, while the free surface will be permeable to both momentum and heat. We will also consider Rayleigh-Benard convection, where the upper and lower boundaries will both be heat conductive, but stress-free. In all cases, we neglect boundary mass flux (e.g., evaporation and precipitation) as well as penetrative radiation. The results are

$$\frac{\partial}{\partial t}\int_V \rho K = -\int_V \boldsymbol{u}\cdot\nabla P dV - \int_V \rho\epsilon dV - \int_V \rho w g dV + \int_S \boldsymbol{u}\cdot\underline{\underline{\tau}}_o\cdot\hat{\boldsymbol{n}} dS \quad (8)$$

where $\hat{\boldsymbol{n}}$ is the unit outward pointing normal at the free surface S, $\underline{\underline{\tau}}_o$ is the surface wind stress and

$$\frac{\partial}{\partial t}\int_V \rho e dV = -\int_V P\nabla\cdot\boldsymbol{u} dV + \int_V \rho(\epsilon + Q) dV - \int_{\hat{S}} \boldsymbol{F}_Q\cdot\hat{\boldsymbol{n}} dS \quad (9)$$

where $\hat{S}$ bounds the volume V. The above equations show that the first term on the right hand side of (8) denotes the exchange of energy between internal and kinetic energies, essentially by means of 'piston work'. The quantity usually related to potential energy appears in (8) as the third term on the right hand side.

Multiplying mass conservation by z and integrating yields

$$\frac{\partial}{\partial t}\int_V \rho z dV = \int_V w\rho dV \quad (10)$$

Note that combining (8), (9) and (10) leads to

$$\frac{\partial}{\partial t}\int_V \rho(K + e + gz) dV = -\int_S \boldsymbol{F}_Q\cdot\hat{\boldsymbol{n}} dS + \int_S \boldsymbol{u}\cdot\underline{\underline{\tau}}_o\cdot\hat{\boldsymbol{n}} dS + \int_V \rho Q dV \quad (11)$$

Equivalently, the total energy budget in the basin is controlled by momentum and generalized heat exchanges with the environment.

For the remainder of the paper, we will consider time averages of (8), (9), (10) and (11), with the time average denoted by an overbar. The results are

$$\begin{aligned}
0 &= -\overline{\int_V \boldsymbol{u}\cdot\nabla P dV} - \overline{\int_V \rho\epsilon dV} + \overline{\int_S \boldsymbol{u}\cdot\underline{\underline{\tau}}_o\cdot\hat{\boldsymbol{n}} dS} \\
0 &= -\overline{\int_V P\nabla\cdot\boldsymbol{u} dV} + \overline{\int_V \rho(\epsilon + Q) dV} - \overline{\int_S \boldsymbol{F}_Q\cdot\hat{\boldsymbol{n}} dS} \\
0 &= \overline{\int_V w\rho dV} \quad (12) \\
\overline{\int_{\hat{S}} \boldsymbol{F}_Q\cdot\hat{\boldsymbol{n}} dS} &= \overline{\int_S \boldsymbol{u}\cdot\underline{\underline{\tau}}_o\cdot\hat{\boldsymbol{n}} dS} + \overline{\int_V \rho Q dV}
\end{aligned}$$

respectively. Note that (12c) has been used in (12a) and has the effect of removing so-called potential energy from the mean energetics of the circulation, which are instead seen to be entirely confined to exchanges between kinetic and internal energy. This view departs somewhat from that obtained from the Boussinesq equations, where we are tempted to view heat as distinct from kinetic energy.

Energy Consequences

Eq. (12d), equating net mechanical forcing and buoyancy fluxes, at first perhaps surprising, emerges upon reflection as obvious. The assumption that the circulation is in statistical steady state requires total energy fluxes into the system must be exactly compensated by total energy fluxes out. In a general statement of energy, the only available fluxes are those of kinetic energy and heat, hence they must balance. Equivalently, forcing by stresses at the surface requires the ocean to seek a state where surface heat fluxes with the atmosphere account for the mechanical energy input.

It is instructive to think through the application of (12d) to the well known problem of Rayleigh-Benard convection. In this case, without mechanical stresses or internal heat sources, (12d) insures that net heat fluxes vanish

$$\int_A \boldsymbol{F}_Q\cdot\hat{\boldsymbol{n}} dA = 0 \quad (13)$$

where the overbar has been dropped, a notational convenience that will be employed from here on.

The steady internal energy equation (12b) becomes

$$\int_V P\nabla\cdot\boldsymbol{u}dV = \int_V \rho\epsilon dV \qquad (14)$$

Rayleigh-Benard flow is generally highly turbulent, and thus highly dissipative. As a result, heat is generated within the convection, yet (13) insures that the heat introduced at the bottom is identical to that lost through the top. A reasonable question is what happens to the dissipatively generated heat.

The right hand side of (14) is positive definite, implying that strong dissipation requires the ability of the fluid to correlate low pressures with fluid contractions and high pressures with fluid expansions. Rayleigh-Benard convection, either heated from below or cooled from above, meets this condition by contracting fluid at low pressures high in the fluid and expanding it at high pressures deep in the fluid. The net work needed to achieve this comes from the fluid reserves of internal energy, effectively ‘cooling’ the internal energy. However, in steady state, as assumed here, this ‘cooling’ of internal energy is returned by the heat production driven by dissipation. Net heat fluxes into the system from the environment are unaware of, and need not account for, this dissipatively generated heat. The primary, and in fact only, energy exchanges are between internal and kinetic; gravity plays an indirect role in generating a gravitationally aware pressure field that increases with increasing depth. The presence of gravity enables a mechanism by which internal energy can be exchanged with kinetic energy. The oppositely forced case of cooling from below and heating from above eventually arrives at a steady state, and meets (13), but cannot tap into internal energy to drive flows that dissipate. Indeed this case places fluid contractions at high pressures and expansions at low pressures, which results in a negative value for the left hand side of (14). The right hand side must be positive definite arguing that the eventual steady state must be one of no flow so that dissipation and fluid contraction both vanish. Instead, molecular conduction will result in a temperature distribution that conducts heat from source to sink in the absence of fluid flow.

Returning to the oceanic setting, we assume for the moment the surface is the only boundary where heat and momentum fluxes are permitted and neglect internal heat sources. Modern ocean circulation is characterized by net wind work on the general circulation (see Wunsch (1998), and Zhai et al. (2012)), on the order of $<$1 TW, although the net work need not be positive (Hazewinkel et al. (2012) describe circulations in which wind stress acts counter to the surface flow). However, more importantly, winds transfer energy to the surface wave field at a rate that apparently dwarfs the work on the general circulation. Wang & Huang (2004) estimate wind work on waves at 60 TW, while acknowledging the uncertainties in this estimate. The fate of this energy is unknown: some is no doubt lost on ocean boundaries in the surf zone, but much of it enters the subsurface ocean through breaking waves and white capping. We can speculate that transfers of this sort have characterized the stress at the ocean-atmosphere interface throughout the geological history of the planet, leading to the suggestion that wind work on the oceans has always been positive. This leads to the first conclusion of this paper, namely that the fundamental role of buoyancy fluxes in the modern ocean is to energetically damp the flow, a balance that has probably existed for the lifetime of the oceans (12d).

The pressure appearing in all the equations so far is total pressure. Normally the Boussinesq equations involve dynamic pressure, p, which is the pressure associated with fluid movements and forcing. These various pressure forms are related via

$$P = P_s + p = P_o - \rho_o gz + p \qquad (15)$$

where P_o is an average constant reference surface pressure, ρ_o a reference density and ‘static’ pressure is given by $P_s = P_o - \rho_o gz$. Note that static pressure is aware of gravity; in fact in the integral balances considered previously, this is the only place where gravity appears.

Introducing this pressure decomposition into (12a)

$$0 = \rho_o g\int_V wdV - \int \boldsymbol{u}\cdot\nabla p dV - \int_V \rho\epsilon + \int_S \boldsymbol{u}\cdot\underline{\underline{\tau}}_o\cdot\hat{\boldsymbol{n}}dS \qquad (16)$$

Performing some algebra on (12c)

$$\rho_o g\int_V wdV = \int_V \rho w b dV \qquad (17)$$

where

$$b = \frac{-g(\rho - \rho_o)}{\rho} \tag{18}$$

is the buoyancy variable introduced in Young (2010).

Regarding the integral in (16) involving dynamic pressure, global heat imbalances are typically $O(0.5W/m^2)$ based on in-situ observations (Meyssignac et al. 2019). If we assume a global imbalance of $1W/m^2$ is distributed uniformly throughout the water column and estimate dynamic pressure by the anomaly associated with a one meter high sea surface anomaly,

$$\int_V p\nabla \cdot \boldsymbol{u} dV \approx 10^4 \frac{\text{Kg}}{\text{m s}^2} \frac{1\text{W}\,\Gamma}{\text{m}^2 \rho C_p} A = 0.18 \times 10^9 \text{W} = 0.18\,\text{GW} \tag{19}$$

where $\Gamma = 2 \times 10^{-4}$/K is the seawater thermal expansion coefficient, $C_p = 4 \times 10^3$ J/(Kg °K) is heat capacity, $\rho = 10^3$Kg/m^3 and $A = 3.5 \times 10^{14}$m^2. This estimate of dynamic pressure work is very small relative to modern estimates of wind work, leading us to neglect this quantity in (16) from here on (see also the appendix in Wang & Huang (2004)).

The equation for volume integrated and time averaged $\rho w b$ is

$$\int_V \rho w b dV = \rho_o \int_{\hat{S}} z \boldsymbol{F_b} \hat{\boldsymbol{n}} dS + K\rho_o \int_S (b_s - b_b) dA - \int_V \frac{z g \alpha_T \rho_o \rho}{C_p} (\epsilon + Q) dV - \int_V \frac{g^2 \rho_o^2 w z}{\rho c_s^2} dV \tag{20}$$

where K is molecular diffusivity, $b_{(s,b)}$ is the (surface, bottom) fluid buoyancy and α_T is the partial derivative of specific volume with respect to temperature at constant entropy and salinity. The derivation of this equation is rather lengthy and contained in the appendix, where it is also argued that the last contribution can be neglected. In any case, the present discussion is more focussed on the first several integrals on the right hand side of (20).

For an ocean without wind forcing ($\underline{\underline{\tau}}_o = 0$) or internal heat sources and an insulating bottom, i.e. the setting envisioned by Sandström's theorem, (16) becomes

$$\int_V \rho \epsilon dV = K \int_S (b_s - b_b) dA + \int_S z \boldsymbol{F_b} \cdot \hat{\boldsymbol{n}} dS \tag{21}$$

Total dissipation is determined by two integrals, one proportional to molecular diffusivity and one proportional to the correlation of surface fluxes and sea surface height anomalies. Approximating the free surface by a geopotential, the latter integral vanishes. Then as viscosity vanishes, for fixed Prandtl number, net dissipation must vanish, which is the statement of the 'Anti-Turbulence' theorem of Paparella & Young (2002). Adding wind stress only mildly modifies (21), in which case, it is possible to estimate buoyantly generated turbulence as compared to wind generation. With a thermal diffusivity of $K = 10^{-7} m^2/s$, an average surface to bottom buoyancy difference of

$$b_s - b_b \approx g \frac{\Delta \rho}{\rho_o} = 0.03 m/s^2 \tag{22}$$

the integral is roughly

$$\rho_o K \int_A (b_s - b_b) dA = 10^3 \frac{\text{Kg}}{\text{m}^3}\, 10^{-7} \frac{\text{m}^2}{s}\, .03 \frac{\text{m}}{s^2}\, 3.5 \times 10^{14}\,\text{m}^2 \approx 10^9\,\text{W} = 1\,\text{GW} \tag{23}$$

which is quite small compared to both typical wind work energy fluxes of O[60] TW, and the perhaps more important wind work on the general circulation of 1 TW. Wang & Huang (2004) arrive at a comparable estimate of this term. We note that at global scales under these conditions, the ocean will be stably stratified, and so the positive sign of this integral is reliable. Its small value can be interpreted to mean buoyant effects generate negligible, if non-zero, turbulence in the real ocean.

Of course, the free surface is not a geopotential, but instead varies by roughly one meter relative to a flat surface over the globe. Thus the other integral in (21) does not vanish, and reflects instead the correlation of sea surface height anomaly (SSA) with heat flux. This quantity represents a possible internal energy input to dissipation that is not proportional the molecular diffusivity, and cannot be shown to vanish in the limit of vanishing viscosity, thereby violating the conditions leading to the 'Anti-Turbulence' theorem. We have calculated the global integral from satellite based observations of both SSA and energy flux, and find

$$\int_S z \boldsymbol{F_b} \cdot \hat{\boldsymbol{n}} = -1.5\,\text{GW} \tag{24}$$

In view of the measurement uncertainties, this is not significantly different than zero, and in any case is small compared to wind input. Thus we find this violation of the 'Anti-Turbulence Theorem' does no practical damage to its ultimate message. But, if we take the value in (24) literally, in particular its negative sign, the surprising interpretation emerges that this buoyancy flux contribution actually resists the development of turbulence. Net dissipation appearing on the left hand side of (21) is positive definite. If turbulence is to exist at all, the sources on the right hand side must also be positive. The first integral, for a stably stratified ocean, is positive definite, and apparently of an absolute value comparable to that of the second. The negative value in (24) thus reduces the net input to internal energy, rendering the first integral as an upper bound on turbulent generation.

In the absence of wind forcing, but with a surface buoyancy flux, one can envision that heating leads to positive SSAs and cooling negative SSAs due to the steric effect, such that a negative value of (24) would be expected. The interesting result in this case is that this buoyant flux contribution to ocean energetics would work against the positive definite value from (23), thus reducing the energy available for turbulence. Even though contributions to global energy exist that are not bounded by the molecular diffusivity, we find support for Paparella & Young (2002) and Sandström's theorem that horizontal convection is non-turbulent.

Also, the real ocean differs from the Sandström and Paparella & Young (2002) setting in that the bottom is not insulating, but rather hosts geothermal heat fluxes at a global average rate of approximately 100 mW/m^2 (1 mW = 10^{-3}W; (Pollack et al. 1993)). The distribution of these fluxes varies greatly in the ocean, with a preference for mid-ocean spreading centers. Although the global ocean is rather poorly sampled for its geothermal flux, global maps do exist. Coupling these with the much better known deep ocean topography allows for an estimate of the contribution of geothermal heat flux to global ocean circulations. Using the $2^\circ x 2^\circ$ maps provided by Davies (2013), we find

$$\int_{S_b} z\boldsymbol{F_b}\cdot\hat{\boldsymbol{n}}dS = 62\,GW \tag{25}$$

where S_b is the bottom surface of the domain. The above value is one to two orders of magnitude larger than all the contributions from surface fluxes.

Finally, it is worth remembering that the marine biosphere acts as a source of heating internal to the ocean by means of kinetic activities. Marine creatures in the course of living consume energy, all of which eventually ends up in the ocean in the form of heat. The total energy expended by the consumers is given by net primary production, a quantity estimated to be 63 TW (Dewar et al. 2006). This energy expenditure is biased toward the surface where the greater fraction of the biota live. If we assume 50 TW of the biosphere energetics is invested uniformly in the upper 500 m, 8 TW from 500 m to 1500 m and 2 TW over the remainder of the ocean, the energy provided to the circulation by the biosphere is roughly

$$-\int_V z\frac{g\alpha_T\rho_o\rho}{C_p}QdV \approx 13\,\mathrm{GW} \tag{26}$$

which is an order of magnitude larger than the contribution from surface buoyancy flux.

Surface Fluxes

Rearranging (12d), the energy flux through the ocean surface can be written

$$\int_S \boldsymbol{F_Q}\cdot\hat{\boldsymbol{n}}dS = \int_S \boldsymbol{u}\cdot\underline{\underline{\tau}}_o\cdot\hat{\boldsymbol{n}}dS + \int_V \rho QdV - \int_{S_b}\boldsymbol{F_Q}\cdot\hat{\boldsymbol{n}}dS \tag{27}$$

The size of the net surface flux needed to balance the inputs can be estimated. Ocean heating due to wind stress is $60TW$. This is matched by the biospheric heating of the ocean at a comparable value. The net geothermal heat flux over the global ocean is

$$\frac{0.1\mathrm{W}}{\mathrm{m}^2}3.5x10^{14}\mathrm{m}^2 \approx 35\mathrm{TW} \tag{28}$$

leading to a total of 155 TW, or a normalized ocean surface cooling flux of

$$\frac{155\,\mathrm{TW}}{3.5\times10^{14}\mathrm{m}^2} = 0.45\frac{\mathrm{W}}{\mathrm{m}^2} \tag{29}$$

Satellite based estimates of heat flux contain errors of O[10] W/m^2, which overwhelms this signature. In-situ methods provide tighter bounds and suggest a net ocean warming heat flux imbalance of O[0.4-1]W/m^2 (Meyssignac et al. 2019), which is comparable, if of the opposite sign to the dissipative contribution. This imbalance is usually interpreted as a sign of global warming, and reflective of a heat content trend. Given the uncertainties, it is unclear how oceanic processes contribute to the imbalance, but future estimation exercises should bear in mind the load in dissipative heat generation due to wind forcing, the biosphere and geothermal flux.

CONCLUSIONS AND DISCUSSION

We have considered the thermodynamics of fluids subject to mechanical and buoyant forcing using the full, compressible equations of motion. Perhaps the most important result to emerge from the analysis is that the net surface heat flux must include the heat generation due to a variety of ocean processes; in particular, the three non-standard energy sources of net wind work, geothermal flux and total metabolic activity of the biosphere. Each of these adds to the heat load the ocean must deposit to the atmosphere and are of the same order as the ocean heat imbalance expected due to anthropogenic climate change. It is necessary for these to be involved in any assessment of time dependent ocean heat content change.

Beyond this, buoyant forcing involves an input which is not directly proportional to molecular diffusivity, and therefore is not subject to the scale arguments used to justify the ‘Anti-Turbulence’ theorem of Paparella & Young (2002). However, estimates of the size of this input under modern conditions render it small, so that in any practical sense this does not represent a departure from the conclusions drawn in that paper. Also, under the setting envisioned by Sandström of a fluid forced only by buoyant fluxes at the surface, it seems likely that the correction will work counter to the production of turbulence, and hence strengthen the idea that horizontal convection is non-turbulent. Thus we support the implications of the Anti-Turbulence Theorem that the primary driver of abyssal mixing is the wind. Finally, we point out that geothermal flux and the net heat generated by the marine biosphere, even though small in an absolute sense, are far more efficient drivers of the general circulation than surface buoyancy fluxes.

THE BUOYANCY EQUATION AND RELATED ISSUES

The internal energy equation, rewritten in terms of enthalphy, $h = e + P\alpha$ is

$$\frac{d}{dt}h = \alpha\frac{d}{dt}P + T\frac{d}{dt}\eta \tag{30}$$

where for conveniece we will ignore salinity. Considering enthalpy as a function of entropy and pressure, and using (30), one can show

$$\begin{aligned} h_P &= \alpha \\ h_\eta &= T \end{aligned} \tag{31}$$

leading to the Maxwell relation

$$T_P = \alpha_\eta \tag{32}$$

where T_P is recognized as the adiabatic lapse rate. It is straightforward to show

$$T_P = \frac{T\alpha_T}{C_p} \tag{33}$$

From (18)

$$\frac{d}{dt}b = g\rho_o\frac{d}{dt}\alpha = g\rho_o\left(\frac{T\alpha_T}{C_p}\frac{d}{dt}\eta + \alpha_P\frac{d}{dt}P\right) \tag{34}$$

Using (6) and with some algebra

$$\frac{d}{dt}b = \frac{g\alpha_T\rho_o}{C_p}\left[-\frac{1}{\rho}\nabla\cdot\boldsymbol{F_Q} + \epsilon + Q\right] + \frac{g^2\rho_o^2}{\rho^2 c_s^2}w \tag{35}$$

where c_s is the speed of sound in water and P has been approximated by static pressure. Multiplying by $-z$, using mass conservation, volume integrating and time averaging leads to

$$\begin{aligned} \int_V \rho w b dV = &\int_{\hat{S}} z\rho F_b\cdot\hat{\boldsymbol{n}}dS + K\rho_o\int_S(b_s - b_b)dS - \\ &\int_V \frac{zg\rho_o\alpha_T\rho}{C_p}(\epsilon + Q)dV - \int_V z\frac{g^2\rho_o^2}{\rho^2 c_s^2}\rho w dV \end{aligned} \tag{36}$$

where $\boldsymbol{F_b} = \frac{\rho_o g\alpha_T}{\rho C_p}\boldsymbol{F_Q}$ and we have ignored the spatial variation of several thermodynamic terms. Regarding the last integral, mass conservation implies

$$\int_z \rho w dA = 0 \tag{37}$$

for any depth z. Therefore

$$\int_V z\frac{g^2\rho_o^2}{\rho^2 c_s^2}\rho w dV \approx \frac{g^2}{c_{so}^2}\int_z z\int_A \rho w dA dz = 0 \tag{38}$$

where c_{so} is a representative sound speed. The above argument suggests the last integral is small.

Author Contributions

W.K.D. : Conceptualization, Analysis, Interpretation, Writing, Editing & Computing;
B.D. : Conceptualization, Interpretation, Writing, Editing & Computing.

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