AN ATTEMPT TO LINK THE BRAZILIAN HEIGHT SYSTEM TO A WORLD HEIGHT SYSTEM

Um ensaio de conexão da Rede Altimétrica Brasileira a um Datum Vertical Global

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ABSTRACT

This paper deals with the geopotential approach to investigate the present Brazilian Height System (BHS). Geopotential numbers are derived from Global Positioning System (GPS) satellite surveying and disturbing potential on selected benchmarks. A model for the disturbing potential can be obtained by an existing set of spherical harmonic coefficients such as the Earth Gravity Model 2008 (EGM08). The approach provides absolute evaluation of local normal geopotential numbers (aka spheropotential numbers) related to a so-called World Height System (WHS). To test the validity of the proposed methodology, a numerical experiment was carried out related to a test region in Southern Brazil. The accuracy of the derived geopotential numbers was tested versus local normal geopotential numbers based on 262 GPS/leveling points. The root mean square error (RMSE) value for metric offset of BHS derived from geopotential numbers and the disturbing potential modeling in the test area was estimated to be near 0.224 meters in the absolute view. Therefore, since these spheropotential numbers are referred to a local datum, these results of comparisons may be an indicator of the mean bias of local network due to the effect of local Sea Surface Topography (SSTop) and possible offset between the unknown reference for the BHS and the quasigeoid model in the region.

Keywords: **G**eopotential Numbers; Vertical Datum Connections; Normal-Orthometric Heights; Quasigeoid.

RESUMO

Este trabalho apresenta uma investigação realizada a respeito da Rede Altimétrica Fundamental do Brasil (RAFB) com base em posicionamento GPS (*Global Positioning System*) e determinação do distúrbio do potencial sobre referências de nível. Um modelo para o distúrbio do potencial pode ser obtido com base nos coeficientes do desenvolvimento em séries de funções harmônicas esféricas do potencial gravitacional como os do *Earth Gravity Model* 2008 (EGM08) por exemplo. A abordagem provê a base para avaliação absoluta de números geopotenciais normais (ou números esferopotencias) locais relativamente a um Sistema Global de Altitudes (SGA). Para verificar a validade desta metodologia proposta um experimento numérico foi desenvolvido em relação a uma área teste no sul do Brasil. As acurácias dos números geopotenciais obtidos foram testadas em relação a números geopotenciais normais com base em 262 pontos GPS sobre a rede de nivelamento. O valor do erro médio quadrático (EMQ) em termos de um desvio métrico absoluto do Sistema Altimétrico Brasileiro (SAB) obtido a partir dos números geopotenciais e do distúrbio do potencial na área teste foi estimado em cerca de 0,224 metros. Uma vez que os números esferopotenciais são referidos ao *datum* local, os resultados das comparações efetivadas podem ser um indicador do desvio da rede local em vista da topografia do nível médio do mar (TNMM) e a possível inconsistência entre a origem do SAB e o modelo do quase-geóide para a região.

Palavras-chave: Números Geopotenciais; Conexão de *Data* Verticais; Altitudes Ortométricas-Normal; Quase-Geóide.

1. INTRODUCTION

The heights of the Brazilian Height System (BHS) were obtained by using spirit leveling as static system, and reduced for the gravity effect only by the normal gravity field. The BHS is not, *ipsis litteris*, a normal-orthometric height system in full because it was derived by using a truncated version of relative normal-orthometric corrections to the leveled height differences. However, from the practical point of view, the BHS could be considered as a normal-orthometric height system. The BHS does not have a complete physical meaning, *i.e.*, a defined reference surface does not exist, and the heterogeneities of Earth's crust are not considered.

The normal-orthometric heights are not capable of supporting the physical height determination departing from ellipsoidal heights obtained with Global Navigation Satellite System (GNSS) technologies (*e.g.*, Global Positioning System – GPS). It is well-know that the height determination using GNSS is based in a simple operative relation that links GNSS's geodetic height, h, above a given ellipsoid and the orthometric height, H^O. Here, these heights are counted along the

same normal to the ellipsoid, which is considered to be normal to the geoid as well without loss of precision. Applying this permissible simplification it is possible to write:

$$H^{O} = h - N \,,\tag{1}$$

where N is the geoid height related to the reference ellipsoid (*e.g.* GRS80 – Geodetic Reference System 1980). The relation (1) holds for normal height, H^{N}, and height anomaly (aka quasigeoid height), ζ. The geoid height (height anomaly) is determined from gravimetric data obtained on or near the Earth's surface.

A height system defined in the context of real Earth's gravity filed, *e.g.* normal and orthometric heights, is important for some set of scientific and practical tasks. However, in this paper we have the interest in vertical datums unification to a World Height System (WHS), *cf.* Ihde and Sánchez (2005). Following Rummel (2002), the main point for connecting two vertical datums is to determine their potential difference at a *datum* point or along the mean offset in the network, which implies in height difference between them. Our aim is to assess the normal-orthometric heights without an *a priori* treatment for connecting a regional normal-orthometric height system to a WHS.

Recently, the Global Geodetic Observing System (GGOS) under the umbrella of the International Association of Geodesy (IAG) proposed in its program of activities to study regional vertical systems and their relations to a WHS. This proposal is in agreement with Geodetic Reference System of Americas (SIRGAS) statements for linking South American vertical networks. A unified vertical datum in South American is important for data exchange in a continental or in a global scale.

Most of the South American vertical datums are based on a determination of Mean Sea Level (MSL) at different tide gauges over a varying range of time intervals and at different epochs (Bolivia and Paraguay are landlocked countries). Therefore, each vertical datum is referred to a particular equipotential surface, associated with the MSL at tide gauge and fixed for a specific epoch. In general, these surfaces are not coincident with some conventional global geoid. The mean ocean surface does not coincide with a level surface (*e.g.*, the geoid) of Earth's gravity field; the deviations are called Sea Surface Topography (SSTop, and if considered its time variations, it is called Dynamic Ocean Topography, DOT). The SSTop depends on both oceanographic and meteorological factors. The quasi-stationary geostrophic level is conditioned by several parameters such as salinity, pressure, temperature, water streams, etc., and its magnitude ranges between about ±2 m.

In this paper, the geopotential numbers – and thus the physical heights – related to any point on the Earth's surface could be derived from the state of the art Global Geopotential Models (GGMs) and GPS satellite surveying. Results shown

the height differences between the BHS and the WHS defined in a so called holonomic system. However, since these normal-orthometric heights are referred to a local datum, these comparisons may be biased due to the effect of localized SSTop and possible offset between the unknown reference for the BHS and the local quasigeoid model. In the sequence we describe the data and the computation methods used to calculate the potential difference between the BHS and the WHS.

2. METHODS

2.1 Derived Geopotential Numbers

Geopotential numbers are still very important for the calculation of physical heights for practical and scientific use. Many authors (*cf.* Moritz, 2000; Ihde and Sánchez, 2010; Bedada, 2010; Ferreira, 2011 and others) have suggested to apply the disturbing potential on the Earth's surface associated with the normal potential at the same point to calculate the geopotential numbers. Technical details on how to develop geopotential numbers from such a model must be worked out, so as to develop other types of heights (ROMAN *et al.*, 2010). In this modern concept, to carry out time-consuming leveling such as spirit and trigonometric leveling is not required any more.

Nonetheless, from Molodenskii's theory, which provides the disturbing potential T at point P on the Earth's surface, we have:

$$W(\varphi, \lambda, h) = U(\varphi, \lambda, h) + T(\varphi, \lambda, h) , \qquad (2)$$

where $W(\varphi, \lambda, h)$ is the geopotential required by:

$$C(\varphi, \lambda, h) = W_0 - W(\varphi, \lambda, h) , \qquad (3)$$

where the geopotential number $C(\varphi, \lambda, h)$ is computed in a direct way from gravity data. It is more general than the geometric determination of the orthometric height from ellipsoidal height associated with the geoid model according to the relation (1). The geopotential number contains the physical measure of height above an equipotential surface associated with the global MSL, conventionally obtained by the cumbersome method of leveling (and gravity observations along the leveling lines).

Following Ihde and Sánchez (2010) the WHS can be realized from points of leveling network in the form:

$$C(\varphi, \lambda, h) = C^j(\varphi, \lambda, h) - W_0 + W_0^j , \qquad (4)$$

where $C^j(\varphi,\lambda,h)$, local geopotential number, will be transformed from the regional level W_0^j to the conventional global level W_0. The difference W_0 and W_0^j can be determined by GPS/leveling in selected co-location points by (IHDE and SÁNCHEZ, 2010):

$$\delta W(\varphi,\lambda,h) = W_0 - T(\varphi,\lambda,h) - U(\varphi,\lambda,h) - C^j(\varphi,\lambda,h) \,. \tag{5}$$

However, in the Brazilian context the directly application of the relation (5) is not possible because $C^j(\varphi,\lambda,h)$ are not available. Therefore, from the definition of the normal-orthometric height H^{NO} (*cf.* HECK 2003), we can get from BHS:

$$C'^{j}(\varphi,\lambda,h) = \overline{\gamma}'(\varphi,H^{NO})H^{NO}(\varphi,\lambda) \,, \tag{6}$$

where C'^{j} is the normal geopotential number (aka spheropotential number) and $\overline{\gamma}'$ is the mean value of the normal gravity along the normal plumb line between the level ellipsoid and the normal equipotential surface $U(H^{NO})$.

The difference between geopotential numbers and the spheropotential numbers can be given as:

$$\delta C(\varphi,\lambda,h) = C^j(\varphi,\lambda,h) - C'^{j}(\varphi,\lambda,h) \,. \tag{7}$$

Applying (7) in (5) we obtain:

$$\delta\Gamma(\varphi,\lambda,h) = \delta W(\varphi,\lambda,h) + \delta C(\varphi,\lambda,h) \,. \tag{8}$$

Then, following Ferreira and de Freitas (2011) we can re-write the (5) as:

$$\delta\Gamma(\varphi,\lambda,h) = W_0 - T(\varphi,\lambda,h) - U(\varphi,\lambda,h) - C'^{j}(\varphi,\lambda,h) \,. \tag{9}$$

The Equation (9) can be simplified in the following way:

$$\delta\Gamma(\varphi,\lambda,h) = C(\varphi,\lambda,h) - C'^{j}(\varphi,\lambda,h) \,, \tag{10}$$

where

$$\delta\Gamma = \overline{\gamma}H^N - \overline{\gamma}'H^{NO} \,, \tag{11}$$

i.e.

$$\delta\Gamma = \overline{\gamma}(h - \zeta) - \overline{\gamma}'(h - \eta)\,, \tag{12}$$

where $\overline{\gamma}$ is the mean value of the normal gravity between the level ellipsoid and the normal equipotential surface $U(H^N)$. The term η is the difference between ellipsoidal height and normal-orthometric height over generic benchmarks as (FERREIRA and DE FFREITAS, 2011):

$$\eta = h - H^{NO}\,. \tag{13}$$

In (12) ζ is the height anomaly related to a "global" vertical datum after applying Bruns's formula in the disturbing potential as:

$$\zeta_P = \frac{T_P}{\gamma_Q}\,. \tag{14}$$

Considering that $\overline{\gamma} \approx \overline{\gamma}'$, we can re-write the (12) as:

$$\frac{\delta\Gamma}{\overline{\gamma}'} = \eta - \zeta = \delta H\,, \tag{15}$$

that represents the height differences between the "global" vertical datum and the local vertical datum. The reference ellipsoid and normal gravity field are defined by Somigliana-Pizzetti theory and the normal gravity potential and all its derivatives can therefore be regarded as known in the exterior space to the reference ellipsoid.

2.1 Disturbing Potential from a GGM

The disturbing potential T in spherical harmonics is (TORGE, 2001):

$$\begin{aligned} T(\varphi, \lambda, h) = T_0(r) + \frac{GM_{GGM}}{r(\varphi, \lambda, h)} \times \\ \sum_{n=0}^{n_{max}} \left(\frac{a_{GGM}}{r(\varphi, \lambda, h)} \right)^n \sum_{m=0}^{n} \left(\overline{C}_{nm}^T \cos m\lambda + \overline{S}_{nm} \sin m\lambda \right) \overline{P}_{nm}(\cos\vartheta) \end{aligned} \tag{16}$$

with degree n and order m of the harmonic coefficients and n_{max} indicating the maximum degree of the series expansions; GM_{GGM} is the geocentric gravitational

constant for the GGM; a_{GGM} semi major axis for the GGM; $\overline{P}_{nm}(\cos\vartheta)$ are the fully normalized associated Legendre functions. The spherical polar coordinates of computation point (r, ϑ, λ) geocentric polar coordinates of radius, geocentric co-latitude and longitude and, which are computed from the geodetic coordinates (φ, λ, h) for each computation point.

The term $\overline{C}_{nm}^{T}$ is the coefficients of the disturbing potential which denotes that the even zonal harmonics of the reference ellipsoid (*e.g.*, GRS80) are removed from the GGM coefficients $\overline{C}_{nm}$. The residual zonal harmonics are computed by:

$$\overline{C}_{nm}^{T} = \overline{C}_{n0}^{GGM} - \frac{GM_{GRS}}{GM_{GGM}}\left(\frac{a_{GRS}}{a_{GGM}}\right)^{n}\overline{C}_{n0}^{GRS} \ . \tag{17}$$

where the GM_{GRS} is the geocentric gravitational constant for the related Geodetic Reference System (GRS) and a_{GRS} is the semi major axis for the GRS. The zero degree term $T_0(r)$ in (16) is given by:

$$T_0(r) = \frac{GM_{GGM} - GM_{GRS}}{r(\varphi,\lambda,h)} - (W_0 - U_0) \ . \tag{18}$$

An important topic is related to the choice of the W_0 which has been the subject of several studies and several discussions, *cf.* Burša *et al.* (2007) and Sánchez (2007). We adopted the geopotential value $W_0 = 62,636,856.0$ m²s⁻² for definition of the WHS according to Burša *et al.* (2004). The normal potential value on the GRS80 ellipsoid is $U_0 = 62,636,860.850$ m²s⁻² (MORITZ, 1980).

2.2 The Normal Gravity Potential
The normal gravity potential U required in (9) is computed at the Earth's surfaces using Somigliana's formula (HOFMANN-WELLENHOF and MORITZ, 2006, Equation 2-126):

$$U(u,\beta) = \frac{GM}{E}\arctan\frac{E}{u} + \frac{1}{2}\omega^2 a^2\frac{q}{q_0}\left(\sin^2\beta - \frac{1}{3}\right) + \frac{1}{2}\omega^2(u^2 + E^2)\cos^2\beta \ . \tag{19}$$

The only constants that occur in this formula are the semi-major axis a, the semi-minor axis b, the geocentric gravitational constant GM, and the angular velocity of the Earth's rotation ω. In (19), the coefficient E stand for the linear

eccentricity $E = \sqrt{a^2 - b^2}$. The ellipsoidal-harmonic coordinates u, β, λ can be computed from φ, λ, h, for more information see (HEISKANEN and MORITZ, 1967, Equations 6-8a and 6-8b).

The coefficients q and q_0 are given by (HOFMANN-WELLENHOF and MORITZ, 2006, Equation 2-113):

$$q = \frac{1}{2}\left[\left(1 + 3\frac{u^2}{E^2}\right)\arctan\frac{E}{u} - 3\frac{u}{E}\right], \tag{20}$$

$$q_0 = \frac{1}{2}\left[\left(1 + 3\frac{b^2}{E^2}\right)\arctan\frac{E}{b} - 3\frac{b}{E}\right]. \tag{21}$$

The mean value of the normal gravity $\bar{\gamma}$ in (6) and (11) is evaluated using (HOFMANN-WELLENHOF and MORITZ, 2006, Equation 4-60):

$$\bar{\gamma} = \gamma\left[1 - \left(1 + f + m - 2f\sin^2\varphi\right)\frac{H}{a} + \left(\frac{H}{a}\right)^2\right], \tag{22}$$

where H can be the normal height H^N for $\bar{\gamma}$ and H^{NO} for $\bar{\gamma}'$; f is the flattening as $f = \frac{a-b}{a}$; the term m stands for the abbreviation $m = \frac{\omega^2 a^2 b}{GM}$; γ is the normal gravity on the ellipsoid given by (HOFMANN-WELLENHOF and MORITZ, 2006, Equation 2-140):

$$\gamma = \frac{GM}{a\sqrt{a^2\sin^2\beta + b^2\cos^2\beta}} \times$$
$$\left[\left(1 + \frac{m}{3}\frac{e'q_0'}{q_0}\right)\sin^2\beta + \left(1 - m - \frac{m}{6}\frac{e'q_0'}{q_0}\right)\cos^2\beta\right]. \tag{23}$$

In the Equation (23) the coefficient q_0' is given by (HOFMANN-WELLENHOF and MORITZ, 2006, Equation 2-113):

$$q_0' = 3\left(1 + \frac{b^2}{E^2}\right)\left(1 - \frac{b}{E}\arctan\frac{E}{b}\right) - 1. \tag{24}$$

The normal gravity γ_Q is computed in the point at the telluroid using (HOFMANN-WELLENHOF and MORITZ, 2006, Equation 2-115):

$$\gamma_Q = \gamma\left[1 - \frac{2}{a}\left(1 + f + m - 2f\sin^2\varphi\right)H^N + 3\left(\frac{H^N}{a}\right)^2\right]. \tag{25}$$

All numerical constants related to the GRS80 can be found in (MORITZ, 1980).

2.3 Test Area and Applied Data Set

A large area covering three states, São Paulo, Paraná and Santa Catarina with elevations from 0 to 1,600 meters was selected as a test area. This was chosen because 262 reasonably precise GPS/leveling ($\varphi, \lambda, h, H^{NO}$) were available for comparison according to relation (15). From the Figure 1 we can see the distribution of the GPS/leveling stations.

Figure 1 – The distribution of 262 GPS/leveling points over the target area in Southern of Brazil. The graphical scale is related to the parallel -22.5°.
SOURCE: Data courtesy from Prof. Nelsi Côgo de Sá.
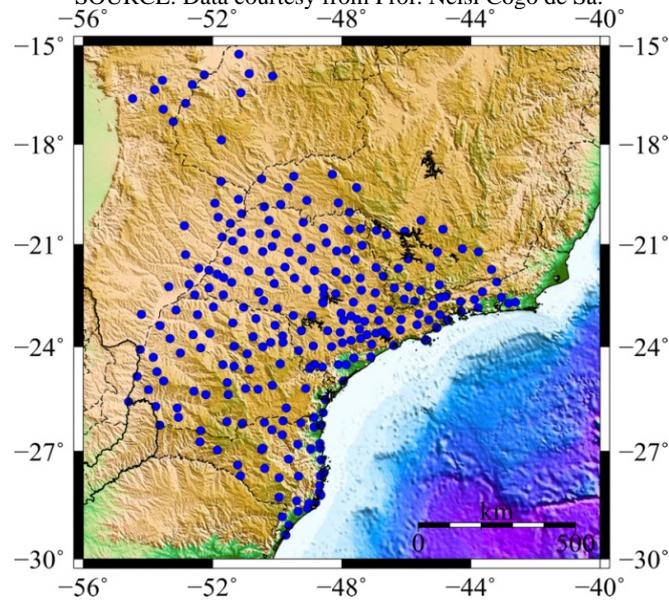
For estimating the height differences in according with (15) the normal-orthometric heights were transformed from the mean-tide system to the tide-free

system. The expression for height conversion between mean-tide to tide-free system with sub-centimeter accuracy is given by (TENZER *et al.*, 2011):

$$H_{mean-tide} = H_{tide-free} - \left\{ (1+k-h)\left[-0.198\left(\frac{3}{2}\sin^2\varphi - \frac{1}{2}\right)\right]\right\}, \qquad (26)$$

where *k* and *h* are the tidal Love numbers and its values are 0.3 and 0.6 respectively. This was necessary because the GPS points and the applied GGMs are related to a tide-free system.

The used GGMs for this research were the Earth Gravity Model 2008 (EGM08) (PAVLIS *et al.*, 2008) and the Gravity Observation COmbination 02S (GOCO02S) (GOIGINGER *et al.*, 2011). In the Table 1 we can see the constant related to these models as well as GRS80 necessary for solve (16).

Table 1. The defining constants of the EGM08, GOCO02S and GRS80.

Model	a (m)	GM (m^{3}s^{-2})
EGM08	6378136.3	3.986004415×10^{14}
GOCO02S	6378136.3	3.986004415×10^{14}
GRS80	6378137.0	3.986005000×10^{14}

In Figure 2 the spectra of the signal as well as the error for the two models are shown in terms of the disturbing potential signal and error degree variances. From the Figure 2 it is shown that the signal degree variances of EGM08 and GOCO02S are very close up to degree 200, after degree 200 the GOCO02S present a decrease. However, the difference between the error degree variances of the models we can see that the EGM08 present an increase after degree 165. The error degree variances of EGM08 between 2 and 164 degree are higher than the GOCO02S.

3. RESULTS AND DISCUSSION

262 points of the height differences were computed over the target area with basis in (15). The solution is based in two geopotential models (GOCO02S and EGM08) for the potential disturbing on the Earth's surface as per (16). The GOCO02S model was applied up to degree and order 250 and the EGM08 in two degree of expansion, *i.e.*, degree and order 720 and degree 2,190 and order 2,158. This selection is not random; the GOCO02S is the latest GGM based on data of the satellite gravity missions CHAMP, GRACE, and GOCE, terrestrial gravity field, satellite altimetry, and SLR data. The EGM08 is the state of the art high resolution GGM of the Earth's external gravity field. The choice of to apply the degree and order 720 was took in consideration for to test the fill-in technique adopted in the EGM08 solution for regions with a poor gravity data set coverage such as the area of study under consideration (see details in PAVLIS *et al.* 2008). The Table 2 shows

the numerical statistics for selected spectral window of the EGM08 as well as the GOCO02S.

The Table 2 shows the rate of the improvement between EGM08 up to degree and order 720 and GOCO02S up to degree and order 250 is about the 36%. This improvement is due to the accurate medium-wavelength (55 km to 200 km) contribution of ground data (gravity anomalies). The rate improvement between the GOCO02S and the EGM08 in its maximum degrees and orders is about the 36%. This value given some indication of the signal omission of GOCO02S ($n = 250$) disturbing potentials in the region of the study. This means that the local information used in the EGM08, *i.e.* gravity anomalies in 15'x15' in the region of study, provides enhanced signal over that presented in the GOCO02S. However, no significant differences between the EGM08 up to degree and order 720 and its maximum degree and order were found.

Figure 2 - Degree variances in terms of the disturbing potential for the EGM08 (red) in comparison to GOCO02S (black). Error degree variances of EGM08 (red dashed) and GOCO02S (black dashed).
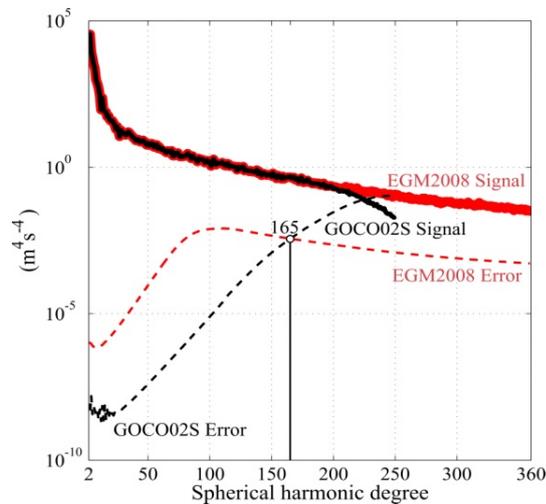

Figure 3 shows the results (δH) in space domain for the EGM08 up to degree 2,190 and order 2,158. The estimated average offsets δH is summarized in the Table 2; however, the estimated standard deviations of the δH is mainly due to the commission errors of the GGM coefficients. The errors due to the leveling networks, inaccuracies within the GPS surveying and the omission error of the GGMs were not taking into account.

Table 2 – Descriptive statistics of residual potential differences.

GGM	Statistic (meters)				Improvement rate
	Mean	SD	Min	Max	
EGM08 (2190)	+0.013	0.224	-0.731	+0.718	+36%
EGM08 (720)	-0.001	0.223	-0.766	+0.676	+36%
GOCO02S (250)	-0.026	0.350	-1.029	+1.156	-
EGM08 (250)	-0.040	0.332	-0.755	+1.002	+5%
GOCO02S (200)	-0.033	0.387	-1.124	+1.292	-11%
EGM08 (200)	-0.047	0.389	-1.096	+1.239	-11%
GOCO02S (165)	-0.057	0.506	-1.332	+1.637	-45%
EGM08 (165)	-0.060	0.525	-1.442	+1.701	-50%

Note: Improvement rates are given in terms of the standard deviation (SD) related to GOCO02S geoid model.

Figure 3 - Residual height differences (δH) represented by scale bar related to the Equation (15) in the target area.
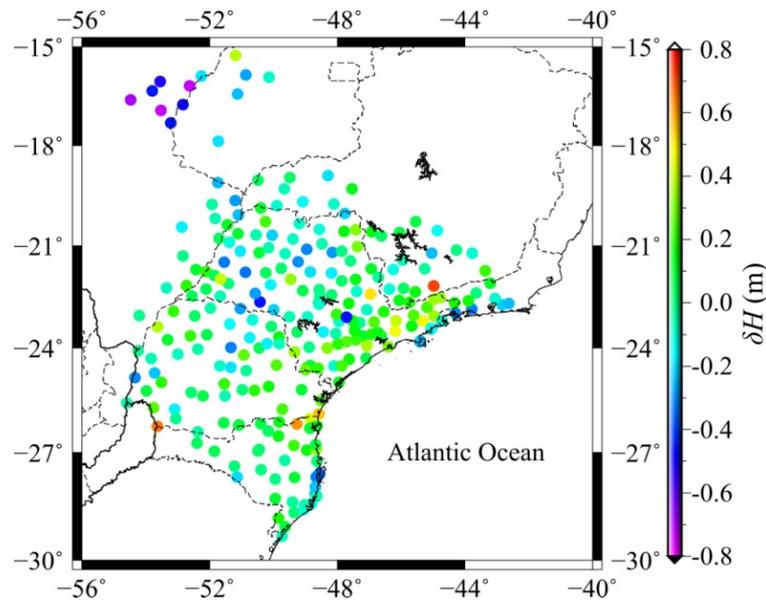
In agreement with Wang *et al.* (2011) no matter how good a GGM becomes, it is always possible to design and compute a local geoid model with higher frequency content if gravity anomalies and Digital Terrain Model are available. However, this is not exactly the case of the official geoid model in Brazil, MAPGEO2010 (IBGE, 2010 and DE MATOS *et al.,* 2012), where the ground gravity anomalies were

available for its determination. The Table 3 shows that the EGM08 slight better fit to the GPS/leveling in the region under consideration. In the Table 3 the EGM08 stands for the height anomaly ζ and it was determined as per (16) and (14) respectively. The EGM08 fits better in the study area indicating that the official geoid model in this region (see Figure 1) should be revised. Other important conclusion is that for the so called normal-orthometric heights in the region of the study, the best reference is related to a quasigeoid model for GPS height determinations.

Table 3 – Descriptive statistics of residual potential differences.

Model	Statistic (meters)					Improvement rate
	Mean	SD	Min	Max	RMSE	
MAPGEO (N)	-0.002	0.263	-0.907	0.825	0.263	-
EGM08 (ζ)	0.013	0.224	-0.731	0.718	0.224	+15%

Note: Improvement rate is given in terms of the root mean square error (RMSE) related to MAPGEO2010 geoid model.

4. SUMMARY AND CONCLUSIONS

The Brazilian Height System (BHS) is defined as a normal-orthometric height system, this means that the difference between the ellipsoidal heights and normal-orthometric heights provides η as showed in (13). In this study, a modern vision of height systems and leveling approach were presented. It was considered the possibility of unification of vertical datums within geopotential-value approach. The methodology applied makes it possible to unify the Brazilian Vertical Datum (BVD) based on normal-orthometric heights to a World Height System (WHS) without an *a priori* treatment. Validated tests presented in this paper show that the EGM08 outperforms the GOCE satellite derived model GOCO02S significantly for the degree greater than 250. From the long wavelengths (>200 km) point of view no significant differences was noted between both models. These tests indicated that EGM08 fits the GPS/leveling to better than 0.224 meters and a bias of 1.3 cm implying that any future changes to W_0 should be minor for the BVD. In doing so, all future improvements of BHS based on W_0 and more precise GGM will cause all the heights to be offset by a small quantities.

However, for allow the use of the GNSS positioning for obtaining orthometric height (or normal) it is necessary a physical model such as a geoid (or quasigeoid) instead of the single model described by (13). For provide BHS with physical meaning, the separation between the local geopotential numbers and the local spheropotential numbers δC, *cf.* (7), should be determined. Due to the quality of the official Brazilian geoid model, in terms of the RMSE, in the Southern of Brazil region (*cf.* Table 3) as well as its official evaluation in the study region where it has its better resolution in the whole country, it is clear that MAPGEO2010 not supply

the needs for vertical positioning by using GNSS. We recommend the revision of this model. Instead of using the official geoid model, in the region of study, the available high-quality geopotential model could be used as a slight better alternative (15% of improving) for obtaining orthometric or normal height from GNSS leveling.

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