Correlations Between Total Cross Sections and Slopes

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We investigate correlations between the total cross section and the slope of the elastic differential cross section for proton-proton and antiproton-proton scattering at the highest energies. Based on the empirical behavior of these quantities as function of the energy, we select two different analytical parametrizations connecting them, and obtain the correlations through fits to the experimental data available. We present and discuss practical uses of extrapolations and interpolations of the results. In the former case we refer to the estimation of the proton-proton total cross sections from the proton-air cross sections (obtained from cosmic-ray experiments), and in the later case, we critically discuss the recent measurement of the slope parameter at the BNL RHIC at 200 GeV by the pp2pp Collaboration.

1 Introduction

For particle-particle and antiparticle-particle interactions at high energies (c.m. energy $\sqrt{s}$ above 10 GeV), proton-proton (pp) and antiproton-proton ($\overline{p}p$) are the processes corresponding to the highest energy values with available data. In accelerator experiments pp scattering has been investigated in detail up to $\sqrt{s} \sim 63$ GeV (CERN ISR) and $\overline{p}p$ up to $\sqrt{s} \sim 2$ TeV (Fermilab Tevatron). Currently, at BNL RHIC, pp collisions at $\sqrt{s} = 200$ GeV are taking place, and the first measurement of the elastic slope parameter has already been reported [1]. However, the differences between the highest energies reached, namely $(\sqrt{s})^{\overline{p}p}_{max} \sim 30(\sqrt{s})^{pp}_{max}$ in the past and, presently, $(\sqrt{s})^{\overline{p}p}_{max} \sim 9(\sqrt{s})^{pp}_{max}$, rend difficult a comparative investigation of both reactions.

On the other hand, experimental information on the proton-proton total cross section exist from cosmic-ray experiments, in the interval $\sqrt{s} = 6$ - 40 TeV. However, the procedure to obtain this quantity is strongly model dependent and different models lead to different and discrepant results [2]. The steps from the experimental determination of the shower attenuation length (through analysis of extensive air showers) up to the extraction of the pp total cross section are discussed in detail in Ref. [3]. For our purposes, we only recall that, in the last step, the pp total cross section is obtained from the $p-air$ total cross section by means of the Glauber multiple diffraction formalism, which connects the corresponding scattering amplitudes. The pp amplitude is parametrized by

$$F^{pp}_{el}(s,t) \propto \sigma^{pp}_{tot}(s) \exp\left\{\frac{B(s)t}{2}\right\}, \qquad (1)$$

where $B(s)$ is the slope of the forward differential cross section,

$$B(s) = \frac{d}{dt}\left[\ln\left(\frac{d\sigma^{pp}_{el}}{dt}\right)\right]_{t=0}$$

and from the Optical Theorem the total cross section is expressed by

$$\sigma^{pp}_{tot}(s) = \frac{ImF^{pp}_{el}(s,t=0)}{s}.$$

One of the sources of the model dependence is the connection between σ^{pp}_{tot} and B, in Eq. (1) and this is the point we are mainly interested in. For example, as discussed in detail in Ref. [2], the Fly's Eye Collaboration using the geometrical scaling model ($\sigma^{pp}_{tot} \propto B$), obtained $\sigma^{pp}_{tot} = 120 \pm 15$ mb at $\sqrt{s} = 30$ TeV; on the other hand, Gaisser, Sukhatme and Yodh, making use of the Chou and Yang prescription connecting σ^{pp}_{tot} and B, estimated $\sigma^{pp}_{tot} \sim 175 \pm 34$ mb at $\sqrt{s} = 40$ TeV. Obviously, these two results are in complete disagreement [2].

In this communication we present and discuss correlations between σ^{pp}_{tot} and B whose *extrapolations* may allow a better determination of the σ^{pp}_{tot} from σ^{p-air}_{tot}. We also make use of *interpolation* of these parametrizations in order to critically investigate the recent measurement of the slope parameter by the pp2pp Collaboration.

The manuscript is organized as follows. In Sec. II we discuss the experimental data to be analyzed and also the strategies used in the selection of the parametrizations. In Sec. III we present the results of the fits, together with extrapolations to cosmic-ray energies. In Sec. IV we critically discuss the recent measurement of the slope parameter at $\sqrt{s} = 200$ GeV, showing the discrepancies with the parametrizations and experimental data available. The conclusions and final remarks are the contents of sec. V.

2 Experimental data and parametrizations

Fig. 1 shows the experimental data on $\sigma_{tot}(s)$ and $B(s)$ *at the same energy*, for pp and $\overline{p}p$ elastic scattering, from ac-

celerator experiments and above $\sqrt{s} = 10$ GeV [4, 5]. We have selected the data above the region of Coulomb-nuclear interference and below the "break" in the hadronic slope at the diffraction peak (localized at $|t| \sim 0.2$ GeV^2 at the ISR and Collider energies), namely $0.01 < |t| < 0.20$ GeV^2. In this region, the differential cross section data are well fitted by a single exponential and therefore there is no change in the slope associated with the t-dependence, but only with the energy [5].

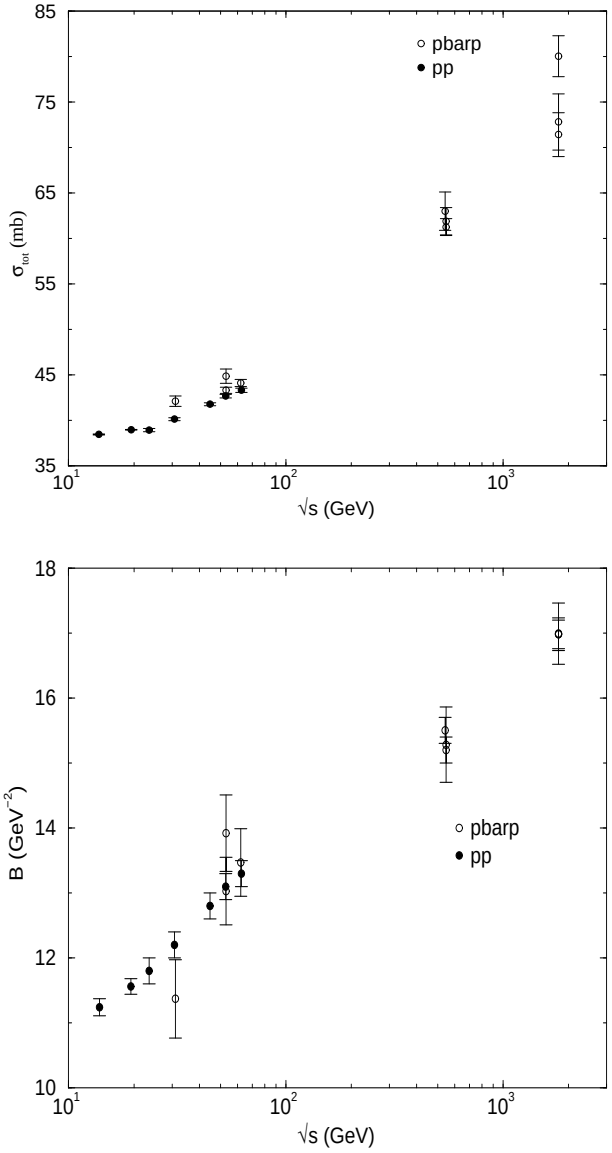


Figure 1. Experimental data on total cross section and the corresponding values of the slope at each energy, for pp and $\overline{p}p$ elastic scattering above 10 GeV [4].

Our choices for the parametrizations are based on the following observations. From Fig. 1, it is empirically evident that the slope parameter increases linearly with the logarithm of the c.m. energy and the total cross section follows some second degree polynomial in $\ln s$. Besides agreement with the above results, suitable analytical parametrizations, connecting $\sigma_{tot}(s)$ and $B(s)$, should also be efficient in the individual description of σ_{tot} as function of the energy. For these reasons, we consider two analytical forms, associated with two different parametrizations, one introduced by Donnachie and Landshoff (DL) [6], $\sigma_{tot}^{pp}(s) = Xs^{\epsilon} + Ys^{-\eta}$, $\sigma_{tot}^{\overline{p}p}(s) = Xs^{\epsilon} + Zs^{-\eta}$ and the other by Kang and Nicolescu (KN) [7] : $\sigma_{tot}^{pp}(s) = A_1 + B_1 \ln s + k \ln^2 s$, $\sigma_{tot}^{\overline{p}p}(s) = A_2 + B_2 \ln s + k \ln^2 s + 2Rs^{-1/2}$. The differences between these parametrizations are well known and have been recently reviewed in [2].

Based on the above observations, we consider analytical forms connecting $\sigma_{tot}(s)$ and $B(s)$ that follow the DL and KN parametrizations, that is, we identify $B(s)$ with $\ln s$ in these equations. Several tests are possible, with different constraints for the free fit parameters. In this communication we shall consider the results with the reasonable assumption that, asymptotically, pp and $\overline{p}p$ scattering follow the same dependence on the energy. However, the differences in the structure of the parametrizations allow different asymptotic behaviors, as we shall show. Specifically, we introduce the following forms and notation connecting $\sigma_{tot}(s)$ and $B(s)$:

- Parametrization *a*

$$\begin{aligned} \sigma_{tot}^{pp} &= a_1 e^{a_2 B} + a_3 e^{-a_4 B} \\ \sigma_{tot}^{\overline{p}p} &= a_1 e^{a_2 B} + a_5 e^{-a_6 B} \end{aligned} \qquad (2)$$

- Parametrization *b*

$$\begin{aligned} \sigma_{tot}^{pp}(s) &= b_1 + b_2 B + b_3 B^2 \\ \sigma_{tot}^{\overline{p}p}(s) &= b_1 + b_2 B + b_3 B^2 + b_4 e^{-B/2} \end{aligned} \qquad (3)$$

Obviously, these are strictly mathematical choices, having nothing to do with the physics behind the DL and KN parametrizations.

3 Fitting results and extrapolations to cosmic-ray energies

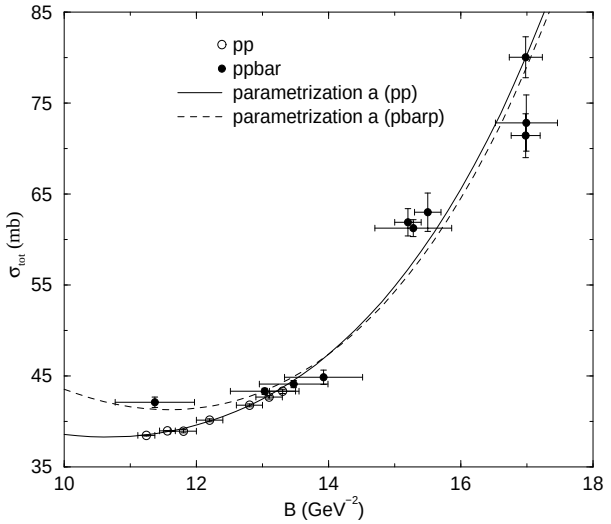
Fits to the experimental data, displayed in Fig. 1, through Eqs. (2) and (3) have been performed with the CERN-Minuit program. Table I shows the results of simultaneous fits to pp and $\overline{p}p$ data, and Fig. 2 shows the parametrizations *a* and *b* together with the experimental data.

Due to the large error bars (mainly in the B parameter) and the small number of points, the χ^2/DOF is a bit high. However, Fig. 2 shows that the data are well described in both cases.

In Fig. 3 we plot the extrapolations of both parametrizations up to values of the total cross sections typical of cosmic-ray experiments, $\sigma_{tot} \sim 200$ mb (Sec. I). We see that above $\sigma_{tot} = 70$ - 80 mb (typical of Tevatron energies), the parametrizations diverge fast. Since the DL parametrization (*a*) eventually violates the Froissart-Martin bound, we understand that the most truthfull and also conservative result for extrapolations concerns that associated with the KN parametrization (*b*).

TABLE I. Fit results

parametrization a		parametrization b	
a_1 (mb)	0.674 ± 0.014	b_1 (mb)	163.5 ± 0.3
a_2 (GeV^2)	0.272 ± 0.001	b_2 (mbGeV^2)	-22.65 ± 0.03
a_3 (mb)	101 ± 1	b_3 (mbGeV^4)	1.027 ± 0.002
a_4 (GeV^2)	0.128 ± 0.001	b_4 (mb)	631 ± 127
a_5 (mb)	179 ± 17	-	-
a_6 (GeV^2)	0.168 ± 0.008	-	-
No. DOF	11	No. DOF	13
χ^2/DOF	7.9	χ^2/DOF	8.0



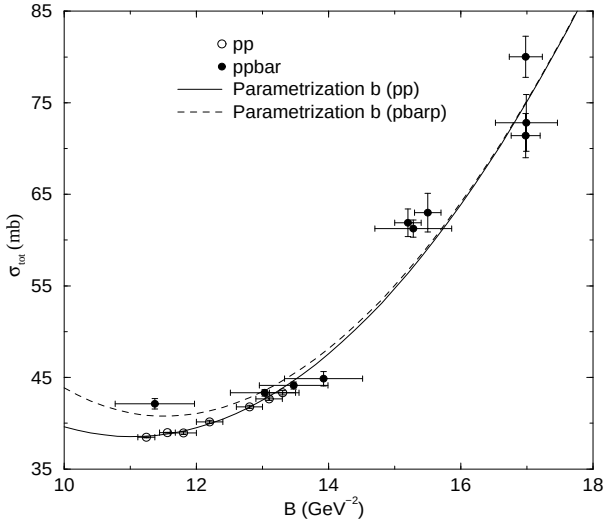


Figure 2. The total cross section in terms of the slope and the fitted parametrizations *a* (above) and *b* (below).

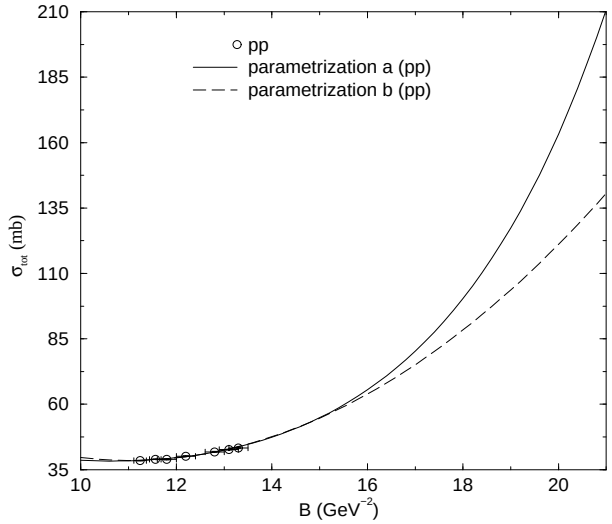


Figure 3. Extrapolation of the parametrizations up to values of the total cross section typical of cosmic-ray energies.

4 The slope parameter at 200 GeV

In this section we discuss the recent measurement of the slope parameter by the pp2pp Collaboration at $\sqrt{s} = 200$ GeV [1]. The B value has been extracted from fit to the differential cross section data in the t range $0.010 \leq |t| \leq 0.019$ GeV^2. The corresponding amplitude has contributions from the Coulomb amplitude, nuclear amplitude and the interference between them. It is parametrized by [1]

$$\begin{aligned} \frac{d\sigma}{dt} &= 4\pi(\hbar c)^2 \left(\frac{\alpha G_E^2}{t}\right)^2 + \frac{1+\rho^2}{16\pi(\hbar c)^2}\sigma_{tot}^2 e^{-B|t|} \\ &- (\rho + \Delta\Phi)\frac{\alpha G_E^2}{|t|}\sigma_{tot} e^{-\frac{1}{2}B|t|} \end{aligned} \quad (4)$$

The fit parameters are the slope B and a normalization constant. The input values for σ_{tot} and ρ used by the authors were 51.6 mb (obtained from the Donnachie-Landshoff model) and 0.13 (fit by the UA4/2 Collaboration), respectively. The resulting slope parameter was $B = 16.3 \pm 1.6$ (stat.) $\pm$ 0.9 (syst.) GeV^{-2}. Adding in quadratures the error reads $\pm$ 1.8 GeV^{-2}.

According to the above result, at $\sqrt{s} = 200$ GeV, the value of the total cross section corresponding to $B = 16.3 \pm 1.8$ GeV^{-2} is $\sigma_{tot} = 51.6$ mb. On the other hand, if we use this B value as input in our parametrizations, we infer: $\sigma_{tot}^{pp}(200\ \mathrm{GeV}) = 69^{+33}_{-19}$ mb from parametrization *a* and $\sigma_{tot}^{pp}(200\ \mathrm{GeV}) = 67^{+23}_{-16}$ mb from parametrization *b*.

These three "peers" of points are plotted in Fig. 4, together with the experimental data. The above values of $\sigma_{tot}^{pp}(200\ \mathrm{GeV})$ from our parametrizations are displayed in Fig 5, together with all the experimental information presently available on total cross sections, above 10 GeV (see [2] for a review and numerical values concerning the cosmic-ray estimates).

From Fig. 2 and 4 we see that the point from the pp2pp Collaboration is in disagreement with our parametrizations and also with the general behavior of the experimental data. If this value of the slope is correct, then new physics is suggested: the pp correlation between σ_{tot} and B follows a linear dependence and the $\overline{p}p$ correlation a quadratic dependence, with a crossing nearly $B \sim 14$ GeV^{-2} and $\sigma_{tot} \sim 45$ mb. These Figures also show that if $B(200\ \mathrm{GeV})$ is used as input, our parametrization predicts cross sections that are in agreement with the general trend of the experimental data on σ_{tot} and B, suggesting, in this case, no need for new physics. However, as shown in Fig. 5, these inferred values for the total cross sections are much higher than generally expected for pp scattering. Moreover, looking at the estimations from cosmic-ray experiments, these values are in qualitative agreement with the fastest increasing scenario for $\sigma_{tot}(s)$, namely, the results by Gaisser-Sukhatme-Yodh (GSY) and Nikolaev [2]. In this case, the experimental information indicate a crossing, with σ_{tot}^{pp} becoming greather than $\sigma_{tot}^{\overline{p}p}$ above $\sqrt{s} \sim 70$ - 80 GeV, and, once more, new physics is suggested.

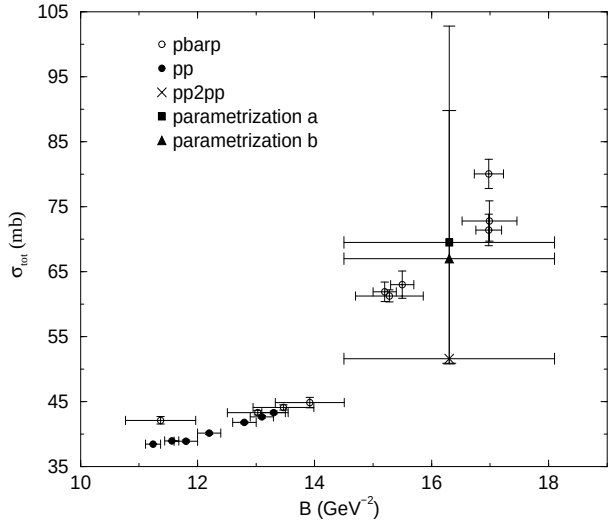


Figure 4. Experimental data on total cross section and the corresponding slope together with the input cross section value and the extracted slope by the pp2pp Collaboration.

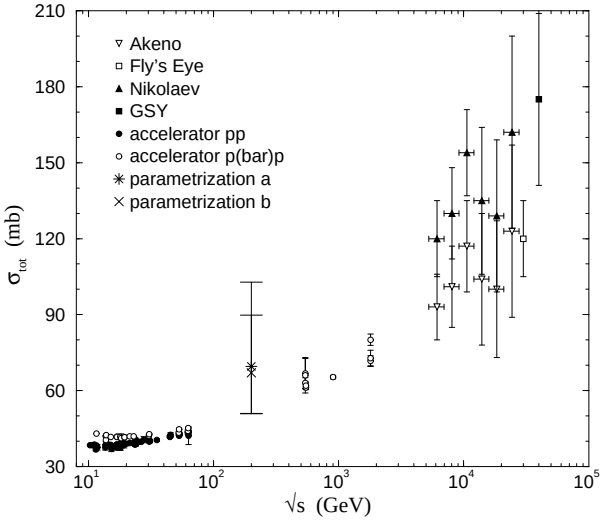


Figure 5. The total cross section and the limits for σ_{pp}^{tot} at $\sqrt{s} = 200$ GeV using parametrizations a and b, Eqs. (2) and (3) (see Ref. [2] for detailed information on the estimations at cosmic-ray energies).

5 Conclusions

In this communication we made use of two different parametrizations connecting the experimental data on the total cross section and the slope parameter. Extrapolations to cosmic-ray energies with parametrization *b* (KN) may be useful in the determination of the pp total cross section from p-air cross section, allowing to connect $\sigma_{tot} - B$ in an almost model independent way. We are presently investigating this subject.

We showed that the combination $B = 16.3 \pm 1.8$ GeV^{-2} and $\sigma_{tot} = 51.6$ mb, indicated by the pp2pp analysis, is in disagreement with the general trend for the behaviors of σ_{tot} and B. If this "peer" is correct, new physics is necessary. Using the above B value as input in our parametrizations, the corresponding values of the total cross sections show agreement with the σ_{tot} versus B data. However, these inferred values for σ_{tot} indicate new physics when plotted as function of the energy.

It should be noted that the slope parameter at 200 GeV has been determined in a relatively small region of the momentum transfer ($0.010 \leq |t| \leq 0.019$ GeV^2), just above the region of Coulomb-nuclear interference, and may be that its value has been overestimated. However, if the B(200 GeV) measurement is correct and represents an hadronic quantity, its high value may indicate a "break" in the slope near 0.02 GeV^2, a phenomenon that was never observed in both pp and $\overline{p}p$ scattering, at $\sqrt{s} \leq 62.5$ GeV and $\sqrt{s} \leq 1.8$ TeV, respectively. Once more, we conclude that new physics is necessary.

Acknowledgments

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