# Application of Anharmonic Vibrational Models for Energy Spectrum Studies of Superheavy Nuclei

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The recent resumption of the synthesis of new superheavy nuclei has provided relevant results for nuclear physics and chemistry. These nuclei provide experimental confirmation of theoretical calculations that predict the existence of two islands of stability, one with maximum stability in Z=114 and N=184 and the other in Z=120 or Z=126 and N=184 or N=228. Through cold fusion, recently synthesized nuclei with Z=114, 116 and 118 have confirmed the theoretical predictions, providing a possibility of reaching the maximum of these islands of stability. Because of the low production rate and the extremely short half-life of these nuclei, it has not yet been possible to determine their energy spectra. However, it has been theoretically predicted that the nuclei should acquire a spherical form in these islands of stability, thus supplying characteristic vibrational energy spectra. The predictions also indicate the existence of deformed superheavy nuclei in the region between the islands, which can possess rotational energy spectra. In this work, the hydrodynamic equation of energy was corrected by inserting terms referring to coulombian effects, shell closure and nuclear deformation, enabling us to predict the energies of the first excited state for superheavy nuclei of quadrupole deformation, the quadrupole phonon state and an octupole phonon for spherical superheavy nuclei. Four anharmonic vibrational models were then used to describe higher energy states of the nuclei in question. The energy spectra of the $^{266}104$, $^{268}106$, $^{270}108$, $^{274}110$, $^{278}112$, $^{298}114$, $^{304}120$, $^{348}120$, $^{354}126$ nuclei and their closest neighbors were determined.

## 1 Introduction

Theoretical predictions reconciled to recent experimental indications of the existence of islands of stability for superheavy elements (SHEs) have been testing to the limit the concepts about nuclear structures and heavy ions.

Experiments conducted in Dubna[1] through nuclei reactions of evaporation channels of 4 and 5 neutrons have indicated a deformed nuclei region of great stability close to the $^{270}Hs_{108}$.

In the case of doubly magic spherical nuclei, theoretical predictions over the last twenty-five years have indicated that the next island of stability, after the closure of layers of 126 neutrons and 82 protons of $^{208}Pb$, is around the region of Z=114 and N=184, which forms the element $^{298}114$ [2-5]. In addition to this island, recent calculations have predicted a second region of stability close to Z=120 or Z=126 with N=184 or N=228, which refers to the $^{304}120$, $^{348}120$, $^{310}126$ and $^{354}126$ elements [2-5]. Fig. 1 is an extension of Segrè's graph for superheavy nuclei, indicating the possible regions where doubly magic spherical nuclei can exist.

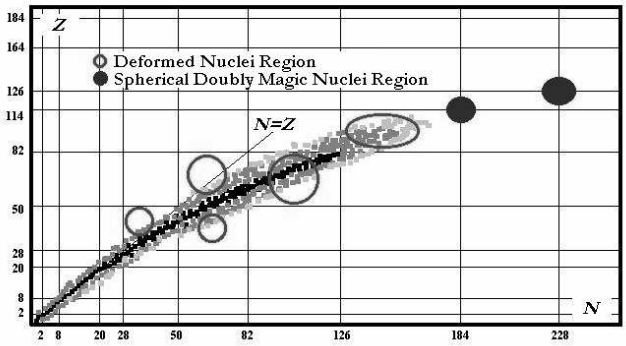

Figure 1. Predictions of islands of stability for superheavy nuclei. The first island occurs in Z=114 and N=184 and the second in Z=120 or Z=126 and N=184 or N=228. The presence of deformed nuclei is predicted in the regions among these islands.

The most recent member of the SHEs family was synthesized in early 1999 in Berkeley[6, 7] through so-called cold fusion[7, 8], whereby the bombardment of $^{86}Kr$ ions on $^{208}Pb$ doubly magic nuclei produces the compound nucleus with Z=118. In a low excitation state, evaporating only one neutron, the final product obtained is the $^{293}118$ element and its immediate decay product, the $^{289}116$ element.

Despite advances in the accelerator, detector and target technology, the production rate of these superheavy elements is extremely low, making difficult an analysis of their nuclear properties, a situation further aggravated by their instability and the resulting short half-life. Thus, only indirect confirmation based on analyses of the decay chain of these nuclei can indicate their existence. A recent analysis in Berkeley, made possible by the sufficiently long half-life of the isotope of the transactinide bohrium[9] element, with Z=107 and N=160, indicated that this element is a member of group VII of the periodic table.

Due to the current impossibility of determining these new nuclei properties experimentally, several theoretical models successfully used in the well-known description of nuclei have been extrapolated in order to predict some of their nuclear characteristics. In this work, the application of hydrodynamic collective and anharmonic vibrational models was extrapolated to predict the energy spectra of deformed and spherical superheavy nuclei.

With the purpose of predicting the energy of the states of an $E(2^+)$ quadrupole phonon and an $E(3^-)$ octupole phonon of superheavy nuclei, three dependences involving the nuclear structure were introduced into the hydrodynamic equation of nuclear excitation energy(HEE)[10]. The first was a dependence on Z involving the coulombian effects in the nucleus in question. The second involved the shell closure effect in doubly magic nuclei[11]. The third dependence concerned the deformation of superheavy nuclei[11] among the aforementioned islands of stability. Four anharmonic vibrational models were used, starting from the $E(2^+)$ and $E(3^-)$ energies, to predict the energy of the superior excitation states.

The energy spectra in the deformed superheavy nuclei region should present rotational characteristics. The rotational states of some well known nuclei described by Yrast's lines are predicted in Casten and Zamfir's(CZM)[12, 13, 14] model. Through its applicability in the construction of the energy spectrum of some well-known rotational nuclei, the CZM may yield the energy spectra of the $^{266}104$, $^{268}106$, $^{270}108$, $^{274}110$, $^{278}112$ nuclei and their closest neighbors.

The spectra of spherical superheavy nuclei, whose existence is predicted in the proximities of the aforementioned islands of stability, should display characteristic excitation levels of vibrational modes, which can be predicted by the application of vibrational models of anharmonic terms in the hamiltonian of the nuclear system. The applicability of the anharmonic vibrational models of Hadermann et al.(HAM)[15, 16], Brink et al.(BRM)[15, 17] and DAS et al.(DAM)[15, 18] was verified by the construction of the energy spectrum of well-known vibrational nuclei[19, 20]. After calculating the $E(2^+)$ and $E(3^-)$ energies, the energy spectra of the $^{298}114$, $^{304}120$, $^{348}120$, and $^{354}126$ nuclei and their neighbors were predicted through successive applications of the three anharmonic vibrational models in question.

## 2 Hydrodynamic equation of energy

The final form of the HEE for $n$ phonons of multipolarity $\lambda = 2$ or 3 with the coulombian, shell and nuclear deformation corrections is given by:

$$\begin{aligned} E(I^{\pm}) &= \epsilon \frac{5}{A}\left\{ \frac{\lambda(\lambda-1)}{(\lambda+1)}[6.25(\lambda+1)(\lambda+2)A - 1.5Z^2] \right\}^{\frac{1}{2}} \\ &- 0.58 s(N,Z) e^{-\frac{\alpha^2}{\alpha_o}} + 8(MeV) \end{aligned} \qquad (1)$$

The first term of Eq. (1) is obtained through the liquid drop model, introducing a total charge Z uniformly distributed over its volume[10]. The analogy between the liquid drop model and the collective modes of the nuclear vibrations has its limitations, since it is the first excited state of nuclear vibration in the order of some hundreds of $keV$, whereas, for a liquid drop, this state reaches some $MeV$. To find lower energy values, the first term of the HEE was parameterized through a constant multiplier $\epsilon$ called the alometric parameter. The $\epsilon$ parameter was obtained by adjusting the experimental $E(2^+)$ and $E(3^-)$ data for well-known nuclei, which yielded, respectively, $\epsilon(2^+) = 0.15$ and $\epsilon(3^-) = 0.30$.

The correction for shell closure is represented by the function $s(N, Z)$, whose mathematical form is the same as the shell correction term for the semi-empirical mass form described by W.D. Myers and W.J. Swiatecki[11] as:

$$s(N,Z) = \frac{F(N) + F(Z)}{(\frac{1}{2}A)^{\frac{2}{3}}} - 0.26A^{\frac{1}{3}} \qquad (2)$$

where $F(N)$ and $F(Z)$ have the same dependence on $N$ and on $Z$, respectively:

$$\begin{aligned} F(N) &= \frac{3}{5}\left(\frac{M_i^{\frac{5}{3}} - M_{i-1}^{\frac{5}{3}}}{M_i - M_{i-1}}\right)(N - M_{i-1}) \\ &- \frac{3}{5}(N^{\frac{5}{3}} - M_{i-1}^{\frac{5}{3}}), \text{ with } M_{i-1} < N < M_i \end{aligned} \qquad (3)$$

and $M_i$ corresponds to the i-th magic number.

The nuclear deformation[11] is described by the exponential $e^{\frac{-\alpha^2}{\alpha_o}}$ where:

$$\alpha^2 = \beta^2 \frac{R_o^2}{4\pi} \qquad (4)$$

and $\beta$ is the nuclear deformation parameter.

Figure 2 illustrates the good agreement between Eq. (1) and the $E(2^+)$ and $E(3^-)$[20] experimental data of mass number nuclei between 8 and 214. The peaks in the magic number regions are predicted by the closely approximated and corrected HEE. Although nuclear deformation parameters were not used in this adjustment, the deformed rotational nuclei region is very closely approximated by the corrected HEE. The extrapolation of the superheavy nuclei region is represented in the same figure. The deformation parameters calculated by J. Meng and N. Takigawa[21] were employed for the Z=104, 106, 108, 110 and 112 nuclei.

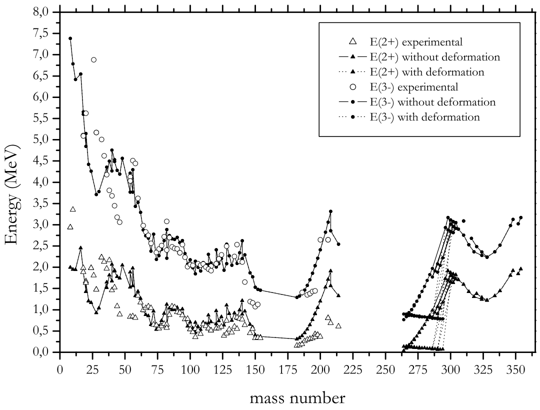

Figure 2. Results obtained by the corrected HEE for the $E(2^+)$ and $E(3^-)$ energies of well-known nuclei.

## 3 Energy spectrum of deformed superheavy nuclei

Casten and Zamfir's(CZM) model is given by the energy equation for an anharmonic vibrator:

$$E_n(I) = nE_1(2^+) + \frac{n(n-1)}{2}\epsilon_4 \tag{5}$$

where the first term corresponds to the harmonic vibrational model expression of an $n$ phonons state and the second term corresponds to the anharmonicity introduced. This model, according to the authors[12, 13, 14], correctly predicts the excited states of collective characteristic nuclei, including rotational, vibrational and vibrational-rotational transition nuclei, presenting discrepancies in the order of $1\%$ from the $I \sim 28$ nuclear spin states.

To verify the applicability of CZM to predict the energy spectrum of deformed nuclei, the $^{232}Th$, $^{234}U$ and $^{242}Pu$ spectra were built and compared with their respective experimental spectra. The results obtained were satisfactory, with greater deviations observed only for spin-parity states above $10^+$.

The values of the $\epsilon_4$ parameter were calculated for rotational nuclei in the region of $224 <$A$< 256$, based on their experimental values of $E(2^+)$ and $E(4^+)$ energies. A function with the form: $\epsilon_4 = c_1 + c_2 exp[-(A - c_3)/c_4]$ was used to fit these values in order to determine the $c_1$, $c_2$, $c_3$ and $c_4$ constants and, by extrapolation, to predict the $\epsilon_4$ value for superheavy nuclei in the $^{270}Hs_{108}$ region. The values obtained were: $c_1 = 0.0576$, $c_2 = 0.0315$, $c_3 = 224$, and $c_4 = 4.2386$. The deformed superheavy nuclei region in question is located close to A=270, not very far from the last nuclide considered in this calculation. Thus, the $\epsilon_4$ values predicted by the adjusted equation should not differ substantially in this region, which justifies an extrapolation of this parameter.

The $E(2^+)$ energy of the nuclei in the $^{270}Hs_{108}$ region was found by extrapolating the corrected HEE, Eq. (1). The $\epsilon_4$ values were determined based on the calculations described in the previous paragraph. Thus, using the CZM, we built the rotational energy spectra of the $^{266}104$, $^{268}106$, $^{270}108$, $^{274}110$, and $^{278}112$ deformed nuclei and their closest neighbors. These spectra are illustrated in Fig. 3.

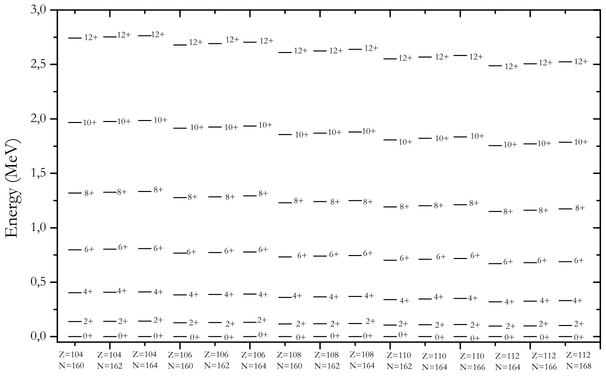

Figure 3. Predicted spectra for deformed superheavy nuclei.

## 4 Energy spectra of spherical superheavy nuclei

As mentioned in sec. 1, the first step in the construction of the energy spectrum of a vibrational nucleus is to identify the $E(2^+)$ energy of a quadrupole phonon state, from which, after the successive application of the three anharmonic vibrational models, HAM, BRM and DAM, one can describe the subsequent excited states. This energy was obtained directly from the HEE extrapolation, taking as zero the deformation parameters of the nuclei in question, since they are probably spherical. In order to verify the three models' applicability, they were used only to describe the energy spectrum of ten nuclei in the 60 <A< 196[19] region. The results obtained showed good agreement with the respective experimental spectra, confirming the applicability of the three anharmonic vibrational models.

Starting from the $E(2+)$ energies of the supposed spherical superheavy nuclei obtained through the corrected HEE extrapolation, the HAM gives the energies of the $0^+$, $2^+$ and $4^+$ states of two-quadrupole phonon triplet of these nuclei. Based on the values of these energies, the BRM provides the states of the three-quadrupole phonon quintuplet. Through the $E(2^+)$ and $E(4^+)$ energies, the DAM provides the $6^+$ quintuplet state, in agreement with the BRM and hence, through this same model, one can predict the $8^+$ energy of the four quadrupole phonon state. The $3^-$ energy state of an octupole phonon for superheavy nuclei is obtained directly from the corrected HEE extrapolation. Hence, the energy spectrum of the aforementioned supposed spherical superheavy nuclei can be built based only on knowledge of the $E(2^+)$ and $E(3^-)$ energies.

Studies on the $^{110}Cd$, $^{112}Cd$, $^{114}Cd$ and $^{110}Pd$ vibrational nuclei by J. Kern and J. Jolie J.[22] have shown that the $E(2^+)$ energy should be renormalized, that is, increased by $15\%$, so that the BRM and the DAM can supply lower states of energies coherent with the experimental spectrum of the nuclei in question. Thus, after the energies of the two-quadrupole phonon triplet for the spherical superheavy nuclei were calculated, the $E(2^+)$ energy was renormalized and BRM and DAM then applied to predict the three and four quadrupole phonon states.

The energy spectra predicted for $^{298}114$, $^{304}120$, $^{348}120$, and $^{354}126$ doubly magic superheavy nuclei and their neighbors are illustrated in Fig. 4.

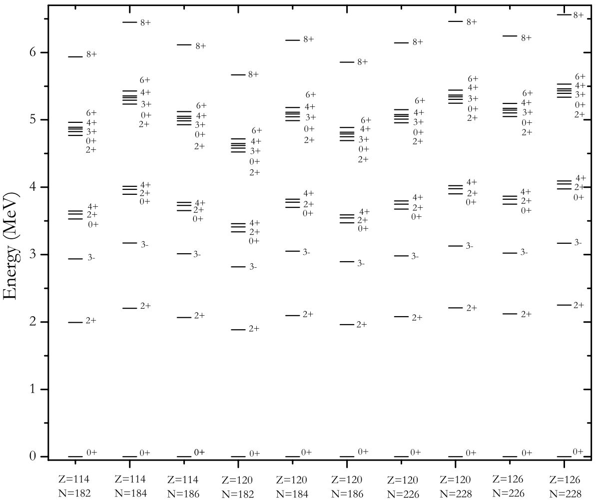

Figure 4. Energy spectra predicted for spherical superheavy nuclei.

# 5 Gamma transitions in superheavy nuclei

The amount of gamma ray energy emitted in the de-excitation process of superheavy nuclei after their formation can be measured today. Thus, the $E2$ transitions, which are the most probable, were calculated for the superheavy nuclei, whose spectra were predicted. In the calculations for spherical superheavy nuclei, the quadrupole phonon state was not renormalized. Table I lists the results for the deformed nuclei while Table II lists the results for the spherical nuclei.

TABLE I. Prediction for the $E2$ transitions in deformed superheavy nuclei.

| nucleus $^AZ$ | $E2(2^+ \rightarrow 0^+)$ | $E2(4^+ \rightarrow 2^+)$ |
|---|---|---|
| $^{264}104$ | 0.1387 | 0.2661 |
| $^{266}104$ | 0.1405 | 0.2679 |
| $^{268}104$ | 0.1423 | 0.2696 |
| $^{266}106$ | 0.1281 | 0.2554 |
| $^{268}106$ | 0.1302 | 0.2575 |
| $^{270}106$ | 0.1322 | 0.2595 |
| $^{268}108$ | 0.1166 | 0.2440 |
| $^{270}108$ | 0.1190 | 0.2464 |
| $^{272}108$ | 0.1213 | 0.2487 |
| $^{272}110$ | 0.1068 | 0.2341 |
| $^{274}110$ | 0.1095 | 0.2368 |
| $^{276}110$ | 0.1120 | 0.2394 |
| $^{276}112$ | 0.0963 | 0.2237 |
| $^{278}112$ | 0.0994 | 0.2267 |
| $^{280}112$ | 0.1022 | 0.2296 |
| average | 0.1197 | 0.2470 |

TABLE II. Prediction for the $E2$ transitions in spherical superheavy nuclei.

| nucleus $^AZ$ | $E2(2^+ \rightarrow 0^+)$ | $E2(4^+ \rightarrow 2^+)$ |
|---|---|---|
| $^{296}114$ | 1.7342 | 1.9142 |
| $^{298}114$ | 1.9171 | 2.0971 |
| $^{300}114$ | 1.7973 | 1.9773 |
| $^{302}120$ | 1.6387 | 1.8187 |
| $^{304}120$ | 1.8213 | 2.0013 |
| $^{306}120$ | 1.7051 | 1.8851 |
| $^{346}120$ | 1.8077 | 1.9877 |
| $^{348}120$ | 1.9217 | 2.1017 |
| $^{352}126$ | 1.8440 | 2.0240 |
| $^{354}126$ | 1.9573 | 2.1373 |
| average | 1.8144 | 1.9944 |

As can be seen in Table I, the average value of the $E2$ transitions, from the $2^+$ state to the fundamental state of the deformed superheavy nuclei, is $0.1197MeV$. For the spherical superheavy nuclei, the average value of this transition is $1.8144MeV$, as indicated in Table II. For $E2$ transitions from the $4^+$ state to the $2^+$ state, the average value of the energy of the emitted gamma ray is $0.2470MeV$ for deformed nuclei and $1.9944MeV$ for spherical nuclei. Therefore, the centroid measurement of the emitted gamma rays can indicate whether the nucleus is spherical or deformed, since the values of the $E2$ transitions are quite different in the two cases.

## 6 Conclusion

As can be observed in Fig. 2, the HEE[10], corrected by the insertion of the coulombian, shell and deformation dependences, offers an excellent description of the experimental data of the $E(2^+)$ and $E(3^-)$ excitation energies. This good agreement with experimental data in such a wide band of measurements makes it possible to extrapolate the HEE for superheavy nuclei.

The applicability of CZM[12, 13, 14] in the construction of the energy spectrum of rotational nuclei was verified through the prediction of the energy spectra of some well-known rotational nuclei and through Yrast's line study of rotational, vibrational and rotational-vibrational transition nuclei. As for the extrapolation of the $\epsilon_4$ parameter for the deformed superheavy nuclei region, it appears acceptable because of its very well-defined functional variation, which supplies a substantially constant value for the $\epsilon_4$ parameter due to the adjusted exponential saturation. Thus, the predicted energy spectra are acceptable for the ${}^{266}104$, ${}^{268}106$, ${}^{270}108$, ${}^{274}110$, and ${}^{278}112$ nuclei and their closest neighbors.

In the case of the prediction of energy spectra for supposed spherical superheavy nuclei, the applicability of HAM[15, 16], BRM[15, 17] and DAM[15, 18] was verified by the construction of the energy spectrum of ten well-known vibrational nuclei. These three models were applied sequentially, starting from the $E(2^+)$ energy value supplied by the HEE extrapolation, to predict the energy spectra of ${}^{298}114$, ${}^{304}120$, ${}^{348}120$, and ${}^{354}126$ spherical superheavy nuclei and their neighbors. The $E(2^+)$ was renormalized, as proposed by J.KERN and J.JOLIE[22], allowing the BRM to provide lower states of energy and, consequently, more coherent results, as verified for some well-known nuclei.

The predictions obtained by CZM, HAM, BRM and DAM for well-known nuclei and the coulombian, shell and nuclear deformation correction terms inserted in HEE provided a solid theoretical base. This allowed for the extrapolation performed with the HEE, as well as the application of the aforementioned models, to describe the spectra of energies of the superheavy nuclei under study. However, confirmation of these predictions will only be possible through direct confrontation with experimental data, which are still inexistent today due to the low production rate, high excitation energy and extremely short half-life of these nuclei.

A form of verifying such predictions would be by directly measuring the gamma ray energy of these nuclei's $E2$ transitions. As discussed in sec. 5, due to the considerable differences in the values of predicted energy of deformed and spherical nuclei, these transitions can also provide important informations on the nuclear deformation of superheavy nuclei. Thus, it is expected that the energy levels obtained from the rotational spectra of deformed superheavy nuclei, and the order and average energy value of the $n$ phonon multiplets predicted for spherical superheavy nuclei will be coherent with future experimental data.

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