MATHEMATICAL MODELING APPLIED TO THE GROWTH OF TILAPIA IN NET CAGES IN THE SUB MIDDLE OF THE SÃO FRANCISCO RIVER

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ABSTRACT: This study aimed to apply mathematical models to the growth of Nile tilapia (*Oreochromis niloticus*) reared in net cages in the lower São Francisco basin and choose the model(s) that best represents the conditions of rearing for the region. Nonlinear models of Brody, Bertalanffy, Logistic, Gompertz, and Richards were tested. The models were adjusted to the series of weight for age according to the methods of Gauss, Newton, Gradiente and Marquardt. It was used the procedure "NLIN" of the System SAS® (2003) to obtain estimates of the parameters from the available data. The best adjustment of the data were performed by the Bertalanffy, Gompertz and Logistic models which are equivalent to explain the growth of the animals up to 270 days of rearing. From the commercial point of view, it is recommended that commercialization of tilapia from at least 600 g, which is estimated in the Bertalanffy, Gompertz and Logistic models for creating over 183, 181 and 184 days, and up to 1 Kg of mass , it is suggested the suspension of the rearing up to 244, 244 and 243 days, respectively.

KEYWORDS: economy, equations, *Oreochromis niloticus*.

MODELAGEM MATEMÁTICA APLICADA AO CRESCIMENTO DE TILÁPIAS EM TANQUES-REDE NO SUBMÉDIO DO SÃO FRANCISCO

RESUMO: Este estudo teve como objetivo aplicar modelos matemáticos ao crescimento de tilápias-do-nilo (*Oreochromis niloticus*) criadas em tanques-rede no Submédio do São Francisco e escolher o(s) modelo(s) que melhor representa(m) as condições de criação para a região. Foram utilizados os modelos não lineares de Brody, Bertalanffy, Logístico, Gompertz e Richards. Os modelos foram ajustados às séries de peso por idade, de acordo com os métodos iterativos de Gauss, Newton, Gradiente e Marquardt. Foi utilizado o procedimento "NLIN" do Sistema SAS® (2003) para a obtenção das estimativas dos parâmetros a partir dos dados disponíveis. O melhor ajuste dos dados foi realizado pelos modelos de Bertalanffy, Gompertz e Logístico, que são equivalentes para explicar o crescimento dos animais até 270 dias de criação. Sob o ponto de vista comercial, recomenda-se a comercialização das tilápias com, pelo menos, 600 g, que é estimado nos modelos de Bertalanffy, Gompertz e Logístico para criações superiores a 183; 181 e 184 dias, e para massa de até 1 kg, sugere-se a suspenção da criação em até 244; 244 e 243 dias, respectivamente.

PALAVRAS-CHAVE: economia, equações, *Oreochromis niloticus*.

INTRODUCTION

The rearing of Nile tilapia, Oreochromis niloticus, for commercial purposes is developed practically, in all states of Brazil, in rearing usually made in net cages, and the products are intended for different market niches (KUBITZA , 2007). The production of this species, according to the

MPA (2010), represents 39% of the Brazilian production of freshwater fish culture, being the most important farmed fish.

For a successful rearing, it is important to monitor the animal growth, because it is an important ecological parameter that can serve as an indicator of life conditions of these organisms: those living in better environmental conditions will grow faster than those that are living in non-ideal or degraded environments (SIANGAS et al., 2012).

In this context, the São Francisco valley is characterized by bringing together the ideal conditions for the development of fish farming: there is plenty of water, both in quantity and quality and also presents warm weather with little temperature variation through the whole year (CODEVASF, 2010).

The net cages are enclosed structures of screen or net, that holds the fish and enable full water exchange in the form of continuous flow, facilitating the visualization of fish and simplifies the handling of animals. The development of farming in net cages must be managed with planning, technical and legal knowledge, aiming the sustainability of the business.

Growth data adjustment based on weight-age functions that describes the whole period of animal life becomes more informative: it allows to obtain accurate information about the mass gain of the animals (TEXEIRA et al, 2012.). A lot of authors studies the development of the fish body. According to VANDER et al. (2007), to study the growth by adjusting a function that describes the entire life of the animal becomes more informative: it condenses the information from a series of data in a small set of biologically interpretable parameters.

In this conception, the purpose of this study was to apply mathematical growth models of Nile tilapia (Oreochromis niloticus) created in net cages in the Sub middle of the São Francisco river and choose the model/models that best represents the conditions of rearing for this region.

MATERIALS AND METHODS

Data collection was performed in CODEVASF/ Embrapa Semi-Arid - CPATSA and covers the production of Nile tilapia in net cages of a commercial property in a complete cycle of rearing, in Juazeiro, BA. The region climate is type Bswh, according to Köeppen classification.

In total, there were 270 days, 9 biometrics, with 83 fish weighed in each measurement, resulting in a total of 747 fish. The rearing period occurred on September 13th of 2010 to May 9th of 2011. Measurements were performed monthly. The fingerlings used had an average of initial weight 10.53 ± 2.17 g.

Water analyzes for monitoring physico-chemical parameters were performed. For fish feeding, commercial extruded feed was offered, initially to 44% crude protein (CP), and according to fish evolution, CP content had a decrease to 36% and later to 32% . The amount of feed was calculated, for each enclosure, based on biomass: proportion of 4% and 3%.

Five non-linear models were used to choose the one that best described the fish growth curve. They are: Brody (Brody, 1945), Bertalanffy (BERTALANFFY, 1957), Gompertz (LAIRD, 1965), Logistics (Nelder, 1961), Richards (RICHARDS, 1959), which have their mathematical representation according to the equations: Eq.1, Eq.2, Eq.3, Eq.4 and Eq.5 respectively.

$$Y = A(1 - Be^{-kt}) + \varepsilon; \tag{1}$$

$$Y = A(1 - Be^{-kt})^3 + \varepsilon; \tag{2}$$

$$Y = Ae^{-Be^{-kt}} + \varepsilon; \tag{3}$$

$$Y = A(1 + Be^{-kt})^{-1} + \varepsilon; \tag{4}$$

$$Y = A(1 - Be^{-kt})^M + \varepsilon; \tag{5}$$

where,

Y = the fish weight to a certain age (t);

A = the asymptotic value of Y (average weight at maturity);

B = the integration constant related to the initial weights (animal maturity degree at birth);

k = the variation rate of the exponential function (speed that the animal is approaching to the adult body weight), and

M = the parameter that shapes the curve, which corresponds to 1 to the Brody model, 3 to the Bertalanffy, -1 for the Logistic, tends to ∞ for the Gompertz and is variable to the Richards.

In nonlinear models, it is not executable solving the equations system formed directly, since that the system resolution depends directly of the specific parameters to be estimated. You must use the iterative process for the acquisition of estimated parameters. Iterative methods are those that define an infinite sequence of operations, determining a sequence of approximations, whose limit is the exact solution of the system (BARD, 1974).

The iterative Newton method (MACLEOD, 1984), allows the approximation of the solution of a nonlinear equation, starting from a initial value x_1, generating a sequence of values $x_i (i = 1, 2, ...)$. So, up to when the desired approximation is achieved. The line that passes through $(x_1, y_1 = f(x_1))$ is given by: $y = f'(x_1)(x - x_1) + f(x_1)$, that intersects the axis of x on the abscissa point $x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$. This value allows to generate, by the same method, a new value $x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$ from the abscissa of the intersection point of the tangent to $f(x)$ on the point $(x_2, y_2 = f(x_2))$ and so on. It is a method based on the calculation of the function derivative $f(x)$.

The iterative gradient method requires the calculation of first and second derivatives, i.e. partial ones. The numerical minimization procedure is finished according to a convergence criterion. The gradient method or the Maximum Descent uses the negative of gradient of the objective function as a search direction. This method relies on the fact that, at a certain point , the gradient opposite direction of the objective function is the direction of the maximum decrease of the objective function (BARD, 1970).

The iterative Gauss method is a method used for solving problems of nonlinear least squares. It can be seen as a modification of the Newton method to find the minimum of a function (HARTLEY, 1961). Unlikely the Newton method, the Gauss algorithm can be used only for minimizing a sum of squared values of the function, but has the advantage that the second derivatives, which can be hard to calculate, are not necessary (JENNRICH. 1969).

The iterative Marquardt method in practice, has global convergence characteristics: converges to the local minimum from any approximate value, being the limit of the step size done through the use of trust region (MARQUARDT, 1963). Although it is very optimized, this procedure depends on several factors that may influence its convergence or not, such as: data number, the type of equation to be adjusted and the initial estimates of the coefficients (WU, 2000).

The NLIN procedure estimates the parameters of nonlinear models by the method of the least squares. The non-linear models are more difficult to estimate parameters than the linear models (RATKWOSKY, 1990). The regression expression must be written, to declare the parameters, providing initial values. Some models are difficult to adjust, and there is no guarantee that the procedure can successfully adjust the model (JENNRICH & MOORE, 1975).

To start the estimation of the mathematical parameters models, the NLIN procedure first examines the specifications of the parameters initial values already inserted in the model. Thereafter

the NLIN evaluates the sum of the residual squares for each combination of parameter values to determine the set of parameter values that produce the smallest sum of the residual squares.

The models adjustments to the data were made by the procedure "NLIN" SAS 2003 and iterative methods of Gauss, Newton, Gradient and Marquardt. In these methods, the iterative process converges in the j-th iteration when $(SRS_{j-1} - RS_j) / (SRS_j + 10^{-6}) < 10^{-8}$. The 10^{-8} value is the value that determines the convergence of the model. The convergence of a mathematical model ensures that the sum of the residual squares (SRS) of its parameters is minimal, tending to zero.

The best model selection, the one developed aiming the selection of an optimal solution (BAIO et al, 2013) was performed on the basis of simple indicators, considering: the data convergence, the determination coefficient (R^2) and the sum of residual squares (SRS). In this study, weight and mass are considered synonymous.

The table below shows an application of the NLIN procedure of SAS, together with the iteration Gauss method, in the function setting of von Bertalanffy to the tilapia weight in the SAS program.

```
BERTALANFFY;

PROC NLIN METHOD = GAUSS /* uses the iterative Gauss method */

      NOITPRINT /* suppress the result f each iteration */

      CONVERGE=10E-8 /* convergence condition */

      OUTEST=saida1; /* results produced by iteration */

/* _weight_ = expression; */

PARMS A = 0.3294 /* initial value for adult weight, kg */

      B = 0.6800 /* initial value for B constant */

      K = 0.1249; /* initial value for K */

MODEL y = A*(1-B*EXP(-K*T))**3; /* bertalanffy model */

CO =(1-B*EXP(-K*T)); /* part of bertalanffy model */

..

BOUNDS A=2.841346496 1.55722118 to 13.12548161; /* A extend considered */

BOUNDS B=1.008 0.0808 to 10.8089; /* B extend considered */

BOUNDS K=0.133531393 0.013353139 to 10.146884532; /* K extend considered */

DER.A=CO**3; /*partial derivative with respect to A */

DER.B=-3*A*EXP(-k*T)*CO**2; /* partial derivative regarding B */

DER.K= 3*A*B*T*CO**2*EXP(-k*T); /* partial derivative regarding K */

OUTPUT OUT= RA_BERT PREDICTED = Y; /* files with predicted values */

PROC PRINT data=RA_BERT; /* print files with  predicted values */

 run;
```

RESULTS AND DISCUSSION

The parameters estimates for each type of animal growth used to predict growth, in mass (kg), of Nile tilapia raised in net cages are shown in Table 1.

TABLE 1. Estimated values for the parameters A, B and k for all models analyzed.

Model	Parameter			
	A	B	K	M
Brody	13.4853	0.9812	0.0001	-
Bertalanffy	4.122	0.9257	0.0037	-
Logistic	1.521	43.7867	0.0183	-
Gompertz	2.383	5.3516	0.0075	-
Richards	0.9728	0.4483	0.0086	4.8917

Figure 1 shows the behavior of the mass accumulation of tilapia, for Y-observed (observed mass) and Y-estimated (estimated weight) to 270 days of rearing.
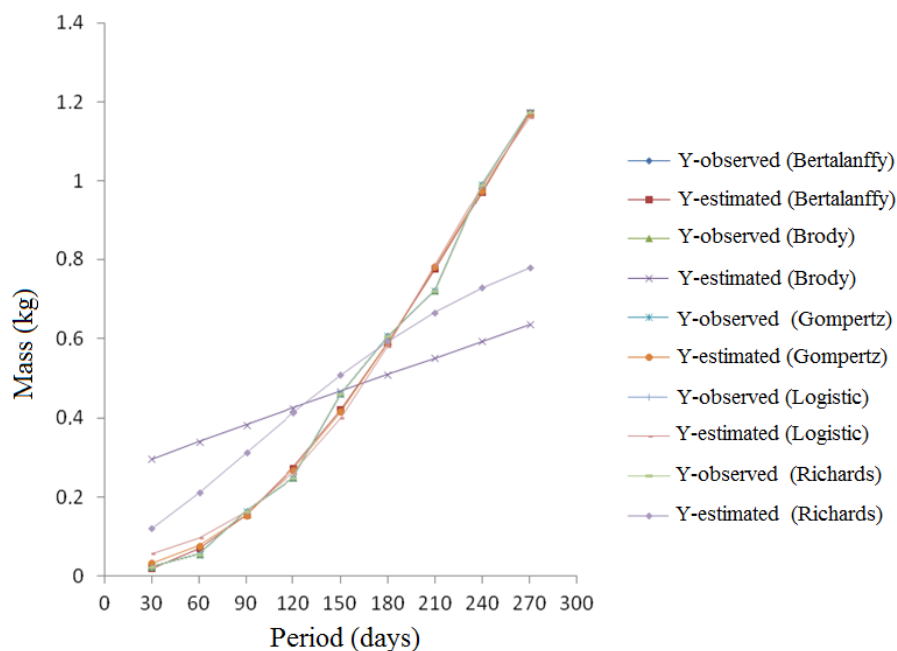

FIGURE 1. Growth curves observed and estimated according to the models, for 270 days of rearing: Bertalanffy, Brody, Logistic, Gompertz and Richards.

Below are shown the set of adjusted equations from Brody, Bertalanffy, Gompertz, Logistic and Richards models, according to the equations: Eq.6, Eq.7, Eq.8, Eq.9 e Eq.10, respectively:

$$Y = 13.4853(1 - 0.9812e^{-0.000108t}); \tag{6}$$

$$Y = 4.122(1 - 0.9257e^{-0.00369t})^3; \tag{7}$$

$$Y = 2.383e^{-5.3516e^{-0.00747t}}; \tag{8}$$

$$Y = 1.5210(1 + 43.7867e^{-0.0183t})^{-1}; \tag{9}$$

$$Y = 0.9728(1 - 0.4483e^{-0.00856t})^{4.8917}; \tag{10}$$

In the evaluation of this species, the tilapia, studies of growth curves by adjusting the weight prediction equations based on the animal age has been important. The requirements for achieving success in animal production are becoming increasingly stringent (CORDEIRO et al., 2012).

In the present study, Brody and Richards models overestimated values of initial mass and underestimated the final values for the 270 days of rearing (Figure 1). FREITAS (2007) modeling the goats growth through nonlinear models, concluded that, Brody, Logistic and Bertalanffy were adequate to estimate the goats growth, in all ages, being the only exception, the birth weight, because the three models overestimated it in the magnitude of 10%.

Taking the results of the logistic model, in particular, it is observed that is one of the tested models that best adjust the observed weights, as obtained by MCMANUS et al. (2003): the adult weight estimated was also lower than those other tested curves (Brody, Richards) for bergamasca sheep in the region of Brasilia.

The parameter A (Table 1) represents the estimate of the asymptotic weight, which is interpreted as adult weight. The estimates of the parameter A, in kg, of tilapia was higher for Brody (13.4853), Bertalanffy (4.1220), and Gompertz (2,383), followed by Logistics model (1.521) and Richards (0.9728). This result differs from that found by MANRIQUE (2012), when modeling the Aracá bandeira growth (Ptherophyllum scalare), where the Logistic model had the lowest asymptotic weight (A) 13.98 g and the highest value was obtained by Bertalanffy, with 66.62 g.

The average weight of tilapia searched by the consumer market is around 600g/850g to 1 kg, in other words, the total tilapia fattening cycle occurs, approximately, in 180 days in net cages (NOGUEIRA, 2007). In this context, the fish in the present study could be marketed from 183, 247, 181, 184, 183 days for Bertalanffy, Brody, Gompertz, Logistic and Richards models, respectively, when they reach the minimum weight marketing: 600g. And to limit mass of 1kg, would require 244, 546, 244 and 243 days for Bertalanfy, Brody, Gompertz and Logistic models.

There was an inability to predict the time required to the estimative time (days) required to the tilapias to achieve 1 kg of body weight on Richards model: the value of A (Table 1) is less than the desired estimate (Y-estimated). So far, this model is not recommended for calculations of Y-estimated for the cultivation of tilapia in net cages in the Sub middle of São Francisco river, when the desired weight is greater than or equal to 972g. It is reiterated here that, this result is due to the lack of data convergence, which will be further explored.

The parameter k (Table 1) represents the rate of maturity of the animal and indicates the growth velocity to achieve asymptotic weight. According to MALHADO et al. (2009a), the lower k value, slower the animal will approach its asymptotic weight, in other words, lower is the growth velocity of the animal in order to reach its adult weight.

The largest value of k in this study was obtained by the Logistic model (0.0183), followed by Richards (0.00856), Gompertz (0.00747), Bertalanffy (0.00369) and Brody (0.00011). A similar result was obtained by MANRIQUE (2012), when modeled the Aracá bandeira growth (Ptherophyllum scalare) with Bertalanffy, Logistic, Gompertz models, where it was found that the Logistic model showed the highest growth velocity (parameter k equal to 0.025), being followed by Richards (0.02), Gompertz (0.0088) and Bertalanffy (0.0034).

In Bertalanffy model, as k is equal to 0.00369 and its asymptotic value is 4.1220 kg, would be necessary, theoretically (since intrinsic and extrinsic variables are influencing the fish growth) just over 4 years and 7 months of rearing to obtain tilapia of, at least 4 Kg. If, hypothetically, the values of A were kept and changed the value of k to 0.0369, it would take 169 days to get tilapia with 4 kg. For Gompertz model, k is equal to 0.00924, asymptotic value of 1.8499 kg, and theoretically would take 570 days for the tilapias obtain 1.8 kg of mass.

Serving only to promote adjustment of the data to the models, the parameter B, will not be analyzed in this study. In Table 2, are shown the criteria used for comparison of the mathematical models reviews.

TABLE 2. Coefficient of determination, Convergence and Sum of deviations according to the models studied.

Model	Convergence	R^2	SQD
Brody	No	0.4688	0.722
Bertalanffy	Yes*	0.9983	0.00614
Logístico	Yes*	0.9967	0.0117
Gompertz	Yes*	0.998	0.00708
Richards	No	0.9116	0.3147

*Converged by Gauss method.

Based on convergence, only 3 models converged by Gauss method. None of the models converged by other iterative methods: Newton, Gradient and Marquardt. The convergence of a model ensures that its estimated parameters better adjusts to the data analyzed, in other words, the sum of residual squares (SRS) is the minimum possible.

The lack of convergence of Brody model, in this study, resembles that occurred in the ESPINGOLAN et al. (2009), when they tried to adjust this model to analyze the weight of Hereford steers. As for Richards, the non-convergence of data is consistent with those described by KARKACH (2006), in their review of the way and growth (mass), states that the adjustment of Richards model may fail in convergence, in some non specific occasions, which does not happen so obviously for other models.

COSTA et al. (2009) who studied the tilapia growth, discarded Bertalanffy and Brody models because they did not adjust into any lineage. Such variations in the choice of the best model are consequences of several factors, in addition to the statistical aspect, but also the need to assess it from the point of biological view of the parameter estimates produced by them (MENDES et al., 2008).

Based on the coefficient of determination (Table 2), there is similarity between the Bertalanffy, Gompertz and Logistic models, because they have higher R^2 values of 0.99. FREITAS & COSTA (1983) and RODRIGUES et al. (1992) studying the different functions (Bertalanffy, Gompertz and Logistic) to estimate pigs growth rates, from birth to slaughter, concluded that the models had good data adjusts, with high values of R².

Similar results were obtained in several studies that aimed to model the weight growth in relation to the length of animal species for commercial purposes: cattle and goats (FREITAS, 2007), sheep (GUEDES et al., 2004), shrimp (SILVA et al., 2007) and rabbit (FREITAS, 2007), for the Bertalanffy model, as presented R² greater than 0.99.

Other studies argue that the Gompertz model best describes the growth of the species analyzed, outweighing other existing models, because of the high value of the coefficient of determination: frogs (RODRIGUES, 2007), cattle (MENDES et al, 2009), horses (MCMANUS et al. 2010). In addition to the Gompertz model obtain high value of R² for Gompertz, the other discarded models shown value of R² less than 0.30.

And for the Logistic model, are examples of good adjusts and high R^2(greater than 0.99) to the following studies: fish (VANDER et al, 2012.), plants (LYRA et al., 2008), cattle (MALHADO et al., 2009b), among others, all with the goal of modeling the growth of the species analyzed with respect to t.

In the comparison between Brody, Logistic, Richards, Bertalanffy and Gompertz models, in a study of beef cattle growth, pointed to the Gompertz model as the most appropriate for estimating

the growth of Nellore cattle (PAZ et al,. 2004). However, PEROTTO et al. (1992) suggests the Richards model as the most appropriate for estimating the growth of Zebu females.

The Bertalanffy model provided a suitable curve for Santa Ines sheep specie, showing the moment of growth stabilization (FREITAS, 2005). In another study of Santa Inês sheep originating from experimental herd in Paraíba, the Gompertz model showed higher average setting, and therefore is indicated to describe the average growth curve (SARMENTO et al., 2006). In the same study, observed that Santa Inês sheep, males and females born from single or double births, showed divergent pattern growth, indicating the need of different curves to describe their growth.

As the sum of squared deviations (Table 2), the Bertalanffy model and Gompertz are similar, because SRS is equal to 0.00614 and 0.00708, respectively. The logistic model has SRS value larger than the other models: 0.0117. Figure 2 represents tilapia mass growth analyzed, for Y-estimated (estimated weight), of Bertalanffy and Gompertz models, for theorists 1000 days of rearing.
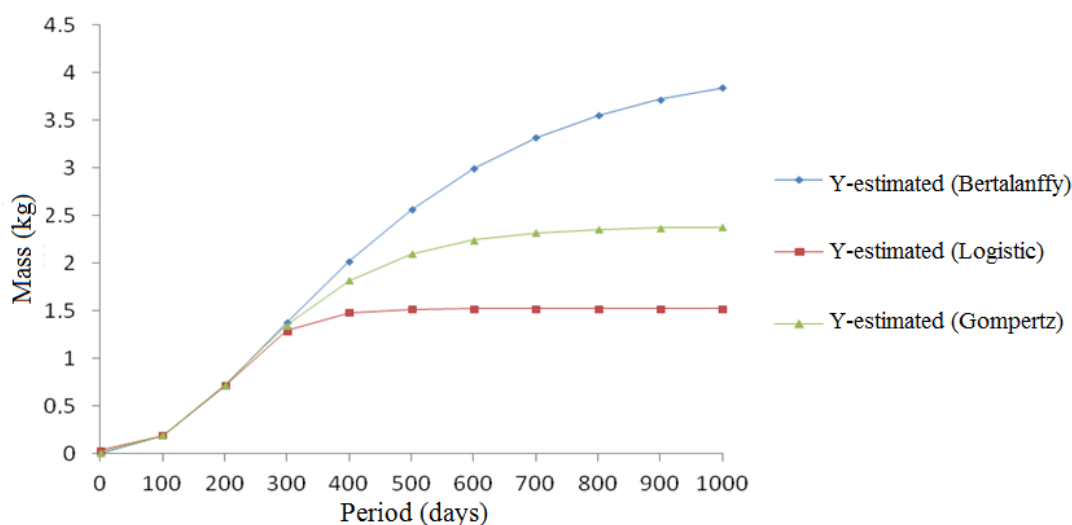

FIGURE 2. Growth curves estimated according to the models: Bertalanffy, Logistics and Gompertz to 1000 days of rearing.

Based on Figure 2, it is observed that even by passing theoretical 1000 days of rearing, the Bertalanffy model did not stabilized, unlike the Gompertz and Logistic models. Although there are reports in the literature of tilapia with up to 5 kg mass (FAO, 2012), this value is far from the national average, which is 600g to 1 kg (NOGUEIRA, 2007). Growth stages have different biological significance, being their analysis recommended separately (PASSOS et al., 2012), as the case of tilapia mass above the average. In such cases, should take into account the growing point can be considered almost constant, in other words, when the curve is sufficiently close to the asymptote, indicating no significant difference (MISCHAN et al., 2011).

Bertalanffy, Gompertz and Logistic models are only indicated for tilapia rearing with crops of up to 270 days for net cages. Furthermore, it would be necessary data of rearing higher than 270 days to validate the models for projections beyond the period of this study, which is not available in this study.

CONCLUSION

Good adjusts are obtained with the Bertalanffy, Gompertz and Logistic models, up to 270 days of rearing. These models are recommended for monitoring growth, in weight, of Nile tilapia reared in net cages in the Sub middle of São Francisco river.

The adjusted models in this study can be used by growers of Nile tilapia, allowing to obtain accurate information about mass growing of animals, besides allowing information for making

future decisions regarding sales/management, for animals of the same species with the same environmental conditions (rearing conditions be similar and that the rearing happens in the Sub middle of São Francisco river).

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