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APPLICATION OF CHICAGO HYETOGRAPH METHOD TO HEAVY RAINFALL EQUATIONS OF AN ALTERNATIVE MODEL OBTAINED BY DISAGGREGATING DAILY RAINFALL

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KEYWORDS

design rainfall, hydrograph, intense rains, drainage.

ABSTRACT

Determining hydrographs for hydraulic works projects such as dams and reservoirs requires the definition of the design rainfall hyetograph. The Chicago method stands out as one of the most used methods, with the advantage of being easy to apply. However, the dependence on traditional and updated IDF equations can be pointed out as a limitation of the method. This study aimed to adapt and apply the Chicago hyetograph method with the intense rainfall equations of the alternative model, which stands out for its ease of obtaining and updating. The equations for estimating rainfall intensities for the duration before and after the peak of the hyetograph were presented. The equations were also adapted to obtain the accumulated depths or volumes of rainfall before and after the peak of the hyetograph. This information allows us to easily obtain the rainfall blocks for each interval of the hyetograph. The method was applied to determine the hyetograph based on the maximum daily rainfall, demonstrating each calculation step. The equations presented here can be implemented in electronic spreadsheets or programming routines, allowing Engineering professionals to apply methods that are more appropriate to local data.

INTRODUCTION

Intense rains cause flooding and waterlogging problems, causing economic losses and damage to infrastructure in urban and rural areas (Piadeh et al., 2022). Hydraulic works, such as drainage channels, culverts, detention reservoirs, or dams, must be designed to mitigate these problems (Ewea et al., 2018).

The sizing of these works is carried out based on a design rainfall, characterized by its duration, intensity, and frequency, which can be represented by IDF curves (Ewea et al., 2016). Bara et al. (2009) reported that IDF curves emerged from studies by Bernard (1932) and were subsequently presented in different regions around the world. IDF equations have gained even more importance with the advancement of information technology applied to engineering, as they allow the implementation of computational routines to obtain rainfall information according to duration and frequency.

The distribution of rainfall intensity (or height) during its duration must be determined when defining the design rainfall (Abreu et al., 2017; Back & Nurnberg, 2022). The Chicago method stands out among the most used models (Keifer & Chu, 1957), with wide application in estimating design rainfall for urban drainage, with a duration of up to three hours (Chen et al., 2023). Several studies have indicated that the Chicago method is the simplest and most efficient (Soldevila et al., 2019; Su et al., 2019; Liao et al., 2021, Yang et al., 2022).

The ease of application and the requirement for a few parameters, such as the rainfall duration and the heavy rainfall equation, stand out among the advantages of the Chicago method. Moreover, the inclusion of the retardation factor allows for changing the hyetograph format. Familiarity with the use and ease of obtaining IDF equations facilitated the acceptance of the Chicago method in Engineering, becoming one of the most used methods (Krvavica & Rubincic, 2020; Wittmanová et al., 2021). Silveira (2016) described the equations for obtaining the hyetograph using the Chicago method by the traditional IDF equation in the format presented by Bernard (1932).

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However, dependence on the updated IDF equation may limit the use of the Chicago method. Back (2020) presented an alternative heavy rainfall equation model based on the daily rainfall disaggregation coefficients for shorter-duration rainfall. Back & Cadorin (2021) surveyed IDF equations in Brazil and found that 81% out of the 3096 registered equations were obtained by disaggregating daily rainfall.

Lima Neto et al. (2021) observed that the proposed model has the advantage of quickly updating intensity, as it only requires daily precipitation and rainfall duration. Oliveira et al. (2021) compared the performance of different heavy rainfall equations and concluded that the alternative model proposed by Back (2020) for the methodology of relationships between durations presented good results, standing out relative to the traditional model regarding the quality of adjustment of IDF equations to the disaggregated rainfall intensities. The model has advantages related to the greater ease of finding values of rainfall intensity associated with a duration and a return period. It uses only the daily rainfall value associated with the corresponding return period without the need to use more complex methods for adjusting IDF equation parameters and without the need to perform the disaggregation procedure. This model can also be useful in situations in which resources are not available to apply the traditional method.

The Chicago method is difficult to apply in locations where updated traditional IDF equations are not available or when there is only information on maximum daily rainfall. Therefore, this study aimed to adapt and apply the Chicago method hyetograph to be used as the alternative heavy rainfall equation model proposed by Back (2020).

Development

Chicago method

The method is based on two analytical equations to obtain rainfall intensities, one equation for the duration before the peak and the other valid for the time after the peak. These equations were deduced based on the IDF equation presented by Sherman (1931), given by:

$$I_m = \frac{a}{(t+b)^n} \quad (1)$$

Where:

I_m is the maximum mean rainfall intensity (mm h^{-1});

t is the rainfall duration (minutes), and

a, b, and n are the equation parameters determined for each location.

The use of the IDF equation is very common in Brazil considering:

$$a = K\,T^m \quad (2)$$

in which

T is the return period (years), and K and m are coefficients determined for each location.

Keifer & Chu (1957) presented the equations considering the rain asymmetry defined by the parameter r ($0 < r < 1$), also called the rainfall advance coefficient. Thus, the peak for rainfall with a t_d duration occurs at time $t_p = r\ t_d$. The equations to estimate rainfall intensity in the time before the peak (t_b) and time after the peak (t_a) are given by eqs (3) and (4), respectively:

$$\mathbf{i_b} = \frac{\mathbf{a}\left[(\mathbf{1-c})\frac{\mathbf{tb}}{\mathbf{r}}+\mathbf{b}\right]}{\left[\frac{\mathbf{tb}}{\mathbf{r}}+\mathbf{b}\right]^{\mathbf{1+c}}} \quad (3)$$

$$\mathbf{i_a} = \frac{\mathbf{a}\left[(\mathbf{1-c})\frac{\mathbf{ta}}{(\mathbf{1-r})}+\mathbf{b}\right]}{\left[\left(\frac{\mathbf{ta}}{\mathbf{1-r}}\right)+\mathbf{b}\right]^{\mathbf{1+c}}} \quad (4)$$

Where:

i_b is the rainfall intensity before the peak (mm h^{-1});

i_a is the rainfall intensity after the peak (mm h^{-1});

t_b is the time before the peak (min), and

t_a is the time after the peak (min).

Silveira (2016) described the Chicago method and presented applications with the IDF equations.

Alternative equation

Back (2020) proposed the alternative heavy rainfall equation model, which can be expressed by:

$$I_m = \left(\frac{60}{a+bt^c}\right) P_{1day} \quad (5)$$

in which:

I_m is the maximum mean intensity (mm h^{-1});

t is the rainfall duration (min);

P_{1day} is the maximum rainfall in one day (mm), and

a, b, and c are constants based on rainfall disaggregation coefficients.

The relationships between durations established for Brazil (Cetesb, 1986) are given by:

$$I_m = \left(\frac{60}{27{,}9327+\ ,8346t^{0{,}7924}}\right) P_{1day} \quad (6)$$

Back & Wildner (2021) adjusted the alternative method for the mean relationships observed in Santa Catarina:

$$I_m = \left(\frac{60}{16{,}5297+7{,}5911t^{0{,}7033}}\right) P_{1day} \quad (7)$$

The precipitation volume can be calculated by:

$$V = i_m\, t = \left(\frac{t}{a+bt^c}\right) P_{1day} \quad (8)$$

It can also be calculated as the integral of the intensity function, given by:

$$V = t_d \int_0^{td} i\, dt \quad (9)$$

Rainfall intensity is distributed over time as shown in the blue curve in Figure 1, with maximum intensity (I_{max}) at the beginning of the rainfall (t = 0) and decreasing exponentially over time, according to a function f(t).

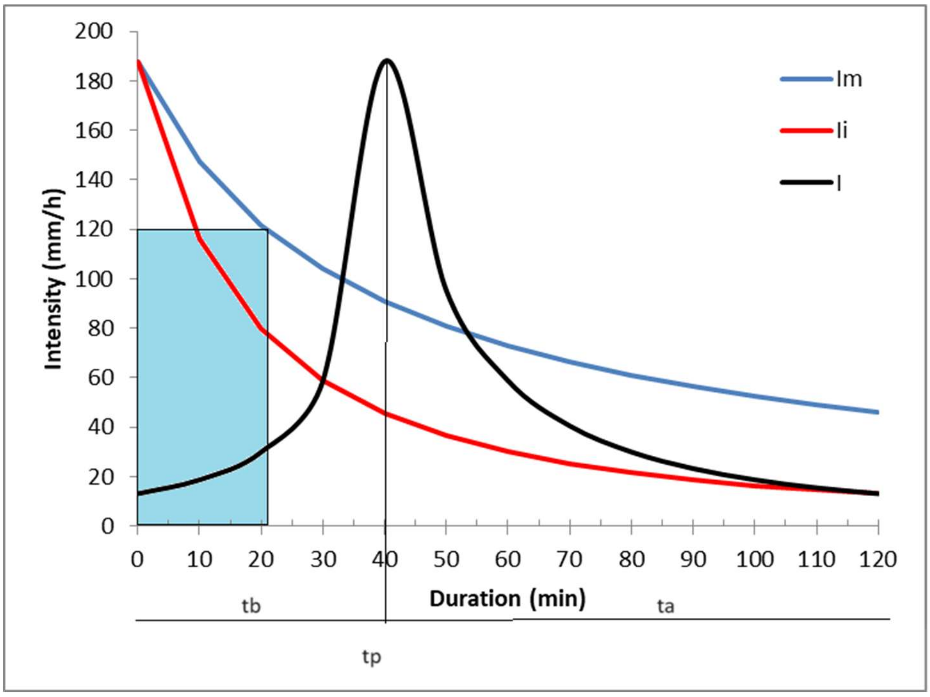

FIGURE 1. Representation of maximum intensity (I_m), instantaneous intensity (I_i), and adopted intensity (I) in the Chicago method.

The rainfall volume for a rainfall duration t_d is represented by the area under the curve from t = 0 to t = t_d. The mean rainfall intensity (I_m) can be estimated by the relationship between volume and duration, that is:

$$I_m = \frac{V}{t_d} \quad (10)$$

The method considers a function of mean rainfall intensity I_m different from instantaneous intensity (i_t), whose integral over time corresponds to the rainfall height of [eq. (9)]. The derivative of equation dv_t/dt results in:

$$I_i = \frac{60P_{1day}[(a+bt^c)-(cbt^c)]}{(a+bt^c)^2} \quad (11)$$

This equation has a maximum intensity for t = 0, as the heavy rainfall equation. However, except for t = 0, when intensities are equal, we can observe that (Figure 1):

$$I_i < I_m \quad (12)$$

Rainfalls with different durations (t_d) but with the same intensity distribution will produce I_m values that decrease as t_d increases. This is how it can be expressed:

$$I_m = \frac{V}{t_d} = \frac{1}{t_d}\int_0^{t_d} f(t)dt \quad (13)$$

The mean intensity I_m over time t can be described by an empirical function using [eq. (14)]:

$$I_m = \left(\frac{60}{a+bt^c}\right) P_{1day} \quad (14)$$

The function (ft) can be obtained by differentiation by combining with [eq. (13)]:

$$f(t) = \frac{d}{dt}\left\{\left(\frac{60}{a+bt^c}\right) P_{1day} t\right\} \quad (15)$$

That results in:

$$i_m = f(t) = \frac{60P_{1day}[(a+bt^c)-(cbt^c)]}{(a+bt^c)^2} \quad (16)$$

Considering the retardation factor r (0 < r <1), the time of the intensity peak for a given duration t_d is given by t_p = r t_d.

The rainfall distribution relative to the time before the peak (0 < t_b < r t_d) is given by:

$$i_b = \frac{60P_{1day}\left[(a+b(t_b/r)^c) - \left(cb(\frac{t_b}{r})^c\right)\right]}{(a+b(t_b/r)^c)^2} \quad (17)$$

n which

i_b is the rainfall intensity before the peak (mm h^{-1});

P_{1day} is the maximum daily rainfall (mm);

t_b is the duration of peak rainfall counted from the peak (min);

r is the retardation coefficient or peak factor (dimensionless), and

a, b, and c are constants of the equation (Back, 2020).

The rainfall distribution relative to the time after the peak t_a ((1 − r) t < t_a < t_d) is given by:

$$i_a = \frac{60P_{1day}\left[(a+b(t_a/(1-r))^c) - \left(cb(\frac{t_a}{(1-r)})^c\right)\right]}{(a+b(t_a/(1-r))^c)^2} \quad (18)$$

Where:

I_a is the rainfall intensity after the peak (mm h^{-1}),

P_{1day} is the maximum daily rainfall (mm),

t_a is the rainfall duration after the peak counted from the peak (min),

r is the retardation coefficient or peak factor (dimensionless), and

a, b, and c are the constants of the equation (Back, 2020).

The calculation of the blocks of the discretized rainfall hyetograph is performed by integrating these equations to obtain an accumulated volume curve. For convenience, this curve is calculated so that the volume V is zero at $t = t_p$ and is defined in terms of the time elapsed before and after t_p. The expressions for volume before and after t_p are given by eqs (19) and (20), respectively:

$$V_b(t_b) = \left\{\left[\left(\frac{P_{1day}}{a+b\left(\frac{t_b}{r}\right)^c}\right)\right] t_b\right\} \quad (19)$$

$$V_a(t_a) = \left\{\left[\left(\frac{P_{1day}}{a+b\left(\frac{t_a}{1-r}\right)^c}\right)\right] t_a\right\} \quad (20)$$

Adapting to the model indicated by Silveira (2016), the equations can be written to obtain the accumulated rainfall heights from the beginning of the hyetograph for the time before the peak:

$$P_t = rP_{tot} - \left\{\left[\left(\frac{P_{1day}}{a+b\left(\frac{t_p-t}{r}\right)^c}\right)\right](t_p - t)\right\} \quad (21)$$

in which

P_t is the accumulated rainfall until time t (mm) of the beginning of the hyetograph;

r is the retardation coefficient or peak factor (dimensionless);

P_{Tot} is the total design rainfall t (mm);

P_{1day} is the maximum daily rainfall (mm);

t_p is the peak time (min);

t is the rainfall duration (min), and

a, b, and c are the constants of the equation (Back, 2020).

The equation for the time after the peak is given by:

$$P_t = rP_{tot} + \left\{\left[\left(\frac{P_{1day}}{a+b\left(\frac{t-t_p}{1-r}\right)^c}\right)\right](t - t_p)\right\} \quad (22)$$

Where:

P_t is the accumulated rainfall until time t (mm) of the beginning of the hyetograph;

r is the retardation coefficient or peak factor (dimensionless);

P_{Tot} is the total design rainfall t (mm);

P_{1day} is the maximum daily rainfall (mm);

t_p is the peak time (min);

t is the rainfall duration (min), and

a, b, and c are the constants of the equation (Back, 2020).

Method application

As an example of application, the hyetograph for the design rainfall was determined for a duration of 120 minutes with a return period of 25 years, considering an advance coefficient r = 0.333. The hyetograph was determined at 10-minute intervals. The maximum daily rainfall with a 25-year return period was estimated as 125.8 mm.

Therefore, the following can be defined:

• Peak time: $t_p = t_d\ r = 120 \times 0.333 = 40$ min

• Rainfall intensity

$$I_m = \left(\frac{60}{27.9327 + 3.8346(120)(120)^{0.7924}}\right) 125.8 = 38.07 \text{ mm/h}$$

• Rainfall height

$$h = \left(\frac{120}{27.9327 + 3.8346(120)(120)^{0.7924}}\right) 125.8 = 76.14 \text{ mm}$$

• Rainfall intensity: obtained from I_m (Table 2) using eqs (17) and (18).

• Rainfall volume before the peak (V_b)

$$V_b(t_b) = \left\{\left[\left(\frac{125.8}{27.9327 + 3.8346\left(\frac{40}{0.333}\right)^{0.7924}}\right)\right] 40\right\} = 25.4 \text{ mm}$$

• Rainfall volume after the peak (V_a)

$$V_a(t_a) = \left\{\left[\left(\frac{125.8}{27.9327 + 3.8346\left(\frac{120}{1-0.333}\right)^{0.72924}}\right)\right] 80\right\} = 50.7 \text{ mm}$$

• Total volume: $(V_b + V_a) = 25.4 + 50.7 = 76.1$ mm

• Hyetograph blocks: we have the rainfall heights before and after the peak with eqs (21) and (22). Thus, [eq. (21)] can be applied for times of 10 minutes and determine:

$$P_t = 0.333 * 76.1 - \left\{\left[\left(\frac{125.8}{27.9327 + 3.8346\left(\frac{40-10}{0.333}\right)^{0.7924}}\right)\right](40-10)\right\} = 2.30 \text{ mm}$$

For a time of 60 min, applying [eq. (22)], we have:

$$\mathrm{P_t} = 0.333 * 76.1 + \left\{\left[\left(\frac{125.8}{27.9327 + 3.8346\left(\frac{60-40}{1-0.333}\right)^{0.7924}}\right)\right](60-40)\right\} = 55.07 \text{ mm}$$

The blocks (Table 1) are obtained by differing the Pt values, represented in Figure 2.

TABLE 1. Determination of the Chicago hyetograph.

Interval	t – duration (min)	t_b (min)	t_a (min)	I_m (mm h^{-1})	Pt (mm)	Block (mm)
0	0	40	0	12.15	-	-
1	10	30	0	15.83	2.30	2.30
2	20	20	0	22.88	5.46	3.15
3	30	10	0	41.77	10.53	5.07
4	40	0	0	270.19	25.38	14.85
5	50	0	10	71.12	46.10	20.72
6	60	0	20	41.77	55.08	8.98
7	70	0	30	29.55	60.90	5.83
8	80	0	40	22.88	65.22	4.32
9	90	0	50	18.70	68.67	3.44
10	100	0	60	15.83	71.53	2.86
11	110	0	70	13.74	73.98	2.46
12	120	0	80	12.15	76.14	2.15

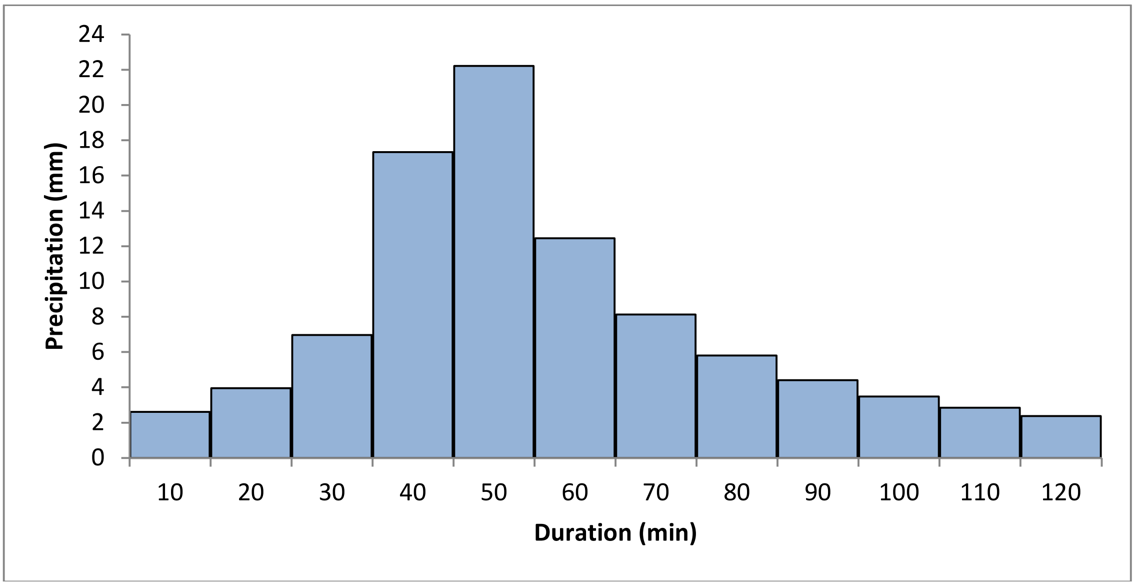

FIGURE 2. Chicago hyetograph method.

CONCLUSIONS

The equations allowed determining the Chicago hyetograph method for locations where only information on maximum daily rainfall is available. The combination of the alternative heavy rainfall equation model by disaggregating daily rainfall with the equations for adapting to the Chicago method represents an important contribution to Engineering, facilitating the use of these methods. These equations can be implemented in electronic spreadsheets or programming routines, allowing engineering professionals to apply methods that are more appropriate to local data.

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