SECTION: ARTICLES

Developing the prospective mathematics teachers' didactical suitability analysis competence*[1]

Belén Giacomone[2]
Juan D. Godino[2]
Pablo Beltrán-Pellicer[3]

Abstract

In this article we describe analyse and evaluate the implementation of an educational design to develop the prospective mathematics teacher's didactical analysis and reflection competence. Planning of the experience, its implementation, and evaluation are based on the application of theoretical tools of the onto-semiotic approach. In several studies supported within this theoretical framework, the notion of didactical suitability as a tool of reflection on a study process has been developed and applied. In this sense, the main objective of the learning processes described here is that prospective teachers know and be competent in the application of this tool. The research scenario is a Master course for Secondary education mathematics teachers in which 27 students participated. The information gathered is based on the analysis of: observer-researcher and teacher-researcher's annotations on the debate in the classroom, audio recordings and the participants' responses throughout the design cycle. The *a priori* analysis of the educational situation reveals a high epistemic and ecological suitability; nevertheless, time constraints have conditioned an adequate learning achievement. Finally, the analysis of class episodes, using the tools described in this article, is revealed as an ideal educational strategy, but it should be complemented with situations focused on the development of other didactical competencies.

Keywords

Mathematics education – Teacher education – Didactical suitability.

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2- Universidad de Granada, Granada, España. Contactos: belen.giacomone@gmail.com; jgodino@ugr.es
3- Universidad de Zaragoza, Zaragoza, España. Contacto: pbeltran@unizar.es

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Introduction

There has been a proliferation of research in mathematics education that proposes reflection on the teaching practice as a key competence for professional development and the improvement of teaching. Thus, new perspectives and new theories have focused on the *reflexive teacher* as a relevant research stream, elaborating diverse and sophisticated research methods (GELLERT; BECERRA; CHAPMAN, 2013). Clear examples of these proposals are *Lesson Study* (FERNÁNDEZ; YOSHIDA, 2004), *Looking with a professional sense* (MASON, 2002) and *Concept Study* (DAVIS, 2008), in which the aim is to promote the teacher's reflection on the action, individually or in peer-interaction.

The Didactic-Mathematical Knowledge and Competence model (DMKC) proposed by Godino et al. (2016) focuses, among others, on the competence of didactical suitability analysis, as the competence for global reflection on the teaching practice, its assessment and its progressive improvement. Also, these authors suggest the importance of designing and implementing training resources that promote the realization of this type of macro-analysis by teachers.

Starting from considering the teacher as a reflective professional (SCHÖN, 1983; ELLIOT, 1993), we have designed a part of a course for the training of prospective secondary teachers. In this work, a training device is described to develop the mentioned competence using the possibilities offered by episodes of video-recorded lessons. The participants in these types of experiences have:

> [...] the opportunity to develop a different kind of knowledge for teaching– knowledge not of "what to do next", but rather, knowledge of how to interpret and reflect on classroom practices. (SHERIN, 2004, p. 14).

However, in this work, video recording of classes remains in the background (STOCKERO, 2008). These should be a mere resource that may facilitate access to the teacher educator and future teachers *fragments of educational reality* in all its complexity, and develop in the training students specific teaching skills through the systematic didactic analysis of the various facets, components and conditioning factors.

To follow, the research problem and theoretical framework is synthesized. In the *Didactic Design* section, the didactical design, which has been planned and implemented as part of a master's course is described. The *Results* section is devoted to present the *results* in terms of the achieved learning level, also showing also prototypical examples of answers that allow us to highlight difficulties, challenges and advances in their reflection competence. In the *Analysis* section, the retrospective analysis of the didactic design is presented. Finally, the last section includes the conclusions.

Research problem and theoretical framework

The contributions in the mathematics teacher education field show a tendency to focus research on aspects that allow the teacher, from certain information, to describe what happens and why it happens, in specific educational contexts (RAMOS-RODRÍGUEZ; FLORES; PONTE, 2016; PONTE, 2011; CLIMENT et al., 2013; GARCÍA; SÁNCHEZ; ESCUDERO, 2007).

Many of these contributions do not determine guidelines for participants, whether they are prospective teachers, teachers or teacher trainers, so they could assess the relevance and suggest improvements to the situation to which they refer, be it an observed or implemented class, a course syllabus, an educational program, a textbook, or the curriculum itself. Basically, they do not address instruments that specifically guide the systematic professional reflection on a teaching and learning process. Hodgen and Johnson (2004) describe *reflection* as a difficult concept to achieve. In practical terms, "[...] It is unclear how teachers' reflection can be facilitated or encouraged. Indeed, there is considerable evidence that enabling teachers to reflect is a far from simple task" (HODGEN; JOHNSON, 2004, p. 224).

Facing this problem, the didactic suitability criteria proposed by Godino (2013) are presented as a resource for teacher and researcher reflection in the design, implementation, and assessment phases of teaching experiences (SECKEL; FONT, 2015).

Didactical suitability: a theoretical-methodological tool for systematic reflection

The notion of *Didactical Suitability* (DS) is part of the *Onto-Semiotic Approach* (OSA) to mathematical knowledge, a theoretical framework introduced by Godino, Batanero, and Font (2007) within mathematics education field. This notion, its components and indicators, allow the systematic analysis and assessment of mathematics teaching and learning processes. It is understood as the degree to which an educational process (or a part of it) combines certain characteristics in order to be classified optimal or appropriate for the adaptation between the personal meanings achieved by students (learning), and the intended or implemented institutional meanings (teaching), taking into consideration the circumstances and the available resources (environment). This assumes the coherent and systemic articulation among the six following facets or dimensions (GODINO; BATANERO; FONT, 2007); each of them is a specific facet of the didactic suitability:

- *Epistemic suitability*: it refers to the degree of representativeness and interconnection of institutional meanings implemented (or intended) regarding to a reference meaning. The tasks/situations-problems are an important component in this facet, and they should include various types of mathematical objects and processes.
- *Ecological suitability*: the extent to which the process of study is adapted to the educational/curricular project, scholar norms, and social environment.
- *Cognitive suitability*: the extent to which intended and implemented meanings are within the students' zone of proximal development, as well as the correlation between students' achieved meaning and the intended and implemented meanings.
- *Affective suitability*: it refers to the degree of the students' involvement (interest, emotions, motivation, attitudes, and beliefs) in the study process.
- *Interactional suitability*: it is the degree to which the didactic configurations and classroom discourse served to identify and solve semiotic conflicts that appeared throughout the instructional process.

• *Mediational Suitability*: is the extent to which the teaching process fit the school and society educational process, and took into account other factors influencing the setting in which it was developed.

For each of these six facets, Godino (2013) identifies a system of associated components and general empirical indicators that constitute a guide for the analysis and systematic reflection; thus, this theoretical model provides criteria for the progressive improvement of the teaching and learning processes. A clear example is the research presented by Ramos and Font, who analyze the role that the criteria of didactic suitability have in the teachers argumentation about an educational process; these authors conclude that:

> The suitability criteria are tools that can be very useful not only to organize and analyze the discursive practices of the teacher on what the instructional process should be, but also to assess the practices involved in the determination of the intended, implemented, and evaluated meanings. (RAMOS; FONT, 2008, p. 262).

Breda and Lima (2016) point out that, a way to carry out a more elaborated reflection, and thus allowing the improvement of the teaching of mathematics, consists of the use of explicit guidelines, such as those that have already been applied in various research projects and teacher training within the framework of the theory of didactic suitability. Specifically, from the OSA, it is considered that the mathematics teacher should know and understand this tool, and acquire competence for its relevant use. This refers to the competence of didactical suitability analysis of mathematical study processes (GODINO et al., 2016); thus, implementing design cycles or educational interventions becomes the next step to promote the professional development of reflective mathematics teachers.

Didactic design

This study is part of an exploratory interpretive research approach. The design research method (KELLY; LESH; BAEK, 2008) is applied in a real class context based on design/planning, implementation, and retrospective analysis of a first educational cycle, supported by the OSA tools. Although this first cycle of design is oriented towards the development of *specific competencies for the analysis and didactic intervention* in prospective mathematics teachers (GODINO et al., 2016), in this article, we analyze and inform on the formative experience of initiating the participants in developing their didactical suitability analysis competence. In order to improve each class session, iterative micro-cycles of designing and reflecting on the resources, strategies, and didactic interactions were implemented, during the experience.

Context of research, participants, and data collection

The formative experience was carried out as part of a Master's course on *Educational Innovation and Initiation to Research in Mathematics Education* aimed at teacher training in Spain (academic year 2015-2016). This course corresponds to the last training period, before the prospective teachers carry out their professional teaching practice; in addition, the face-to-face implementation was applied during the last three class sessions, of two hours each.

The participants involved in this research process are: the professor who is also a researcher, observer researcher (with active participation), and twenty-seven prospective high school teachers who have––or should have, a consolidated mathematical knowledge.

The data collection is obtained from the researcher-observer and researcher-teacher's annotations of the whole-class sharing and dialogues among participants, audio recordings, and students' answers to the proposed task––delivered by them with two weeks deadline. The data analysis is oriented to identify relevant facts about the initial state of students' personal meaning, recognition of conflicts, and progress in developing the intended competence.

Resources and implementation phases

The implementation is organized in six phases, which include different didactic resources as well as moments of autonomous and group work, and final evaluation.

Initial exploration phase

Reading and discussion of a highly ambitious document about the characteristics of an ideal mathematics class, taken from the curriculum orientations of the NCTM – National Council of Teachers of Mathematics (2000, p. 3): *A Vision for School Mathematics*.

The goal is that students develop a first reflection on possible ideal characteristics of a math class. The students worked individually upon a reflection guide, which played a key role in motivating the discussion about previous ideas, beliefs and conceptions that prospective teachers may have about mathematics and the complex processes of their teaching and learning. Likewise, it is intended to involve them in a reflection on the different ways of teaching and the different didactic-mathematical conceptions, in order to finally provoke an evolution of their ideas. This reflection gives rise to concerns, and motivates us to ask ourselves: what would a high-quality education be like? Which criteria can help me to assess my own mathematical practices? How should I carry out a systematic reflection on the teaching and learning processes? How to assess the suitability of the implemented process? How the analysis of these processes can be supported and improved, controlled and managed, with some models (didactic) that help us to look at the reality of our classroom? According to Schoenfeld (1998, p. 3): "Something has happened. What will the teacher do next, and (more importantly) why?"

The discussion made the epistemic, cognitive, affective, interactional, mediational, and ecological components to emerge and how they articulate each other and how they affect the development of a study process. In addition, the prospective teachers' meanings of possible suitability criteria of teaching and learning processes were highlighted. This phase ends with a reflection on the need to know about specific tools that allow the teacher to assess the teaching practice in a systematic way and to be competent using them. It's not just about describing and explaining what is happening in that ideal class, but also to reflect on what aspects could be improved.

Introduction phase to a tool for reflection

This phase started with the reading and discussion of the research article: *Indicadores de idoneidad didáctica de procesos de enseñanza y aprendizaje de las matemáticas* [Indicators of didactic suitability of mathematics teaching and learning processes] (GODINO, 2013).

In the second-class session the article, previously read by students, is jointly discussed. The notion of didactic suitability is presented in this article, in addition to a system of didactic suitability indicators for each of the different facets involved, and the concordance between this system and other proposals from various authors.

Implementation phase

After discussing the article, it is proposed that students watch a fragment of a high school mathematics class. This episode was selected from the Internet, being free open-access material, in which it is possible to observe 10 minutes of a class taught in Mexico. Two stages can be distinguished within the video: in the first one, the students work in groups solving problems related to the calculation of inaccessible heights, followed by a discussion in the whole class. In the second stage, they work with real objects (trees, posts, etc.) measuring their shadows. Table 1 includes the transcription of the video to facilitate the analysis of the answers.

After watching the class episode, the reflection task shown in Table 1 was delivered and the prospective teachers worked on it in teams of two or three. The important point of this phase is not the video as an instrument, but the class that is observed through the technological resource of video recording, and even more important are the theoretical-methodological tools that help to identify significant didactic practices in the evolution of the teaching-learning process.

Discussion phase and sharing

Peer discussion took place during the development of the whole task. The sharing of sections 1 (*description*) and 2 (*explanation*) was done during the second session. From the information gathered in these two sections, students worked in teams during the last class session, in section 3 (*evaluation*) and then the final sharing was done. The *a priori* analysis carried out by the research team allowed to support the sharing done within the class, as well as to prevent possible learning conflicts.

Table 1- Transcription of the video-episode focused on the participants' voices.

1T	Good afternoon, everyone
2Ss	Good afternoon
3T	Look, today we are going to work with a new task. From the curriculum content: shape, space and measurement, under the topic geometric shapes and under the sub-topic (emphasis) Similarity
4T	We will work normally, as always, as we have been doing
5T	Professor Martín Eduardo Martínez Morales is here and will take evidence of the classes, of what we do and how we do it. You all have to work in a normal way, as usual.
6T	We hope all of you solve this task
DISTRIBUTION OF TASKS [minute 00:52]	
7T	Now, you all can turn over the tasks sheet and start reading
READING INSTRUCTIONS [01:07]	
8T	Attention boys and girls. Have you all read the problem?
9T	Who can tell me, what does the task ask?
10T	Mr. Legarre
11S	Based on the drawing that is there, calculate the height
12T	Good. What do the others say? Do you agree?
13Ss	Yes!!!
VERBALIZATION [01:49]	
14T	You have to calculate the height of the tree that appears in a drawing.
15T	Okay?
16Ss	Yes!!!
17T	Go ahead. Calculate the height of the tree according to the information.
18T	Now. Now. Look here
USE OF ICT [02:18]	
19T	There on the blackboard, we can see the projected problem that we are solving
20T	Use the knowledge acquired in the previous problems, because there, you have calculated the value of the measurements of some triangles with their homologous sides
21T	You have also previously calculated the value of proportionality
DIDACTIC SITUATIONS [02:52] STUDENTS SPEAKING SPANISH [03:18]	
22T	Understood?
STUDENTS SPEAKING NAHUATL DIALECT [03:40]	
23T	Here you have two possibilities. To solve the problem, you can use one of the two methods, ok, but also you can verify the solution using the other method.
24T	The most correct thing is to be "like that" (the teacher points out the student's sheet).
SHARING [03:44]	
25S	The answer to the problem is 5.23 (She explains the procedure used and writes it on the board)
26S	Then we apply a rule of three, and X is 5.23
27T	You got the same results by both methods. Good
28T	So, the height of the tree is 5.23
29	The students go to study outside, into the schoolyard
30T	'This' times 'this' divided 'this other' is equal to the height of the post
31S	Ah!
32T	Now you have to do the same procedure. You are going to choose a small tree and measure its shadow with the measuring tape.
ADDITIONAL ACTIVITIES [06:41]	
33I	Teacher, we have to present to the school supervision evidence of the problems that are carried out according to the secondary reform. Could you briefly comment on what are you doing, the students educational level, the kind of instruction, and what mathematical knowledge you are studying in this moment?
34T	These students are grade 3 (course A)
35T	We are studying the similar triangles. So, the reform involves exercises applying similarity. So we are solving some problems about that.
36T	We are working now in the schoolyard; in this way, the students have practical experience to calculate the height of some trees/poles, which are difficult to measure.
37T	This problem is solved using similarity of triangles
38T	They measure the shadow of some objects, and based on that data, they calculate their heights
39I	Okay teacher. Thank you so much. These are the problems currently proposed by the reform. In this moment, are you developing any particular task?
39T	Of course, Similarity triangles

S: Student - Ss: Students - I: Interviewer - T: Teacher

Source: by authors.

Chart 1- Didactic reflection task (guide for prospective teachers).

At the following link we find a video of a math class: http://www.youtube.com/watch?v=60s_0Ya2-d8. After watching the video, work in teams and prepare a report answering the questions below:

1) Description: *What is happening*?

a. What mathematical content is studied?

b. Which meanings characterize the content studied?

c. What are the context and the educational level in which the class takes place?

d. What does the teacher do?

e. What does the student do?

f. What resources are used?

g. What prior knowledge should students have in order to tackle the task?

h. What learning difficulties/conflicts are manifested?

i. What norms (regulations, habits, customs) make possible and condition the development of the class?

2) Explanation: *Why is it happening*?

a. Why is that content studied?

b. Why is a realistic problem used to study the content?

c. Why does the teacher act the way he does?

d. Why do students act the way they do?

3) Evaluation: *What could be improved*?

Issue a reasoned judgment on the teaching observed in the following aspects, indicating some changes that could be introduced to improve it:

a. Epistemic (mathematical content studied)

b. Ecological (relations with other subjects, curriculum)

c. Cognitive (previous knowledge, learning, ...)

d. Affective (interest, motivation, ...)

e. Interactional (modes of interaction between teacher and students)

f. Mediational (resources used)

4) *Limitations of the available information*:

What additional information would be necessary to make the analysis carried out more accurate and reasoned?

Source: Done by authors. Adaptation of the task proposed by Godino and Neto (2013).

Evaluation phase

As mentioned above, the evaluation processes of educational interventions are complex, since they involve different dimensions and components. Thus, to reach a reasoned judgment on the epistemic, ecological, cognitive, affective, interactional, and mediational facets, as requested in Table 1, we considered as necessary for students to have two weeks to complete the task in a portfolio format. That is, presenting a complete document that shows the development of the entire design.

Assessment phase

At the end of the design cycle, the prospective teachers were asked to complete an anonymous survey on the following five aspects:

1. Clarity of the task and language.
2. Adequacy of the methodology followed (kind of work, teacher's explanation, and resources).
3. Degree of motivation and interest aroused by experience.
4. Achieved level of learning.
5. Degree of global relevance of the workshop for your training as a teacher of mathematics.

They were asked to assess the items according to the scale of [1-5], being 1: minimum value and 5: maximum value. There was also room for them to add any comments they considered relevant to improve the activity.

Twenty-six participants out of twenty-seven completed the survey; the median score in all the items was 4; the minimum score was 2 and maximum 5. The analysis of this survey provides valuable information for the retrospective analysis, being a means of reflection for the researchers.

Results: learning achievement indicators

When students watch the video for the first time, they focus on specific elements, which are known to them as *good practices*. In this way, positively valued issues appear, such as the use of problems with context, collaborative work, classroom arrangement, the use of technological resources, sharing, respect, class dynamics and fieldwork. However, the first analysis they perform is based on superficial characteristics and without connections between the information collected by the items in the reflection guide.

The group discussion aims "[...] to help the prospective teachers to acquire professional teaching competencies" (LLINARES, 2012, p.24). In this case, these competencies are focused on implementing the system of indicators and components previously studied. In this way, it is possible to find more elaborate and organized analysis in the answers (portfolios), where the students seek to establish key connections between those elements that seemed important to them. It should be highlighted that of 24 portfolios delivered in the final stage, 20 of them presented an analysis where possible improvements are proposed to increase the suitability of the observed study process. However, not every participant wanted to give a low, medium or high assessment of each facet considering that a lot of additional information is required in order to assess them.

The following subsections show prototypical examples of significant responses collected from the experience, considering the participants' reflection on each of the six facets. In this sense, we have considered how their answers contributed in the first and second part of the task (description and explanation of the teaching situation) to make the assessment of the third part (Chart 1). The aim is to confront the analysis of the participants with the *a priori* analysis of the researchers. The latter allows opening a range of possible *expert responses* and thus to highlight the importance of knowing and being competent in the use of the didactic suitability tool and training as reflective professionals capable of assessing and improving their own practice.

Epistemic facet

A key point to assess the epistemic facet (institutional mathematical knowledge) is to reflect on the type of situations-problems implemented in the video-recorded class episode. Although the participants notice that it is not possible to observe a representative and articulated sample of contextualization, exercising and application tasks in only nine minutes, the presence of a problem guide on inaccessible height calculation stands out, as well as the field work in the school courtyard. From a mathematical point of view, the prospective teachers notice that the studied content allows us to put into practice significant and relevant mathematical practices (knowledge, comprehensions and competences): geometric proportionality, linear function, similarity of triangles, calculation of inaccessible heights and distances. They assess positively the type of problems that allow to explore this content, as well as the type of languages that they mobilize.

The first problem that is considered is to calculate the height of a tree from a drawing that models the situation. We highlight the following student's response (R)[4]:

> R1. Calculate the height measurement is useful to contextualize, apply and exercise the contents of similarity of triangles and proportionality. In addition, this type of situations uses different methods of expression: verbal-visual (in the yard); graphic (interpretation of the data); symbolic-arithmetic (because it requires calculations to solve the task).

However, the prospective teacher highlights important aspects such as: *Lack of precision in the teacher's language and concepts referred* to (Table 1, items 20T and 30T); likewise, they identify aspects that should be improved, such as the lack of didactic situations to argue and generate definitions or propositions:

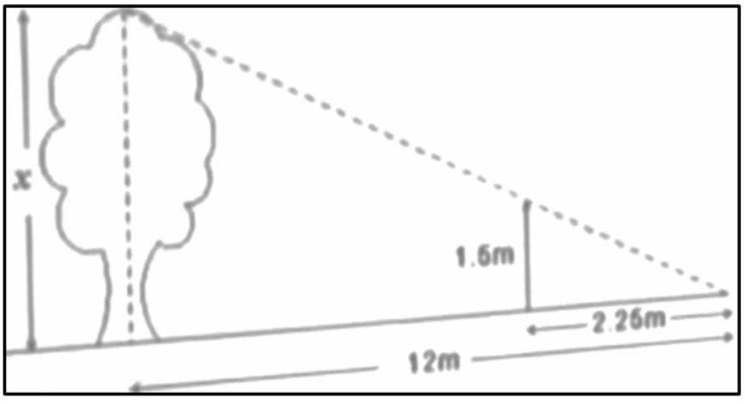
> R_2. Although the problematic situations that appear seems to enhance the connections between the different concepts, propositions and procedures, the absence of moments of argumentation or justification, make the task itself become a mere exercise of application of a rule. It does not mean that it is incorrect, but it would be appropriate to add statements as 'justify your answer', in this way students can establish relationships between previously studied concepts and the teacher can evaluate their knowledge on the subject.

4- R1, R2, … refer, respectively, to each answer given by prospective teachers.

The next participant has previously considered the meanings that characterize the similarity of triangles to argue why such improvements should be made. This means that he/she has taken into account the analysis obtained from part 1 of the reflection task.

> R_3. If this is a problem of contextualization [Figure 1] it can be improved, because the problem as it is posed is a bit artificial. There are applications of the similarity to real situations better posed, such as, for example, not only to calculate inaccessible lengths, but also how is reality represented? (representation on maps, plans, etc.); How are images reproduced? (projections); also using diverse material resources, dynamics, or the history of mathematics itself: how did Thales use knowledge about the similarity of triangles to calculate the height of the Egyptian pyramid, or ethnomathematics taking advantage of the variety of cultures of the country and within the classroom. Nor is it problematized, for example, why this is a context of similarity of triangles.

Figure 1- Mathematical problem posed to the class-episode students.



Source: Image retrieved from video-recorded episode (2:11 minutes).

Ecological facet

The participants were able to identify components and indicators that characterize this facet by articulating their responses with the previous analyzes obtained from part 1 and 2 of the task. They evaluated the adequacy of the content and its implementation according to the curricular guidelines that mark the new reform of Mexico[5] (2011), which conditions the development of the class.

Some aspects to be improved are the implementation of problems emphasizing intra/interdisciplinary connections, as well as situations of innovation and reflective practice:

5 - The curricular reform (2011) is not a radical reform. The main points of attention are in the teaching performance, in what refers to the design of teaching situations, reorganization of the teaching environment and ways of working in the classroom, giving rise to different relationships between students, teacher, and content, in addition to the use of reading, audiovisual and computer resources. Some of those critical points are mentioned at the end of the proposed video.

> R_4. Although the teacher tries to make way for the opening of new technologies [Table 1, item 19T], the observation of the video-recorded class does not allow showing its real use. It is not just about innovating as suggested by the reform; it must be done in a justified way with respect to the different activities [statements] proposed.

Cognitive facet

The prospective teachers focused their attention on the prior knowledge needed to address the calculation of inaccessible heights. If we focus on the section: *what prior knowledge students should have to approach the task?* (Chart 1), only three of twenty-four prospective teachers do not answer 'the simple rule of three'. These participants are aware that 'the rule of three' is a procedure to solve a task and not the objective itself; their reflections are coherent and highlight the importance of justifying procedures. For instance, the following prospective teacher reflects:

> R_5. During fieldwork they [students] collect information and apply Thales' theorem when the conditions of the theorem are not met (e.g., parallelism). At least they should consider certain assumptions to solve them [tasks], or the teacher could take advantage to do it, and thus generate instances of institutionalization.

The same student adds:

> R_6. The teacher must know: the role of argumentation and validation when mathematical justification is carried out; the students' difficulties about proportional reasoning and the generalization of the contents explained.

The remaining twenty-one participants indicate that the simple rule of three is *a priori* knowledge necessary to solve the tasks. Among these answers, nineteen of them believe that the similarity of triangles and Thales's theorem are not prior knowledge. In addition, they value positively the cognitive facet, since they consider that the students are capable of satisfactorily applying the rule of three, or at least it is an accessible objective.

Wilhelmi (2017) assesses the epistemic suitability of teaching proportionality in the secondary education curriculum. The author points out that, although the rule of three (a particular form of cross-product) is not included homogeneously in international curricula, the teaching of this rule has persisted in the school as prototypical model of the proportionality. While this was discussed in class, it seems that the rule of three is a procedure that is deeply rooted in its formation, as well as the deliberate use of proportionality relations:

> R_7. Students use the rule of three because the segments are proportional; that is easily demonstrated from the measurement of the sides [pointing to Figure 1].

The two remaining prospective teachers emphasize that the use of the simple rule of three is important as *a prior knowledge necessary* to solve the task. However, their later reflections are in contradiction since they value negatively the learning linked to the application of rules and mechanical procedures. That is, how is it possible to consider this rule as an important knowledge and, at the same time, to consider learning using this rule as erroneous? In fact, the habitual use of the rule of three in the schools is purely instrumental "[...] concealing in a certain way the intervention of the ratios and proportion, which may imply a degenerate meaning of the arithmetical proportionality" (GODINO et al., 2017, p. 6-7).

Finally, the participants have not made a value judgment about the learning achieved, certainly because in the video it is not possible to observe it, but they did highlight possible learning difficulties:

> R_8. Not all students start from the same initial level of concepts and learning. For example, during the fieldwork, it seems that not everyone works and that not everyone understood the statements. It could be due to the fact that in the sharing phase the different results found are not discussed.

Others highlight cognitive difficulties such as to find a *unit of measurement, difficulties with the notion of direct proportionality*, etc. Assuming the possible learning conflicts is an advance in the process of professional reflection.

Affective facet

The assessments that are identified as related to this facet are very superficial, such as: fieldwork generates motivation; it allows assessing mathematics in everyday life. Only one prospective teacher negatively assessed this facet:

> R_9. The students are not interested in the task. Not everyone works and, surely, not everyone learns, since there is no argumentation in the mathematical work. You could think about smaller work groups, involve other materials, motivate them with questions and discuss the results.

Interactional facet

The participants' reflections are related to the information obtained from the previous questions. In general, competences about reflection on the different modes of interaction are observed: between students and teachers, between students and about the autonomous study. The participants identify rules established in class such as the classroom arrangement, raising a hand to call the teacher, the role of the teacher as an observer in the class. They also make reasonable judgments about these. Regarding the teacher's role, the prospective teachers classify him as the protagonist of the class. They admit that the purpose of sharing is to present the answer to the problem:

> R_{10}. It seems that the student who goes to the blackboard already knows that he has the right results and also will not be questioned. It would be ideal to give everyone the opportunity to confront their results.

Four answers show superficial analysis, such as: *There is a fluid dialogue between the students and the teacher, and between the students among themselves.* We consider it a false impression created by the collaborative work dynamics.

Mediational facet

The prospective teachers referred to the use of different manipulative materials, as scarce and unproductive, valuing this facet as not very suitable; while recognizing the importance of problem guidance and the use of calculators, participants emphasize that computer resources are very valuable in this type of content but they are not present:

> R_{11}. It would be advisable to use dynamic software to show, for example, how the shadow of a tree varies as the sun passes through different points, thus generating moments where students should estimate, test hypothesis and search for relationships between height and shadow, without needing to calculate it.

During fieldwork, the students use graduated rules, their feet, hands and other elements to measure the shadows, so the measurements are not precise. In this sense, they suggest incorporating moments where the teacher could problematize the situation, as for example, to discuss the problem of the measurement precision and standardization in the fieldwork and to acquire competence in the correct measure of lengths.

Regarding the number of students and the time devoted to the task, the prospective teachers consider that they could be adequate, but there is a lack of data to assess these aspects and others such as the time in which this subject is taught and the distribution of time to each task.

Interaction among the different facets

In addition to the evaluations of each facet, both in class interactions and in written responses, prospective teachers stated that the facets analyzed are articulated with each other and that it is often difficult to differentiate them:

> R_{12}. [About cognitive suitability] It would be positive for all the students to make a sharing by not only showing the results obtained, but also the difficulties encountered during the development of the activity, where the teacher can use various resources to question the students and include them in an environment where they have [the students] to justify their results. In this case, speaking about the cognitive dimension also implies the affective and interactional dimension.

> R_{13}. It is impossible to focus the analysis on only one facet. This makes me think about the complexity of teaching and learning processes, particularly, of mathematics. A clear example for me is about the time spent. Although in this case it is not possible to know, it is undoubtedly a factor that affects the entire study process: in the ecological: how is the program of the subject designed? Epistemic: how is the content organized? Cognitive: is the intended knowledge within the reach of the students? Interactions: are individual, group, discussion, etc., taken into account? When reading the article [Godino, 2013] I realized how this model allows us to become aware of these relationships.

As shown in this section, the results indicate the effectiveness of the theoretical model put into practice, as well as awareness of the importance of incorporating reflective learning in university teaching (ALSINA, 2010).

Retrospective analysis of the design cycle

As reflective researchers, a retrospective look at the design and its implementation is necessary to become aware of the limits and challenges that remain to be overcome.

Firstly, an *a priori* analysis of the didactic situation reveals a high *epistemic-ecological suitability.* The implementation stages are articulated to each other and appropriate to the formative level involved. In each educational situations proposed, the prospective teachers are faced with moments in which they have to investigate, interpret, relate meanings, discuss, and argue. In addition, this didactic design shows openness to innovation based on research and reflective practice. The video-recorded class offers a small window into the world of education; in this sense, it is fundamental to consider that the information shown in it is limited (SHERIN, 2004); therefore, the participants’ answers could be superficial or simplistic. To address this problem, the task in Figure 1 proposes to assess the observable aspects and reflect on those not available to achieve more detailed answers.

Regarding the formative course itself, the educational objective was to *initiate* prospective teachers in the development of their reflective competence. The final productions (portfolio) could have been more elaborate, supported by complementary information search and new specific readings; nevertheless, the intended contents have been achieved assessing as suitable the cognitive facet. More time was dedicated to the moments of group discussion; these moments played a major role and were of great interest for the participants, who presented their arguments and critical analysis in the sharing. They also expressed their commitment to the resolution and homework submission. On the other hand, Cooney (1994) argues that, in these processes to develop reflection, it is necessary that teachers/participants feel the motivation to reflect. In this experience, the observation of a real classroom was the main focus of attention for participants, as indicated by the results of research on this topic (PONTE, 2011; CLIMENT et al., 2013), concluding that the affective facet is highly suitable. In the assessment phase (described in the *Evaluation phase* section), regarding the degree of motivation raised by the experience, a prospective teacher writes:

> I would have loved to have known this tool [didactical suitability] before. It is very useful for example to reflect on: how to look at what is wrong in a class, and what aspects could be improved.

On the other hand, the low mediational suitability is attributed mainly to the limitations of the time allotted. While this type of design research occurs in real class environments, where it is not possible to have a greater workload, the short period of time between the implemented tasks reveals a great limitation of this study. The discussion of the final answers, delivered in the portfolio, did not take place within the class. In this sense, we consider that an exchange of final answers would have provided a greater opportunity for the participants to develop ways of reflecting on the different facets and appropriating the theoretical framework offered by this design. According to other research (AMADOR, 2016), the inclusion of additional experiences, or thinking about continuous cycles in teacher training, would be beneficial for prospective teachers to acquire greater competence in reflecting on the practice.

Regarding the quality of the interactions in the classroom, we consider that it has been high, highlighting the dialogue and discussions in the classroom, the inclusion of the prospective teachers in the class dynamics, the appropriate presentation of the topic using various resources. In addition, moments of autonomous study and continuous evaluation were contemplated.

Conclusions

In this article, two main questions have been raised:

1) What theoretical tools could be available to prospective teachers of mathematics, helping them to reflect, in a systematic way, on the educational processes that take place?
2) What kinds of strategies are feasible to educate reflective professionals?

In relation to the first question, we have shown that the system of facets, components and indicators of didactical suitability allows putting into action key decision points for reflection and innovation towards the introduction of substantiated changes. Didactical suitability, as a theoretical and methodological construct, is a tool that has been implemented in the training of teachers in various Spanish and Latin American universities (BREDA, FONT, LIMA, 2015). Other authors have also highlighted its use for identifying improvement of teaching units, as shown by Castro and other authors (2013).

Considering the second question, this work proposes an example of design research in which this theoretical tool is made operational in the different stages of implementation. Although the different factors affecting the educational processes are complex, the participants of this study have positively evaluated this type of didactic situations for their formation, highlighting them as necessary; moreover, highlighting their usefulness for the next stage of his/her professional work: lesson planning and implementation of professional practices in a school institution. It is worth noting that three students have continued their master's thesis using the didactic suitability tool to reflect on their own teaching practice.

On the other hand, the use of video recordings as a resource has been widely recognized in teacher training (ALSAWAIE, ALGHAZO, 2010), and has undoubtedly proved to be a suitable training strategy, as it allows prospective teachers "[...] to view a lesson from a perspective of an observer" (SHERIN, 2004, p. 22). However, we must not forget that "[...] it is the reflection on and analysis of the practice of teaching mathematics that creates the conditions for the construction of useful knowledge to teach mathematics" (LLINARES; VALLS, 2009, p 9). This reflection can be supported by episodes of video-recorded lessons, transcribed fragments, simulated situations, or the teaching experiences themselves.

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Belén Giacomone is PhD candidate at the University of Granada and master's degree at the same university.

Juan D. Godino is doctor from the University of Granada, professor at the same university.

Pablo Beltrán-Pellicer is doctor by the National University of Distance Education (UNED), assistant professor at the University of Zaragoza.