SECTION: ARTICLES

The haughtiness of mathematical ignorance: *Superbia Ignorantiam Mathematicae*

João Paulo Attie[I]
Manoel Oriosvaldo de Moura[II]

Abstract

In this article, we explore a few aspects associated with two phenomena, namely, aversion to mathematics and the consequent refusal to learn the subject, both originated in the relationship between society and the process of teaching and learning mathematics. Based on a supposed binariness which is historically believed to exist and defines only the opposed poles, i.e., knowing everything and knowing nothing of mathematics, the conditions are created for triggering a mechanism in which, first, there is the perpetuation of the current view that this subject is only meant for special, illuminated beings and, later, there is an inversion of categories between what is 'in' and what is 'out' regarding the set of those who know mathematics and the set of those who do not know mathematics. In respect to this binariness, we point its impossibility in today's society, due to the fact that there are no longer individuals who know all the existing mathematics – the last one of them, according with some historians, would have been Poincaré, in the early 20th century – nor individuals without any mathematical knowledge, even if only a non-formal knowledge. Because of the nature of the phenomena studied and the specificities of our approach, we chose a qualitative methodology. Moreover, we discuss in this article the range and intensity of these phenomena and consider some of the possible causes and consequences thereof, concerning the process of teaching and learning the discipline.

Keywords

Aversion to mathematics – Knowing mathematics – Refusal to learning.

I- Universidade Federal de Sergipe, São Cristóvão, SE, Brazil.
Contact: attiejp@gmail.com
II- Universidade de São Paulo, São Paulo, SP, Brazil.
Contact: modmoura@usp.br

A altivez da ignorância matemática: Superbia Ignorantiam Mathematicae

João Paulo Attie[I]
Manoel Oriosvaldo de Moura[II]

Resumo

Neste artigo, exploramos alguns aspectos associados a dois fenômenos, a aversão à matemática e a consequente renúncia em aprender a matéria, originários da relação entre a sociedade e o processo de ensino e aprendizagem de matemática. A partir de uma suposta binariedade que historicamente se acredita existir, definindo apenas os polos opostos, saber tudo *e* não saber nada *em matemática, criam-se as condições para o disparo de um mecanismo em que, de início, se perpetua a visão corrente de que o assunto é feito somente para seres especiais e iluminados para, mais adiante, criar uma inversão de categorias entre o que é* in *e o que é* out*, relacionadas ao conjunto dos que sabem matemática e ao conjunto dos que não sabem matemática. A propósito dessa binariedade, apontamos sua impossibilidade na sociedade atual, pelo fato de não existirem mais indivíduos que saibam toda a matemática existente – o último deles, segundo alguns historiadores, teria sido Poincaré, no início do século XX – nem indivíduos sem nenhum conhecimento matemático, ainda que seja apenas um conhecimento não formal. Pela natureza dos fenômenos estudados e pelas particularidades da abordagem realizada, optamos por uma metodologia qualitativa. Além disso, discutimos no artigo o alcance e a intensidade desses fenômenos e consideramos algumas das possíveis causas e consequências dos mesmos, localizadas no processo de ensino e aprendizagem da disciplina.*

Palavras Chave

Aversão à matemática – Saber matemática – Renúncia à aprendizagem.

I- Universidade Federal de Sergipe, São Cristóvão, SE, Brasil.
Contato: attiejp@gmail.com
II- Universidade de São Paulo, São Paulo, SP, Brasil.
Contato: modmoura@usp.br

Introduction

When somebody asks, “Do you know mathematics?”, or perhaps, “Who knows mathematics?”, he can expect to get few types of answers. Knowing mathematics seems to be a domain in which most people feel that means are not possible and there are only two possibilities. Either one knows mathematics or one does not, this is what many believe and propagate. It is an area that is still “...seen by the great majority as something to be mastered only by a few illuminated” (SAKAY, 2007, p. 119), a belief that would be linked to questions that are “deep-rooted in, and experienced by, all of us” (SAKAY, 2007, p. 119).

In a news story about the Instituto de Matemática Pura e Aplicada [Institute of Pure and Applied Mathematics] (IMPA), the remark of a cab driver about the institution reveals in a crystal-clear way the viewpoint of some people about mathematicians: “Nothing but loonies in there!” (POLONI, 2012, p. 82). However, despite emphasizing, after this remark, that “it is one of the world’s most celebrated and respected research centers”, the writer does not seem to strongly disagree from the image initially expressed, as “the cab driver’s joke has a pinch of truth to it” (POLONI, 2012, p. 82).

Besides the representations usually made about mathematics, such as it being incomprehensible, or that it is made only for very intelligent people, another noteworthy element is the legitimacy that mathematical discourse, or mathematical knowledge, provides to arguments in many circles. In a re-elaboration of the well-known “argument from authority” in which the conclusion relies solely on the supposed authority of the one who states it, rather than on the power, validity and logic consistency of the thought, in this case, when one wants to lend truthfulness to an allegation, one resorts to mathematical concepts.

> A few circumstances, such as the increasing use of mathematics to describe and model natural, social and other phenomena, on the one hand, and the coherence and power of the axiomatic method of demonstration, on the other, endorse and justify, in part, this situation. However, there are cases where the use of the mathematical discourse or knowledge occurs without the necessary consistency or coherence. Besides the classic cases where advertisements use a figure (almost always a male) in a white coat who is vaguely referring to formulas and charts to justify to the consumer the purchase of a given product, the validation of arguments by means of mathematics, particularly in the scientific field, is widely known and admitted (ATTIE, 2013, p. 69-70).

In a study conducted with secondary education students in France, one of the characteristics attributed as fundamental about mathematics was its universality; the justification was that mathematics would be “above all sciences because it is mathematics which gives them validity” (TRABAL, 1997, p. 126).

In this arena – and here, the word ‘arena’ is intentionally used, as we believe we are before one of the *fields of struggle* of reality concerning the imposition of meanings (FOUCAULT, 2007) – the dichotomy posed between the *knowing everything* and the *not knowing anything*, with no shades of gray between both poles, can be pointed as one of the causes of one of the phenomena we want to discuss in this article, which is an immediate, intense, and seldom conscious reaction, one that is loaded with meanings and consequences.

The question we refer to is the phenomenon of *aversion to mathematics,* experienced in an almost spontaneous way by a large number of individuals and it can develop into a second phenomenon, which we will call a *refusal to learn mathematics*. We consider it important to analyze at length how, in our view, the mechanism that triggers both phenomena operates.

It may not be fundamental, although not excessive either, for us to stress that there is not an individual today who knows all the mathematics produced. Some of the most respected historians of mathematics consider that the French Mathematician Henri Poincaré was, in the turn of the 20th century, the last "universalist" in mathematics (BELL, 2003; BOYER, 1974; EVES, 2005). Anyhow, the first few decades of the 20th century are normally considered as the historical point when the amount of mathematics produced reached a level "far beyond the understanding of any one person" (DAVIS; HERSH, 1995, p. 35).

Likewise, we also consider it impossible for there to be an individual who does not know anything of the matter, since in addition to the traditionally known "elementary calculation competencies, particularly the ones necessary to perform algorithms of the so-called four operations" (SÃO PAULO, 2014, p. 3), and the ability to establish geometric relations, both of which are usually the only abilities attributed to mathematical thought, the abilities to compare, ordinate, classify, estimate, relate and generalize, among others, are also part of what can be considered mathematical knowledge. Therefore, we can say that an individual's very existence would be impossible without any of these abilities, even if he ignored any traces of formal mathematics.

So, considering that, indeed, there are neither individuals who *know everything* nor individuals who *do not know anything*, all the more shocking (and embarrassing) are the intensity and social range of the thought that an individual must belong to only one of these mutually excluding sets. It is believed that either one 'knows' mathematics and, in this case, one is considered an intelligent, 'elected', 'in' person who is part of the select group of the almost genius, or else, in case the individual does not belong in that group of 'special' beings, i.e., in case he is included among those who *do not know mathematics,* he is considered, whether consciously or not, an inferior, ignorant, 'out' person, comparable with a handicapped person in the derogatory sense that often accompanies the term. In school and even social environments,

> [...] good performance in mathematics is generally considered a display of wisdom and intelligence. People who can easily learn mathematics are considered special people, who possess some extraordinary gift: mathematical knowledge enjoys prestige... This 'prestige', in turn, generates in those who have difficulties a very strong aversion to mathematics (MARKARIAN, 2004, p. 276-277).

In our analysis, the theoretical sources we rely on – with regard to the phenomena categorized as aversion, refusal, and some of their consequences, such as pride in ignorance, for example – are particularly the writings of Attie (2013), Foucault (2007) and, to a lesser degree, also the writings of Silva (2004) and Hardy (2007). With regard to elements such as the banalization and naturalization of these phenomena, our main sources were Freire (1983), Marx (1987) and, again, Foucault (1995). As to the methodology we applied, because of the nature of the phenomena studied and some specificities of our work, such as taking the social environment (in addition to bibliography) as a direct source of data, as well as the descriptive character of the analysis and its inductive focus as elements that constitute the study, we chose a qualitative methodology, as we consider that it would not be admissible to express quantitatively the connections between the subjective and the objective in the cases we approached.

Aversion

The phenomena we address here cannot by any means be considered isolated or unknown, as "our society seems to be full of individuals who have developed an aversion to this discipline and who irremediably proceed to transmit a derogatory image of mathematics to those around them" (SOUSA, 2005, p. 3).

Among the factors that could underlie this wave of aversion, considering that the most important of them, is what we would call a naturalizing persistence to conceive the teaching of mathematics as a teaching directed towards methods and techniques rather than processes and arguments. Several situations can be listed as examples of this pattern, but some of the classic models we can point appear in the teaching of operatory algorithms, or in the divisibility rules, to cite just two cases.

In the first of them, the quick and efficient way with which the algorithms determine the result of an operation supposedly justify a teaching relying on the purest 'this is how you do it' technique, ignoring the historical process that culminated in the predominance of the current procedures for solving arithmetic operations. The multiplication algorithm, for example, whose current constitution is owing to necessities that occurred in the human development process, has had several resolution forms –from the Egyptian duplication method to the Indian and Greek methods, to the Arab 'grid' method – until it consolidated in the process we use today. However, this process, is taught to students in the first stage of basic education, and then they (and, sometimes, even teachers) end up viewing it as the only process possible, disregarding the different modes in which multiplication was carried out in various places and historical periods.

At this point, it may be necessary to reinforce that we consider it imperative to teach the usual algorithms, which seems fully justified to us, due to how easy they make calculations. However, assuming methods to be historically invariant has been the norm in pedagogical practices, disregarding the fact that the procedures currently employed were the consequence of a development process. Thus, particularly with regard to operatory algorithms, what we advocate, for the sake of a greater significance in the learning process, is that this historical evolution might be considered.

In the second case, with regard to divisibility criteria, it seems even worse, as the only argument supporting the rules seems to rely on the fact that the "account is right", despising the thought that certain affirmations and rules should have a justification at the corresponding education level, where possible. As to the case of the divisibility rules, the preceding affirmations can be verified by analyzing the collections of didactic books recommended in the last few years by the National Didactic Book Program – PNLD (BRASIL, 2011, 2013, 2014). In all of the books we examined, the divisibility rules are either incomplete or without any justification. Instead of deductions and justifications, there is an attempt at the induction of the criteria based on a few examples. In fact, one of the titles even points the pretext that there is no observable regularity that can justify the divisibility rule for that specific number, making it necessary for results to be demonstrated later, since

> [...] we can't see a pattern that leads to a rule of divisibility by 3. In this case, to deduct a rule, it is necessary to resort to knowledge that you still haven't acquired. Therefore, we will now present only the rule. You will see the explanation in the next few years, when you become more familiar with algebra" (IMENES; LELLIS, 2009, p. 60).

Indeed, this collection presents the explanation in the 9th grade, but only at an algebraic level. It is as though the rules came into being from illuminated, superior minds, a discourse that actually appears in an implicit way in one of the collections, when, instead of providing a justification that is compatible with the knowledge on that grade, the text prefers to point that "mathematicians have managed to prove that both divisions have always the same remainder" (JAKUBOVICH; LELLIS; CENTURION, 2008, p. 97).

At this point, we consider it necessary to note that there are demonstrations for each divisibility rule, and to advocate that most of the justifications can be provided at a level that is suitable to the understanding of basic education students and, therefore, they could be included in didactic books for that level. We are aware that, in every case, the demonstration could be conducted using congruencies, which, however, would not be didactically advisable for students of that level as congruencies are a higher-education level topic. Even the use of 8th- or 9th-grade algebra would be an anticipation we consider artificial for treating the subject on the 7th grade. On this stage, however, with a few of the simplest arithmetic operations, such as addition, multiplication, and the use of the distributive property, it would be possible to, at least, justify the divisibility criteria. However, our student and probably also the teacher who relies on the didactical book as the main class preparation tool (BETTENCOURT, 1993; FREITAG, 1997) are denied the possibility to understand the process that justifies that knowledge to the advantage of a sort of dressage on rules.

Thus, and unfortunately not only in the two cases above, we point how, in spite of the emergence and consolidation of a progressionist movement in the teaching of mathematics, the discoveries and understanding of processes which justify mathematical knowledge continue to be replaced by dressage on techniques to operate procedures.

An undesirable habit is created in the teaching of mathematics, a habit in which there is no need to understand the causes of affirmations, but rather, and strongly, the need to accept the latter without the presence of the former. Learning mathematics will then mean submitting to a stance in which critical sense and reasonable argument do not exist, and surrendering to utilitarian arguments, in the best case. The primacy of technique and adeptness without significance is instituted, granting the good 'exercise solver' the attribute of a good mathematics student. It is hardly surprising to us that, in view of this scenario, a good part of students have no interest in learning a subject that is taught with these characteristics, in which intelligence should be replaced by acceptance, not to say submission.

Refusal and haughtiness

With regard to the binariness between knowing and not knowing mathematics, we believe it would not be excessive to say that the vast majority of humanity considers itself as part of the latter group, and this is one of the elements which, combined to aversion to mathematics, produce a fertile ground for the emergence and growth of the phenomenon of refusal to learn the subject. The feeling of being excluded, of being out of a group that is unconfessedly desired, yet seen as unreachable, drives a number of individuals to react by distaining that desire and that group. Thus, as a secondary consequence, there emerges,

> [...] a large group who ends up haughtily propagating their ignorance of mathematics, a set of individuals who, because they are mandatorily placed in a category of supposed inferiority, react not only by accepting the classification of inferiority imposed on them (as they believe they lack the means to refute it), but also by inverting the classification and, in a haughtily, even proud way, publically rejoicing to embrace that fortune, i.e., the fortune of belonging to the set, to the group of those who do not know mathematics (ATTIE, 2013, p. 15).

It is important that this propagation be public, that it be boasted everywhere, since in a private, reserved way, this joy would not be able to hide the initial resentment, which is in the origin of this type of reaction. Only in the social body can this false pride thrive and survive. Even among so-called learned people, it is common for one to feel embarrassed if one "is caught being unable to distinguish concretism from futurism, but the same person takes pride in 'hating' mathematics," (SILVA, 2004, p. 99).

The mathematician Godfrey Hardy, who lived in the 20th century, uses some sarcasm to comment the phenomenon:

> The fact is that there are few more "popular" subjects than mathematics. Most people have some appreciation of mathematics, just as most people can enjoy a pleasant tune; and there are probably more people really interested in mathematics than in music. Appearances may suggest the contrary, but there are easy explanations. Music can be used to stimulate mass emotion, while mathematics cannot; and musical incapacity is recognized (no doubt rightly) as mildly discreditable, whereas most people are so frightened of the name of mathematics that they are ready, quite unaffectedly, to exaggerate their own mathematical stupidity. (HARDY, 2007, p. 74).

Thus emerges, as we can see, the other face of that same coin, one of the complementary characteristics of the refusal to learn mathematics, i.e., an intended, yet fake, 'pride in ignorance', a certain joy in not being part of a group, in an obvious attempt to disparage it. Paraphrasing Aesop's fable, *The Fox and the Grapes* (ESOPO, 1997), in which the former, after several failed attempts to reach the fruits that hang high on the grapevine, goes away saying it did not really want the grapes as they were not ripe, we could also claim that the grapes were really sour, and not for a moment did they really attract us. We find it noteworthy that, while in the fable we mentioned, the fox lacked better instruments, skills, or, perhaps, a little more luck in reaching the grapes, the situation does not prove so different out the fantasy world, with regard to the group we refer to. After all, if the animal lacks the consciousness that it could be able to catch the food if it had the appropriate tools, the individual who believes it *does not know mathematics* also lacks the consciousness that there are suitable instruments to allow it to understand how it already uses mathematics in its life (and how it can use it better). What could differentiate the human from the animal, i.e., the disposition to be able to change its own history, to overcome the limitations imposed by existing conditions, is unfortunately absent in these individuals. Therefore, both man and the fox lack the consciousness the Spanish philosopher refers to when he affirms that "man reaches his fullest capacity when he is in complete consciousness with himself and his circumstances" (ORTEGA Y GASSET, 1981, p. 21)[1].

The individual's supposed incapacity is something which it believes is within it (and which is immutable to it, almost like a genetic mark that this individual will carry with it throughout life after having been defined as incapable). The analogy we allow ourselves to establish with the fox is that it is as though the fox had an uncorrectable 'defect' which did not allow it to even dream about the juicy meal. For even if the grapes were ripe and within reach, the fox would never be allowed to savor them and, what is worse, because of its own incapacity.

Banal and Natural Facts

In our view, the perception of these phenomena occurs, in our case, in a locus at once peculiar and comprehensive, which is the position of a mathematics teacher. Peculiar because being mathematics teachers (and having been so for various years) is the specificity, the particularity that allows us to see signs of the phenomena in question in students, parents, colleagues of other disciplines, school coordinators, etc. And comprehensive because the perception of the phenomena does not occur only in the classroom, to the contrary, it seems to us to occur both within the school and out.

1- And, in another passage: "I am I and my circumstance" (ORTEGA Y GASSET, 1981, p. 25).

Even though these phenomena lack statistic verification, we believe they occur so often in the various social fields besides school itself, that we can assign to them a phrase coined by Foucault, i.e., that they are one of these 'banal facts'. The French philosopher uses the phrase 'banal fact' not in the sense of a fact that is not something important, but rather to refer to a fact that occurs "all the time" and that "everybody is aware of". And it is precisely because it occurs so often that this event can end up going totally unnoticed by individuals, then indeed causing the impression that it can be a fact of little importance, i.e., banal in the most current sense of the term in everyday language. Aversion to mathematics and the consequent refusal to learn it are, in our view, included in these events about which we would affirm, as the author, that "the fact that they're banal does not mean they don't exist" (Foucault, 1995, p. 232). What is necessary to do with these facts is to "discover – or try to discover – which specific and perhaps original problem is connected with them" (FOUCAULT, 1995, p. 232).

The existence of this phenomenon is equally ratified by the historian Paul Veyne, to whom certain human facts are not obvious, and yet "they seem so evident to their contemporaries' eyes and even to their historian eyes, that neither the former nor the latter notice them" (VEYNE, 1982 apud FISCHER, 2001, p. 222).

Thus, a process of naturalization of the phenomenon manifests, a concept pointed and criticized by Marx (1987) as part of a procedure that makes the individual unable to understand the historical and social process of its own formation. This artifice points us to the attempt to justify inequalities by means of supposed natural causes. With regard to the naturalization of the phenomenon we treat here – the individual's animosity relationship with mathematics – this process means erasing from the context this relationship, which thus turns into a relationship viewed not only as natural, but also permanent and immutable.

Various historical notes, among which we highlight those of Bell (1986), Davis and Hersh (1995), and Attie (2013), show how, on the one hand, the institutional valuing of mathematics, and on the other, an increase in the invisibility of that knowledge, developed simultaneously to shape a scenario where mathematics' increasing abstraction implied driving individuals away from understanding mathematics' significance processes, thus producing a mechanism of increasing alienation. It is worth considering that the maintenance of this framework and alienation process for political and economic necessities makes it strictly indispensable for the teaching of mathematics to continue privileging memorization and repetition to the detriment of understanding. It is in the context of these elements, and obviously as a result thereof, that the negative representations about mathematics are produced and consolidated.

Magical Thinking

In face of this context, we dare to conjecture that, based on episodes as the ones exemplified earlier, a consistent justification could be provided for the current and permanent association that is made between the mathematics teaching-learning process and what Paulo Freire called "magical thinking" (FREIRE, 1983), in which the justification for events would lie somewhere beyond our possibilities to understand.

> This is the reason why, when he perceives a concrete fact of reality without admiring it in critical terms so he might see it from within, perplexed at the appearance of the mystery, insecure of himself, man becomes magical. Unable to capture the challenge in its authentic relations with other facts, mesmerized before the challenge, his understandable tendency is to seek, beyond true relations, the explicative reason for the perceived given. (FREIRE, 1983, p. 29).

Therefore, it is hardly surprising that individuals (even educated ones) relate results and skills in mathematics to natural individual gifts or even to unexplainable, magical characteristics, making success in this area "a matter of faith" (WATTERSON, 1996, p. 38), After all, based on an attitude in which neither the causes nor the process of development of concepts is considered, "the acceptance or non-acceptance of these rules are not a function of a rational analysis of concrete necessities. Rather, it is a matter of passivity, i.e., accepting them, or mere rebelliousness, i.e., opposing to them, without knowing why" (DUARTE, 1986, p. 08).

This separation between technique and knowledge obviously encourages representations of mathematical knowledge as not only an incomprehensible subject, but also one that relies on non-logically verifiable dogmas, being thus founded on properties seen as uncontestable and accepted based on the belief that they are correct. Thus, the representation of what mathematics might be progressively consolidates into the representation of a knowledge far from the student's concrete reality. It is as though the results of operations between mathematical elements and also the existence of good students in the subject occurred "by magic", since there is nobody who can tell how a certain result was obtained, why the result is that one instead of another, or why and how some people can decipher the "secrets" and others cannot. Thus, the individual eventually infers that, in mathematics, results (and the procedures that produce them) are unexplainable, incomprehensible, and, like the existence of God, with these results and procedures, the individual is in a situation where it simply "believes it or it does not".

This analogy between mathematics and religion may sound unreasonable at first, since nothing suggests any affinity between such distinct human ways of understanding reality, as they rely on such contrary elements as reason and faith. However, it is worth considering that if, in religious terms, the individual is in no position to know or to discuss the processes that have led to the creation and acceptance of the dogmas of a given religion and, what is more, there is the agreement that, perhaps, these processes can only be known by an 'elected', illuminated few, due to a greater power, this same vision can be transferred onto individuals who point out how mathematics can be seen as a set of 'things' whose meanings and processes seem mysterious and enigmatic to them, and therefore can only be accepted as a matter of faith, since (even disregarding the humorous tone), "math is not a science, it's a religion. All these equations are like miracles. You take two numbers and when you add them, they magically become one new number. No one can say how it happens. You either believe it or you don't" (WATTERSON, 1996, p. 38).

If we admit that the attitude of submitting to certain dogmas is considered a socially acceptable stance when it comes to religious matters, then there is the strong emergence, besides learning-specific questions, of one of the negative consequences of this possible (and undesirable) analogy between mathematics and religion, when the principles of mathematics are perceived as dogmas, with the necessity of subordination to rules. Anyway, we find it necessary to stress that this similarity should be much more characterized as an affinity between stances, attitudes and procedures regarding the element (mathematics or religion) than an affinity between the elements themselves.

Final reflections

With regard to the two main phenomena approached in this text, i.e., aversion and refusal to learn, one given which accompanies the phenomena and should be viewed as fundamental in this process is that no mobility is considered to exist between the two sets in the binariness described in the beginning of the article, i.e., *those who know and those who do not know* mathematics. No change seems possible to the initial state of the not-knowers.

With regard to this element of immobility in itself, we wish to offer a few considerations.

The first of them concerns who gets to assign the categories. It is a well-known fact that, in the case of mathematical knowledge, the classification in the categories of those who know mathematics and those who do not know it ('in' and 'out') is done by individuals or groups situated in a position which authorizes and allows them to do so. It is important to stress that occupying this position can be either a self-conferred condition which is then accepted by the other groups, or it can be an extension conferred by those who already possess this power. Therefore, those within this position get to decide who can be part of it and be 'in' too. And those who are 'out', excluded, have no choice in terms of position assignment, as the power to assign does not belong to them.

Another point we believe it is worth considering concerns the possible responses or reactions to this assignment of exclusion. In general terms, we can see two fundamental types of alternatives and responses the individuals (who were put in the set of the excluded) resort to. We stress that they are not the only reactions that can be found, but rather the ones that interest us most in view of the consequences (for the teaching-learning process) of the phenomena we proposed to discuss here. The first response, which a good part of the group initially tries to adopt as their own, is to make honest, diligent and disciplined efforts to enjoy the opportunity to be included in the privileged place, which, in practical terms, means to study mathematics harder in order to try to have oneself included in the 'in' group, an event that will only occur if the latter is convinced that the individual is "reliable" and possesses the necessary requisites to be part of it, which, in general terms, would be discipline, brilliance, intelligence, perseverance, and responsibility (ESPÍNDOLA, 2009; MARKARIAN, 2004; MESQUITA, 2004; ATTIE, 2013; SANTOS, 1989).

However, besides the fact that this response fails to prove fruitful to everybody, as these attempts do not always succeed, even in the majority of cases where it results in what is desired (i.e., inclusion), it can hardly be called a quick-effect response, as both the learning process and the validation process in mathematics are usually considered slow processes. Thus, what ends up occurring is that a great part of these ranks gives up and goes, instead, for a second type of reaction, which is often, but not always, a consequence of failure in the first response. This reaction, which we consider a likely reflex of a succession of unsuccessful attempts, consists precisely of giving up trying to belong to the 'in' group, and definitely refusing to be part of that privileged position. This refusal can vary as to its level of intensity. It can range from the previously referred pseudo-resignation about the sour grapes to the strong, explicit pride not to belong to that group, thus inverting the order of who the insiders and the outsiders would be, since the qualification of both groups would also be inverted by this discourse.

This inversion of groups, i.e., turning what is 'out' into what is 'in', takes clever advance of the fact that being pushed out[2] of a set can mean being within another group, the very set of those who have been pushed out. From then on, it is a matter of valuing the set of those who are out and diminishing the set of those who are in. A real *field of struggles* is set therein, and it emerges with a frequency that is intense, even crystal clear (for those willing to see it) and, perhaps for this reason, scary. One can consider this type of response as a truly legitimate reaction by a group who is, from its perspective, arbitrarily pushed into an outside zone because it cannot figure out or legitimize the logic existing in the assignment of categories. And it is from this position-taking, denying the desire and the intention to belong to a class considered privileged, that the group can free itself from the imprisonment of categories attributed by others, and can invert

2- Can there be a better synonym for 'excluded' than the phrase "being pushed out"?

the values of what is considered ‘in’ and what is considered ‘out’. It is important to stress that this process includes the public propagation of what we are calling “pride of ignorance”.

The consequences of this stance should no doubt be considered relevant for improving individuals’ self-esteem at first. However, this liberation process leads to a new imprisonment process, as it almost mandatorily induces a separation from any traces of what might be a formal mathematical thought. Although it works as a perfectly understandable emotional mechanism, it proves a perverse one in the sense that it thus deprives the individual from any power an explicit use of the simplest mathematics could provide to its life. Here, we are talking about a more systematic use of the skills mentioned earlier, such as abstracting, ordaining, estimating, classifying, in addition to the very elementary mathematical operations, solely in benefit of an expansion of the being’s autonomy capacity, the possibility for knowledge to provide the individual with alternatives to choose from. In this terms, even for one who considers oneself unfit and unadapted to the use mathematical thinking, it is no small power (nor is it news) what the use of mathematical tools and thought could provide. The consequences of this knowledge refusal mechanisms for the mathematics teaching-learning process are obvious and can be considered, at least, disastrous.

Finally, we can say that the description of these phenomena brings along, at once and paradoxically, a characteristic that is both discouraging and auspicious. Discouragement appears when we consider a major, if not the greatest, consequence of the stance we described, i.e., the reproduction of a status and a way of living and thinking that does not help to increase the individual’s ability to live with more autonomy, as, to the contrary, it only strengthens that individual’s submission to whatever it is shown to, because of the mere incapacity to criticize and assess what is more advantageous or correct to it. A huge possibility emerges, however, when the characteristics of an imprisoning behavior are more properly described and looked into. For this is where possibilities open up for the transformation and overcoming of it. This is what we work towards and this is what we count on.

References

ATTIE, João Paulo. **Relações de poder no processo de ensino e aprendizagem de matemática**, 2013. Tese (Doutorado) – Faculdade de Educação da Universidade de São Paulo, São Paulo, 2013.

BELL, Eric Temple. **Historia de las matematicas.** Mexico, DC: Fondo de Cultura Económica, 2003.

BELL, Eric Temple. **Men of mathematics.** New York: Simon & Schuster, 1986.

BITTENCOURT, Circe Maria. **Livro didático e conhecimento histórico:** uma história do saber escolar, 1993. Tese (Doutorado) – Faculdade de Filosofia, Letras e Ciências Humanas da Universidade de São Paulo, São Paulo, 1993.

BOYER, Carl Benjamin. **História da matemática.** São Paulo: Edgar Blucher, 1974.

BRASIL. Ministério da Educação e Cultura. Secretaria de Educação Fundamental. **Guia de livros didáticos – 5ª a 8ª séries.** Brasília, DF: MEC, 2011.

BRASIL. Ministério da Educação e Cultura. Secretaria de Educação Fundamental. **Guia de livros didáticos – 5ª a 8ª séries.** Brasília, DF: MEC, 2013.

BRASIL. Ministério da Educação e Cultura. Secretaria de Educação Fundamental. **Guia de livros didáticos – 5ª a 8ª séries.** Brasília, DF: MEC, 2014.

DAVIS, Philip; HERSH, Reuben. **A experiência matemática.** Lisboa: Gradiva, 1995.

DUARTE, Newton. Como somar para não diminuir. **Jornal do Professor de 1º Grau,** Brasília, DF, n. 8, p. 8, 1986.

ESOPO. **Fábulas.** Porto Alegre: L&PM, 1997.

ESPÍNDOLA, Elisângela Bastos de Melo. **Profissão professor de matemática:** um estudo sobre as Representações Sociais, 2009. Dissertação (Mestrado) – Universidade Federal de Pernambuco, Recife, 2009.

EVES, Howard. **Introdução à história da matemática.** Campinas: Unicamp, 2005.

FISCHER, Rosa Maria Bueno. Foucault e a Análise do Discurso em Educação. **Cadernos de Pesquisa**, São Paulo, n. 114, p. 197-223, nov. 2001.

FOUCAULT, Michel. **A microfísica do poder.** São Paulo, Graal, 2007.

FOUCAULT, Michel. Dois ensaios sobre o sujeito e o poder. In: DREYFUS, Hubert; HABINOW, Paul. **Michel Foucault:** uma trajetória filosófica. Rio de Janeiro: Forense, 1995. p. 231-239.

FREIRE, Paulo. **Extensão ou comunicação?** Rio de Janeiro: Paz e Terra, 1983.

FREITAG, Bárbara; COSTA, Wanderley Ferreira; MOTTA, Valéria Rodrigues. **O livro didático em questão.** São Paulo: Cortez, 1997.

HARDY, Godfrey Harold. **A apologia de um matemático.** Lisboa: Gradiva, 2007.

IMENES, Luiz Márcio e LELLIS, Marcelo. **Matemática.** 7º ano. São Paulo: Moderna, 2009.

JAKUBOVICH, José; LELLIS, Marcelo; CENTURIÓN, Marília Ramos. **Matemática na medida certa.** 6º ano. São Paulo: Scipione, 2008.

MARKARIAN, Roberto. A matemática na escola: alguns problemas e suas causas. In: BRASIL: Secretaria de Educação Básica. **Explorando o ensino de matemática.** v. 1. Brasilia, DF: MEC/SEB, 2004. Cap. 6.

MARX, Karl. **Elementos fundamentales para la crítica de la economía política.** México: Siglo XXI, 1987.

MESQUITA, Carla Gonçalves Rodrigues de. O professor de matemática no cinema: cenários de identidades e diferenças. **Educação Matemática em Revista,** São Paulo, v. 11, n. 16, p. 56-62, 2004

ORTEGA Y GASSET, José. **Meditaciones del Quijote.** Madrid: Alianza, 1981.

POLONI, Gustavo. Impa se destaca em trabalhos científicos e produtos inovadores. **Revista INFO**, São Paulo, n. 317, p. 81-88, jul. 2012.

SAKAY, Lady. **Análise das contribuições de uma pesquisa-ação de reeducação matemática para a formação de professoras dos anos iniciais**, 2007. Dissertação (Mestrado) – Faculdade de Educação da Universidade de Brasília, Brasília, 2007.

SANTOS, Vinício de Macedo. **A matemática no 1º grau:** o significado que pais, alunos e professores conferem à matemática, 1989. Dissertação (Mestrado) – Faculdade de Ciências Exatas e Tecnologia da Pontifícia Universidade Católica de São Paulo, São Paulo, 1989.

SÃO PAULO (Estado). Secretaria da Educação/CGEB. **Orientações curriculares do estado de São Paulo:** anos iniciais do ensino fundamental. São Paulo: SE, 2014.

SILVA, Erondina Barbosa. O impacto do curso pie na reconstrução de representações sociais da matemática e do seu processo de aprendizagem e ensino. In: ENCONTRO NACIONAL DE EDUCAÇÃO MATEMÁTICA, 8., 2014, Recife. **Anais do...** Recife: [s. n.], 2004. GT 7. p. 1-18.

SOUSA, Pedro Miguel Lopes de. O ensino de matemática: contributos pedagógicos de Piaget e Vigotsky. **Portal da psicologia**. Porto: [s. n.], 2005. Disponível em: <http://www.psicologia.pt/artigos/textos/A0258.pdf>. Acesso em: 17 fev. 2012.

TRABAL, Patrick. **La violence de l'enseignement des mathematiques et des sciences:** un outre approche de la sociologie des sciences. Montreal: L'Harmattan, 1997.

WATTERSON, Bill. **Felino selvagem psicopata homicida n. 2**. São Paulo: Best News, 1996.

Received on July 22nd, 2015.

Approved on May 17th, 2016.

João Paulo Attie is an assistant professor for the mathematics department at the Universidade Federal de Sergipe. He heads the Grupo de Pesquisas Processos de Argumentação no Ensino de Matemática.

Manoel Oriosvaldo de Moura is a full professor at the Faculdade de Educação da Universidade de São Paulo. He heads the Grupo de Estudos e Pesquisas sobre a Atividade Pedagógica (GEPAPe).