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The Carreau-Yasuda Fluids: a Skin Friction Equation for Turbulent Flow in Pipes and Kolmogorov Dissipative Scales

In this work the turbulent flow of the Non-Newtonian Carreau-Yasuda fluid will be studied. A skin friction equation for the turbulent flow of Carreau-Yasuda fluids will be derived assuming a logarithmic behavior of the turbulent mean velocity for the near wall flow out of the viscous sub layer. An alternative near wall characteristic length scale which takes into account the effects of the relaxation time will be introduced. The characteristic length will be obtained through the analysis of viscous region near the wall. The results compared with experimental data obtained with Tylose (methyl hydroxil cellulose) solutions showing good agreement. The relations between scales integral and dissipative obtained for length, time, velocity, kinetic energy, and vorticity will be derived for this type of fluid. When the power law index approach to unity the relations reduces to newtonian case.
***Keywords**: turbulence; non-newtonian fluids; skin friction equation*

Introduction

The turbulent flow of non-Newtonian fluids appears in many situations in mechanical and chemical engineering. Despite this importance, many questions related to the nature of such type flows remains unanswered. One important and not well understood phenomenon is the turbulent drag reduction, which was discovered by Toms (1948). The occurrence of the drag reduction is related to certain viscoelastic and shear thinning properties of some non-Newtonian fluids, and according to Hinch (1977), Tabor, Durning, and O´Shaughnessy (1989) and other workers, it is connected to the strong strain imposed elongation to the molecules and its effects on the viscosity. In some important works with diluter polymer flows Virk, Mickley and Smith (1970) developed empirically, a velocity law of the wall model, and proposed a limit for the drag reduction.

The experimental work of Pinho and Withelaw (1990) and Escudier, Jones and Gouldson (1992) of turbulent flow of shear-thinning aqueous polymeric solutions with viscosity power law indices between 0.39 and 0.9 also showed the validity of Virk´s asymptote for these variable viscosity fluids. Although, many works have been done on the non–Newtonian fluids turbulent flow analysis, some important parameters are not yet well defined. In a recent work, Cruz et. al. (2000) defined a Kolmogorov dissipation length valid for the turbulent flow of power law fluids. More recently, Cruz and Andrade (2001) developed a Kolmogorov dissipation length and a skin friction equation, both valid for the turbulent flow of a particular case of Carreau fluids (null viscosity at infinite shear rate). A extension of the former work will be shown here for the Carreau –Yasuda fluids.[1]

For the general case of the turbulent flow of non-Newtonian fluids some near wall parameters, like the characteristic velocity and the characteristic length are also still not established. In his work, Virk adopted the classical friction velocity as the appropriate characteristic velocity for the near wall region, considering that the viscous sub-layer of the non-Newtonian fluids is similar to the viscous sub-layer of the Newtonian fluids. This assumption is very controversial, and probably is not valid for all types of non-Newtonian fluids.

Since the drag reduction phenomenon is mainly restricted to the near wall turbulent flow region, the development of a procedure to analyze the non Newtonian turbulent flow in that region is of great importance to the development of turbulent models that can describe the drag reduction occurrence. In order to obtain some information on the deduction of a general treatment for the near wall turbulent flow region of drag reducing fluids, it is important to analyze first the pure viscous situation.

The objective of this work is to obtain a skin friction equation for the turbulent flow of Carreau-Yasuda fluids in pipes. In this equation a new near wall characteristic length is defined, using a relation similar to the developed by Dodge and Metzner (1959). The implicit equation is derived from viscous sub layer analysis employing the Carreau-Yasuda rheological model. The results are compared with the experimental data of turbulent pipe flows of very low molecular weight polymer solutions of variable viscosity, showing good agreement. Additionally, the relations between scales integral and dissipative for length, time, velocity, kinetic energy, and vorticity will be developed to Carreau-Yasuda fluids, generalizing the results obtained for Carreau fluids (Cruz and Andrade, 2001) with zero viscosity at infinite shear rate.

Nomenclature

A = total frontal area of the flow, area, m^2
D = pipe diameter, m
f = Fanning friction factor, dimensionless
Lc = near wall characteristic length scale of the flow, L
ℓ =dissipative length scale, L
Re = Reynolds number of the water flow, Reynolds number, dimensionless
U = mean velocity, m/s
We = Weissenberg number, dimensionless

Greek Symbols

ΔP = pressure drop, Pa
η = water dynamic viscosity, dynamic viscosity, $kg/(m\ s)$
ρ = water density, kg/m^3
ν_o = kinematic viscosity L^2/s

Subscripts

τ, w = Relative to wall
∞ = Relative to far from the wall variables

Paper accepted December, 2006. Technical Editor: Atila P. Silva Freire.

The Skin Friction Equation

The Carreau-Yasuda rheological model defined by the following equations,

$$\tau = \eta \dot{\gamma} \tag{1}$$

with

$$\eta = \eta_\infty + (\eta_o - \eta_\infty)\left[\ 1 + (\lambda\dot{\gamma})^a\right]^{\frac{n-1}{a}} \tag{2}$$

is a generalization of the Newtonian model, and describe with more accuracy the variation of viscosity η with shear rate $\dot{\gamma}$ than the power law rheological model. The fundamental difference between these two formulations is that the Carreau-Yasuda equation involves five parameters ($\eta_o, \eta_\infty, \lambda, n, a$) to describe the fluid rheology, instead of the two parameters formulation (k, n) of the power law relation. To formulate a skin friction equation for turbulent flow of Carreau-Yasuda fluids it is necessary first to analyze the near wall region to obtain the characteristic length scale of the phenomenon. In the viscous sub layer, neglecting the turbulent stresses, the mean momentum equation can be written on the following form:

$$\frac{\partial}{\partial y}\left(\eta\,\frac{\partial \overline{u}}{\partial y}\right) = 0 \tag{3}$$

Integrating an applying the boundary condition at the wall (in $y = 0$) we obtain:

$$\frac{1}{\rho}\tau_w = \frac{1}{\rho}\eta\,\frac{\partial \overline{u}}{\partial y} \tag{4}$$

where τ_w is the shear stress at wall and ρ is the density of solvent (water). The friction velocity is now defined as $u_\tau = (\tau_w/\rho)^{1/2}$. Substituting the definition of friction velocity and equation (2) into equation (4) one have:

$$u_\tau^2 = \nu_o \left[\frac{\eta_\infty}{\eta_o} + \left[1 - \frac{\eta_\infty}{\eta_o}\right]\left[1 + \left(\lambda\,\frac{\partial \overline{u}}{\partial y}\right)^a\right]^{\frac{n-1}{a}}\right]\frac{\partial \overline{u}}{\partial y} \tag{5}$$

where ν_o denotes kinematic viscosity.

Considering that in viscous sub layer the velocity profile is nearly linear the following approximation for the velocity gradient will be used:

$$\dot{\gamma} = \frac{\partial \overline{u}}{\partial y} \cong \frac{u_\tau}{L_c} \tag{6}$$

Inserting (6) into (5) we have:

$$u_\tau^2 = \nu_o \left[\frac{\eta_\infty}{\eta_o} + \left[1 - \frac{\eta_\infty}{\eta_o}\right]\left[1 + \left(\lambda\,\frac{u_\tau}{L_c}\right)^a\right]^{\frac{n-1}{a}}\right]\frac{u_\tau}{L_c} \tag{7}$$

Defining the Reynolds number and the Fanning friction factor as $Re = UD/\nu_o$ and $f = 2(u_\tau/U)^2$ where U is the mean velocity and D is the diameter of the tube, and inserting these definitions into equation (7) one finds:

$$\frac{f}{2} = \frac{1}{Re}\frac{D}{Lc}\left(\frac{f}{2}\right)^{1/2}\left[\frac{\eta_\infty}{\eta_o} + \left[1 - \frac{\eta_\infty}{\eta_o}\right]\left[1 + \left(\lambda\,\frac{1}{Re}\frac{U^2}{\nu_o}\frac{D}{L_c}\left(\frac{f}{2}\right)^{1/2}\right)^a\right]^{\frac{n-1}{a}}\right] \tag{8}$$

Here, Lc represents the near wall characteristic length scale of the flow.

Equation (8) represents a relation between the friction factor and the near wall characteristic length thus, in order to obtain a equation for the friction factor it is necessary to produce other relation involving those parameters. Such a relation can be obtained form the formulation developed by Dodge and Metzner (1959) for power law fluids and is given by (9):

$$\frac{1}{Lc} = \frac{1}{D}\frac{3n+1}{4n}8^{\frac{n-1}{n}}2^{\frac{n-2}{2n}}1.\mathrm{E}\left[\frac{1}{4n^{0.25}}\left(\frac{1}{\sqrt{f}} + \frac{0{,}4}{n^{1{,}2}}\right)\right] \tag{9}$$

where $1.E$ denotes power of 10.

The equation (9) was obtained assuming that the turbulent mean velocity in the turbulent flow region, out of the viscous sub layer, has a logarithmic behavior similar to the Newtonian fluid case. The influence of the non Newtonian behavior should be considered through the introduction of a proper characteristic length obtained by the viscous sub layer analysis. The set composed by eqs. (8) and (9) can now be solved for the Fanning friction factor. The existence of a logarithmic velocity profile at the near wall region, seems to be an invariant characteristic of attached turbulent flows. It can de observed even for turbulent flows of viscoelastic fluids.

The Kolmogorov Dissipation Lenght

In an important work, Kolmogorov (1941) showed how to obtain the smallest scales of turbulence for Newtonian fluids using dimensional arguments. In that scale the kinetic energy dissipates into heat and the convective effects have the same order of magnitude of the viscous effects. The correct determination of such dissipation length is of great importance for the correct modeling of non- Newtonian turbulence terms that emerges from the non linearity of the molecular diffusive terms.

The procedure used to determinate the dissipation length is almost the same used by Kolmogorov for Newtonian fluids in the present case however, the viscosity is not constant and the analysis was developed for high Weissenberg numbers $(\lambda\frac{u}{\ell} >> 1)$. After some algebra the following value of the non dimensional dissipation length was obtained:

$$\frac{\ell}{L} = \left[\overline{Re}_A\right]^{\frac{-1}{2(n+1)}}\ A_2\,A_3 \tag{10}$$

with

$$\overline{Re}_A = \frac{\rho\,L^n U^{2-n}}{\mu_o \lambda^{n-1}} = \frac{L^n U^{2-n}}{\nu_o \lambda^{n-1}} \tag{11}$$

$$\Gamma = \frac{\lambda^2\,U^3}{L(\nu_o - \nu_\infty)} \tag{12}$$

$$A_1 = 1 + \frac{n-1}{a}\ \Gamma^{\frac{-a}{n+1}} \tag{13}$$

$$A_2 = \left[\frac{L^{n+1} U^{n+1}}{\nu_o^n \left(\nu_o - \nu_\infty \right) A_1} \right]^{\frac{-1}{2(n+1)}} \tag{14}$$

and

$$A_3 = \left[\frac{\nu_\infty}{\nu_o} + \left(1 - \frac{\nu_\infty}{\nu_o} \right) \Gamma^{\frac{n-1}{n+1}} \left(A_1 \right)^{\frac{2}{n+1}} \right]^{\frac{1}{2}} \tag{15}$$

The relations between smaller and biggest scales of time, velocity, vorticity and kinetic energy are derived of analogous form. It are given respectively by:

$$\frac{t}{T} = \left[\left[\bar{R}e_\Lambda \right]^{\frac{-1}{2(n+1)}} \; A_2 \, A_3 \right]^{\frac{2}{3}} \tag{16}$$

$$\frac{u}{U} = \left[\left[\bar{R}e_\Lambda \right]^{\frac{-1}{2(n+1)}} \; A_2 \, A_3 \right]^{\frac{1}{3}} \tag{17}$$

$$\frac{\omega}{W} = \left[\left[\bar{R}e_\Lambda \right]^{\frac{-1}{2(n+1)}} \; A_2 \, A_3 \right]^{-\frac{2}{3}} \tag{18}$$

and

$$\frac{e}{E} = \left[\left[\bar{R}e_\Lambda \right]^{\frac{-1}{2(n+1)}} \; A_2 \, A_3 \right]^{\frac{2}{3}} \tag{19}$$

Results and Discussion

In figures 1 to 4 the results of the present formulation are compared with the Prandtl-von Kármán equation for Newtonian fluids and experimental data of Pereira and Pinho (1999) for aqueous solutions of Tylose with concentrations varying of 0.2% to 0.6% by weight. In these figures f_d is the Darcy friction factor and Re_w is the Reynolds number calculated using the rheological parameters of Carreau -Yasuda fluids at the wall, viz., with $\eta(\dot{\gamma})$ at $y = 0$.

In figures 1 and 2 due to its low concentration, the influence of the Tylose into the skin friction factor is very small and no significant difference between the present work and the Prandtl-von Kármán equation exists. As the Tylose concentration increases (Figs. 3 and 4) the solution begins to exhibit some drag reduction behavior and the Prandtl-von Kármán equation starts to over predict the skin friction factor. The same is not true for the present formulation which still adequately describes the experimental data.

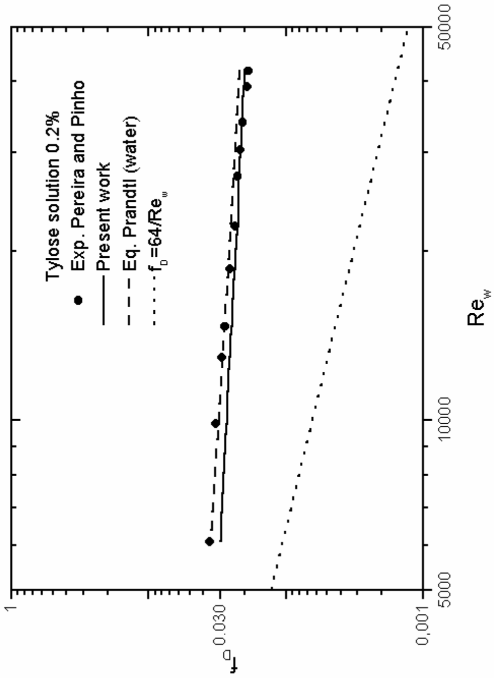

Figure 1. Friction factor of Darcy versus Reynolds number at wall - Tylose solution 0.2%.

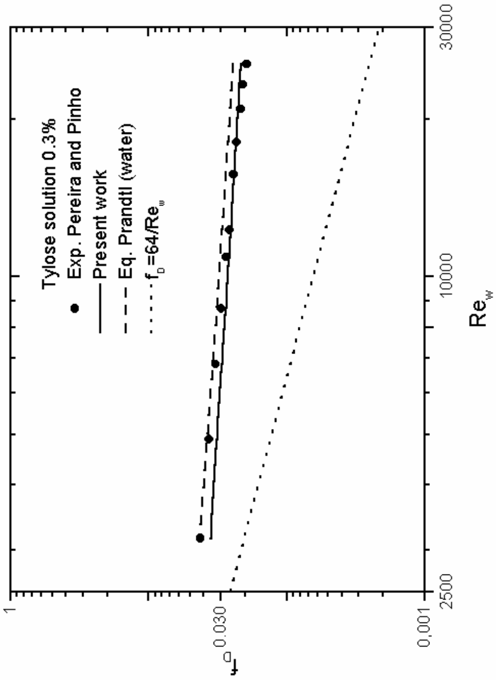

Figure 2. Friction factor of Darcy versus Reynolds number at wall - Tylose solution 0.3%.

In Figure 3 the experimental data exhibits a clear drag reduction and the Newtonian formulation can not be used anymore to calculate the friction factor. The present formulation however, describes accurately the experiments, disregarding scatter of the data. Figure 4 show the experimental values of the skin friction factor for the highest value of the Tylose concentration in water. In this particular case the flow has not achieve the fully turbulent condition, and the laminar-transitional regime is still governing the flow. Although any analysis in this situation should be made with care, it should be noted that the formulation proposed here presents a clear drag reduction, and it is much closer of the experimental data than the Prandtl-von Kármán equation for high values of the Re_w.

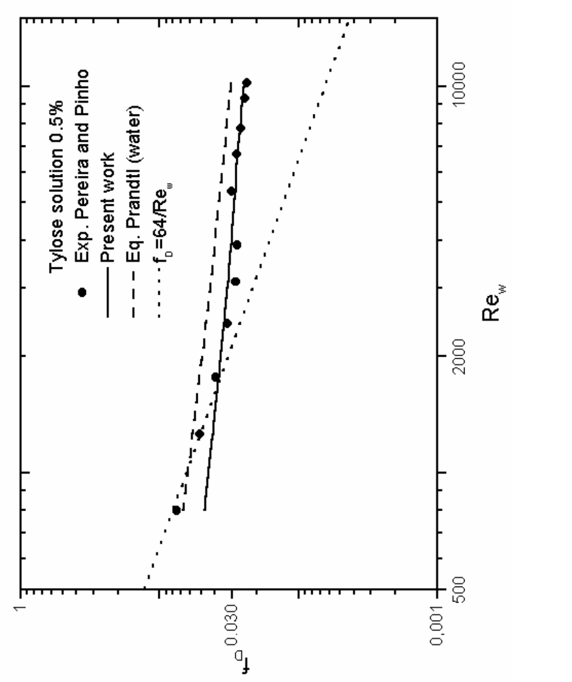

Figure 3. Friction factor of Darcy versus Reynolds number at wall - Tylose solution 0.5%.

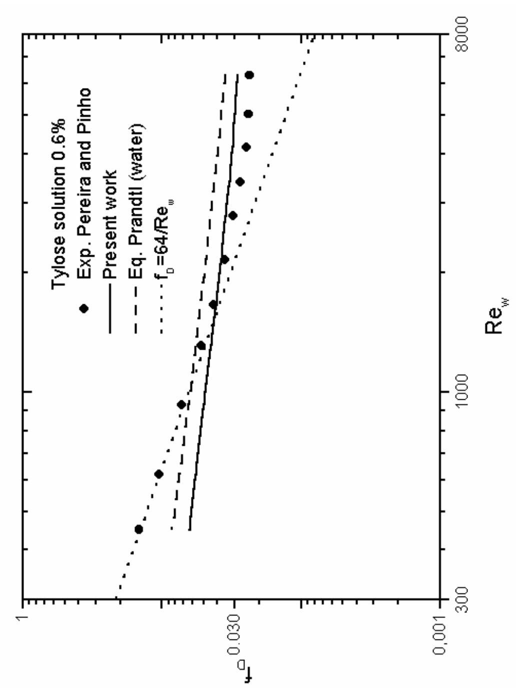

Figure 4. Friction factor of Darcy versus Reynolds number at wall - Tylose solution 0.6%.

The Figures 1 to 4 shows that the present formulation reproduces adequately the experimental data giving satisfactory results, taking into account the experimental uncertainty. This fact indicates that, although in the case of turbulent flows the velocity gradient is very high in the near wall region, the influence of the relaxation time (small for all concentrations) can not be discarded.

One of main differences between the present analysis and the power law approach is that the characteristic length scale used is different in each formulation. This fact suggests that the near wall turbulent non-Newtonian flow can be analyzed in the same way as the Newtonian case. Since the logarithmic velocity profile can be obtained using dimensional arguments alone, and that in such analysis, the viscous effects are present only through the wall shear stress (Landau and Lifshitz, 1987) it indicates that even for the turbulent flow of non-Newtonian fluid the velocity profile in the turbulent region outside the viscous sub layer has a logarithmic behavior. This conclusion is confirmed by the works of Virk, Mickley and Smith (1970) and Dodge and Metzner (1959), among others. Therefore the influence of the non-Newtonian behavior is restricted to the constant that appears in the analysis, as the von Kármán's constant κ; and to the characteristics length and velocity of the near wall flow.

The assumption that the characteristics parameters of the near wall flow of non-Newtonian fluids are the same of the Newtonian case is very popular. In this work, however a different approach was adopted showing good results, and indicating that at least some other alternatives should be tried. The choice of the adequate near wall parameters is very important for the near wall modeling of the non-Newtonian turbulent flow. Not only if the logarithmic law approach is used with some k-ε or other turbulence model to avoid the viscous sub layer, but also if the low Reynolds number formulation is adopted.

The use of a "correction factor" into eq. (7) similar the one that was introduced in the work of Cruz and Andrade (2001) for Carreau fluids, in order to adjust the Dodge and Metzner power law relation to the Carreau-Yasuda fluids flow, proved to be not necessary in the present formulation. However it should be noted that the expression which define the characteristic length, eq. (7), does not take into account the parameter a of the rheological model. If it had been considered into the analysis, probably the correlation between analytical and experimental results would be even better.

The good results obtained here for the Tylose solutions turbulent flow using a pure viscous formulation, indicates perhaps that the viscoelastic effects are, in this case (concentrate weakly solutions of low molecular weight), not very strong and that the present formulation can be used to calculate the friction factor. However, more comparisons with others experimental data should be made to confirm this conclusion.

The influence of the non-Newtonian character of the flow on the dissipation length is show in figure 5. It is clear that shear thinning produces a great change on the non dimensional dissipation length in the sense that the increase of the shear thinning behavior causes a diminish of the smallest scales of the turbulent flow. This phenomenon can be explained considering the fact that the increase of the shear thinning causes a similar effect of a decrease of the molecular viscosity on a Newtonian fluid. The influence of the non-Newtonian behavior on the turbulent flow smallest scales, must be carefully considered in the turbulent flow numerical calculations and modeling.

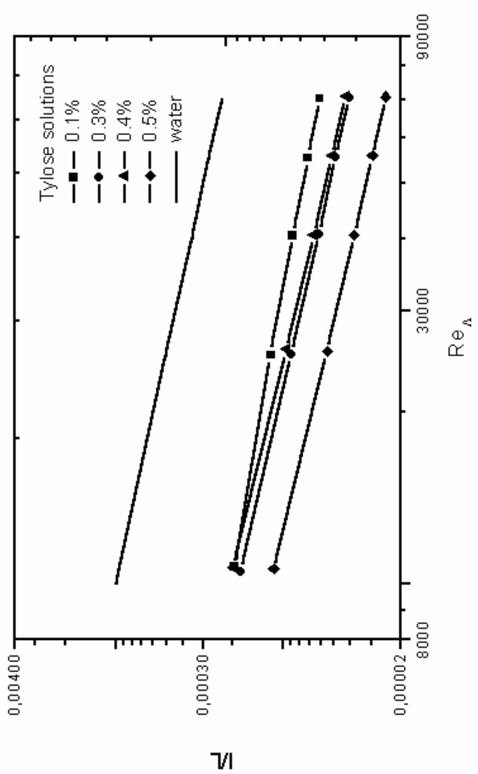

Figure 5. Relation between scales integral and dissipative – length.

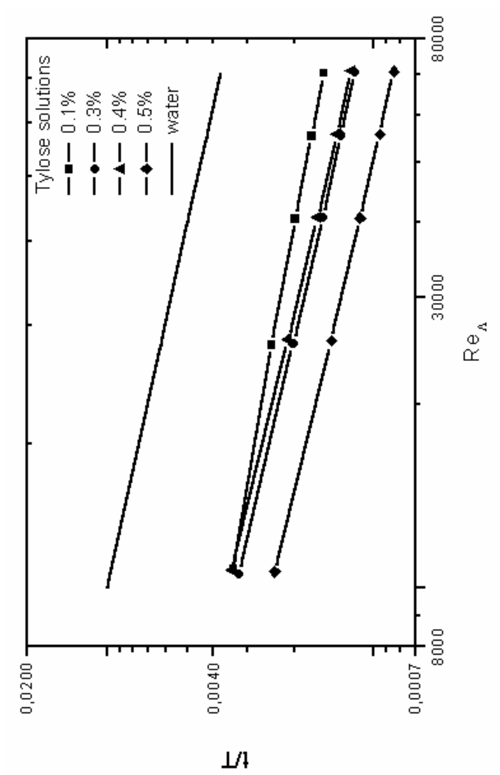

Figure 6. Relation between scales integral and dissipative – time.

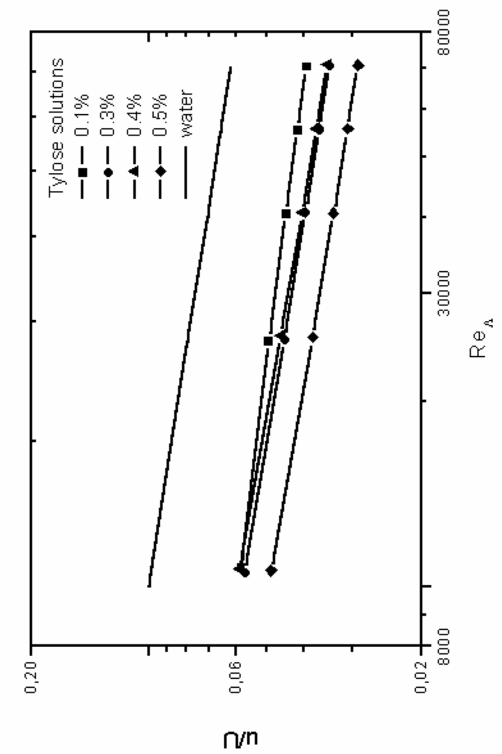

Figure 7. Relation between scales integral and dissipative – velocity.

Conclusion

In this work, a type of skin friction equation for Carreau-Yasuda fluids was deduced. The equation was derived using an alternative formulation for the near wall characteristic length, instead of the classical Newtonian characteristic length given by ν/u_τ. A comparison with experimental data of four concentrations of low molecular weight Tylose solutions was made, and the proposed formulation proved to be consistent, reproducing the drag reduction behavior adequately. This fact suggests that the near wall turbulent flow analysis of non-Newtonian should be made in a similar manner as the one used to describe the turbulent flow of Newtonian fluids. This also indicates that the logarithmic behavior of the turbulent mean velocity profile in the near wall region, out of the viscous sub layer, is a consequence of the convective character of the turbulent transport and is probably not affected by the non-Newtonian nature of the viscous stress. The influence of the Non-Newtonian character of the viscous stress on the turbulent mean velocity profile, should be considered through the constants that emerges from the analysis, and the characteristic length of the near wall flow which is obtained from the viscous sub layer analysis.

The classical Kolmogorov dissipation length expression was extended for the non-Newtonian case of Carreau-Yasuda fluids. It was show that the non-Newtonian character of the fluid exerts a major influence on the size of the turbulent flow smallest scales. Therefore, the direct numerical simulations of the turbulent flow of shear thinning non-Newtonian fluids should be done, with the refinement of the meshes more intense than the Newtonian case.

Acknowledgements

The authors are in debt with professors A. S. Pereira and F.T Pinho who kindly provided the experimental data.

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