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An Adaptive Fuzzy Dead-Zone Compensation Scheme and its Application to Electro-Hydraulic Systems

The dead-zone is one of the most common hard nonlinearities in industrial actuators and its presence may drastically compromise control systems stability and performance. Due to the possibility to express specialist knowledge in an algorithmic manner, fuzzy logic has been largely employed in the last decades to both control and identification of uncertain dynamical systems. In spite of the simplicity of this heuristic approach, in some situations a more rigorous mathematical treatment of the problem is required. In this work, an adaptive fuzzy controller is proposed for nonlinear systems subject to dead-zone input. The boundedness of all closed-loop signals and the convergence properties of the tracking error are proven using Lyapunov stability theory and Barbalat's lemma. An application of this adaptive fuzzy scheme to an electro-hydraulic servo-system is introduced to illustrate the controller design method. Numerical results are also presented in order to demonstrate the control system performance.
Keywords: adaptive algorithms, dead-zone, electro-hydraulic actuators, fuzzy logic, nonlinear control

Introduction

Dead-zone is a hard nonlinearity, frequently encountered in many actuators of industrial control systems, especially those containing some very common components, such as hydraulic or pneumatic valves and electric motors. Dead-zone characteristics are often unknown and it was already observed that its presence can severely reduce control system performance and lead to limit cycles in the closed-loop system.

The increasing number of works dealing with systems subject to dead-zone input shows the great interest of the engineering community in this particular nonlinearity. The most common approaches are adaptive schemes (Tao and Kokotovi¢, 1994; Wang et al., 2004; Zhou et al., 2006; Ibrir et al., 2007), fuzzy systems (Kim et al., 1994; Oh and Park, 1998; Lewis et al., 1999), neural networks (Šelmić and Lewis, 2000; Tsai and Chuang, 2004; Zhang and Ge, 2007) and variable structure methods (Corradini and Orlando, 2002; Shyu et al., 2005). Many of these works (Tao and Kokotović, 1994; Kim et al., 1994; Oh and Park, 1998; Šelmić and Lewis, 2000; Tsai and Chuang, 2004; Zhou et al., 2006) use an inverse dead-zone to compensate the negative effects of the dead-zone nonlinearity, even though this approach leads to a discontinuous control law and requires instantaneous switching, which in practice cannot be accomplished with mechanical actuators. An alternative scheme, without using the dead-zone inverse, was originally proposed by Lewis et al. (1999) and also adopted by Wang et al. (2004). In both works, the dead-zone is treated as a combination of a linear and a saturation function. This approach was further extended by Ibrir et al. (2007) and by Zhang and Ge (2007), in order to accommodate non-symmetric dead-zones.

This paper presents an adaptive fuzzy controller for nonlinear systems subject to dead-zone input. An unknown and non-symmetric dead-band is assumed. The dead-zone nonlinearity is also considered as a combination of linear and saturation functions, but an adaptive fuzzy inference system is introduced, as universal function approximator, to cope with the unknown saturation function. Based on a Lyapunov-like analysis using Barbalat's lemma, the convergence properties of the closed-loop system are analytically proven. To show the applicability of the proposed control scheme, an electro-hydraulic system is chosen as an illustrative example. Simulation results of the adopted mechanical system are also presented to demonstrate the control system efficacy.

Problem Statement

Consider a class of nth-order nonlinear and non-autonomous systems:

$$x^{(n)} = f(\mathbf{x},t) + b\upsilon \tag{1}$$

where the scalar variable x is the output if interest, $x^{(n)}$ is the nth derivative of x with respect to time t, $\mathbf{x} = \left[x, \dot{x}, ..., x^{(n-1)} \right]$ is the system state vector, $f: \mathsf{R}^n \to \mathsf{R}$ is a nonlinear function, b represents a constant input gain and υ states for the output of a dead-zone function, as shown in Fig. 1.
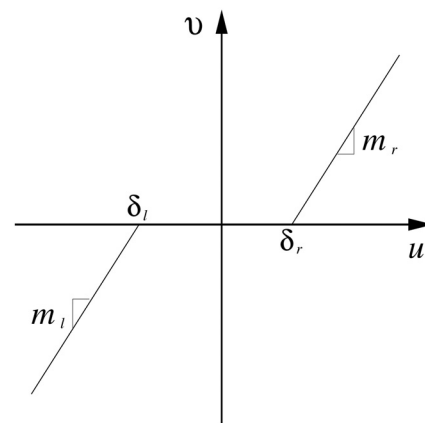
Figure 1. Dead-zone nonlinearity.

The dead-zone nonlinearity presented in Fig. 1 can be mathematically described by:

$$\upsilon = \begin{cases} m_l\left(u - \delta_l\right) & \text{if } u \le \delta_l \\ 0 & \text{if } \delta_l \le u \le \delta_r \\ m_l\left(u - \delta_1\right) & \text{if } u \le \delta_r \end{cases} \tag{2}$$

where u represents the controller output variable.

In respect of the dead-zone model presented in Eq. (2), the following physically motivated assumptions can be made:

Assumption 1: *The dead-zone output υ is not available to be measured.*

Assumption 2: *The slopes in both sides of the dead-zone are equal and positive, i.e., $m_l = m_r = m > 0$.*

Assumption 3: *The dead-band parameters δ_l and δ_r are unknown, but bounded and with known signs, i.e., $\delta_{l\,min} \le \delta_l \le \delta_{l\,max} < 0$ and $0 < \delta_{r\,min} \le \delta_r \le \delta_{r\,max}$.*

In this way, Eq. (2) can be rewritten in a more appropriate form (Lewis et al., 1999; Wang et al., 2004):

$$\upsilon = m\left[u - d\left(u\right)\right] \tag{3}$$

where $d(u)$ can be obtained from Eq. (2) and Eq. (3) as:

$$d\left(u\right) = \begin{cases} \delta_l & \text{if } u \le \delta_l \\ u & \text{if } \delta_l \le u \le \delta_r \\ \delta_r & \text{if } u \le \delta_r \end{cases} \tag{4}$$

Remark 1: Considering Assumption 3 and Eq. (4), it can be easily verified that $d(u)$ is bounded: $|d(u)| \le \delta$, that is, $\delta = max\{-\delta_{l\,min}, \delta_{r\,max}\}$.

Adaptive Fuzzy Dead-Zone Compensation

As demonstrated by Bessa and Barrêto (2009), adaptive fuzzy algorithms can be properly combined with nonlinear controllers in order to improve the trajectory tracking of uncertain nonlinear systems. It has also been shown that such strategies are suitable for a variety of applications ranging from remotely operated underwater vehicles (Bessa et al., 2008) to chaos control (Bessa et al., 2009a).

The proposed control problem is to ensure that, even in the presence of an unknown dead-zone input, the state vector $\mathbf{x}$ will follow a desired trajectory $\mathbf{x}_d = \left[x, \dot{x}_d, ..., x_d^{(n-1)}\right]$ in the state space.

Regarding the development of the control law, the following assumptions should also be made:

Assumption 4: *The state vector $\mathbf{x}$ is available.*

Assumption 5: *The desired trajectory $\mathbf{x}_d$ is once differentiable in time. Furthermore, every element of vector $\mathbf{x}_d$, as well as $x_d^{(n)}$, is available and with known bounds.*

Let $\tilde{x} = x - x_d$ be defined as the tracking error in the variable x, and

$$\tilde{\mathbf{x}} = \mathbf{x} - \mathbf{x}_d = \left[\tilde{x}, \dot{\tilde{x}}, ..., \tilde{x}^{(n-1)}\right]$$

as the tracking error vector. Now, consider a combined tracking error measure:

$$\varepsilon = \mathbf{c}^T \tilde{\mathbf{x}} \tag{5}$$

where $\mathbf{c} = [c_{n-1}\lambda^{n-1}, ..., c_1\lambda, c_0]$, λ is positive constant and c_i states for binomial coefficients, i.e.,

$$c_i = \binom{n-1}{i} = \frac{(n-1)!}{(n-i-1)!i!}, \quad i = 0,\,1,\,...,\,n-1 \tag{6}$$

which makes $c_{n-1}\lambda^{n-1} + c_{n-2}\lambda^{n-2}s + ... + c_1\lambda s^{n-2} + c_0 s^{n-1}$ a Hurwitz polynomial in s.

From Eq. (6), it can be easily verified that $c_0 = 1$, for $\forall n \ge 1$. Thus, for notational convenience, the time derivative of ε will be written in the following form:

$$\dot{\varepsilon} = \mathbf{c}^T \dot{\tilde{\mathbf{x}}} = \tilde{x}^{(n)} + \overline{\mathbf{c}}^T \mathbf{x} \tag{7}$$

where $\overline{\mathbf{c}} = \left[0, c_{n-1}\lambda^{n-1}, ..., c_1\lambda\right]$.

Based on Assumptions 4 and 5, the following control law can be proposed:

$$u = \frac{1}{bm}\left(-f + x_d^{(n)} - \overline{\mathbf{c}}^T\tilde{\mathbf{x}} - \kappa\varepsilon\right) + \hat{d}\left(\hat{u}\right) \tag{8}$$

where κ is a strictly positive constant and $\hat{d}\left(\hat{u}\right)$ an estimate of $d(u)$ that will be computed in terms of the equivalent control $\hat{u} = (bm)^{-1}\left(-f + x_d^{(n)} - \overline{\mathbf{c}}^T\tilde{x}\right)$ by an adaptive fuzzy algorithm.

The adopted fuzzy inference system was the zero order TSK (Takagi – Sugeno – Kang), with the rth rule stated in a linguistic manner as follows:

If $\hat{u}$ is $\hat{U}_r$ then $\hat{d}_r = \hat{D}_r$; $r = 1, 2, ..., N$

where $\hat{U}_r$ are fuzzy sets whose membership functions could be properly chosen, and $\hat{D}_r$ is the output value of each one of the N fuzzy rules.

Considering that each rule defines a numerical value as output $\hat{D}_r$, the final output $\hat{d}$ can be computed by a weighted average:

$$\hat{d}\left(\hat{u}\right) = \frac{\sum_{r=1}^{N} w_r \cdot \hat{D}_r}{\sum_{r=1}^{N} w_r} \tag{9}$$

or similarly,

$$\hat{d}\left(\hat{u}\right) = \hat{\mathbf{D}}^T \mathbf{\Psi}\left(\hat{u}\right) \tag{10}$$

where $\hat{\mathbf{D}} = \left[\hat{D}_1, \hat{D}_2, ..., \hat{D}_N\right]$ is the vector containing the assigned value $\hat{D}_r$ to each rule r, $\mathbf{\psi}(\hat{u}) = [\psi_1(\hat{u}), \psi_2(\hat{u}), ..., \psi_N(\hat{u})]$ is a vector with components $\psi_r(\hat{u}) = w_r / \sum_{r=1}^{N} w_r$ and w_r is the firing strength of each rule.

To ensure the best possible estimate $\hat{d}\left(\hat{u}\right)$, the vector of adjustable parameters can be automatically updated by the following adaptation law:

$$\dot{\hat{\mathbf{D}}} = -\varphi\varepsilon\mathbf{\Psi}\left(\hat{u}\right) \tag{11}$$

where φ is a strictly positive constant related to the adaptation rate.

The boundedness and convergence properties of the closed-loop system are established in the following theorem.

Theorem 1: *Consider the nonlinear system* (1) *subject to the dead-zone* (2) *and Assumptions* $1 - 5$. *Then, the controller defined by* (8), (10) *and* (11) *ensures the boundedness of all closed-loop signals*

and the exponential convergence of the tracking error, i.e., $\tilde{\mathbf{x}} \to \mathbf{0}$ *as* $t \to \infty$.

Proof: Let a positive definite Lyapunov function candidate V be defined as

$$V(t) = \frac{1}{2}\varepsilon^2 + \frac{bm}{2\varphi}\boldsymbol{\Delta}^T\boldsymbol{\Delta} \tag{12}$$

where $\boldsymbol{\Delta} = \hat{\mathbf{D}} - \hat{\mathbf{D}}^*$ and $\hat{\mathbf{D}}^*$ is the optimal parameter vector, associated to the optimal estimate $\hat{d}^*(\hat{u}) = d(u)$.

Thus, the time derivative of V is

$$\begin{aligned}
\dot{V}(t) &= \varepsilon\dot{\varepsilon} + bm\varphi^{-1}\boldsymbol{\Delta}^T\dot{\boldsymbol{\Delta}} \\
&= \left(\tilde{x}^{(n)} + \overline{\mathbf{c}}^T\tilde{\mathbf{x}}\right)\varepsilon + bm\varphi^{-1}\boldsymbol{\Delta}^T\dot{\boldsymbol{\Delta}} \\
&= \left(x^{(n)} - x_d^{(n)} + \overline{\mathbf{c}}^T\tilde{\mathbf{x}}\right)\varepsilon + bm\varphi^{-1}\boldsymbol{\Delta}^T\dot{\boldsymbol{\Delta}} \\
&= \left(f + bmu - bmd(u) - x_d^{(n)} + \overline{\mathbf{c}}^T\tilde{\mathbf{x}}\right)\varepsilon + bm\varphi^{-1}\boldsymbol{\Delta}^T\dot{\boldsymbol{\Delta}}
\end{aligned}$$

Applying the proposed control law (8) and noting that $\dot{\boldsymbol{\Delta}} = \dot{\hat{\mathbf{D}}}$, then

$$\begin{aligned}
\dot{V}(t) &= \left[bm(\hat{d} - d) - \kappa\varepsilon\right]\varepsilon + bm\varphi^{-1}\boldsymbol{\Delta}^T\dot{\hat{\mathbf{D}}} \\
&= \left[bm(\hat{\mathbf{D}} - \hat{\mathbf{D}}^*)^T\boldsymbol{\Psi}(\hat{u}) - \kappa\varepsilon\right]\varepsilon + bm\varphi^{-1}\boldsymbol{\Delta}^T\dot{\hat{\mathbf{D}}} \\
&= \left[bm\boldsymbol{\Delta}^T\boldsymbol{\Psi}(\hat{u}) - \kappa\varepsilon\right]\varepsilon + bm\varphi^{-1}\boldsymbol{\Delta}^T\dot{\hat{\mathbf{D}}} \\
&= -\kappa\varepsilon^2 + bm\varphi^{-1}\boldsymbol{\Delta}^T\left[\dot{\hat{\mathbf{D}}} + \varphi\varepsilon\boldsymbol{\Psi}(\hat{u})\right]
\end{aligned}$$

Furthermore, defining $\dot{\hat{\mathbf{D}}}$ according to (11), $\dot{V}(t)$ becomes

$$\dot{V}(t) = -\kappa\varepsilon^2 \tag{13}$$

which implies $V(t) \le V(0)$ and that ε and $\boldsymbol{\Delta}$ are bounded. Considering that $\varepsilon = \mathbf{c}^T\tilde{\mathbf{x}}$, it can be verified that $\tilde{\mathbf{x}}$ is also bounded. Hence, Eq. (7) and Assumption 5 imply that $\dot{\varepsilon}$ is also bounded.

Now, in order to evoke Barbalat's lemma the uniform continuity of $\dot{V}$ must be demonstrated. According to Slotine and Li (1991), a sufficient condition for a differentiable function to be uniformly continuous is that its derivative be bounded. On this basis, the time derivative of $\dot{V}$ should be analyzed:

$$\ddot{V}(t) = -2\kappa\varepsilon\dot{\varepsilon} \tag{14}$$

Equation (14) implies that $\dot{V}(t)$ is also bounded and that, from Barbalat's lemma, $\varepsilon \to 0$ as $t \to \infty$. From the definition of limit, it means that for every $\xi > 0$ there is a corresponding number τ such that $|\varepsilon| < \xi$ whenever $t > \tau$. According to Eq. (5) and considering that $|\varepsilon| < \xi$ may be rewritten as $-\xi < \varepsilon < \xi$, one has

$$\begin{aligned}
-\xi &< c_0\tilde{x}^{(n-1)} + c_1\lambda\tilde{x}^{(n-2)} + \ldots + \\
&+ c_{n-2}\lambda^{n-2}\dot{\tilde{x}} + c_{n-1}\lambda^{n-1}\tilde{x} < \xi
\end{aligned} \tag{15}$$

Multiplying Eq. (15) by $e^{\lambda t}$ and noting that

$$\begin{aligned}
\frac{d^{n-1}}{dt^{n-1}}\left(\tilde{x}e^{\lambda t}\right) &= \left(c_0\tilde{x}^{(n-1)} + c_1\lambda\tilde{x}^{(n-2)} + \ldots + \right. \\
&\left. + c_{n-2}\lambda^{n-2}\dot{\tilde{x}} + c_{n-1}\lambda^{n-1}\tilde{x}\right)e^{\lambda t}
\end{aligned}$$

one has

$$-\xi e^{\lambda t} < \frac{d^{n-1}}{dt^{n-1}}\left(\tilde{x}e^{\lambda t}\right) < \xi e^{\lambda t} \tag{16}$$

Thus, integrating Eq. (16) $n-1$ times between 0 and t gives

$$\begin{aligned}
&-\frac{\xi}{\lambda^{n-1}}e^{\lambda t} + \left(\left|\frac{d^{n-2}}{dt^{n-2}}\left(\tilde{x}e^{\lambda t}\right)\right|_{t=0} + \frac{\xi}{\lambda}\right)\frac{t^{n-2}}{(n-2)!} + \\
&+ \ldots + \left(\left|(\tilde{x}(0))\right| + \frac{\xi}{\lambda^{n-1}}\right) \le \tilde{x}\varepsilon^{\lambda t} \le \frac{\xi}{\lambda^{n-1}}e^{\lambda t} + \\
&+ \left(\left|\frac{d^{n-2}}{dt^{n-2}}\left(\tilde{x}e^{\lambda t}\right)\right|_{t=0} - \frac{\xi}{\lambda}\right)\frac{t^{n-2}}{(n-2)!} + \ldots + \\
&+ \left(\left|(\tilde{x}(0))\right| - \frac{\xi}{\lambda^{n-1}}\right)
\end{aligned} \tag{17}$$

Furthermore, dividing (17) by $e^{\lambda t}$, it can be easily verified that the values of $\tilde{x}$ can be made arbitrarily close to 0 (within a distance ξ) by taking t sufficiently large (larger than τ), i.e., $\tilde{x} \to 0$ as $t \to \infty$. Now, considering the $(n-2)^{\text{th}}$ integral of (16), dividing again by $e^{\lambda t}$ and considering that $\tilde{x}$ converges to zero, it follows that $\dot{\tilde{x}} \to 0$ as $t \to \infty$. The same procedure can be successively repeated until the convergence of each component of the tracking error vector is achieved: $\tilde{\mathbf{x}} \to \mathbf{0}$ as $t \to \infty$.

As previously reported in the literature, the dead-zone nonlinearity is frequently encountered in many industrial actuators, especially those containing hydraulic (Knohl and Unbehauen, 2000; Bessa et al., 2009b) or pneumatic (Guenther and Perondi, 2006) valves. Considering that instantaneous switching cannot be accomplished with mechanical actuators and the proposed adaptive fuzzy approach does not require a dead-zone inverse, this scheme is perfectly suitable for hydraulic or pneumatic actuators. On this basis, an application of the proposed adaptive fuzzy scheme to an electro-hydraulic servo-system is introduced in the next section to illustrate the controller design method.

Electro-Hydraulic System

Electro-hydraulic actuators play an essential role in several branches of industrial activity and are frequently the most suitable choice for systems that require large forces at high speeds. Their application scope ranges from robotic manipulators to aerospace systems. Another great advantage of hydraulic systems is the ability to keep up the load capacity, which in the case of electric actuators is limited due to excessive heat generation.

However, the dynamic behavior of electro-hydraulic systems is highly nonlinear, which in fact makes the design of controllers for such systems a challenge for the conventional and well established linear control methodologies. In addition to the common nonlinearities that originate from the compressibility of the hydraulic fluid and valve flow-pressure properties, most electro-hydraulic systems are also subjected to hard nonlinearities such as dead-zone due to valve spool overlap.

In order to design the adaptive fuzzy controller, a mathematical model that represents the hydraulic system dynamics is needed.

Dynamic models for such systems are well documented in the literature (Merritt, 1967; Walters, 1967).

The electro-hydraulic system considered in this work consists of a four-way proportional valve, a hydraulic cylinder and variable load force. The variable load force is represented by a mass-spring-damper system. The schematic diagram of the system under study is presented in Fig. 2.
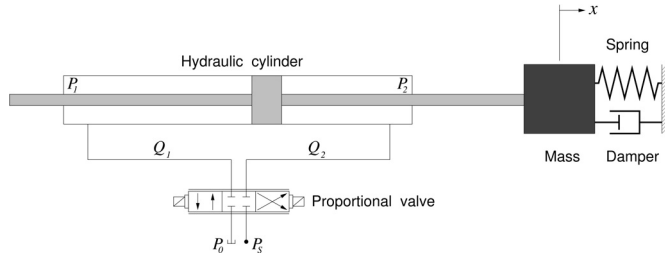

Figure 2. Schematic diagram of the electro-hydraulic servo-system.

The balance of forces on the piston leads to the following equation of motion:

$$F_g = A_1 P_1 - A_2 P_2 = M_t \ddot{x} + B_t \dot{x} + K_s x \qquad (18)$$

where F_g is the force generated by the piston, P_1 and P_2 are the pressures at each side of cylinder chamber, A_1 and A_2 are the ram areas of the two chambers, M_t is the total mass of piston and load referred to piston, B_t is the viscous damping coefficient of piston and load, K_s is the load spring constant, and x is the piston displacement.

Defining the pressure drop across the load as $P_L = P_1 - P_2$ and considering that for a symmetrical cylinder $A_p = A_1 = A_2$, Eq. (18) can be rewritten as

$$M_t \ddot{x} + B_t \dot{x} + K_s x = A_p P_L \qquad (19)$$

Applying continuity equation to the fluid flow, the following equation is obtained:

$$Q_L = A_p \dot{x} + C_{tp} + \frac{V_t}{4\beta_e} \dot{P}_L \qquad (20)$$

where $Q_L = (Q_1 + Q_2)/2$ is the load flow, with Q_1 and Q_2 as the flow in each chamber, C_{tp} the total leakage coefficient of piston, V_t the total volume under compression in both chambers and β_e the effective bulk modulus.

Considering that the return line pressure is usually much smaller than the other pressures involved ($P_0 \approx 0$) and assuming a closed center spool valve with matched and symmetrical orifices, the relationship between load pressure P_L and load flow Q_L can be described as follows

$$Q_L = C_d \omega \bar{x}_{sp} \left[\frac{1}{\rho} \left(P_s - \text{sgn}(\bar{x}_{sp}) P_L \right) \right]^{\frac{1}{2}} \qquad (21)$$

where C_d is the discharge coefficient, ω the valve orifice area gradient, $\bar{x}_{sp}$ the effective spool displacement from neutral, ρ the hydraulic fluid density, P_s the supply pressure and sgn(.) is defined by

$$\text{sgn}(z) = \begin{cases} -1 & \text{if } z < 0 \\ 0 & \text{if } z = 0 \\ 1 & \text{if } z > 0 \end{cases} \qquad (22)$$

Assuming that the valve dynamics is fast enough to be neglected, the valve spool displacement can be considered as proportional to the control voltage. For closed center valves, or even in the case of the so-called critical valves, the spool presents some overlap. This overlap prevents from leakage losses, but leads to a dead-zone nonlinearity within the control voltage.

Considering the voltage as control input u and the valve gain as dead-zone slope m, the valve nonlinearity can be mathematically described by Eqs. (3) and (4), with parameters δ_l and δ_r depending on the size of the overlap region.

Now, combining Eqs. (3), (4), (19), (20) and (21) leads to a third-order differential equation that represents the dynamic behavior of the electro-hydraulic system:

$$\dddot{x} = -\mathbf{a}^T \mathbf{x} + bmu - bmd(u) \qquad (23)$$

where $\mathbf{x} = [x, \dot{x}, \ddot{x}]$ is the state vector with an associated coefficient vector $\mathbf{a} = [a_0, a_1, a_2]$ defined according to

$$a_0 = \frac{4\beta_e C_{tp} K_s}{V_t M_t}; \qquad a_1 = \frac{K_s}{M_t} + \frac{4\beta_e A_p^2}{V_t M_t} + \frac{4\beta_e C_{tp} B_t}{V_t M_t};$$

$$a_2 = \frac{B_t}{M_t} + \frac{4\beta_e C_{tp}}{V_t}$$

and

$$b = \frac{4\beta_e A_p C_d \omega}{V_t M_t} \left\{ \frac{1}{\rho} \left[P_s - \frac{\text{sgn}(u)(M_t \ddot{x} + B_t \dot{x} + K_s x)}{A_p} \right] \right\}^{\frac{1}{2}}$$

Although b states for a constant gain in Eq. (1), it will be shown in the next section, by means of numerical simulations, that the proposed control scheme can also deal with a variable again. In this way, based on Eqs. (8), (10) and (11) and considering $\varepsilon = \dddot{\tilde{x}} + 2\lambda \ddot{\tilde{x}} + \lambda^2 \tilde{x}$, the following adaptive fuzzy controller can be proposed to deal with the dynamic model presented in Eq. (23).

$$u = \frac{1}{bm} \left(\mathbf{a}^T \mathbf{x} + \dddot{x}_d - 2\lambda \ddot{\tilde{x}} - \lambda^2 \dot{\tilde{x}} - \kappa\varepsilon \right) + \hat{d}(\hat{u}) \qquad (24)$$

Simulation Results

The simulation studies were performed with a numerical implementation, in C, with sampling rates of 400 Hz for control system and 800 Hz for dynamic model. The differential equations of the dynamic model were numerically solved with the fourth order Runge-Kutta method.

Regarding the fuzzy inference system, the number of fuzzy rules and the type of the membership functions, as well as how they are distributed over the input space, could be heuristically defined to accommodate designer's experience and experimental knowledge. On this basis, assuming no previous knowledge about d, seven rules (with seven related fuzzy sets $\hat{U}_r$) were arbitrarily chosen and placed within the input space $\hat{u}$. Triangular and trapezoidal membership functions were adopted for $\hat{U}_r$, with the central values defined as C={-5.0; -1.0; -0.5; 0.0; 0.5; 1.0; 5.0} × 10^{-1} (see Fig. 3). It should be emphasized that the input space could be partitioned and

represented in many other ways, and that the system designer may test each one of them in order to improve the output value $\hat{d}(\hat{u})$.

Concerning the vector of adjustable parameters, it was initialized with zero values, $\hat{\mathbf{D}} = \mathbf{0}$, and updated at each iteration step according to the adaptation law presented in Eq. (11).
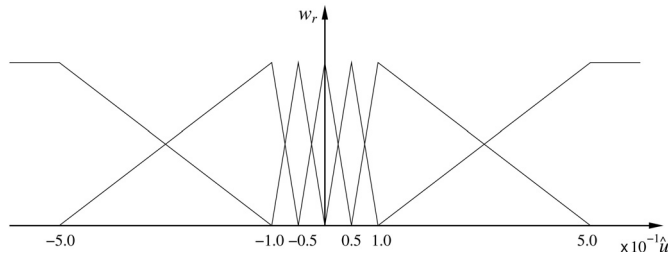

Figure 3. Adopted fuzzy membership functions.

In order to evaluate the control system performance, four different numerical simulations were performed. The obtained results were presented from Figs. 4 to 7.

In the first case, the adopted parameters for the electro-hydraulic systems were P_s = 7 MPa, ρ = 850 kg/m³, C_d = 0.6, ω = 2.5 × 10⁻² m, A_p = 3 × 10⁻⁴ m², C_{tp} = 2 × 10⁻¹² m³/(s Pa), β_e = 700 MPa, V_t= 6 × 10⁻⁵ m³, M_t = 250 kg, B_t = 100 Ns/m, K_s = 75 N/m, m = 4 × 10⁻⁶ m/V, δ_l = -1.1 V and δ_r = 0.9 V. The parameters of the controller were λ = 8, κ = 1 and φ = 0.5. Figure 4 shows the results obtained with x_d = 0.5 sin(0.1t) m.

As observed in Fig. 4, the proposed control law is able to provide trajectory tracking, Fig. 4(a), with a small associated error, Fig. 4(c). Figure 4(d) shows the ability of the adaptive fuzzy scheme to recognize and previously compensate for dead-band characteristics.

In the second simulation study, variations of ±20% in the supply pressure, P_s = 7[1 + 0.2 sin(x)] MPa, were also taken into account. Such variations are very common in real plants but it's very important to emphasize that, to demonstrate the capacity of the control scheme to deal with parametric uncertainties, the supply pressure was considered as 7 MPa for the controller. The other model and controller parameters, as well as the desired trajectory, were defined as before. It can be easily verified in Figs. 5(a) and 5(c) that the adopted controller provides trajectory tracking and is almost indifferent to variations in the supply pressure.
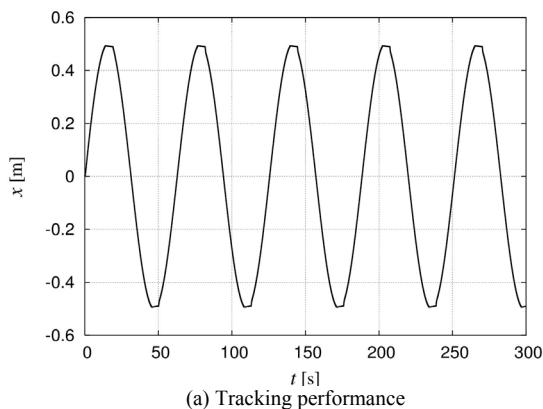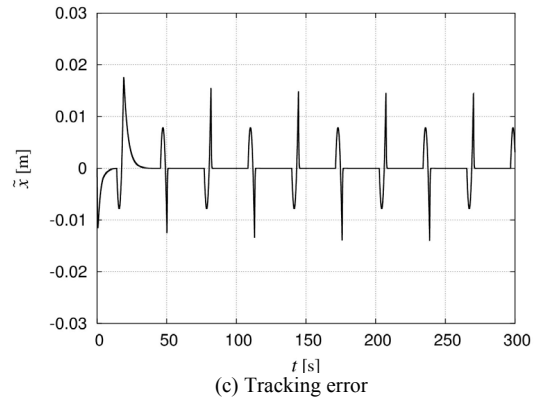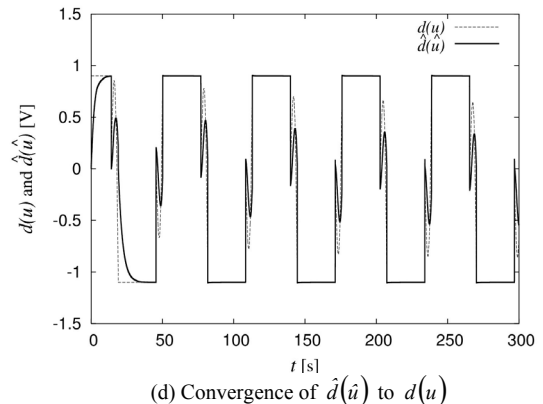

(a) Tracking performance

Figure 4. Tracking of x_d = 0.5 sin(0.1t) m with constant supply pressure.
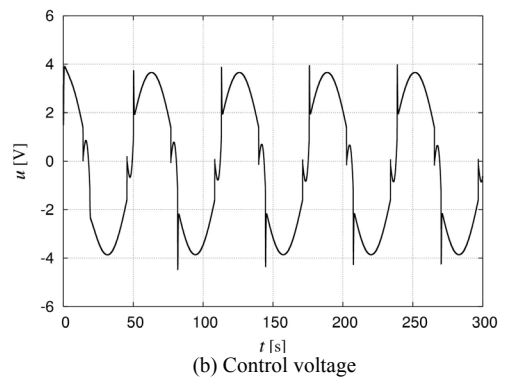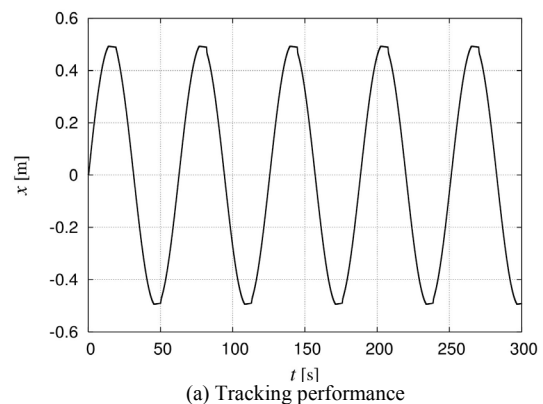

(b) Control voltage

(c) Tracking error

(d) Convergence of $\hat{d}(\hat{u})$ to $d(\hat{u})$

Figure 4. (Continued).

(a) Tracking performance

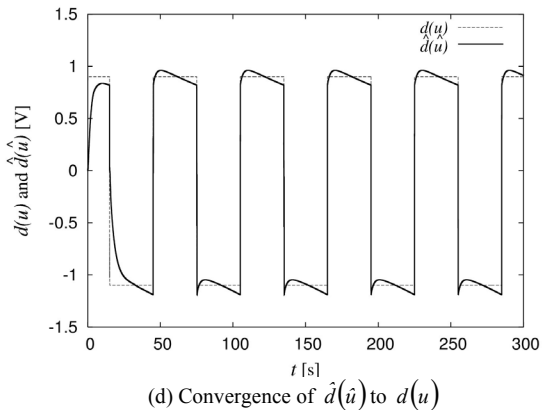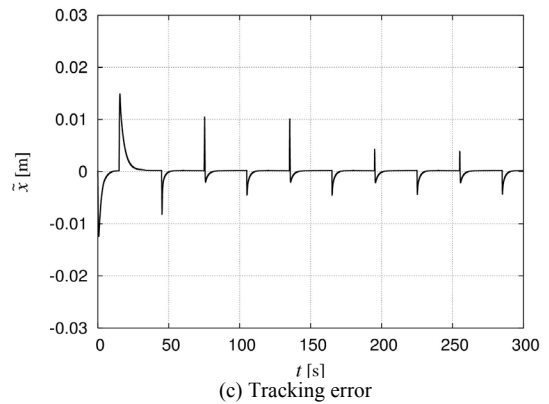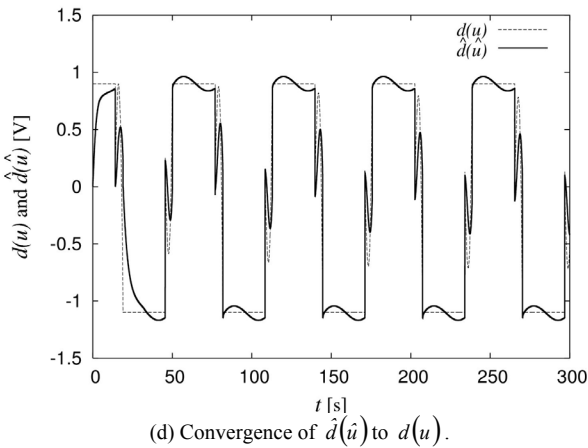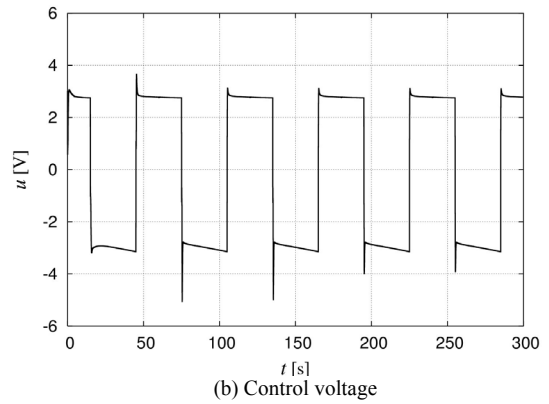**Figure 5. Tracking of x_d = 0.5 sin(0.1t) m with variable supply pressure.**

(b) Control voltage

(a) Tracking performance

(c) Tracking error

(b) Control voltage
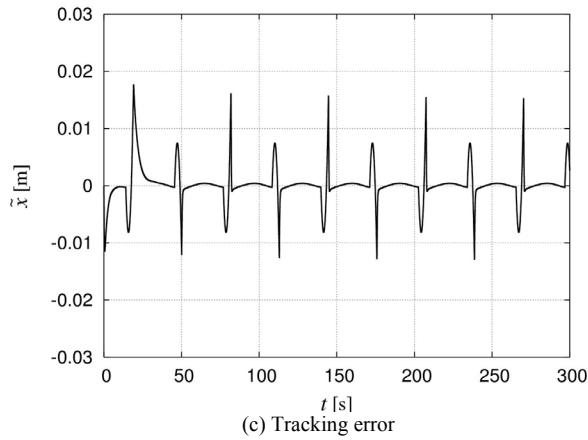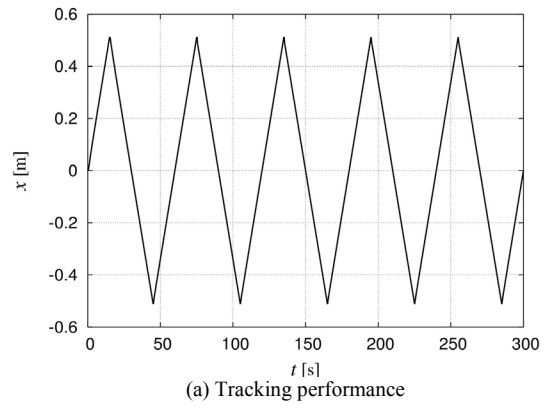

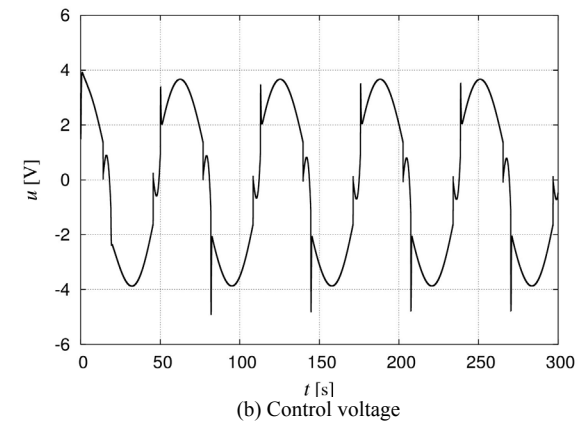(d) Convergence of $\hat{d}(\hat{u})$ to $d(u)$.

Figure 5. (Continued).

In the last two simulations, piece-wise differentiable functions were chosen as desired trajectories. The model and controller parameters were defined as in the second simulation study. The obtained results are shown in Figs. 6 and 7. As observed in Figs. 6 and 7, the proposed control law is able to provide trajectory tracking and stabilization even with non-smooth trajectories.

(c) Tracking error

(d) Convergence of $\hat{d}(\hat{u})$ to $d(u)$

Figure 6. Tracking of a triangle wave trajectory with variable supply pressure.

(a) Tracking performance
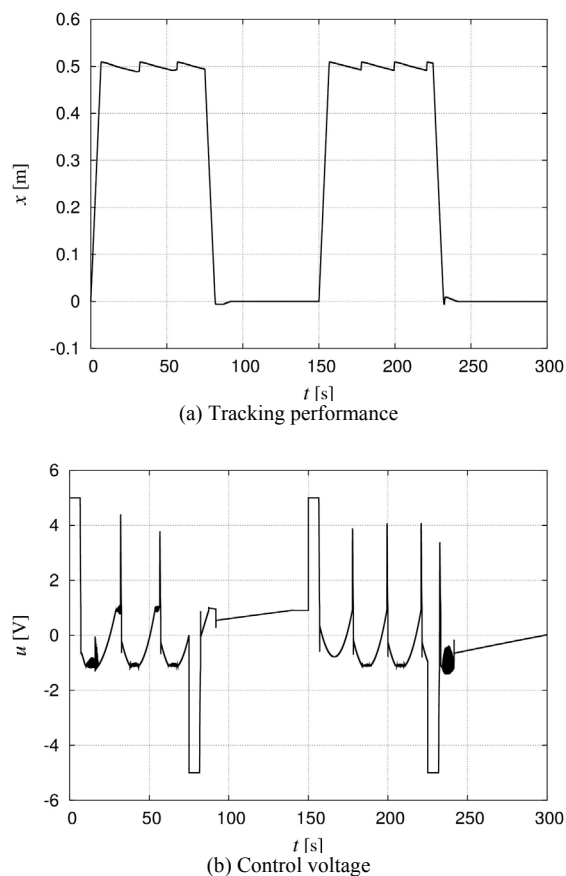

(b) Control voltage

Figure 7. Tracking of a square wave trajectory with variable supply pressure.

Concluding Remarks

The present work addressed the problem of controlling nonlinear systems subject to dead-zone input. An adaptive fuzzy scheme was proposed and combined with a state feedback controller to deal with the trajectory tracking problem. The boundedness and convergence properties of the closed-loop signals were analytically proven using Lyapunov stability theory and Barbalat's lemma. The control system performance was also confirmed by means of numerical simulations with an application to an electro-hydraulic system. The adaptive algorithm could automatically recognize the dead-zone nonlinearity and previously compensate its undesirable effects, even in the presence of parametric uncertainties and considering non-smooth trajectories. Some advantages of the adopted approach could be pointed out: (i) it does not require a dead-zone inverse; (ii) it is able to cope with dead-zones having unknown characteristics, since the fuzzy inference system represents an universal function approximator; (iii) the proposed adaptive fuzzy algorithm is actually independent of the underlying control architecture and could be easily combined with many other control methodologies. The main drawback is the lack of robustness against uncertainties. As a suggestion for future works, one can embed the adaptive fuzzy algorithm within a sliding mode controller in order to confer robustness to both modeling inaccuracies and external disturbances.

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