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A Comparative Study of the Application of Differential Evolution and Simulated Annealing in Radiative Transfer Problems

The radiative transfer phenomenon is modeled by an integro-differential equation known as Boltzmann equation. This equation describes mathematically the interaction of the radiation with the participating medium, i.e., a medium that may absorb, scatter and emit radiation. In this sense, this work presents a study regarding the estimation of radiative properties in a one-dimensional participating medium by using two optimization heuristic methods, namely Simulated Annealing and Differential Evolution. First, a review of these two optimization techniques is presented. The direct radiative transfer problem solution, which is required for both optimization techniques, is obtained by using the Collocation Method. Finally, case-studies are presented aiming at illustrating the efficiency of these methodologies in the treatment of inverse radiative transfer problems.
Keywords: *inverse problem, radiative transfer, Differential Evolution, Simulated Annealing*

Introduction

The problem of parameter identification characterizes a typical inverse problem in engineering. It arises from the difficulty in building theoretical models that are able to represent satisfactorily physical phenomena under real operating conditions. Considering the possibility of using more complex models along with the information provided by experimental data, the parameters obtained through an inverse problem approach may then be used to simulate the behavior of the system for different operation conditions. Traditionally, this kind of problem has been treated by using either classical or deterministic optimization techniques (Baltes et al., 1994; Cazzador and Lubenova, 1995). In recent years, however, the use of non-deterministic techniques or the coupling of these techniques with classical approaches, thus forming a hybrid methodology, became very popular due to the simplicity and robustness of evolutionary techniques (Wang et al., 2001; Silva Neto and Soeiro, 2002; Silva Neto and Soeiro, 2003; Silva Neto and Silva Neto, 2003).

The increasing interest on inverse problems (IP) is due to the large number of practical applications in scientific and technological areas such as tomography (Kim and Charette, 2007), environmental sciences (Hanan, 2001) and parameter estimation (Souza et al., 2007; Alvarez Acevedo et al., 2010), to mention only a few.

When the parameters of the model (radiative properties) are known, it is possible to solve the so-called direct problem. Recently, the application of Monte Carlo Methods and new variations of the Discrete Ordinate Methods, as proposed by Chalhoub et al. (2007a), have opened new possibilities in this field. A review of other methods proposed in the literature can be found in Hansen and Travis (1974) and Lenoble (1977).

The inverse problem consists in the determination of radiative parameters through the use of experimental data for minimizing the residual between experimental and calculated values of the radiation intensity. The solution of inverse radiative transfer problems has been obtained by using different methodologies, namely deterministic, stochastic and hybrid methods. As examples of techniques developed for dealing with inverse radiative transfer problems, the following methods can be cited: Levenberg-Marquardt method (Silva Neto and Moura Neto, 2005); Simulated Annealing (Silva Neto and Soeiro, 2002; Souza et al., 2007); Genetic Algorithms (Silva Neto and Soeiro, 2002; Souza et al., 2007); Artificial Neural Networks (Soeiro et al., 2004; Oliveira et al., 2010); Ant Colony Optimization (Souto et al., 2005); Particle Swarm Optimization (Becceneri et al., 2006); Generalized Extremal Optimization (Souza et al., 2007); Interior Points Method (Silva Neto and Silva Neto, 2003); Particle Collision Algorithm (Knupp et al., 2007); Monte Carlo Method and Three Variations of the Discrete Ordinates Method (Chalhoub et al., 2007a); Artificial Neural Networks and Monte Carlo Method (Chalhoub et al., 2007b); Epidemic Genetic Algorithm and the Generalized Extremal Optimization Algorithm (Cuco et al., 2009); Epidemic Genetic Algorithm and Simulated Annealing Algorithm (Galski et al., 2009); Hybrid Approach with Artificial Neural Networks, Levenberg-Marquardt and Simulated Annealing Methods (Lugon et al., 2009).

The main goal of this paper is to compare the solutions of inverse radiative transfer problems by using two evolutionary techniques: Simulated Annealing and Differential Evolution. These techniques are used as optimization tools to deal with the inverse problem. The Collocation Method is used for solving the corresponding direct problem.

This work presents a review about Heuristic Optimization Methods focusing on the algorithms of Simulated Annealing and Differential Evolution. The direct radiative transfer problem and the formulation of the associated inverse problem are presented. Afterwards, the methodology used in this work is described and applied to four illustrative examples. Finally, the conclusions and suggestions for future work are outlined.

Nomenclature

a	= Lower limit of independent variable
$A1$	= Intensity of the isotropic external source of radiation incident at $\tau = 0$
$A2$	= Intensity of the isotropic external source of radiation incident at $\tau = \tau_o$
b	= Upper limit of independent variable
CR	= Crossover constant
$cputime$	= Computational time
D	= Perturbation rate
DE	= Differential Evolution
g	= Auxiliary function
GA	= Genetic Algorithm
I	= Intensity of the radiation field
IP	= Inverse Problem

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K_b = *Boltzmann constant*
m = *Number of dependent variables*
n = *Number of experimental data points*
N = *Population size*
NF = *Number of function evaluations*
Nt = *Number of temperature levels*
$Ntemp$ = *Cooling number*
p = *Parameter*
P = *Boltzmann probability*
Q = *Objective function*
r = *Random number*
Res = *Difference between experimental and calculated values of the exit radiation intensities*
S = *Auxiliary function*
SA = *Simulated Annealing*
T = *Temperature*
x = *Dependent variable*
y = *Independent variable*

Greek Symbols

ΔE = *Energy variation*
τ_o = *Optical thickness*
τ = *Optical variable*
μ = *Cosine of the polar angle*
ω = *Albedo for a single scattering*
σ = *Standard deviation of the experimental error*
Λ = *Hemispherical reflectivity*
Γ = *Transmissivity*

Subscripts

i, j, k Initial
f Final
cal Calculated
exp Experimental

Heuristic Methods

It is well known that the solution of optimization problems through a deterministic approach requires the objective and constraint functions as well as their derivatives to be continuous. Besides, the optimum solution obtained is dependent on the initial configuration provided. Deterministic methods invest the whole computational effort in a single point (Vanderplaats, 1999; Edgar et al., 2001) that evolves along successive iterations. On the other hand, non-deterministic techniques are based on heuristics that are related to natural processes and genetics of populations or, alternatively, they are formulated according to structural approaches. These methods do not make use of gradient information for updating the search direction. However, they demand a high number of evaluations of the objective function (Coelho, 2003). In general terms this aspect can make the procedure time-consuming.

In the following, a concise presentation of the main characteristics of the two evolutive optimization methods used in the present contribution will be made.

Simulated Annealing

The Simulated Annealing Algorithm (SA) belongs to the same class of methods such as Neural Networks and Genetic Algorithms in the sense that they all seek to mimic nature skills in providing optimal solutions. The method is inspired in the thermodynamic process of cooling (annealing) of molten metals to attain the lowest free energy state (Kirkpatrick et al., 1983). Metropolis et al. (1953) introduced a simple numerical method for representing the state of a set of atoms in equilibrium at a given temperature. Consequently, this optimization method uses an analogy with the annealing process from metallurgy. Annealing means that the metal is heated to a high temperature, bringing the atoms to a higher level of internal and kinetic energy. If the metal is cooled rapidly, the microstructure may be locked into a random unstable state. However, if the temperature is cooled slowly the atoms tend to fall into patterns that are relatively stable for the temperature considered.

In this approach, a small random displacement of an atom is performed and the variation of the energy, ΔE, is calculated. If $\Delta E < 0$ the displacement is accepted and the configuration corresponding to the displaced atom is used as the starting point for the next step. On the other hand, if $\Delta E > 0$, the new configuration can be accepted according to Boltzmann probability:

$$P(\Delta E) = \exp\left(-\frac{\Delta E}{K_b T}\right) \tag{1}$$

A uniformly distributed random number r in the interval [0, 1] is then calculated and compared with $P(\Delta E)$ by using the Boltzmann constant (K_b). The Metropolis criterion establishes that the new configuration is accepted if $r < P(\Delta E)$; otherwise, it is rejected and the previous configuration is used again as a starting point. The design variables are perturbed randomly, being stored the best value of the objective function to each perturbation. The temperature is then reduced and new attempts are executed. Such procedure continues, avoiding the local minima, and, hopefully, at the end of the process the global minimum is obtained (Metropolis et al., 1953).

If T has a magnitude much higher than the standard deviation of the function in the interval, almost all the points are accepted. On the other hand, if T tends to zero, the method becomes a random search of the minimum. Thus, T_i is adopted as being the value of the standard deviation of the objective function in the studied interval and T_f is the expected precision of the optimal point (Corana et al., 1987).

The main control parameters of the SA (cooling procedure) are the initial temperature, T_i, the cooling rate, N_{temp}, the number of times the procedure is repeated before the ''temperature'' is reduced, N_t, and the number of points of minimum (one for each temperature) that are compared and used as the stopping criterion.

Some successful applications of SA methodology can be mentioned: optimization of mechanical systems (Saramago et al., 1999); estimation of the phase function of anisotropic scattering (Silva Neto and Soeiro, 2002); and estimation of radiative properties in an inverse radiative transfer problem (Souza et al., 2007).

Differential Evolution

The Differential Evolution (DE) is a structural algorithm proposed by Storn and Price (1995) for optimization problems. This approach is an improved version of Goldberg's Genetic Algorithm (GA) (Goldberg, 1989) for faster optimization and presents the following advantages: simple structure, easiness of use, speed, and robustness (Storn and Price, 1995).

Basically, DE generates trial parameter vectors by adding the weighted difference between two population vectors to a third vector. The key parameters of control in DE are the following: N, the population size, CR, the crossover constant, and D, the weight applied to random differential (scaling factor). Storn and Price (1995) have given some simple rules for choosing key parameters of DE for any given application. Normally, N should be about 5 to 10 times the dimension (number of parameters in a vector) of the problem. As for D, it lies in the range 0.4 to 1.0. Initially, $D = 0.5$

can be tried, and then D and/or N is increased if the population converges prematurely.

DE has been successfully applied to various fields such as digital filter design (Storn, 1995), batch fermentation process (Chiou and Wang, 1999), estimation of heat transfer parameters in a bed reactor (Babu and Sastry, 1999), synthesis and optimization of heat integrated distillation system (Babu and Singh, 2000), optimization of an alkylation reaction (Babu and Gaurav, 2000), parameter estimation in fed-batch fermentation process (Wang et al., 2001), optimization of thermal cracker operation (Babu and Angira, 2001), engineering system design (Lobato and Steffen, 2007), economic dispatch optimization (Coelho and Mariani, 2007), identification of experimental data (Maciejewski et al., 2007), apparent thermal diffusivity estimation during the drying of fruits (Mariani et al., 2008), besides other applications (Storn et al., 2005).

Mathematical Formulation and Solution of the Radiative Transfer Problem

Consider a one-dimensional gray homogeneous participating medium of optical thickness τ_o, with transparent boundary surfaces that are subjected to external radiation. The mathematical formulation for such a problem considering no emission inside the medium and azymuthal symmetry is given by an integro-differential equation, known as Boltzmann equation (Özişik, 1973; Silva Neto and Moura Neto, 2005; de Abreu, 2005):

$$\mu\frac{\partial I}{\partial\tau}(\tau,\mu)+I(\tau,\mu)=\frac{\omega}{2}\int_{-1}^{1}I(\tau,\mu')d\mu' \tag{2}$$

with $0 < \tau < \tau_o$ and $-1 \le \mu \le -1$ and subject to the boundary conditions:

$$\begin{cases} I(0,\mu)=A_1 & \text{for } \mu>0 \\ I(\tau_o,\mu)=A_2 & \text{for } \mu<0 \end{cases} \tag{3}$$

In this equation, $I(\tau,\mu)$ is the intensity (radiance) of the radiation field, τ the optical variable, μ the cosine of the polar angle, ω the single scattering albedo, and A_1 and A_2 are the intensities of the isotropic external sources of radiation incident at $\tau = 0$ and $\tau = \tau_o$, respectively, according to Fig. 1.

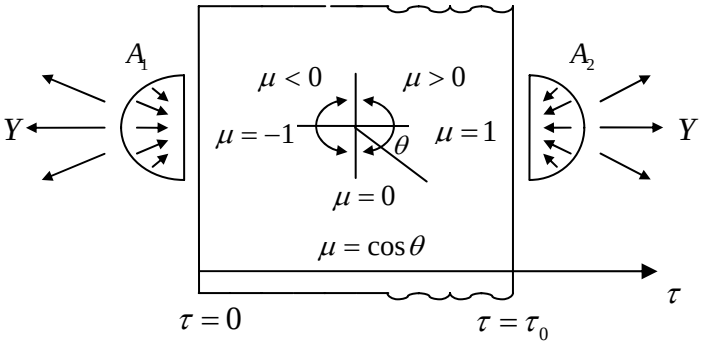

Figure 1. One-dimensional participating medium.

In order to solve the direct problem, the Collocation Method (Villadsen and Michelsen, 1978; Wylie and Barrett, 1985) was used. In this methodology, the general Boundary Value Problem (BVP) is described as:

$$\ddot{y}=f(x,y,p),\qquad a\le x\le b \tag{4}$$

where x is the independent variable, y is a vector of dependent variables and p is a vector of unknown parameters. This BVP, subject to general nonlinear, two-point boundary conditions

$$g(y(a),y(b),p)=0 \tag{5}$$

is approximated by a polynomial function ($S(x)$) on each subinterval $[x_n, x_{n+1}]$ of a mesh $a = x_o < x_1 < ... < x_N = b$. This approximation should satisfy the boundary conditions

$$g(S(a),S(b))=0 \tag{6}$$

and satisfies the differential equations at both ends, and at the midpoints of each subinterval

$$\dot{S}(x_n)=f(x_n,S(x_n)) \tag{7}$$

$$\dot{S}((x_n+x_{n+1})/2)= f((x_n+x_{n+1})/2,S((x_n+x_{n+1})/2)) \tag{8}$$

$$\dot{S}(x_{n+1})=f(x_{n+1},S(x_{n+1})) \tag{9}$$

In the context of this work, the integral terms found in the right hand side of Eq. (2) were substituted by Gauss-Legendre Quadratures (Wylie and Barrett, 1985). The Collocation Method is formally derived by evaluating the governing integro-differential equation at the collocation points, which results in a system of nonlinear ordinary differential-algebraic equations describing the evolution of the solution at the collocation points. This methodology is very attractive due to its easiness of implementation, even when the problem to be solved is highly nonlinear (Villadsen and Michelsen, 1978; Wylie and Barrett, 1985).

It should be emphasized that 20 points were used for the approximation of the variable μ and 10 collocation points were taken into account to solve the direct problem. All case studies were solved by using a personal computer with a 3.2 GHz PENTIUM IV processor and 2 GB of RAM. Both algorithms were executed 10 times for obtaining the values presented in the tables.

Mathematical Formulation of the Inverse Problem

The inverse problem is devoted to the determination of the parameters ω, τ_o, A_1 and A_2 that minimize the difference between the experimental and calculated values for the radiation intensities, I_i^{cal} and $I_i^{\exp}$, respectively, as given by

$$Q=\sum_{k=1}^{m}\sum_{i=1}^{n}\left(I_i^{cal}(\tau,\mu_k)-I_i^{\exp}(\tau,\mu_k)\right)^2 \tag{10}$$

where m represents the number of points considered for the discretization of the angular domain, i.e., the variable μ, and n represents the number of points considered for the discretization of the spatial domain, i.e., the variable τ. In all test case results presented in this work a total of 400 experimental data are considered, i.e., $m = 20$ and $n = 20$ in Eq. (10). For the solution of the inverse problem just described, both the SA and DE algorithms have been used.

Illustrative Examples

As in any experimental procedure, there are uncertainties associated to the measured values. This aspect is taken into account in this work by adding noise to the calculated solution. The radiative properties are known in this case. The standard deviation of experimental errors, σ, is included in the following equation:

$$I = I_{cal} + \sigma R \tag{11}$$

where R is a pseudo-random number, generated with a zero mean Gaussian distribution and standard deviation equal to one, I_{cal} is a vector containing the calculated values for the radiation intensity obtained by using the unknowns that are to be estimated in the inverse problem.

In order to evaluate the performance of the methods of Simulated Annealing and Differential Evolution for the simultaneous estimation of both the single scattering albedo, ω, and the optical thickness, τ_o, of the layer, and also the intensities A_1 and A_2 of the external sources at $\tau = 0$ and $\tau = \tau_o$, respectively, of a given one-dimensional plane-parallel participating media, the four test cases listed in Table 1 have been performed.

Table 1. Parameters used to compose the illustrative examples.

Parameter	Meaning	Case #			
		1	2	3	4
ω	Single scattering albedo	0.1	0.1	0.9	0.9
τ_o	Optical thickness of the layer	0.5	5.0	0.5	5.0
A_1	Intensity of external source at $\tau = 0$	1.0	1.0	1.0	1.0
A_2	Intensity of external source at $\tau = \tau_o$	0.0	0.0	0.0	0.0

The parameters used for the two algorithms are presented in Table 2. These parameters are derived from previous contributions found in the literature (Angira and Babu, 2005; Storn et al., 2005; Kirkpatrick et al., 1983; Saramago et al. 1999).

Table 2. Parameters used in the two evolutionary algorithms.

Parameter		SA	DE	
Generation number	N_{gen}	100	100	
Population size	N	-	10	
Crossover probability	CR	-	0.8	
Perturbation rate	D	-	0.8	
Strategy	-	-	DE/rand/1/bin	
Temperature number	N_{temp}	50	-	
Iterations number for each temperature	N_t	10	-	
Temperature initial/final	T_i / T_f	0.5/0.01	-	
Initial estimate	Case #1	[0.25 0.25 0.5 0.5]	Randomly generated	$0 \le \omega \le 1$; $0 \le \tau_o \le 1$; $1 \le A_1 \le 1.5$; $0 \le A_2 \le 1$
	Case #2	[0.25 0.45 0.5 0.5]		$0 \le \omega \le 1$; $3 \le \tau_o \le 5$; $1 \le A_1 \le 1.5$; $0 \le A_2 \le 1$
	Case #3	[0.75 0.25 0.5 0.5]		$0 \le \omega \le 1.4$; $0 \le \tau_o \le 1$; $1 \le A_1 \le 1.5$; $0 \le A_2 \le 1$
	Case #4	[0.75 0.45 0.5 0.5]		$0 \le \omega \le 1.4$; $3 \le \tau_o \le 5$; $1 \le A_1 \le 1.5$; $0 \le A_2 \le 1$

In this table, it should be emphasized that the initial estimates used by the SA algorithm in each case study were chosen inside of the design space. For the DE algorithm the initial estimate of the population is generated randomly inside the design space.

The present case studies are intended to observe the performance of evolutionary algorithms for different levels of noise with standard deviation of experimental errors of 0%, 0.5% and 5%. For all test case results presented in this section the inverse problem algorithm was run ten times, showing then the worst, average and best results obtained.

In Table 3 the results obtained for case #1 are presented. In this table, *NF* is the number of function evaluations and *cputime* is the running time, given in minutes. It can be observed that when using $\sigma = 0$ (without noise) both algorithms presented good estimates for the unknown parameters. However, if noise is increased, it can be observed that the estimates become poorer. The same behavior was observed for test cases #2-4 whose results are presented in Tables (4)-(6), respectively. The results obtained can be considered satisfactory.

Table 3. Results obtained for case #1.

			ω	τ_o	A_1	A_2	Q (Eq. (10))
Exact	*Error in experimental data*		0.1	0.5	1.0	0.0	-
DE*	0.0	*Worst*	0.1003	0.5002	1.0000	0.0001	1.5578×10^{-6}
		Average	0.0998	0.4999	0.9999	0.0000	5.7702×10^{-7}
		Best	**0.1000**	**0.4999**	**0.9999**	**0.0000**	$\mathbf{4.4564 \times 10^{-9}}$
	0.5%	*Worst*	0.1015	0.4991	0.9980	0.0012	8.4403×10^{-4}
		Average	0.1007	0.4985	0.9976	0.0010	8.4244×10^{-4}
		Best	**0.1006**	**0.4983**	**0.9974**	**0.0011**	$\mathbf{8.4144 \times 10^{-4}}$
	5.0%	*Worst*	0.0876	0.5018	0.9992	0.0058	0.0842
		Average	0.0876	0.5018	0.9992	0.0058	0.0842
		Best	**0.0870**	**0.5017**	**0.9990**	**0.0057**	**0.0842**
SA**	0.0	*Worst*	0.0994	0.4999	1.0001	0.0000	5.3920×10^{-7}
		Average	0.0996	0.4998	0.9999	0.0000	3.4741×10^{-7}
		Best	**0.0999**	**0.4999**	**0.9999**	**0.0000**	$\mathbf{2.1496 \times 10^{-7}}$
	0.5%	*Worst*	0.0944	0.4917	0.9922	0.0001	9.6060×10^{-4}
		Average	0.0962	0.4959	0.9970	0.0000	8.5299×10^{-4}
		Best	**0.0984**	**0.4976**	**0.9974**	**0.0000**	$\mathbf{8.4058 \times 10^{-4}}$
	5.0%	*Worst*	0.0885	0.5012	0.9991	0.0059	0.0849
		Average	0.0880	0.5010	0.9990	0.0059	0.0844
		Best	**0.0879**	**0.5010**	**0.9989**	**0.0056**	**0.0842**

* *NF* = 1010, *cputime* = 4.18 min and ** *NF* = 7015, *cputime* = 30.21 min.

Table 4. Results obtained for case #2.

			ω	τ_o	A_1	A_2	Q (Eq. (10))
Exact	*Error in experimental data*		0.1	5.0	1.0	0.0	-
DE*	0.0	*Worst*	0.1024	4.9982	0.9988	0.0013	6.3559×10^{-6}
		Average	0.1004	4.9976	0.9992	0.0000	2.6107×10^{-6}
		Best	**0.0998**	**5.0036**	**1.0008**	**0.0000**	$\mathbf{1.1856 \times 10^{-7}}$
	0.5%	*Worst*	0.0978	4.9438	0.9844	0.0007	8.0356×10^{-4}
		Average	0.0984	4.9470	0.9847	0.0008	8.0333×10^{-4}
		Best	**0.0983**	**4.9494**	**0.9850**	**0.0010**	$\mathbf{8.0310 \times 10^{-4}}$
	5.0%	*Worst*	0.0453	4.9678	0.9683	0.0000	0.0878
		Average	0.0454	4.9675	0.9682	0.0000	0.0878
		Best	**0.0455**	**4.9674**	**0.9680**	**0.0000**	**0.0878**
SA**	0.0	*Worst*	0.0997	5.0097	1.0026	0.0004	8.6468×10^{-7}
		Average	0.0998	4.9981	0.9995	0.0003	7.7231×10^{-7}
		Best	**0.0994**	**4.9956**	**0.9988**	**0.0005**	$\mathbf{7.1664 \times 10^{-7}}$
	0.5%	*Worst*	0.0929	4.9487	0.9789	0.0009	9.4786×10^{-3}
		Average	0.0971	4.9256	0.9848	0.0005	8.0999×10^{-3}
		Best	**0.0987**	**4.9390**	**0.9841**	**0.0004**	$\mathbf{8.0645 \times 10^{-4}}$
	5.0%	*Worst*	0.0483	4.9578	0.9689	0.0001	0.0892
		Average	0.0484	4.9575	0.9685	0.0001	0.0890
		Best	**0.0485**	**4.9554**	**0.9680**	**0.0001**	**0.0888**

* *NF* = 1010, *cputime* = 21.45 min and ** *NF* = 8478, *cputime* = 62.14 min.

Table 5. Results obtained for case #3.

			ω	τ_o	A_1	A_2	Q (Eq. (10))
Exact	*Error in experimental data*		0.9	0.5	1.0	0.0	-
DE*	0.0	**Worst**	0.8998	0.5001	1.0000	0.0000	$4.0332\text{x}10^{-9}$
		Average	0.8999	0.5000	1.0000	0.0000	$2.1772\text{x}10^{-9}$
		Best	**0.9000**	**0.5000**	**1.0000**	**0.0000**	**$2.0152\text{x}10^{-9}$**
	0.5%	**Worst**	0.9028	0.4978	0.9979	0.0001	$8.9999\text{x}10^{-3}$
		Average	0.9020	0.4980	0.9984	0.0000	$8.8788\text{x}10^{-4}$
		Best	**0.9018**	**0.4988**	**0.9994**	**0.0000**	**$8.6296\text{x}10^{-4}$**
	5.0%	*Worst*	0.9020	0.4700	0.9864	0.0000	0.0776
		Average	0.9022	0.4790	0.9870	0.0000	0.0746
		Best	**0.9032**	**0.4807**	**0.9871**	**0.0000**	**0.0736**
SA**	0.0	*Worst*	0.8998	0.5000	1.0000	0.0001	$8.3002\text{x}10^{-8}$
		Average	0.8998	0.5000	1.0000	0.0000	$4.7782\text{x}10^{-8}$
		Best	**0.8999**	**0.5000**	**1.0000**	**0.0000**	**$2.0152\text{x}10^{-8}$**
	0.5%	*Worst*	0.9039	0.4981	0.9981	0.0000	$8.7988\text{x}10^{-4}$
		Average	0.9025	0.4980	0.9986	0.0000	$8.7744\text{x}10^{-4}$
		Best	**0.9021**	**0.4990**	**0.9994**	**0.0000**	**$8.7014\text{x}10^{-4}$**
	5.0%	*Worst*	0.9049	0.4790	0.9859	0.0000	0.0760
		Average	0.9024	0.4792	0.9860	0.0000	0.0756
		Best	**0.9030**	**0.4800**	**0.9864**	**0.0000**	**0.0738**

* *NF* = 1010, *cputime* = 3.87 min and ** *NF* = 8758, *cputime* = 27.98 min.

Table 6. Results obtained for case #4.

			ω	τ_o	A_1	A_2	Q (Eq. (10))
Exact	*Error in experimental data*		0.9	5.0	1.0	0.0	-
DE*	0.0	**Worst**	0.9000	5.0002	0.9996	0.0000	$2.8555\text{x}10^{-8}$
		Average	0.9000	5.0001	0.9999	0.0000	$2.6683\text{x}10^{-8}$
		Best	**0.9000**	**5.0000**	**0.9999**	**0.0000**	**$2.6203\text{x}10^{-8}$**
	0.5%	**Worst**	0.8985	5.0043	1.0040	0.0008	$7.8547\text{x}10^{-4}$
		Average	0.8990	5.0030	1.0038	0.0009	$7.5553\text{x}10^{-4}$
		Best	**0.8993**	**5.0023**	**1.0028**	**0.0009**	**$7.4263\text{x}10^{-4}$**
	5.0%	*Worst*	0.8999	5.0599	1.0118	0.0001	0.0844
		Average	0.8992	5.0592	1.0117	0.0000	0.0824
		Best	**0.8979**	**5.0562**	**1.0107**	**0.0000**	**0.0804**
SA**	0.0	*Worst*	0.9001	5.0003	0.9998	0.0000	$3.7788\text{x}10^{-8}$
		Average	0.9000	5.0002	0.9999	0.0000	$2.9988.\text{x}10^{-8}$
		Best	**0.9000**	**5.0000**	**0.9999**	**0.0000**	**$2.7245\text{x}10^{-8}$**
	0.5%	*Worst*	0.8988	5.0034	1.0040	0.0009	$7.9877\text{x}10^{-4}$
		Average	0.8989	5.0033	1.0040	0.0009	$7.7747\text{x}10^{-4}$
		Best	**0.8990**	**5.0033**	**1.0041**	**0.0009**	**$7.5245\text{x}10^{-4}$**
	5.0%	*Worst*	0.8999	5.0692	1.0090	0.0001	0.0855
		Average	0.8994	5.0691	1.0189	0.0001	0.0834
		Best	**0.8981**	**5.0566**	**1.0179**	**0.0001**	**0.0811**

* *NF* = 1010, *cputime* = 16.39 min and ** *NF* = 8588, *cputime* = 58.98 min.

Figures 2 to 5 present the noiseless experimental values for the radiation intensities, i.e., $\sigma = 0$ in Eq. (11), and the calculated values obtained by using the estimated radiative properties in the inverse problem solution. The results corresponding to Differential Evolution (DE) and Simulated Annealing (SA) are shown. The radiation intensity values are depicted for the output radiation at both the boundaries of the medium, i.e., at $\tau = 0$ with $\mu < 0$, and $\tau = \tau_o$ with $\mu > 0$.

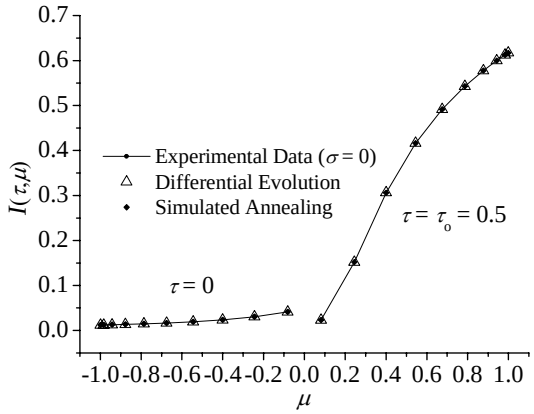

Figure 2. Radiation intensity profile for case #1 ($\omega = 0.1$, $\tau_o = 0.5$, $A_1 = 1$ and $A_2 = 0$).

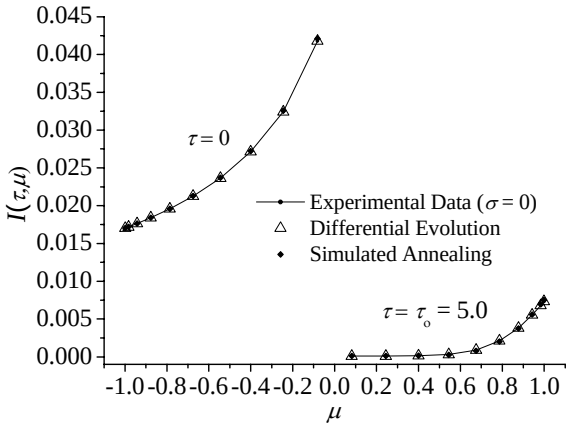

Figure 3. Radiation intensity profile for case #2 ($\omega = 0.1$, $\tau_o = 5.0$, $A_1 = 1$ and $A_2 = 0$).

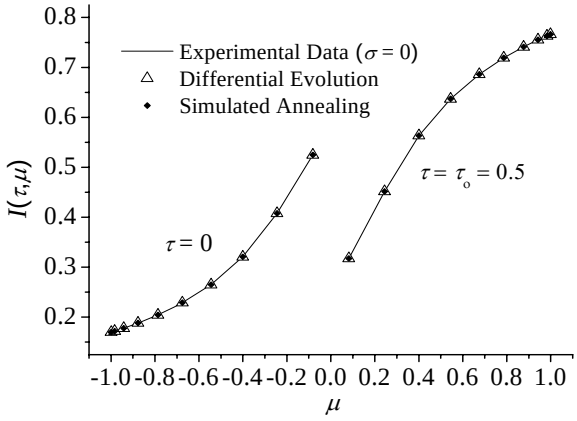

Figure 4. Radiation intensity profile for case #3 ($\omega = 0.9$, $\tau_o = 0.5$, $A_1 = 1$ and $A_2 = 0$).

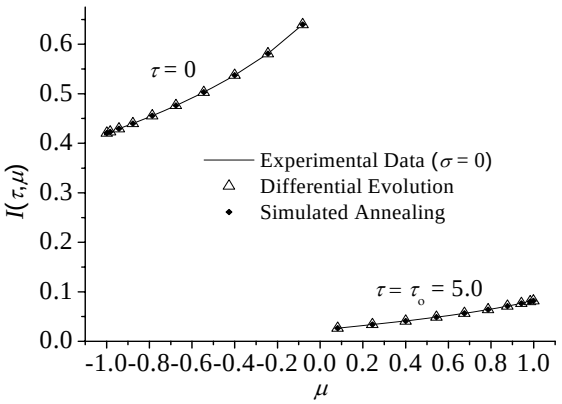

Figure 5. Radiation intensity profile for case #4 ($\omega = 0.9$, $\tau_o = 5.0$, $A_1 = 1$ and $A_2 = 0$).

In Figure 6, for case #1, the residuals (*Res*) between the experimental and calculated values of the output radiation at the boundaries of the medium are shown:

$$Res = I_{cal} - I_{\exp} \tag{12}$$

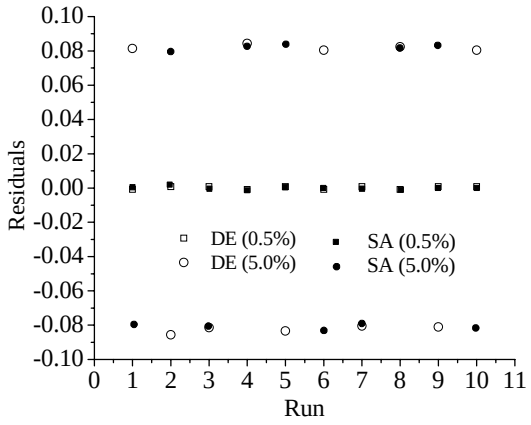

Figure 6. Residuals observed for case #1 with experimental data of 0.5% and 5%.

In this Fig. 6 it is not possible to observe any dependence between the residuals obtained by both algorithms. The same behavior is observed for the other cases.

In Table 7, the values for the hemispherical reflectivity (Λ) and the transmissivity (Γ) calculated with Lii and Özişik (1973), and Özişik and Yener (1982) are shown.

$$\Lambda = \frac{2\pi\int_0^1 I(0,-\mu)\mu d\mu}{2\pi\int_0^1 \mu d\mu} \tag{13}$$

$$\Gamma = \frac{2\pi\int_0^1 I(\tau_o,\mu)\mu d\mu}{2\pi\int_0^1 \mu d\mu} \tag{14}$$

Table 7. Comparison of the values for the transmissivity and hemispherical reflectivity.

		Case #1		Case #2	
		Γ	Λ	Γ	Λ
	Exact	0.17059	0.45631	0.15030	0.00205
DE	0% noise	0.17091	0.45623	0.15006	0.00205
SA		0.17044	0.45636	0.14915	0.00207
DE	0.5% noise	0.17068	0.45634	0.14544	0.00215
SA		0.17044	0.45641	0.14592	0.00217
DE	5% noise	0.14441	0.45281	0.06621	0.00187
SA		0.14595	0.45335	0.07047	0.00191
		Case #3		Case #4	
		Γ	Λ	Γ	Λ
	Exact	1.73085	0.65665	1.13355	0.05750
DE	0% noise	1.73084	0.65665	1.13355	0.05750
SA		1.73081	0.65666	1.13354	0.05749
DE	0.5% noise	1.73706	0.65752	1.13670	0.05763
SA		1.73711	0.65769	1.13658	0.05735
DE	5% noise	1.77045	0.65831	1.14554	0.05536
SA		1.76984	0.65830	1.14574	0.05553

Conclusions

In the present work, the effectiveness of using Differential Evolution and Simulated Annealing for the estimation of radiative properties through an inverse problem approach was analyzed.

In this sense, four benchmark cases were studied and it was possible to conclude that both algorithms led to good results for an acceptable number of generations. It should be pointed out that the Differential Evolution Algorithm led to optimal values that are very similar to those obtained by Simulated Annealing, requiring, however, a smaller number of objective function evaluations. This result was expected, since for the Simulated Annealing Algorithm, for a given iteration, every "temperature" is submitted to a proper number of internal iterations for refinement purposes. This makes the evolutionary process longer, thus increasing the total processing time. On the other hand, as previously mentioned in the works of Storn and Price (1995), Storn (1999) and Angira and Babu (2005), the number of evaluations of the objective function resulting from the Differential Evolution Algorithm is smaller because the evolution scheme is much simpler.

Another interesting aspect is that by adding noise to the synthetic experimental points results in an increase in the objective function values, as observed in Tables 3-6. Such a behavior was previously expected since noise does not permit the convergence of the optimization process to the real experimental values. Consequently, the user should be aware of this behavior when using real experimental data, which is always affected by noise.

As future research work, the authors intend to analyze the influence of changing the parameter values of the optimization algorithms and observe their influence on the performance of the estimation procedure. The inclusion of the conduction heat transfer effect in the inverse problem of combined conduction and radiation effects in semitransparent media is also left for further studies.

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