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Nonlinear vibration of an electrostatically actuated microbeam

Abstract

In this paper, we have considered a new class of critical technique that called the He's Variational Approach (VA) to solve the nonlinear vibration of an electrostatically actuated microbeam. It has been indicated that the Variational Approach (VA) is quickly convergence and does not demand small perturbation and also sufficiently accurate to both linear and nonlinear problems in engineering. The obtained results show that the approximate solutions are uniformly legitimate on the whole solution field.

Keywords

Electrostatically actuated microbeam, Nonlinear vibration, Analytical method.

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1 INTRODUCTION

Studying on the nonlinear oscillators models which are arisen in many areas of physics and engineering is one of the most interesting topics for scientists. Micro electromechanical systems (MEMS) are used in many engineering applications such as: microswitches, transistors, accelerometers,etc (Senturia,2001). Yang et al (2012) consider the electro-dynamic response of an electrically actuated micro-beam. They studied the effects of initial curvature and nonlinear deformation. Nathanson et al. (1967) and Taylor (1968) had an experimental study on the pull-in instability of electrostatically-actuated micro electromechanical systems (MEMS).

Qian et al (2012) the large-amplitude vibration of electrostatically actuated microbeams by using an homotopy analysis method. They used the Euler–Bernoulli beam theory and the Galerkin approach for the MEMS beam model. Fu et al. (2011) applied the energy balance method to investigate the nonlinear oscillation problem arising in the MEMS microbeam model. The problem gets complex in large amplitude of vibration and other physical parameters. It is impossible to prepare an exact solution for such a problem.

Generally, finding an exact solution for nonlinear problems is very difficult. Therefore, many analytical and numerical approaches have been investigated to solve nonlinear equations such as Homotopy perturbation (He,2005; Ganji et al,2006), energy balance method (Pakar and Bayat,2013a), Variational Iteration method (He,1999), Hamiltonian Approach (Bayat et al,2013a), max-min approach (Zeng and Lee ,2009), parameter expansion method (Bayat et al., 2012a), and other analytical and numerical methods (He,2008 ;2007; Ke,2009; Chen,2009;Bayat and Pakar,2013b; Sharma,2011; Pakar and Bayat,2013b;Pakar et al,2012a,b;Bayat,2012b; Bayat,2011;Bayat et al,2013c).

In this study He's Variational Approach is used to find analytical solutions for the large-amplitude vibration of electrostatically actuated microbeams. Variational Approach first was developed by J.H. He (2007) . This approach is used in this study and investigated in different works (Bayat et al,2012a;2013b; Pakar and Bayat,2012b), which have the following advantages, over above-mentioned methods:

1. They provide physical insight into the nature of the solution of the problem.
2. The obtained solutions are the best among all the possible trial-functions

In the first section we describe the basic idea of He's variational approach. The considered system is presented in section 2 and we will study the applications of He's variational approach method, to illustrate the applicability and accuracy of the method, in section 3. Section 4 contains some comparisons between VA solution and energy balance; eventually we show that VA can meet to a precise periodic solution for nonlinear systems.

It is shown that the solutions are quickly convergent and their components can be simply calculated.

2 BASIC CONCEPT OF VARIATIONAL APPROACH

He (2007) suggested a variational approach which is different from the known variational methods in open literature. Hereby we give a brief introduction of the method:

$$u'' + f\left(u\right) = 0 \tag{1}$$

Its variational principle can be easily established utilizing the semi-inverse method (He, 2007):

$$J\left(u\right) = \int_0^{T/4} \left(-\frac{1}{2}u'^2 + F\left(u\right)\right) dt \tag{2}$$

Where T is period of the nonlinear oscillator, $\partial F / \partial u = f$. Assume that its solution can be expressed as:

$$u\left(t\right) = A\cos\left(\omega t\right) \tag{3}$$

Where A and w are the amplitude and frequency of the oscillator, respectively. Substituting Eq.(3) into Eq.(2) results in:

$$\begin{aligned} J\left(A,\omega\right) &= \int_0^{T/4}\left(-\frac{1}{2}A^2\omega^2\sin^2\omega t + F(A\cos\omega t)\right)dt \\ &= \frac{1}{\omega}\int_0^{\pi/2}\left(-\frac{1}{2}A^2\omega^2\sin^2 t + F\left(A\cos t\right)\right)dt \\ &= -\frac{1}{2}A^2\omega\int_0^{\pi/2}\sin^2 t\,dt + \frac{1}{\omega}\int_0^{\pi/2}F\left(A\cos t\right)dt \end{aligned} \tag{4}$$

Applying the Ritz method, He require:

$$\frac{\partial J}{\partial A} = 0 \tag{5}$$

$$\frac{\partial J}{\partial \omega} = 0 \tag{6}$$

But with a careful inspection, for most cases we have:

$$\frac{\partial J}{\partial \omega} = -\frac{1}{2}A^2\int_0^{\pi/2}\sin^2 t\,dt - \frac{1}{\omega^2}\int_0^{\pi/2}F\left(A\cos t\right)dt < 0 \tag{7}$$

Thus, modify conditions Eq.(5) and Eq.(6) into a simpler form:

$$\frac{\partial J}{\partial \omega} = 0 \tag{8}$$

From which the relationship between the amplitude and frequency of the oscillator can be obtained.

3 NONLINEAR VIBRATION OF AN ELECTROSTATICALLY ACUATED MICROBEAM

Figure 1 represents a fully clamped microbeam with uniform thickness h, length l, width b ($b>>5h$), effective modulus $\bar{E} = E/\left(1-\nu^2\right)$, Young's modulus E, Poisson's ratio υ and density ρ. By applying the Galerkin Method and employing the classical beam theory and taking into account of the mid-plane stretching effect as well as the distributed electrostatic force, the dimensionless equation of motion for the microbeam is as follow [6]:

$$(a_1u^4 + a_2u^2 + a_3)\ddot{u} + a_4\,u + a_5\,u^3 + a_6\,u^5 + a_7\,u^7 = 0, \qquad u(0) = A\,, \quad \dot{u}(0) = 0, \tag{9}$$

Where u is the dimensionless deflection of the microbeam, a dot denotes the derivative with respect to the dimensionless time variable $t = \tau\sqrt{\bar{E}I/(\rho bhl^4)}$ with I and t being the second moment of area of the beam cross-section and time, respectively.

In Eq. (9), the physical parameters $a_i\left(i=1-7\right)$ are given by (Fu et al, 2011):

$$a_1=\int_0^1 \phi^6\,d\xi \tag{10}$$

$$a_2=-2\int_0^1 \phi^4\,d\xi \tag{11}$$

$$a_3=\int_0^1 \phi^2\,d\xi \tag{12}$$

$$a_4=\int_0^1\left(\phi''''\phi-N\phi''\phi-V^2\phi\right)d\xi \tag{13}$$

$$a_5=-\int_0^1\left(2\phi''''\phi^3-2N\phi''\phi^3+\alpha\phi''\phi\int_0^1\left(\phi'\right)^2 d\xi\right)d\xi \tag{14}$$

$$a_6=\int_0^1\left(\phi''''\phi^5-N\phi''\phi^5+2\alpha\phi''\phi^3\int_0^1\left(\phi'\right)^2 d\xi\right)d\xi \tag{15}$$

$$a_7=-\int_0^1\left(\alpha\phi''\phi^5\int_0^1\left(\phi'\right)^2 d\xi\right)d\xi \tag{16}$$

In which, the following nondimensional variables and parameters are introduced

$$\alpha=\frac{6g_0^2}{h^2},\ \xi=\frac{x}{l},\ N=\frac{\bar{N}l^2}{\bar{E}I},\ V^2=\frac{24\varepsilon_0 l^4\bar{V}^2}{\bar{E}h^3g_0^3} \tag{17}$$

While a prime ($'$) indicates the partial differentiation with respect to the coordinate variable x. The trial function is $\phi\left(\xi\right)=16\xi^2\left(1-\xi\right)^2$. The parameter $\bar{N}$ denotes the tensile or compressive axial load, g_0 is initial gap between the microbeam and the electrode, $\bar{V}$ the electrostatic load and ε_0 vacuum permittivity. The complete formulation of Eq. (9) can be referred to Ref. (Fu et al, 2011) for details.

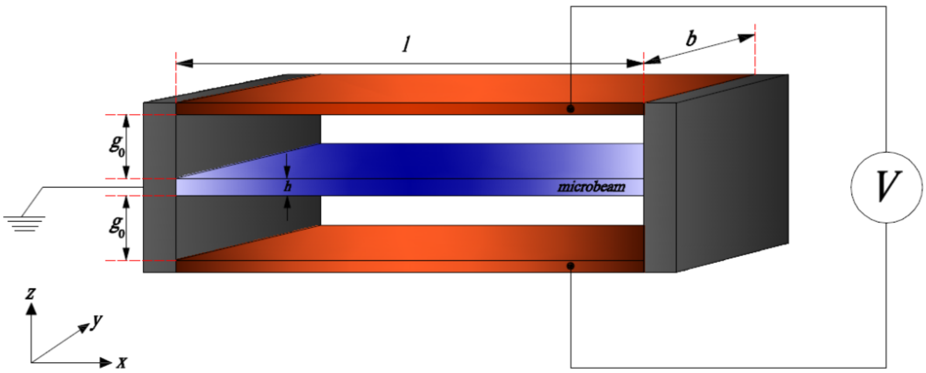

Figure 1 Schematics of a double-sided driven clamped-clamped microbeam-based electromechanical resonator

4 APPLYING VARIATIONAL APPROACH TO MICROBEAM

In Eq. (9), Its Variational principle can be easily obtained:

$$J\left(u\right)=\int_0^t\left(-\frac{1}{2}\left(a_1 u^4+a_2 u^2+a_3\right)\dot{u}^2+\frac{1}{2}a_4 u^2+\frac{1}{4}a_5 u^4+\frac{1}{6}a_6 u^8+\frac{1}{8}a_7 u^8\right)dt \tag{18}$$

Choosing the trial function $u(t)=A\cos(wt)$ into Eq.(18) we obtain:

$$J(A)=\int_0^{T/4}\begin{pmatrix}-\frac{1}{2}A^2\omega^2\sin^2\omega t\left(a_1 A^4\cos^4\omega t+a_2 A^2\cos^2\omega t+a_3\right)\\+\frac{1}{2}a_4 A^2\cos^2\omega t+\frac{1}{4}a_5\ A^4\cos^4\omega t+\frac{1}{6}a_6 A^6\cos^6\omega t+\frac{1}{8}a_7 A^8\cos^8\omega t\end{pmatrix}dt \tag{19}$$

The stationary condition with respect to A leads to:

$$\begin{aligned}\frac{\partial J}{\partial A}&=\int_0^{T/4}\begin{pmatrix}-A\omega^2\sin^2\omega t\left(a_1\ A^4\cos^4\omega t+a_2\ A^2\cos^2\omega t+a_3\ \right)\\+a_4\ A\cos^2\omega t+a_5 A^3\cos^4\omega t+a_6\ A^5\cos^6\omega t+a_7\ A^7\cos^8\omega t\end{pmatrix}dt=0\\&=\int_0^{\pi/2}\begin{pmatrix}-A\omega^2\sin^2 t\left(3a_1 A^4\cos^4 t+2a_2 A^2\cos^2 t+a_3\right)\\+a_4\ A\cos^2 t+a_5\ A^3\cos^4 t+a_6\ A^5\cos^6 t+a_7\ A^7\cos^8 t\end{pmatrix}dt=0\\&=-\omega^2\int_0^{\pi/2}\left(3a_1\ A^4\cos^4 t+2a_2\ A^2\cos^2 t+a_3\right)A\sin^2 t\,dt\\&\quad+\int_0^{\pi/2}\left(a_4\ A\cos^2 t+a_5\ A^3\cos^4 t+a_6\ A^5\cos^6 t+a_7\ A^7\cos^8 t\right)dt=0\end{aligned} \tag{20}$$

Solving Eq.(20), according to w, we have

$$\omega^2 = \frac{\int_0^{\pi/2}\left(a_4 A\cos^2 t + a_5 A^3\cos^4 t + a_6 A^5\cos^6 t + a_7 A^7\cos^8 t\right)dt}{\int_0^{\pi/2}\left(3a_1 A^4\cos^4 t + 2a_2 A^2\cos^2 t + a_3\right)A\sin^2 t\,dt} \tag{21}$$

Then we have

$$\omega_{VA} = \frac{\sqrt{2}}{4}\frac{\sqrt{64a^4 + 48a_5A^2 + 40a_6A^4 + 35a_7A^6}}{\sqrt{4A^2a_2 + 3A^4a_1 + 8a_3}}, \tag{22}$$

According to Eqs.(3) and (22), we can obtain the following approximate solution:

$$u(t) = A\cos\left(\frac{\sqrt{2}}{4}\frac{\sqrt{64a^4 + 48a_5A^2 + 40a_6A^4 + 35a_7A^6}}{\sqrt{4A^2a_2 + 3A^4a_1 + 8a_3}}t\right) \tag{23}$$

5 RESULTS AND DISCUSSIONS

To demonstrate and verify the accuracy of this new approximate analytical approach, some comparisons of the time history oscillatory displacement responses with those of the energy balance method are presented in Figs. 2(a) and 2(b). Table 1 shows the comparison of frequency corresponding to various parameters of system with energy balance method (Fu et al., 2011).

Table 1 Comparison of frequency corresponding to various parameters of system

Constant parameters				Variational Approach solution	Energy Balance solution
A	N	a	V	w_{VA}	w_{EBM} (Fu et al,2011)
0.3	10	24	0	26.3644	26.3867
0.3	10	24	10	24.2526	24.2753
0.3	10	24	20	16.3556	16.3829
0.6	10	24	0	28.5579	28.9227
0.6	10	24	10	26.1671	26.5324
0.6	10	24	20	17.0940	17.5017

Figure 2(a) and 2(b) represent comparisons of the analytical solution of $u(t)$ based on time with energy balance solutions. It could be obtained from the figures that the motion of the system is a periodic motion and the amplitude of vibration is a function of the initial conditions.

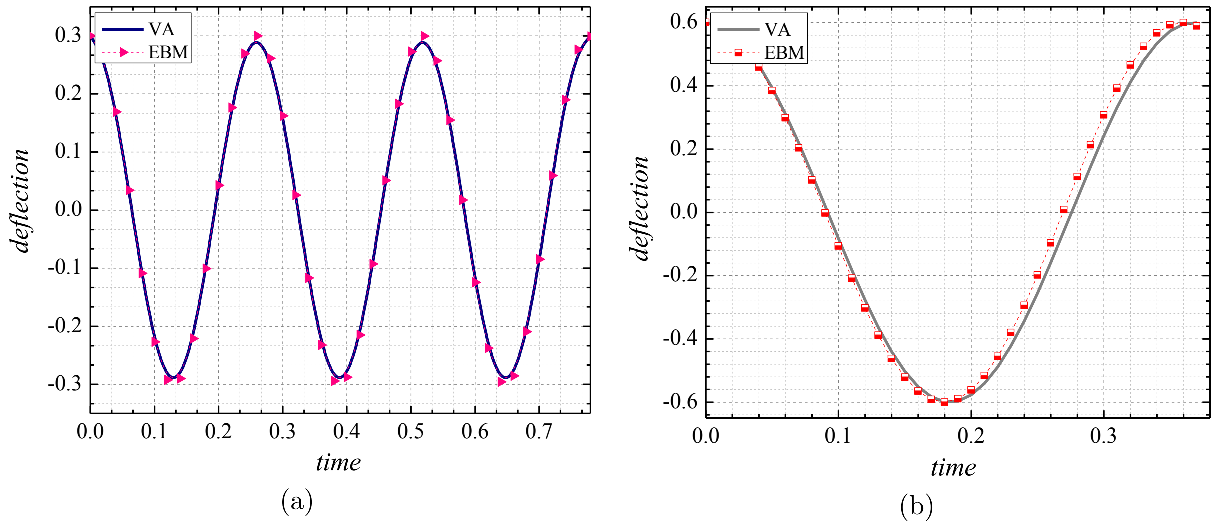

Figure 2 Comparison of variational approach solution of deflection with the EBM solution based on time for cases
(a): $N=10$, $a=24$, $V=10$, $A=0.3$ (b): $N=10$, $a=24$, $V=20$, $A=0.6$

Figure 3(a) and 3(b) show the comparisons of the analytical solution of $\dot{u}(t)$ based on $u(t)$ with the energy balance solution. Figures 4(a) and 4(b) are presented to show the effects of amplitude and V parameter on the phase plan of the system.

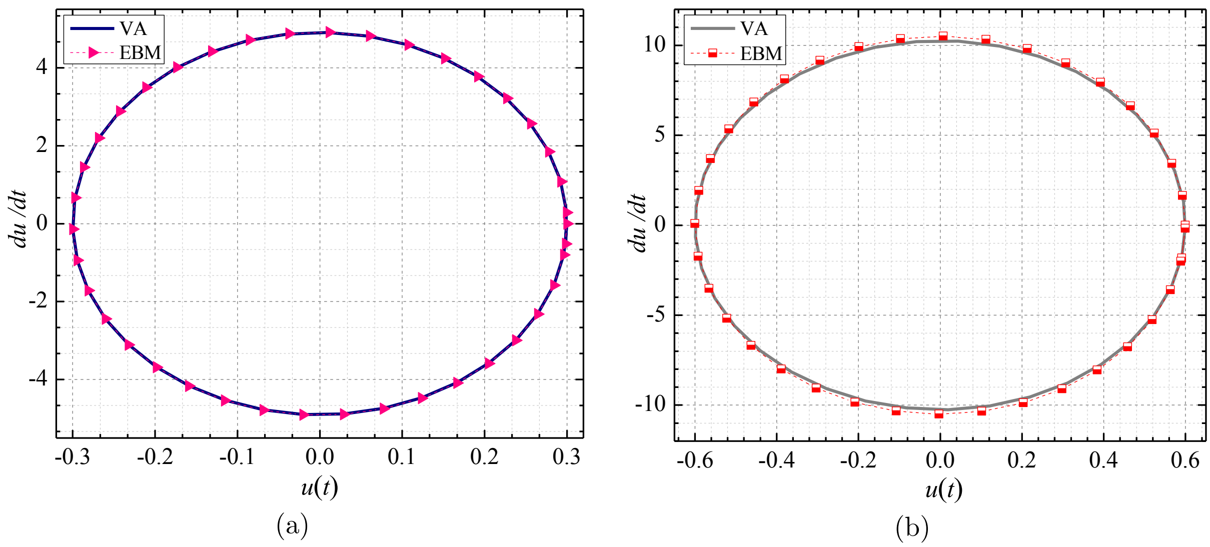

Figure 3 Comparison of variational approach solution of phase plan with the EBM solution for cases
(a): $N=10$, $a=24$, $V=10$, $A=0.3$ (b): $N=10$, $a=24$, $V=20$, $A=0.6$

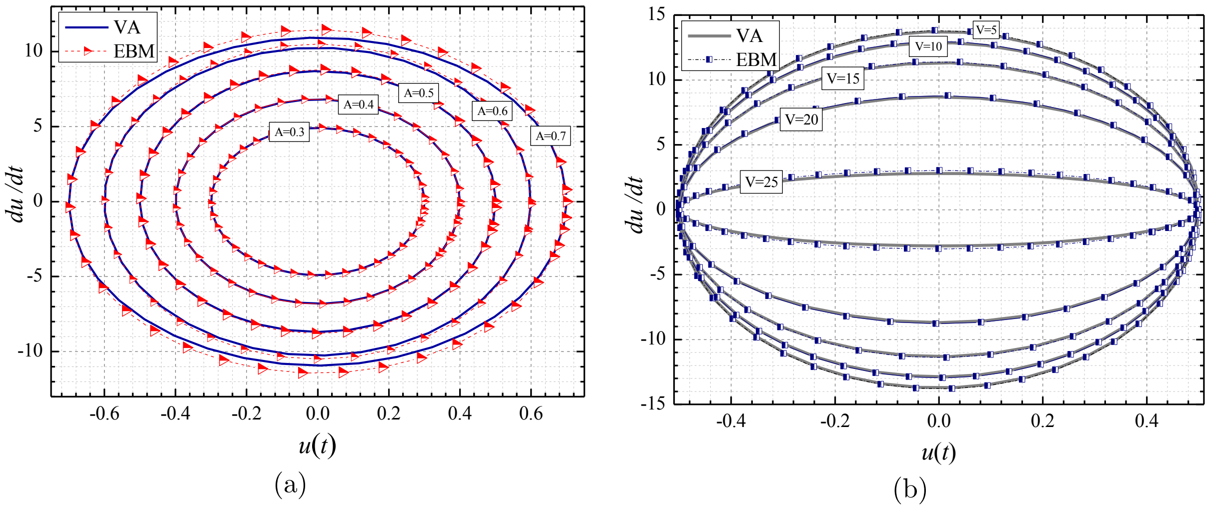

Figure 4 (a): Effect of amplitude on the phase plan of the problem for $N=10,\ a=24,\ V=10$

(b): Effect of V parameter on the phase plan of the problem for $N=10,\ a=24,\ A=0.5$

Figure 5 is the effect of V parameter on nonlinear frequency of electrostatically microbeam base on amplitude. It can be observed that from the figure, the nonlinear frequency is increased when the V parameter is increasing but after reaching to the peak points of amplitude the nonlinear frequency is decreased by increasing V parameter.

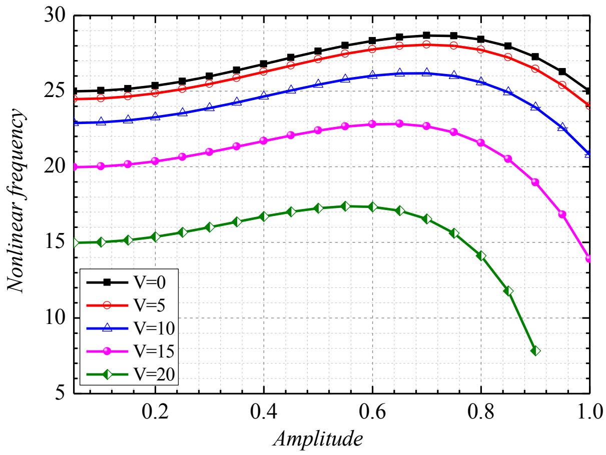

Figure 5 Effect of V parameter on nonlinear frequency of electrostatically microbeam base on amplitude

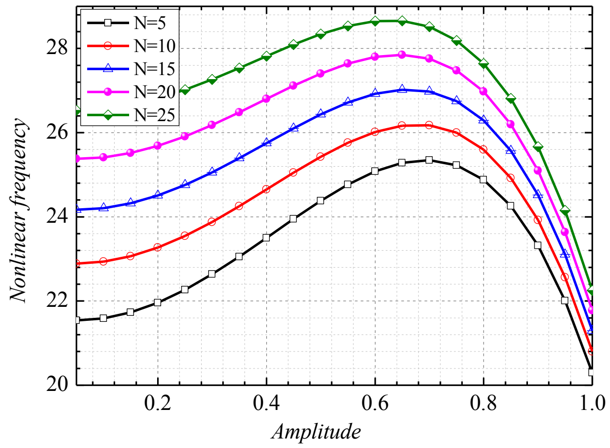

Figure 6 Effect of N parameter on nonlinear frequency of electrostatically microbeam base on amplitude

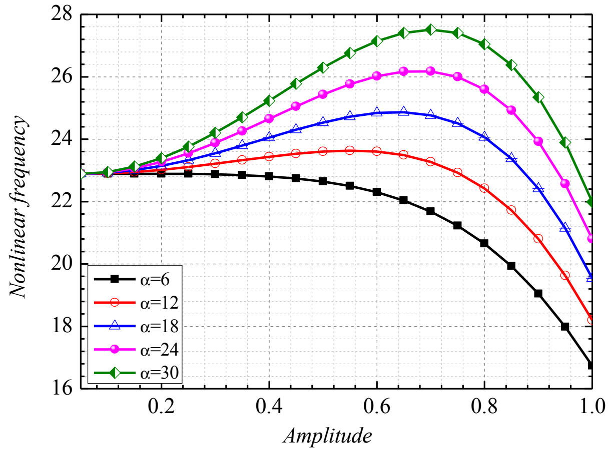

Figure 7 Effect of α parameter on nonlinear frequency of electrostatically microbeam base on amplitude

The figure 6 is the effect of N parameter on nonlinear frequency of electrostatically microbeam base on amplitude. The N parameter has the same behavior of V on the nonlinear frequency of the system by increasing it. The figure 7 is considered the α on the nonlinear frequency of the system for different values of amplitude. It is evident that Variational Approach (VA) shows an excellent agreement with the energy balance solution and quickly convergent and valid for a wide range of vibration amplitudes and initial conditions. The accuracy of the results shows that the VA can be potentially used for the analysis of strongly nonlinear oscillation problems accurately.

6 CONCLUSION

In this paper, the Variational Approach was employed to solve the nonlinear governing equation of an electrostatically actuated microbeam. Excellent agreement between approximate frequencies and the numerical solutions are demonstrated and discussed. The high accuracy of this method demonstrated that the proposed approach can be applied for any strong conservative nonlinear problems without any limitations. We can suggest Variational Approach as strongly nonlinear method as novel and simple method for oscillation systems which provide easy and direct procedures for determining approximations to the periodic solutions.

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