

## A numerical investigation into ultimate strength and buckling behavior of locally corroded steel tubular members

### Abstract

This paper presents the results of a numerical investigation into ultimate strength of locally corroded tubular members under axial compressive loads. A parametric finite element approach was used in order to simulate structural behavior of damaged members. The results were then examined against an available experimental test. Validated models were used to derive a semi-empirical formula for predicting ultimate strength of locally damaged tubes as a function of corrosion dimensions. Geometry of corrosion can be defined by its depth, length, width and location of damage along the tube. In this study it is focused on the effect of some parameters that have not been addressed yet by other researchers, e.g. slenderness of the tubes and location of patch corrosion. It was found that location of corrosion has great effect on reduction of ultimate strength. Effect of corrosion geometry was studied and formulated as well as tubular slenderness and it was shown that tubes with different corrosion dimensions show different behaviors under compressive loads. In cases with severe corrosion damages, the occurrence of local buckling plays an important role on reduction of ultimate strength and deformation of damaged region.

### Keywords

Tubular member, local corrosion, ultimate strength, tubular steel bracing

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## 1 INTRODUCTION

Tubular members are widely used in offshore platforms. They have many advantages such as: isotropic behavior under compressive loads, low drag resistance and ease of handling. Figure 1 shows a fixed offshore platform and its structural elements which are mainly made of tubular members.

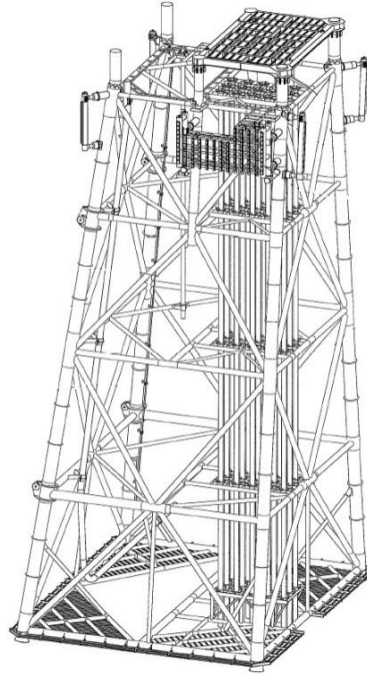
Corrosion of metallic structures due to existence in corrosive marine environment is inevitable. Nevertheless, coatings and cathodic protection are useful means against occurrence and deterioration of corrosion. Corrosion can take place in the shapes of uniform or local damages. Figure 2 shows a locally corroded tube. As a result of loss of material and changing mechanical properties of corroded region, the ultimate strength and behavior of damaged member are directly affected.

Structures suffering expected corrosion damages should maintain their stability and strength during their service life. The importance of this issue becomes clearer when affected structure is expected to have adequate level of reliability, e.g. offshore platforms. Assessment of reliability for

such a complex structure necessitates full knowledge about residual strength of their structural elements.

The corrosion pattern used in this study complies with a common local corrosion of tubes named as corrosion 'patch'. Although the other form of corrosion such as uniform type may exist over the surface of aged tubes, it is mostly observed in the form of locally restricted damages.

The major difference between overall and local corrosion is that as a result of non-uniform reduction of wall thickness, transverse deflections of tube under axial compression is likely to increase and stability of tube is affected.



**Figure 1:** Schematic view of an offshore platform (Courtesy of Petropars Oil & Gas Co.).

Behavior of tubes under axial compression can be divided into 4 distinct stages: development of elastic stresses, formation of plastic region, beginning of local buckling, and plastic deformation of corroded region. The occurrence of these stages may be affected by shape and dimensions of corrosion patch.



**Figure 2:** A pipe with local corrosion damage.

## 2 PREVIOUS RELATED RESEARCHES

Estimating the reduction of strength due to corrosion has been addressed by some guidance notes and design codes. Their proposed approximations give overestimating values and usually consider corrosion depth as a defining parameter for geometry of corrosion e. g. API RP 2A-WSD (2000) and NORSOK N-004 (1998). However, corrosion damages cannot be introduced only by its depth.

Moehlman (2000) conducted a series of experiments in order to study the ultimate strength of corroded tubular members. The specimens were taken from an aged offshore platform and were severely damaged.

Lutes et al. (2001) carried out an analytical study on compressive strength of corroded tubes. They only consider a corrosion mode with a regular pattern and didn't examine other patterns that may exist. Yamane et al. (2006) assessed buckling strength of a corroded tubular member and compared the results with numerical models.

Some researchers studied the effect of corrosion on buckling/ultimate strength of ship structural elements. One can refer to the works of Matsushita et al. (2002), Nakai et al. (2004, 2<sup>nd</sup> and 3<sup>rd</sup> reports). Effect of corrosion on frame structures was also assessed in some studies e.g. Okeda et al. (2000).

Evaluation of ultimate strength for cylindrical tubular members with local corruptions was performed by Nishimura et al. (2001).

Lee and Kim (2000) used numerical simulations in order to predict failure of externally corroded pipelines. They proposed a failure criterion based on the local stress state and plastic collapse failure mechanism.

Effect of corrosion on pressurized tubes has been assessed by Michell et al. (2001) using numerical simulation. They assumed that corrosion has regular pattern and is confined in an arc of tube with uniform loss of thickness.

In all mentioned studies and related investigations carried out on corrosion and strength of tubular members, only very simple corrosion patterns, especially uniform one, have been studied. Therefore there is a need for further investigations on the effect of non-uniform corruptions.

There are also papers that used elliptical shapes for idealizing corrosion pattern e.g. Netto et al. (2007) and Shariati and Rokhi (2008), but some parameters such as  $L/D$ ,  $D/t$ , and location of corrosion have not been included in those studies.

## 3 DESCRIPTION OF NUMERICAL MODEL

Ansys 11 finite element program (2008) was used to perform a series of numerical analyses. Four-noded shell elements, type shell 181 with large strain and 6 degrees of freedom per each node were utilized in order to model the tube.

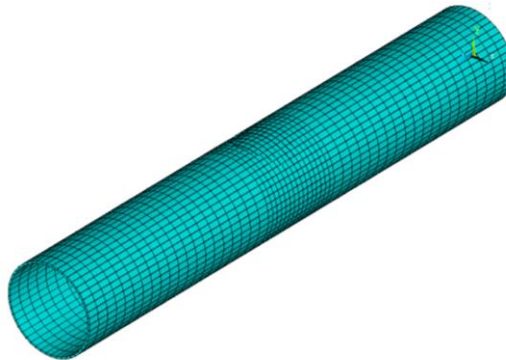
Shell elements are a special class of elements that are designed to efficiently model thin structures. They take advantage of the fact that the only shear on the free surface is in-plane. Mottran and Shaw (1996) proposed them as the best choice for modeling plate made structures.

The shell elements used are of reduced integration type and could consider large strain and strain hardening effects as well. Each element has real constraint at its own nodes; therefore by introducing thickness at each node the tube can possess different distributions of wall thickness.

Forty equally sized shell elements were used in circumferential direction; and depending on the length of the tubes, 120 to 500 elements were located in axial direction of tubular members. Straight lines have been chosen for modeling sides of shell elements connecting any two adjacent nodes.

In the vicinity of corroded region a finer discretisation was made. In addition to precise modeling of geometrical changes of corroded region, applying finer meshes of the elements allows the nu-

numerical model to behave in similar way with real tubular members. This way of meshing enables the numerical model to defeat existing problems due to non-convergences of the solution. The schematic view of the model and finite element discretisation is shown in Fig. 3.



**Figure 3:** Distribution and fine mesh zone over the numerical model.

In order to assess post buckling behavior of tubular members a displacement controlled method was used. In other words, total compression was divided into a number of steps. In each step, axial shortening of tube and deformed shape were calculated based on the history of previous steps. Deformation at the end of each step was being assumed as a starting point for the next step.

Due to nonlinear properties of material and geometric nonlinearities caused by large deformations, a series of nonlinear finite element analyses should be made.

Full Newton-Raphson approach and an adaptive descent technique were included in the solution procedure. Sparse direction solver was also used in the process of analyzing.

The corrosion pattern used in this study consists of a corroded region that can vary in depth, length and location along the tube. The corroded region is in the form of elliptical shape and has smooth slopes at the corrosion patch. This pattern was chosen on the basis of proposed contours from actual specimens and is compatible with studies of Hebor and Ricles (2002), Netto et al. (2005 and 2007) and Ostapenko et al. (1993). Figure 4 shows the idealized corrosion patch and its defining parameters used in the present study.

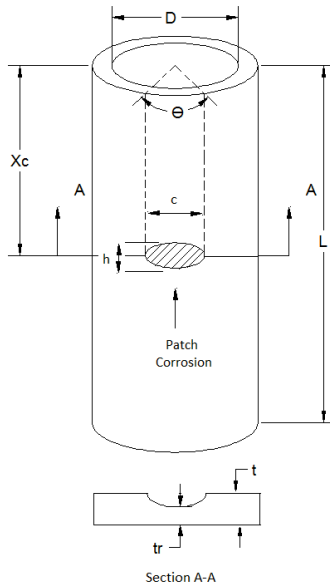
The most important difference between the present model and previous models is that the proposed model includes most of the geometric characteristics of corrosion patch. The compressive strength of a locally corroded tube depends on the dimensions of tube and the size of corrosion defect. The purpose of this paper is to provide an estimate to calculate the ultimate strength of corroded tubes under axial compression; therefore an equation is suggested.

The effect of parameters  $x/L$ ,  $L/D$ , and  $D/t$  on reduction of compressive strength is rarely studied by other researchers. Other characteristics such as corrosion depth and development of corrosion defect over the tube surface (i.e.  $t_r/t$ ,  $c$ , and  $h$ ) are studied in a confined range of parameters. A detailed view of numerical model at corrosion patch is presented in fig. 5.

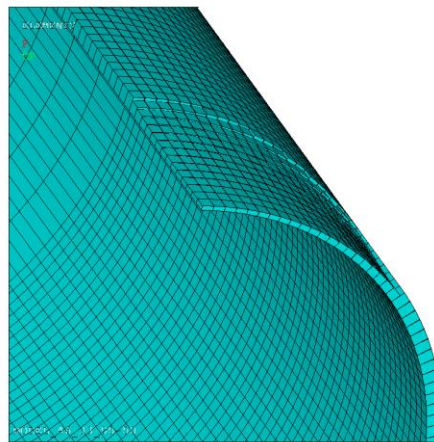
It should be emphasized that different types of boundary conditions have been considered by various researches, e.g. Khedmati and Nazari (2012) and Zeinoddini et al. (2008). Depending on rigidity of tubular joint and strength of adjacent members, several end conditions can be assumed. However, the study of sensitivity on response of tubular members to the end conditions was not included in the scope of the current study.

The boundary conditions used in the present study is of clamped end type with ability of sliding in the axial direction. In order to simulate this type of end condition, a rigid diaphragm is arranged at the ends of the tube. The center of diaphragm is constrained in all directions except axial movement, Fig. 6. This way of simulating, to some extent, reflects the actual behavior of members with

stiff tubular joints, e.g. brace members that connected to platform legs (Khedmati and Nazari, 2012).



**Figure 4:** Corrosion patch and its geometric parameters



**Figure 5:** The cross sectional view of the numerical model at corrosion patch region.



**Figure 6:** End conditions used in the study

### 3.1 MATERIAL PROPERTIES

Models used in this study were of steel material with yield stresses of 265 MPa. The modulus of elasticity was taken as 210 GPa and the Poisson's ratio was chosen 0.3. The models have isotropic material; and a bilinear isotropic hardening with Von Mises failure criteria was introduced to simulate the plastic behavior.

### 3.2 MODEL DIMENSIONS

The dimensions of models are selected so the tubes are representative of scaled brace members of fixed offshore platforms. i.e. the ratios  $L/D$  and  $D/t$  are equal to actual common tubular members. In order to parametrically assess the effect of corrosion dimensions, 16 groups of specimens were analyzed. Each group contains tubes varying in one parameter. The dimensions of models for 3 of the 16 groups are listed in table 1.

In order to identify each specimen a numeric code was used that consisted of the  $D/t$  ratio,  $t_r/t$  ratio,  $\vartheta$ , and  $x/L$  ratio. e.g. as we can see in the table 1, specimen 22-90-75-50 represents a tubular member with  $D/t = 22$ ,  $t_r/t = 0.9$ ,  $\vartheta = 75$  degrees, and  $x/L = 0.5$ .

Group	Specimen	L (m)	D (m)	t (m)	$\frac{D}{t}$	$\frac{t_r}{t}$	$\Theta$ (Deg.)	h (m)	$\frac{x_c}{L}$ (%)
I	22-90-75-50	6.75	0.47	0.021	22	0.9	75	0.25	50
	22-70-75-50	6.75	0.47	0.021	22	0.7	75	0.25	50
	22-50-75-50	6.75	0.47	0.021	22	0.5	75	0.25	50
	22-30-75-50	6.75	0.47	0.021	22	0.3	75	0.25	50
	22-20-75-50	6.75	0.47	0.021	22	0.2	75	0.25	50
II	22-50-15-50	6.75	0.47	0.021	22	0.5	15	0.25	50
	22-50-45-50	6.75	0.47	0.021	22	0.5	45	0.25	50
	22-50-58-50	6.75	0.47	0.021	22	0.5	58	0.25	50
	22-50-75-50	6.75	0.47	0.021	22	0.5	75	0.25	50
	22-50-95-50	6.75	0.47	0.021	22	0.2	95	0.25	50
III	22-20-75-10	6.75	0.47	0.021	22	0.2	75	0.25	10
	22-20-75-20	6.75	0.47	0.021	22	0.2	75	0.25	20
	22-20-75-30	6.75	0.47	0.021	22	0.2	75	0.25	30
	22-20-75-40	6.75	0.47	0.021	22	0.2	75	0.25	40
	22-20-75-50	6.75	0.47	0.021	22	0.2	75	0.25	50

Table 1: Dimensions of a number of specimens (groups I, II, and III).

## 4 NUMERICAL RESPONSES OF THE MODEL

Before presenting the results of analyses, a comparison between present numerical simulations and experimental results conducted by Hebor et al. (2002) is provided. The curves of load-displacement for undamaged and corroded specimens are respectively illustrated in figure 7 and figure 8.

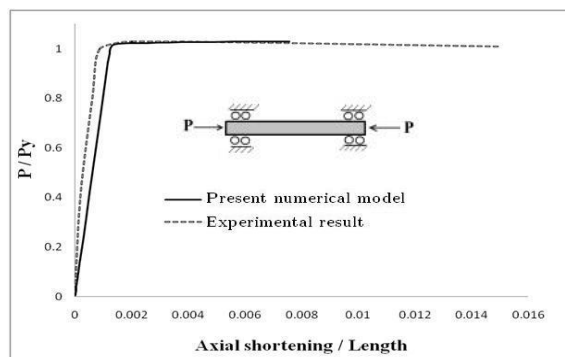
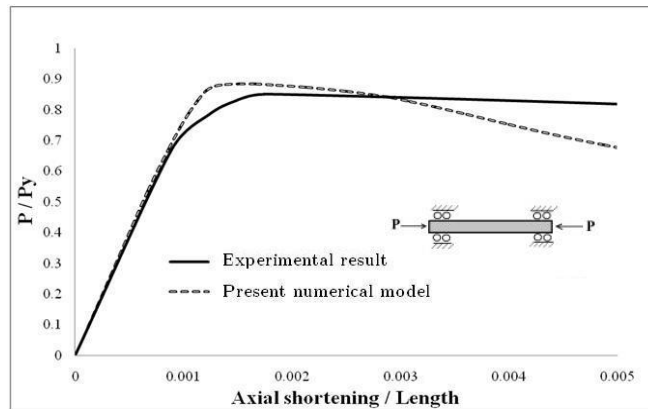


Figure 7: Comparison of experimental (Hebor et al. (2002)) and the present numerical results for non-corroded specimen 46-100-0-0

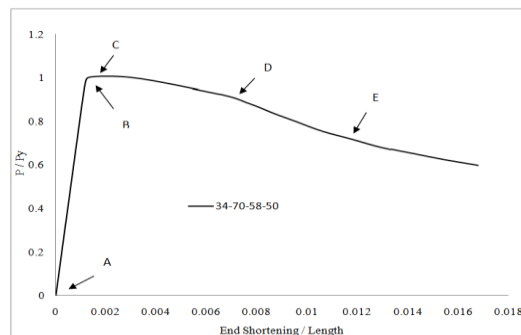


**Figure 8:** Comparison of experimental (Hebor et al. (2002)) and the present numerical results for corroded specimen 46-33-95-50

In general, the curve of load-displacement for compression of tubular members is in the form of presented curve in fig. 9. The behavior of models can be divided into 4 distinct stages.

At early stages of compression, models show elastic deformations, point 'A' to 'B'. Increasing the load causes the stresses to exceed the yielding limit. Due to formation of plastic region in the corroded area, the rate of load-displacement decreases and the ultimate strength state occurs, point 'B' to 'C'.

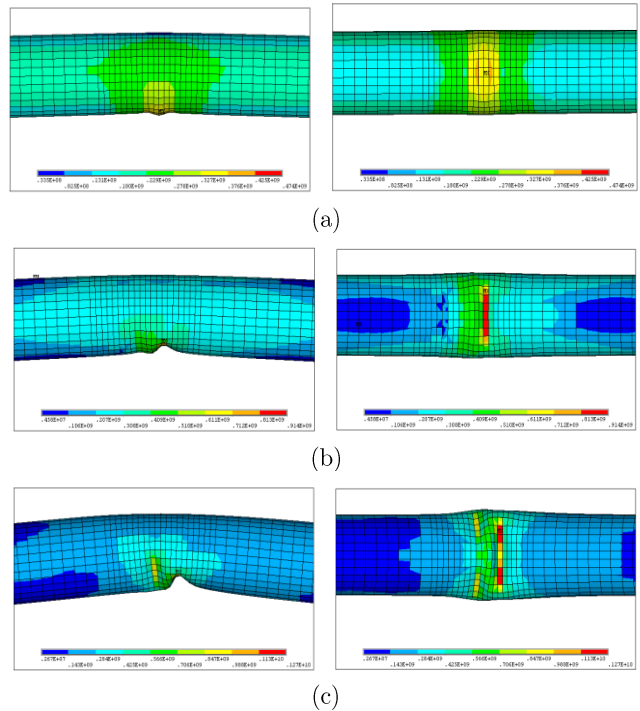
Applying axial force on corroded tubes, leads the tube to deflect in transverse direction. After reaching its ultimate strength, the load decreases as the lateral deflection increases, point 'C' to 'D'. At this stage, further compression will result in development of plastic region until a local buckling is created in the maximum compressive strain region, point 'D'.



**Figure 9:** The curve of compressive load-displacement for a typical tubular member (specimen 34-70-58-50)

After occurrence of local buckling, the rate of load-displacement decreases again and development of local buckling over the corroded region begins, point 'D' to 'E'.

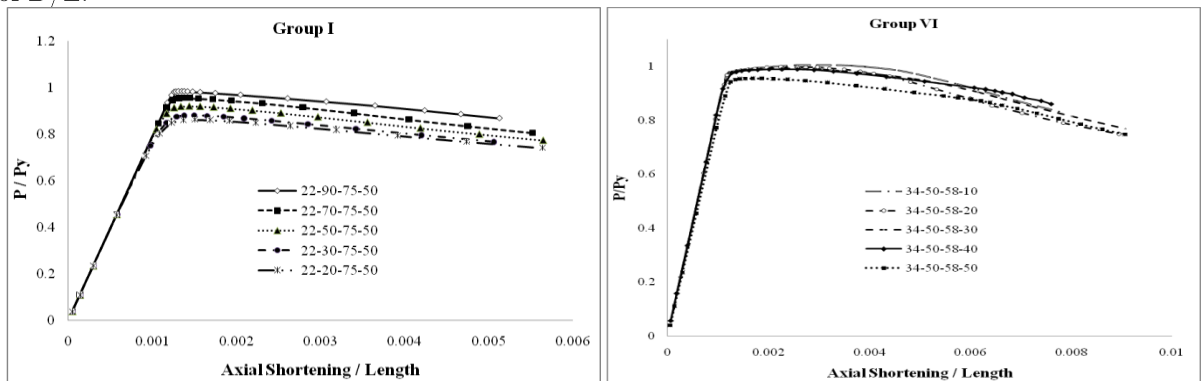
Bai (2003) expressed that for perfect tubes under compression, the local buckling in terms of cross-sectional deformation may be approximated by cosine mode or dent mode. Figure 10 shows, for instance, development of local buckling and stress distribution over deformed shape of corroded region. As it can be seen in the figure, the model is going to buckle under cosine mode.



**Figure 10:** Development of cosine local buckling in corroded region, left: side view, right: front view, a) initial mode, b) subsequent mode 1, and c) subsequent mode 2

The graphs of corresponding results for 8 groups of specimens are presented here. In the fig. 11 behavior of some numerical models are depicted. Each diagram consists of models with one varying parameters, e.g.  $D/t$ ,  $t_r/t$ ,  $\theta$ , or  $x/L$ .

As it is demonstrated, reduction of ultimate strength shows a direct relationship with corrosion depth. This can be seen in the graphs of groups I, IV, and X. Occurrence of local buckling depends on the wall thickness and slenderness of the tubes. Development of local buckling is observed in the region of post ultimate strength for models of group IV. This group contains tubes with high ratios of  $D/L$ .





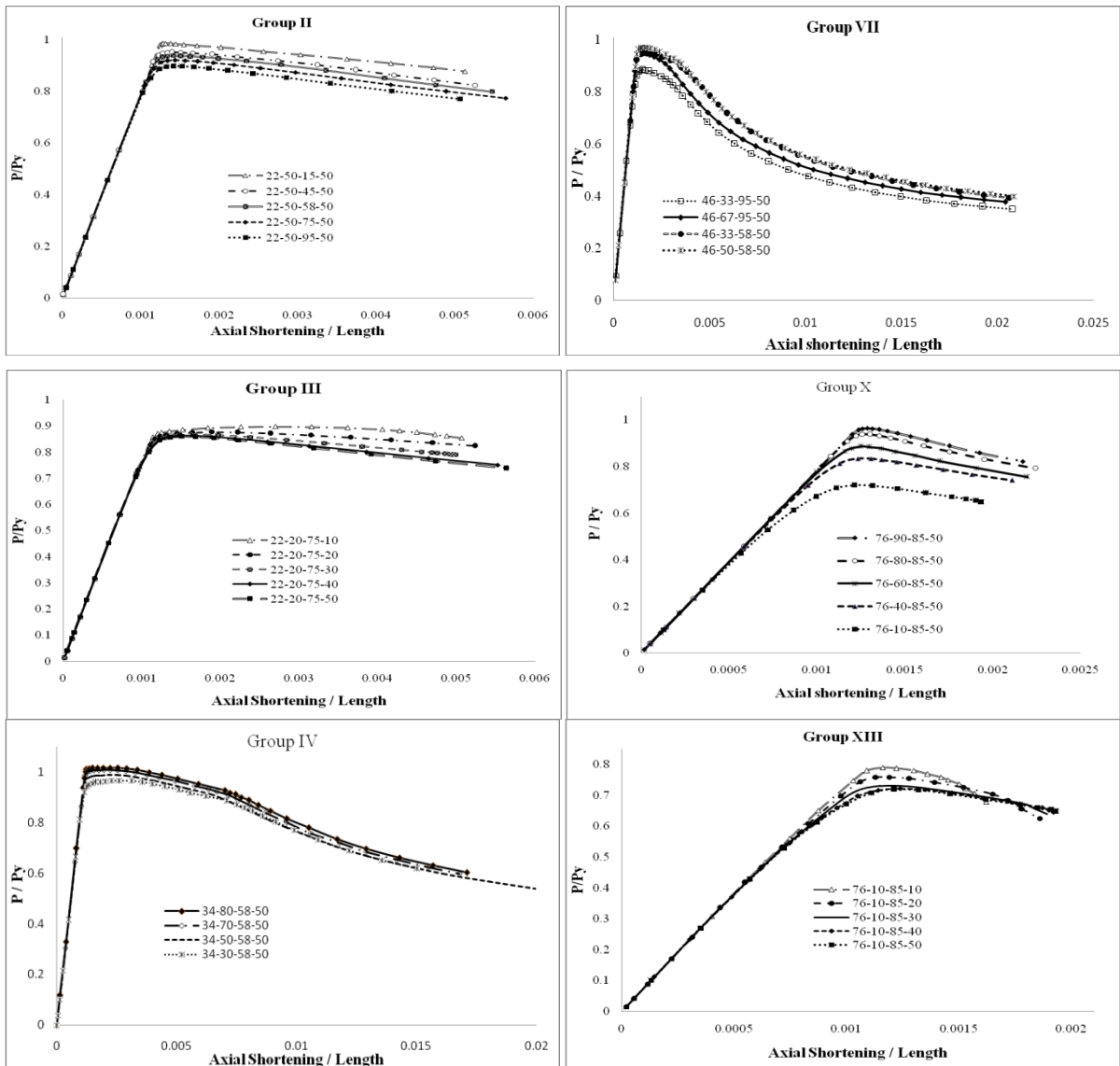


Figure 11: Behavior of numerical models under compression

The effect of damage location on behavior of tubes was studied on specimens of groups III, VI, and XIII. It was found that by approaching the location of corrosion towards the middle of tube, the ultimate strength is being reduced. Therefore the minimum ultimate strength occurs when the tube is centrally damaged.

Group II consists of specimens with subtended angle ranging from 15 to 95 degrees. Distribution of corrosion patch over the surface of tube causes the ultimate strength to be decreased. In other words as the corrosion takes place in the form of a distributed shape, eccentricity increases and combined with amplification of stress due to reduced cross section, ultimate strength decreases. On the other hand, in cases with confined corrosion patches, contribution of undamaged part of cross section is an important factor.

As mentioned earlier, initiation and development of local buckling play an important role on ultimate strength of tubes as well as their behavior at post ultimate strength state.

An important parameter to be investigated is the thickness ratio ( $D/t$ ). The effect of this parameter is demonstrated in diagram of group VII. In general, higher values of  $D/t$  make the tube to be more susceptible to local buckling at the corroded area.

## 5 RECOMMENDED DESIGN EQUATIONS

Estimation of ultimate strength by means of numerical methods is expensive and time consuming. As mentioned, a number of equations have been proposed by some researchers in order to provide an approximation of compressive strength.

A relation between ultimate strength of a locally corroded tube and dimensions of corrosion was proposed by Hebor (2002) as follows:

$$\frac{P_u}{P_y} = 0.052 \left( \frac{t_r}{t} \right) - 0.001 \left( \frac{D}{t} \right) - 0.0026(\theta) + 0.0028(\theta) \left( \frac{t_r}{t} \right) + 0.9998 \quad (1)$$

This equation applies only for tubes with the following characteristics:

$$0 \leq \frac{t_r}{t} \leq 1$$

$$34 \leq \frac{D}{t} \leq 100$$

$$58 \leq \theta \leq 311 \text{ (Deg.)}$$

This equation is based on modeling of short tubular members with length of 1.41 meter and therefore the equation is not comprehensive for a wide range of tubes. In order to obtain relationship between ultimate strength and geometric parameters, a parametric finite element study was performed.

The parameters  $t_r/t$ ,  $D/t$ ,  $\theta$ , and  $L/D$  represent the known regressors as one of them was variable while the other three were held constant. The unknown regression coefficients can be determined by solving linear equations derived from regression analysis; e.g. Fig. 12 and Fig. 13 show a linear relationship between the ultimate strength and the ratios  $L/D$  and  $D/t$ .

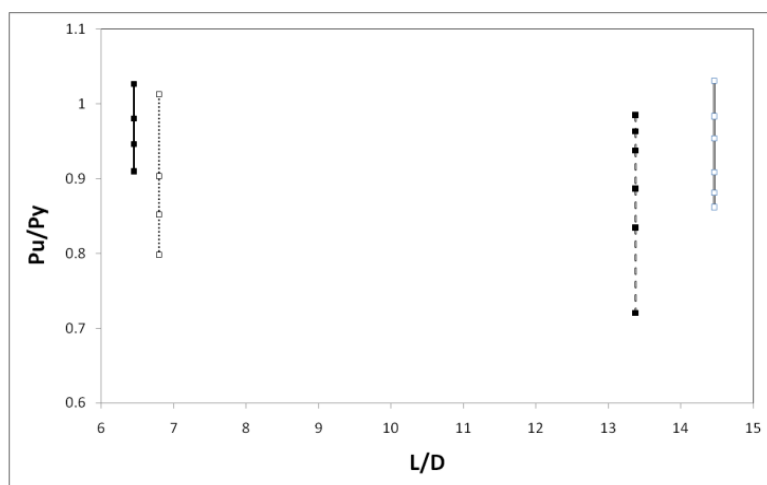


Figure 12: linear relationship between ultimate strength and tube slenderness ratio ( $L/D$ )

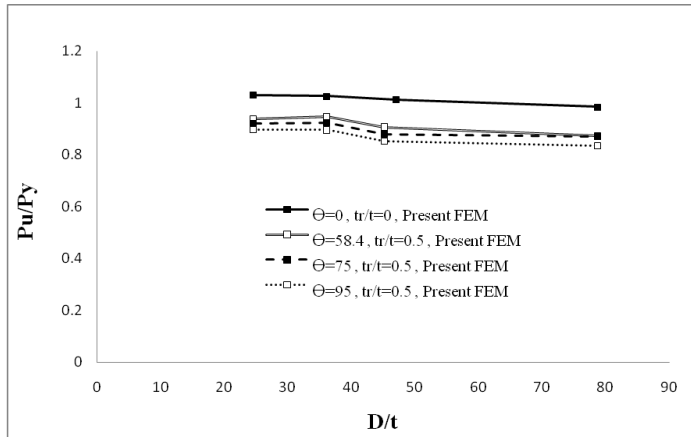


Figure 13: linear relationship between ultimate strength and the ratio D/t

According to numerical modeling and nonlinear analyses of several tubular members, the authors suggest the following equation to predict the residual strength of corroded tubes:

$$\frac{P_u}{P_y} = 0.021 \left( \frac{t_r}{t} \right) - 0.0011 \left( \frac{D}{t} \right) - 0.0007 \left( \frac{L}{D} \right) - 0.0022(\theta) + 0.0018(\theta) \left( \frac{t_r}{t} \right) + 1.0415 \quad (2)$$

Where:

$$0 \leq \frac{t_r}{t} \leq 1$$

$$22 \leq \frac{D}{t} \leq 100$$

$$15 \leq \theta \leq 360 \text{ (Deg.)}$$

$$1.41 \leq L \leq 15 \text{ (m)}$$

It is the aim of the authors to derive a semi-empirical formula that contains all parameters involved in the corrosion geometry, e.g. ratio of tube length to tube diameter. This parameter even has a significant effect on determination of ultimate strength for undamaged tubes. Thus it seems to be important to consider the effects of length in calculations of compressive strength for tubular members.

The accuracy of Eq. (2) for some typical cases is demonstrated in Fig. 14. Some results of present FEM in comparison with the Eq. 1 (Hebor (2002)) is shown in Fig. 15. As can be seen, a relatively good agreement is observed.

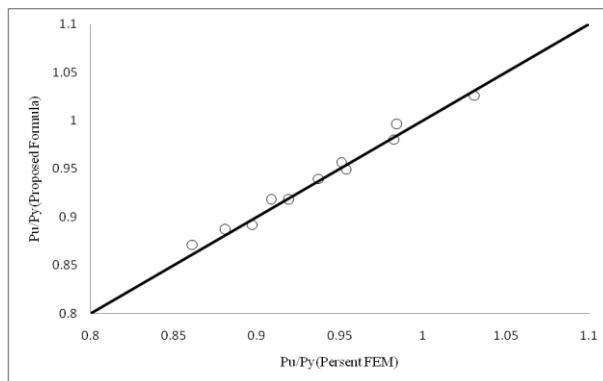
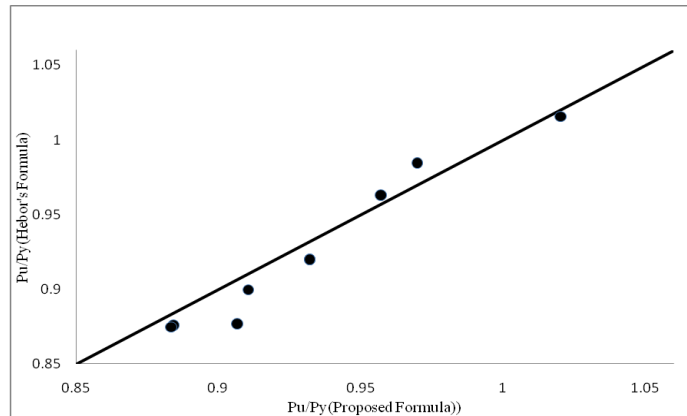


Figure 14: Accuracy of the proposed design equation (Eq. 2) in comparison with the finite element method.

Decreasing of ultimate strength due to corrosion can be expressed by assuming total reduction of wall thickness. In other words, ultimate strength of a tube with patch corrosion is the same as corresponding strength of an undamaged tube with a less wall thickness named as “Effective Thickness” or “Equivalent thickness”.

The authors propose the following equation to calculate effective thickness:

$$\text{Effective Thickness} = 0.278 \left( \frac{P_u}{\sigma_y D} \right) - 0.0162 \left( \frac{D}{L} \right) + 0.28 \left( \frac{P_u}{\sigma_y D} \right) \left( \frac{D}{L} \right) + 0.002461 \quad (3)$$



**Figure 15:** Accuracy of the proposed design equation (Eq. 2) in comparison with the recommended equation by Hebor (Eq. 1).

## 6 CONCLUSIONS

Nonlinear finite element method was adopted in order to investigate the response of locally corroded tubular members under axial compressive loading. First, a number of numerical models were developed and then they were verified against experimental test.

An idealized patch corrosion pattern was used and tubes with different dimensions were modeled. The effects of parameters involved in the patch corrosion were studied by analyzing tubes with different corrosion geometries.

It has been found that a significant reduction in capacity occurs due to formation of local buckling at the corroded region. Occurrence of local buckling is attributed to reduction of wall thickness as well as asymmetric cross section at the corroded region. The effect of corrosion location was studied and reduction of residual strength was reported.

Assessing the effects of corrosion location showed that the maximum reduction in the residual strength occurs when the corrosion is at the middle of tubes. The effect of corrosion depth was also studied as well as other parameters such as height, width, and subtended angle.

An analysis of regression was made and it showed the corroded tube capacity to have linear relation with the tube dimension parameter  $L/D$ . The proposed formulae for predicting ultimate strength and effective thickness apply to a wide range of tube dimensions. These equations give an initial estimation of compressive strength for engineers.

## 7 NOMENCLATURE

c	Corrosion width
D	Tube diameter
h	Corrosion height
L	Length of tubular member
$P_u$	Ultimate compressive strength of tube
$P_y$	Yield strength
$t_r$	Reduced thickness of tube wall
t	Original thickness of tube wall
x	Distance of center of corrosion from tube ends
$\vartheta$	Distribution of corroded region over the circumferential direction in degrees
$\sigma_y$	Yield stress

## References

- N-004. (1998). NORSOK STANDARD – Design of Steel Structure.
- RP 2A-WSD. (2000). API Recommended Practice 2A-WSD. Recommended Practice for Planning, Designing and Constructing Fixed Offshore Platforms-Working Stress Design.
- Moehlman, S., (1990). Testing and evaluation of damaged tubular jacket braces, Master of Science Thesis, Texas A&M University, USA.
- Lutes, L. D., Kohutec, T. L., Ellison, B. K., Konen, K. F. (2001). Assessing the compressive strength of corroded tubular members. *Journal of Applied Ocean Research* 23: 263-268.
- Yamane, M., Tanaka, K., Matsuda, B., Fujikubo, M., Yanagihara, D., Iwao, N. (2006). Residual strength evaluation of corroded steel members in marine environment. International Offshore and Polar Engineering Conference, California, USA.
- Matsushita, H., Yamamoto, T., Arai, H. (2002). Effect of Corrosion on Static Strength of Hull Structural Members. *J. Soc. Naval Arch. Japan* 192: 357- 365.
- Nakai, T., Matsushita, H., Yamamoto, N., Arai, H., (2004). Effect of corrosion on static strength of hull structural members (2<sup>nd</sup> report). *J. Soc. Naval Arch. Japan* 195: 221-231.
- Nakai, T., Matsushita, H., Yamamoto, N. (2004). Effect of corrosion on static strength of hull structural members (3<sup>rd</sup> report). *J. Soc. Naval Arch. Japan*, 195: 233-242.
- Nishimura, N., Takeuchi, S., Murakami, S., Jikuya, K. (2001). Ultimate Strength of Cylindrical Tubular Columns with Corrosive Damage. *JSSC* 9: 489-496.
- Okada, H., Kitaura, K., Sun, K., Yamaguchi, T. (2000). A simplified Method for estimating Deterioration of Collapse Strength of Damaged Structures Including Cylindrical Tubular members with Perforation. *JCOSSAR2000*. pp. 595-602.
- Lee, O. S. and Kim H. (2000). Effect of Corrosion in Pipeline on Failure Prediction. *International Journal of Korean Society of Precision Engineering* 1: 48-54.
- Michell, S., Fatt, H., Xue, J. (2001). Propagating Buckles in Corroded Pipelines. *Journal of Marine Structures* 14: 571-592.
- Bai, Y., (2003). *Marine Structural Design*, Elsevier.
- Hebor, M. F., and Ricles, J. M., (2002). Local buckling strength of patch corrosion damaged steel tubular bracing. *Journal of steel structures* 2: 59-70.
- Netto, T. A., Ferraz, U. S., Estefen, S. F. (2005). Effect of corrosion defects on burst pressure of pipelines. *Journal of construction steel research* 61: 1985-2004
- Netto, T. A., Ferraz, U. S., Botto, A. (2007). On the effect of corrosion defects on the collapse pressure of pipelines. *International journal of solids and structures* 44: 7597-7614.

Melchers, R. E. (2004). Pitting corrosion of mild steel in marine immersion environment – part 1; maximum pit depth, corrosion. *NACE international* 60: 824-836.

ANSYS Inc. (2008). *Ansys 11.0 reference Manual*.

Khedmati, M. R. and Nazri, M. (2012). A numerical investigation into strength and deformation characteristics of preloaded tubular members under lateral impact loads. *Journal of marine structures* 25: 33-57.

Zeinoddini, M., Harding, J. E., Parke G. A. R. (2008). Axially pre-loaded steel tubes subjected to lateral impacts (a numerical simulation). *International journal of impact engineering* 35: 1267-79.

Ostapenko, A., Wood, B., Chowdhury, A., Hebor, M. (1993). Residual strength of damaged and deteriorated tubular members in offshore structures. *ATLSS Reports*, paper 184.

Shariati, M. and Rokhi, M. M. (2008). Numerical and experimental investigations on buckling of steel cylindrical shells with elliptical cutout subjected to axial compression. *Thin Walled Structures* 46: 1251-1261.

Mottran, J.T. and Shaw, C.T. (1996). *Using finite elements in mechanical design*, McGraw-Hill.