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ARTICLES

MATHEMATICAL MODELING OF TOW TRUCKS LOCATION CONSIDERING TRAFFIC CONGESTION QUEUE

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ABSTRACT. We propose a mathematical model including queueing theory to reduce the negative consequences generated by large queues of vehicles as a result of traffic incidents. The mathematical model maximizes the number of incidents' responses, and minimizes the non-recurrent congestion generated by them, considering a predetermined time to clear them, and an average or maximum response time. The model is applied to real-world incident data from Avenida Brasil, Rio de Janeiro, and the results showed that it is possible to concentrate the attendance in the critical areas of the expressway. The mathematical model proposed in this work can be used as a decision tool to help the government defining the acceptable average time for incident response, and the number of tow trucks necessary to achieve a pre-established service level.

Keywords: incident response, location problem, queueing theory.

1 INTRODUCTION

Traffic congestion is routine in large cities and the occurrence of incidents further aggravate this scenario. The adoption of measures aimed to the quick removal and rapid unblocking of roads is essential to reduce the anti-economic effect generated by these problems.

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According to Jeong et al., 2011, in the case of expressways, the effects of incidents are generally more severe, since these roads are characterized by a large flow of vehicles that travel at high speed. It results in accidents with serious injuries or even fatalities, besides causing an extensive traffic congestion and long delays.

The influence caused by traffic incidents is proportional to the time they remain on making the circulation difficult. To clear an incident, it is required, for example, changing flat tires, offering gasoline to drivers and removing broken down vehicles. Incidents occur in uncertain ways in terms of time, location and speed, which result in a stochastic demand being attended by a finite number of available resources.

Traffic incidents are recognized as one of the main sources of traffic jam and, very often, they lead to the reduction of travel time reliability, a key measure of road performance (Hojati et al., 2016), because of the delays generated. Therefore, the assessment of the travel time reliability became important in transportation planning and decision making (Xiao et al., 2017). In cities with consolidated networks, the perspectives of travel time economy, which were used to be the main source of socioeconomic benefits for transport projects, are now limited.

Therefore, re-establishing full capacity of the road in the shortest possible time is one of the main points of incident management. As a consequence, it is important to identify and quantify the factors that have influence on the time to clear the incidents to benefit management and, consequently, mitigate the impact of non-recurrent congestion (Ma et al., 2017). In this context, tow trucks play an essential role in incident management, since they are responsible for removing vehicles from the road, assisting agents and protecting drivers from broken down vehicles.

Incidents, on the other hand, can occur in several sections of an expressway and have different clearance times, then it is necessary to allocate the tow trucks appropriately so broken down vehicles can be attended as soon as possible.

The main objective of this paper is to develop a mathematical model including queueing theory for tow trucks allocation, aiming to reduce the negative consequences produced by the incidents, and considering the capacity reduction of the expressway and the places with longer queues of vehicles. More specifically, the mathematical model maximizes the number of incidents' responses, while minimizing the non-recurrent congestion generated by them, considering a predetermined time to clear the incidents, and an average or maximum response time. As a case study, this model was applied to real incident data from Avenida Brasil, an expressway in the city of Rio de Janeiro, made available by the Traffic Engineering Company of the city (CET-Rio).

In multi-objective problems, we do not have only one solution, but a set of them, which is called the Pareto Front. The weighted function method can be used to solve a multi-objective problem, which consists of converting the original problem into a scalar single-objective problem, using different weights for each objective (Obal et al., 2013). We used this approach to evaluate the objectives presented in the problem faced in this paper.

The contributions presented in this work aim not only to help public managers to use the available resources in the best way, but also to enhance the population's life quality, trying to improve the travel times due to non-recurrent congestion.

The remainder of this paper is organized as follows. Section 2 presents a literature review about the importance of incidents management and about the location problems associated with tow trucks that are used to clear incidents. Section 3 describes the proposed mathematical model. Section 4 presents a case study with the description of the area of study, application of the proposed model and the results obtained. Finally, Section 5 highlights the conclusions and presents some recommendations for future works.

2 LITERATURE REVIEW

An incident can be defined as any occurrence in a road that obstructs the normal traffic flow and its duration can be divided into four phases: detection time; response time (time between incident detection and support arrival); clearance time of the incident (time between arrival of the support team and the end of the clearance service); and recovery time (time between the end of the cleaning service and the reinstatement of the road to the normal condition) (TRB , 2010).

According to Steenbruggen et al. (2013), in practice, incident management is a set of measures that aims to minimize the negative effects on safety conditions and traffic flow. According to Zou et al. (2006), the traffic incidents management is important for transport agencies. Besides the injuries, fatalities and medical costs, congestion, delay and pollution caused by incidents are a problem to travelers and to environment.

The response and clearance times are of particular interest to agencies, because they represent the fraction of the duration of the incident that is directly impacted by the response strategy adopted. Therefore, the factors related to these times are critical in developing a response plan (Zou et al., 2006).

Li and Walton (2013) have presented a simulation model which shows that the size of the patrol area reflects more on the cost-benefit rate of tow trucks than on the number of vehicles.

Thus, there are some initiatives about incident management presented in the literature. According to Nitsche et al. (2016), in United States, recent projects try to reduce clearance time and secondary accidents. In Australia, the emphasis is on developing policies and laws that provide more power to the agencies that take care of the service of clearing the incident, so they can reinstate the road to normal conditions quickly. In areas with heavy traffic networks such as Dubai, Singapore, Hong Kong and Japan, the focus is on remote monitoring, traffic information and the use of intelligent transport systems.

In Netherlands, the incident management depends on the cooperation between Dutch road authority and other emergency response services. Together, they created new guidelines and protocols to shorten the time that is necessary to clean the road after incidents. In 2013, they were on the integration phase, where the focus of this approach is to provide traffic information services to road users in case of incidents (Steenbruggen et al., 2013). This phase continues until today, giving approach to the dissemination of the information, like the case of the City alert that is in implementation in Amsterdam (AmsterdanSmart, 2018).

To reduce the clearance time of the roads affected by incidents, the tow trucks must be well located close to the places that historically present larger problems, with well-defined service areas. This is a well-known characteristic of the location problems, such as p-median and maximum coverage. The original goals of these types of problems have always focused on the economic view of the problem, such as to minimize the number of facilities allocated, to minimize the fixed costs, or to maximize service. Nowadays, the decision-makers are also concerned to consider social and environmental factors, such as land use, traffic jam, noise, and life quality (Zanjiran et al., 2010). Moreover, it is possible to note that location problems that mix other aspects, for example, Queueing Theory, received considerable attention (Hajipour and Pasandideh, 2011).

The p-median problem is a classic facility location problem which consists in locating p medians (centers) in a network, to minimize the sum of the distances of each vertex to the nearest center. The first formulations of this problem were presented by Hakimi (1965). In this context, the covering problems aim to determine the minimum number of facilities in such a way that each demand point is attended by at least one facility.

These two problems are being considered for locating bases of incident response. For example, Yin (2006) sought to maximize the effectiveness of incident response services considering the configuration of accidents and the available number of vehicles. The objective was to minimize the travel time of the service vehicles considering the reduction of road capacity due to the incident.

In Pal and Bose (2009), a model for locating response units for incidents was proposed. The study considered the fixed and variable cost of vehicles and depots, in addition to the incidents of major or minor severity depending on the type and a reliability constraint, defined by probability of the incident being dealt immediately.

In Lou et al. (2011), it is studied the problem of using tow trucks to detect, respond and clear traffic incidents in two scenarios: deterministic and stochastic. The deterministic configuration assumes that there is only one scenario of incidents whereas the stochastic considers several scenarios, each one with a given probability. The main objective of both problems is to minimize the total response time.

Also looking to reduce the incident response time, Ozbay et al. (2013) proposed different scenarios considering the probability of incidents occur and the quantity of available service vehicles. In their mathematical model, the authors considered locations for tow trucks and depots and they introduced a concept related to service quality that offers flexibility to evaluate the possible impacts caused by the incidents.

Considering a stochastic approach, Zhu et al. (2018) studied the tow trucks allocation in a problem that allowed the possibility of occurring up to two incidents at the same time. The authors aimed to reduce the total clearance time which involves both occurrences. In addition, the method seeks to define the service areas of each tow truck.

In Adler et al. (2014), it is considered that the service vehicles must be close to the places with incidents and close to the places where many traffic violations occur, because in this case the tow trucks also supervise the vehicles. The authors considered a mathematical formulation to maximize a value of importance attributed to the place serviced. The tow trucks were located in points close to places with incidents and large traffic flows.

The demand for service can generate queues for the tow trucks, which cause delays in the response time. So, Geroliminis et al. (2009) studied a model for allocation of emergency vehicles in urban roads that considers the fact that the service vehicle is not available when required. The model was based on the maximum coverage problem and considers the difference between incident types. It proved to be effective at peak times when queues are formed. Out of the peaks, the authors used the the p-median model.

The maximum coverage problem with an approximate queueing model is used in Akdogan et al. (2017) to study the location of any service of emergence. Specific service rates were defined depending on the location of the demand and the location of the emergency vehicles.

Dunnet et al. (2019) sought to optimize the incidents response by proposing a method that aims to find an efficient allocation of police patrols considering the severity of the incident and the priority of the occurrence, due to the time restriction, combining mapping and routing algorithms.

Different of the papers published in the literature, the mathematical model proposed in this article aims to reduce the negative consequences produced by the incidents considering the capacity reduction of the road and the places with larger queues of vehicles. Besides, the mathematical model seeks to maximize the quantity of incidents responses providing a medium time to clear the incidents. There is a trade-off involving the minimization of the congestion queues and the maximization of the incidents' responses, since the highest number of incidents cannot occur in the section that generates the larger congestion queues. Note that the size of the congestion queue is related to the hourly traffic flow in the section and to the section capacity. For example, an incident occupying a lane can lead to a small congestion queue if the relationship flow/capacity of the section is small. Therefore, when the model seeks to maximize the incidents' responses, it can attend more incidents in locations presenting lower flow of vehicles and higher speeds to achieve its goal.

3 MATHEMATICAL MODELING

The tow trucks have several uses. Besides towing vehicles, they can protect traffic agents at the scene of the incident, protect drivers with broken down vehicles and provide road assistance services. Their performance must consider some factors such as quantity of vehicles, coverage area, hours of operation, time of arrival to the incident, time of service, and time to reach the depots with the towed vehicle (Li and Walton, 2013; Wu et al., 2014).

While in the private sector the main objectives are to minimize costs and maximize profit, in the public sector it is also important to maximize the benefits to society (Revelle et al., 1970). In this sense, the mathematical model proposed in this paper aims to assist the public sector in the decision process involving the best location of the tow trucks in order to reduce the social and economic impacts generated by the incidents, due to the queues of congestion and loss of quality of life of the affected population. Real situations often include optimization problems with conflicting objectives (Antequara et al., 2020) which consider: minimize the congestion queue and maximize the number of incidents served. The average time in the system, provided by the model $M/M/1$ of Queueing Theory, was considered as a constraint in the mathematical model to define an expected level of service, represented by the average response time. Figure 1 presents a summary of the main aspects presented in the model.

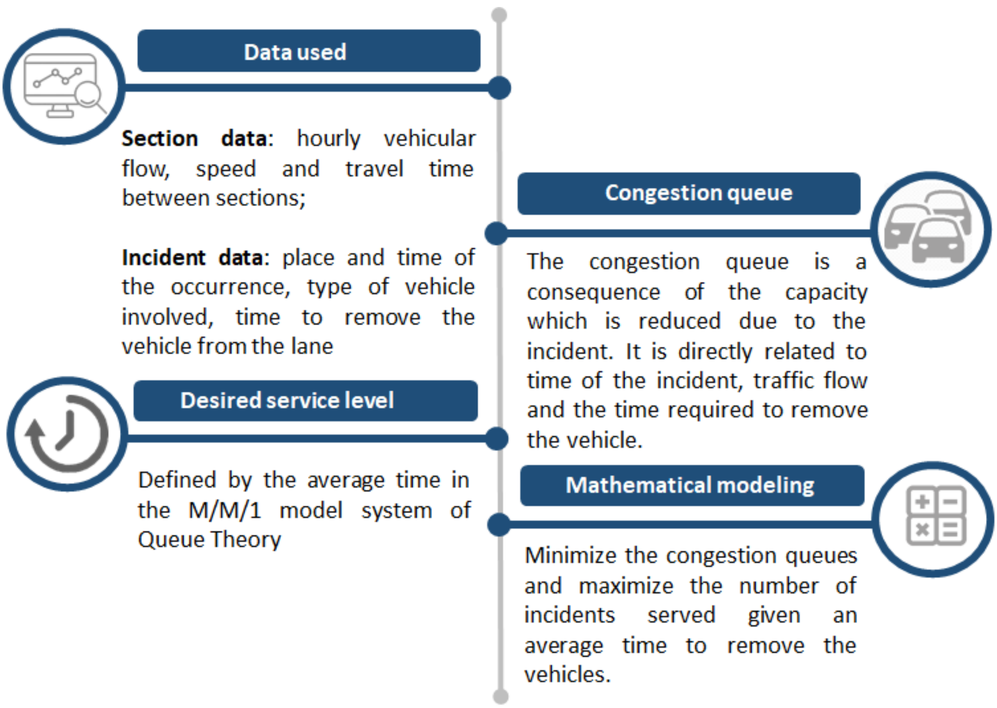

Figure 1 – Summary of the main aspects presented in the model.

Source: the authors.

Thus, consider an expressway divided into homogeneous sections (or places) defined according to the traffic flow and the number of traffic lanes. Let S be the set containing all these sections. Each section $r \in S$ is associated with a set I_r of incidents occurred in the historical period of study, where $|I_r|$ is the total amount of incidents that occurred in section r. Thus, all incidents of the expressway can be represented by the set $\Gamma = \bigcup_{r \in S} I_r$. Let $I_{\max} = \max_{r \in S} \{|I_r|\}$ be the largest number of incidents occurred in any section $r \in S$. Finally, $It = \sum_{r \in S} |I_r|$.

Assuming that the tow trucks can be located in any section $s \in S$, let T_{is} be the duration time of incident $i \in I_r, r \in S$ ($r \neq s$ or $r = s$, it does not matter) when i is serviced by a tow truck located at $s \in S$. Also consider that each section $s \in S$ can only be served by a maximum of one tow truck. The duration time T_{is} is defined by (i) the detection time, (ii) the travel time of the tow truck located at section $s \in S$ until the section $r \in S$ plus (iii) the clearance time of incident i.

Each section $s \in S$ is associated with a flow of vehicles V_s and with a number of incidents ($|I_r|$). The incident can lead to a congestion queue in s due to the reduction of the road capacity, depending on its duration and on the traffic flow. According to TRB (2010), a traffic incident can considerably reduce the capacity of the road even in its free lanes, due to the change of the drivers' behavior. For example, an incident blocking one of the three traffic lanes of an expressway, reduces capacity around the incident by approximately 51%. So, if the traffic flow at the time that occurred the incident is higher than the road capacity during the incident, a long congestion queue is formed.

Therefore, let Q_{rs} be the average size of the queue of congestion caused by the incidents $i \in I_r$ occurred in section $r \in S$, when serviced by a tow truck located at section $s \in S$. The queues are formed due to the lack of capacity of the system to serve the traffic flow that arrives in the section and they increase with the incident duration. Considering C the reduced road capacity in section r, Q_{rs} can be calculated based on Queueing Theory, see Equation 1. In this case, the arrival rate of the traffic flow and the service rate of the road (its reduced capacity) were considered deterministic. Although the congestion queue is linked to the section, the queues were calculated considering the duration of the incident and the hourly flow, so a queue may extrapolate the section size.

$$Q_{rs} = \frac{\sum_{i \in I_r} \lfloor V_s T_{is} \rfloor - \left\lfloor \left(T_{is} - \frac{1}{V_s} \right) C \right\rfloor}{|I_r|}, \tag{1}$$

The largest queue ($Q_{\max}$) generated in the whole system can be obtained as follows: $Q_{\max} = \max_{r,s \in S} \{Q_{rs}\}$. It is important to note that even with a high number of tow trucks there will be queues of congestion due to the time taken to remove the vehicle from the lane, so the queue can be reduced, but it will exist whenever there is a flow greater than the capacity.

However, some sections may not have a tow truck to clear the incident, which causes the occurrence of an incident to have a negative impact on overall traffic with queues of congestion of unknown size. Let I_n, $\forall n \in S$, be a set of not serviced incidents. Thus, the unknown size of this respective queue is approximated by γ_r, which is defined as the highest value among all Q_{rs} calculated for all section $s \in S$ candidate to receive a tow truck. Since this type of event should be avoided, the respective term will be penalized in the objective function of the mathematical model proposed in this article, as it will be seen later.

In order to better serve the population, the tow trucks should be located so as to serve as many incidents as possible. Considering that the incidents I_r, $\forall r \in S$, occur in a stochastic way, and that the processes of incident response by the tow trucks present the same behavior (Ozbay et al., 2013), the whole process can be represented by the $M/M/1$ model of the Queue Theory. In this model, the time between the incidents and the duration time of the incidents can be represented by exponential probability distributions, besides each section can only be serviced by one tow truck.

Let τ be the incidents' average duration time expected as a level of service, λ_r be the average incident rate associated with section $r \in S$, which is calculated by dividing 1 by the product between $|I_r|$ in section $r \in S$ and the analysis time, and $\mu_{rs} = \frac{1}{|I_r|}\sum_{i \in I_r} \frac{1}{T_{is}}$ be the average service rate when the tow is located at section $s \in S$, as shown in Fig. 2. The Queue Theory models allow us to relate the average incident rate per section with the average service rates (which depend on where the tow truck is located) and the incidents' average duration time, as it will be seen later in the mathematical model.

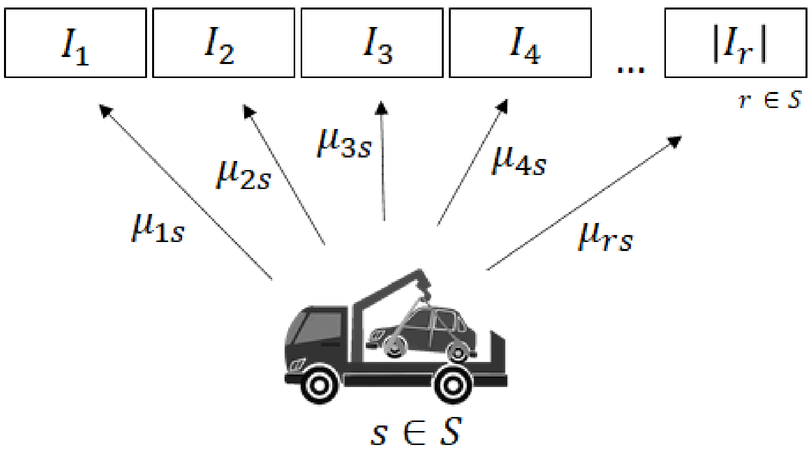

Figure 2 – Incident management scheme.

Source: the authors.

Due to operating and budget constraints, not all sections will receive tow trucks, thus suppose that p sections should be selected to receive the tow trucks. The mathematical decision variables are binary and defined as follows: $y_s \in \{0,1\}$ determines whether a tow truck should ($y_s = 1$) or should not ($y_s = 0$) be located at section $s \in S$; and $x_{rs} \in \{0,1\}$ indicates whether the section $r \in S$ is ($x_{rs} = 1$) or is not ($x_{rs} = 0$) serviced by a tow truck located at section $s \in S$. The notations used in the mathematical problem are shown in Table 1.

The problem of allocating tow trucks for incident handling can be mathematically modeled as follows:

$$Maximize \sum_{r \in S}\sum_{s \in S} \frac{|I_r|}{I_{\max}} x_{rs} - \sum_{r \in S}\sum_{s \in S} \frac{Q_{rs}}{Q_{\max}} x_{rs} - \rho \sum_{r \in S} \frac{\gamma_r}{Q_{\max}} \left(1 - \sum_{s \in S} x_{rs}\right) \tag{2}$$

Table 1 – Model input data.

Notation	Description
I_r	Set of incidents occurred in section $r \in S$ obtained from historical data
$I_{\max}$	Largest number of incidents occurred in any section $r \in S$
It	Total number of incidents of all section $r \in S$
T_{is}	Duration time of a incident $i \in I_r$ serviced by a tow truck located at $s \in S$
τ	Expected incidents' average duration time
V_s	Traffic flow at section $s \in S$.
Q_{rs}	Average size of the congestion queue caused by the incidents occurred in section $r \in S$ served by a tow truck located at section $s \in S$
$Q_{\max}$	Largest congestion queue
λ_r	Average incident rate associated with section $r \in S$
μ_{rs}	Average service rate when the tow truck is located at section $s \in S$
γ_r	Highest value among all Q_{rs} for all section $s \in S$.
p	Number of sections to receive the tow trucks

subject to:

$$\sum_{r\in S} x_{rs} \leq 1, \qquad \forall s \in S \tag{3}$$

$$\sum_{s\in S} y_s = p \tag{4}$$

$$\sum_{r\in S} |I_r|\, x_{rs} \leq \left(\frac{It+1}{p}\right) y_s, \qquad \forall s \in S \tag{5}$$

$$\sum_{r\in S} \mu_{rs} x_{rs} \geq \sum_{r\in S} \frac{x_{rs}}{\tau} + \sum_{r\in S} (\lambda_r x_{rs}), \qquad \forall s \in S \tag{6}$$

$$x_{rs} \in \{0,1\}, \qquad \forall r, s \in S \tag{7}$$

$$y_s \in \{0,1\}, \qquad \forall s \in S. \tag{8}$$

The normalized objective function (2) has three distinct terms, aiming, respectively, at: (i) maximizing the number of serviced incidents, (ii) minimizing the queue of congestion generated by the serviced incidents, and (iii) minimizing the queue of congestion generated by not serviced incidents. The last term is penalized by a weight ($\rho > 1$) since the size of the queues generated by not serviced incidents is unknown.

With respect to the constraints, Constraints (3) ensure that at most one tow truck can be allocated to serve a section. Constraints (4) guarantees that exactly p tow trucks will be located. Constraints (5) are of the knapsack type and allow to balance the amount of incidents among the tow trucks. Constraints (6) are based on M/M/1 model to relate the average incident rate per section with the average service rates (which depend on where the tow truck is located) and the incidents' average duration time, ensuring that this latter is respected. Finally, the Constraints (7)-(8) are related to the domain of the decision variables.

If we consider a probability β for the incident I_r in the homogeneous section $r \in S$ to be serviced in a maximum time Υ, and no longer in the mean time of service, Constraints (6) are now written as in Marianov and Serra, 1998:

$$\sum_{r\in S} \mu_{rs} x_{rs} + \frac{x_{rs}}{\tau}\, ln(1-\beta) \leq \sum_{r\in S} (\lambda_r x_{rs}), \qquad \forall s \in S. \tag{9}$$

Constraints (9) will be used in some scenarios of the case study presented later in this paper.

In multi-objective optimization problems, many objective functions have to be optimized simultaneously and hardly have a single solution that minimize all of them at once, so it is important to consider a set of non-dominated solutions (Pareto Front) (Fukuda and Drumond, 2014).Therefore, when analyzing the mathematical model (2)-(9) from the bi-objective perspective, a weight $0 \leq \alpha \leq 1$ can be considered which generates the mono-objective function (10). Solving a series of optimization problems varying weights systematically, we can obtain the Pareto set (Huang et al., 2002), this is a traditional method for multiobjective optimization (Kim and Weck, 2005). When $\alpha = 1$, it maximizes the response to incidents. In an opposite way, when $\alpha = 0$, it minimizes the congestion queues. For $0 < \alpha < 1$, the trade-off between response to incidents and congestion queues can be analyzed.

Noteworthy, the results obtained through this method are strongly dependent on the adopted weights. Furthermore, it is important that all functions are expressed in such a way as to assume close numerical values and when there are maximization and minimization objectives together, all objectives need to be converted to just one criterion, for example, minimization. The advantage of this method is that it is intuitive and easy to use and guarantees to find the optimal set of Pareto solutions (Deb, 2001).

$$Maximize \quad \alpha \sum_{r\in S}\sum_{s\in S} \frac{|I_r|}{I_{\max}} x_{rs} - (1-\alpha)\left(\sum_{r\in S}\sum_{s\in S} \frac{Q_{rs}}{Q_{\max}} x_{rs} - \rho \sum_{r\in S} \frac{\gamma_r}{Q_{\max}}\left(1 - \sum_{s\in S} x_{rs}\right)\right) \tag{10}$$

4 CASE STUDY

The City of Rio de Janeiro is the second largest Brazilian metropolis and has specific geographical characteristics: its urban center is moved to the east and in the geographic center of the city there is the Tijuca massif. Because of this configuration, the travelled distances are large and there are few connection routes to the Center. Avenida Brasil is the main expressway of the city with 58 kilometers of extension. The four highways that give access to the city (Rio-Santos, Presidente Dutra, Washington Luiz and Ponte Rio-Niterói) converge on Avenida Brasil which is also the main link between the West and North Zones to the center, giving acess to the Port of Rio de Janeiro. The average daily traffic flow is 210,000 vehicles and aproximately 770 incidents are serviced per month. The services are carried out by different types of tow and by motorcyclists who can reach the place faster and assist the roadside in the marking.

Nowadays, the city suffers from the very high traffic of vehicles on its high-capacity roads, especially at peak times. In addition, the large number of incidents directly impacts daily urban mobility. Currently, these incidents are attended by tow trucks, motorcycles and trucks of CET-Rio, the agency responsible for planning, coordinating and controlling the flow of pedestrians and vehicles in the City of Rio de Janeiro.

These incidents directly interfere the travel time of all travelers and increase road congestion. Based on data from CET-Rio and the Transport Master Plan for the Metropolitan Region of Rio de Janeiro, Fig. 3 shows the forecast for queue congestion growth (in km) in the City of Rio de Janeiro, according FIRJAN (2014). Congestion can reach 182 kilometers in 2022 if no new investments in transport infrastructure are made, considering the projections of population growth and vehicle fleet (FIRJAN ,2014). Besides affecting urban mobility, congestion also causes great economic losses, estimated at R$ 40 billion in 2022, as shown in Fig. 4.

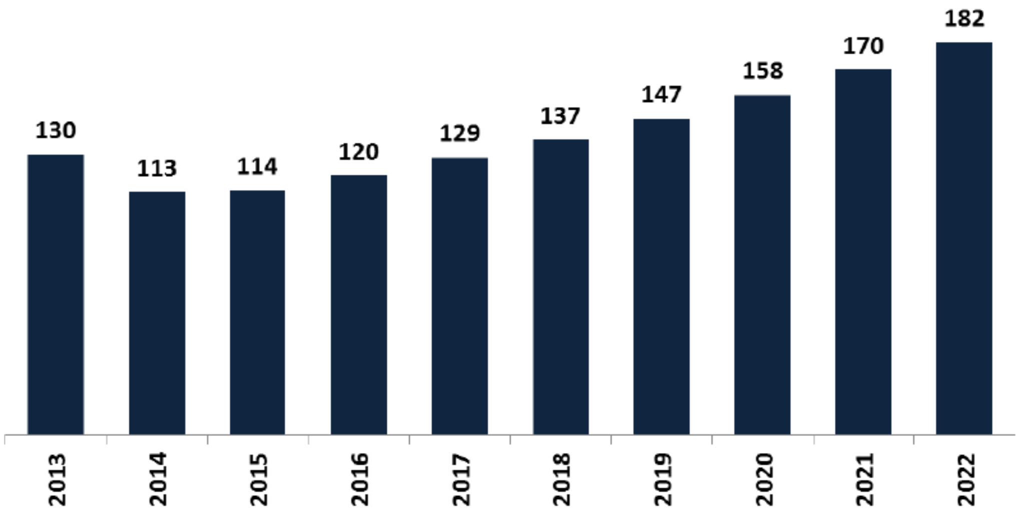

Figure 3 – Congestion evolution in the metropolitan region of Rio de Janeiro (km).

Source: FIRJAN, 2014.

At the moment, the detection of incidents in Avenida Brasil is made in three ways: cameras of monitoring, patrolling, and by means of a telephone number made available to the population. After the incident detection, a vehicle is sent for servicing it. It may be a tow truck, a pickup or the motorcycle itself doing the patrolling, it depends on the type of incident and on the vehicle(s) involved. For example, trucks can only be serviced by super-heavy tow truck, but broken down vehicles can be pushed by a pickup, which makes the traffic reinstatement faster. It is important to emphasize that, as a standard procedure, the first service is performed by the motorcycle, regardless of the type of incident, since motorcyclists are responsible for signaling and protecting the scene of the incident.

In this case study, we are using the database of a public service that has tow trucks to clear the incidents on the main roads of the city, including Avenida Brasil. In the city, there are also private services of incident responses, but we do not have access to their databases. It is important

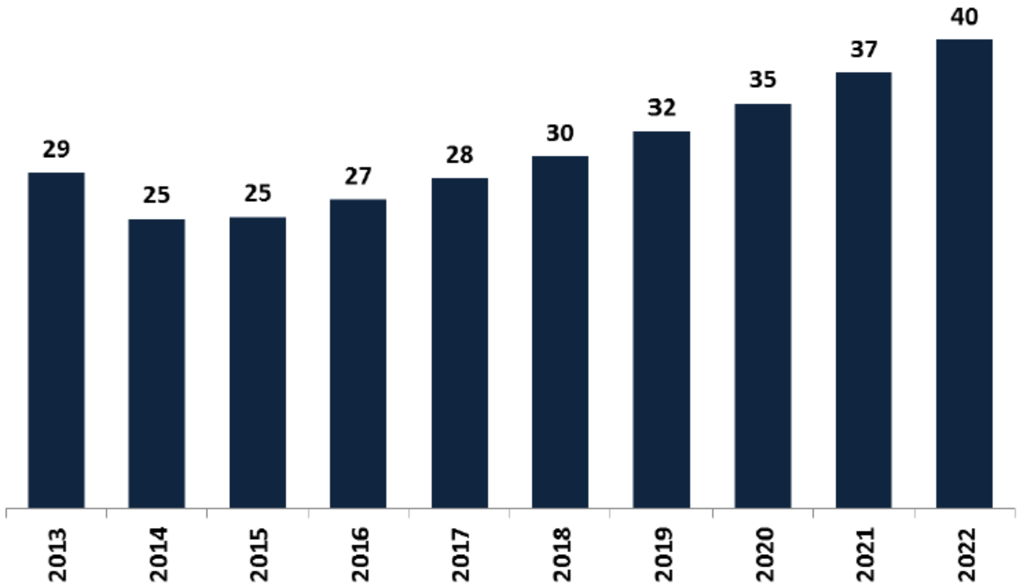

Figure 4 – Congestion evolution cost in the metropolitan region of Rio de Janeiro (R$$\times 10^6$).

Source: FIRJAN , 2014.

to emphasize that most incidents attended are broken down vehicles, so a simplification was made considering that all incidents occupied only one traffic lane and required a tow truck for assistance. In addition, it is possible to service more than one vehicle by the same tow truck.

4.1 Data sampling

The data characterizing the incidents were collected during four months of 2018, from March to June, once the incidents began to be registered with greater detail of location in March 2018, being possible georeferencing them. The data available for the study case were: place of occurrence, time of occurrence, type of incident, time of service and vehicle used in the service.

The concentration of these incidents along the Avenida Brasil is shown in Fig. 5. There is a correlation between the size and the color of the spot: larger and redder spot, greater number of incidents located in that region. According to Fig. 5, closer to the city center, greater the concentration of incidents. In addition, this region is also the one with the highest traffic flow, which corroborates the importance of focusing attention mainly in this region.

There are 3080 incidents distributed along Avenida Brasil during the period of four months, March to June of 2018. These occurrences involve broken down vehicles (61%), accidents (17%), flat tires (9%), vehicle abandonment (1%), lack of fuel (1%) and others (2%).

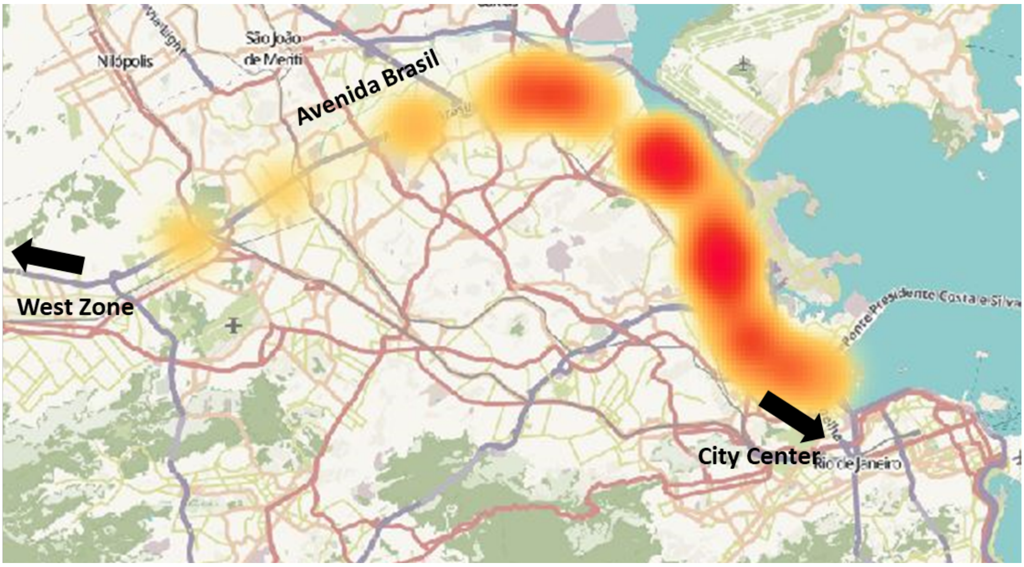

Figure 5 – Heatmap of the incidents occurred in Avenida Brasil - March to June of 2018.

Source: the authors.

In 2018, one light tow truck, two heavy tow trucks and one super heavy tow truck operated at Avenida Brasil. The light tow truck was responsible for the highest number of services, as shown in Fig. 6.

In addition, each tow truck has an average time of service, according to Table 2. The super-heavy tow truck, which services trucks, takes longer time of service than the light, that services passenger vehicles and motorcycles, for example.

Table 2 – Average time of service to an incident, per type of tow truck.

Type of tow truck	Average time to response (minutes)
Light	21
Heavy	25
Super-heavy	34

As stated in CET-Rio (2018), 75% of the vehicle flow in Avenida Brasil is of passenger vehicles, 10% of motorcycles, and trucks and buses correspond to 7% and 8%, respectively.

Data from vehicle counts obtained by the electronic surveillance system were used for the calculation of the congestion queue size. The travel times between the tow trucks and the places of occurrence were calculated by means of the average speed and the geographic distance, both obtained through Google Maps data. The incident rate and the service rate were represented by Poisson distributions, with r-squared bigger than 0.9 and 0.88, respectively.

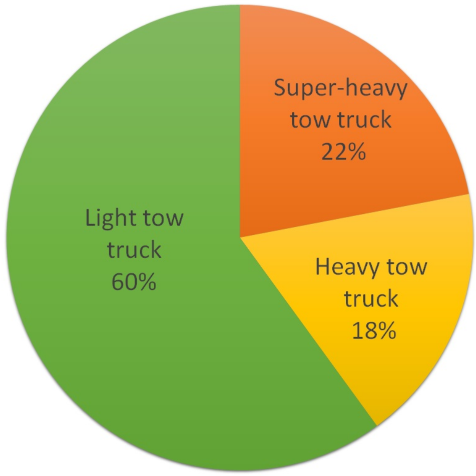

Figure 6 – Numbers of responses per type of tow truck in Avenida Brasil - March to June of 2018.

Source: the authors.

The probability of occurring two or more incidents at the same time in the expressway was calculated based on the M/M/1 queueing model. The highest probabilities calculated considering any possible tow trucks and incident locations were 0.060% for light tow trucks, 0.005% for heavy tow trucks, and 0.012% for super-heavy tow trucks. Therefore, the probability of secondary accidents was disregarded.

As shown in Fig. 5, Avenida Brasil has different characteristics regarding the flow, speed and frequency of incidents, and these characteristics still vary according to the way of the expressway (to the center of the city or to the west zone of the city). This motivated its division in 32 sections, being 16 sections by each way, for the possible locations of the tow trucks: Caju, Benfica, Fiocruz, Bonsucesso, Maré, Olaria, Penha, Trevo das Missões, Parada de Lucas, Margaridas, Irajá, Acari, Coelho Neto, Guadalupe, Deodoro and Transolimpica, as shown in Fig. 7. The number of incidents per section and per way is shown in Fig. 8. The mathematical model proposed in this article pointed the best place to allocate the tow truck to maximizes the quantity of incidents responses and minimizes the non-recurrent congestion generated by the incident, considering 32 sections as possible candidates.

The incident duration is represented by the sum of the detection, response time and the clearance time of the incident. To consider the detection time, after contacts with specialists, it was suggested that 10 extra minutes should be added. The response time was calculated based on all possible locations (any of the 32 sections), the average speed at the time of the incident, and the geographic distance between the possible tow truck location and the location of the incident.

Finally, the clearance time was based on the field notes made by the traffic operators.

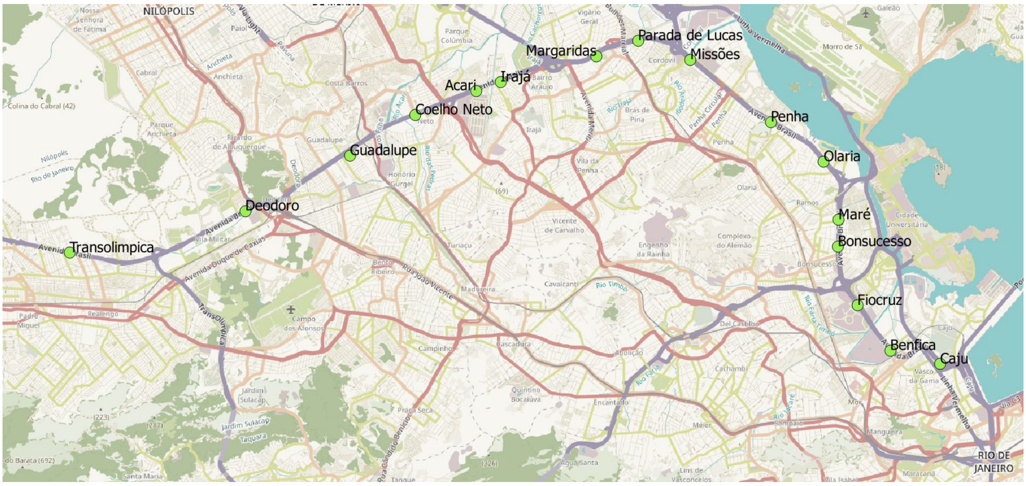

Figure 7 – Possible places to locate the tow trucks in Avenida Brasil.

Source: the authors.

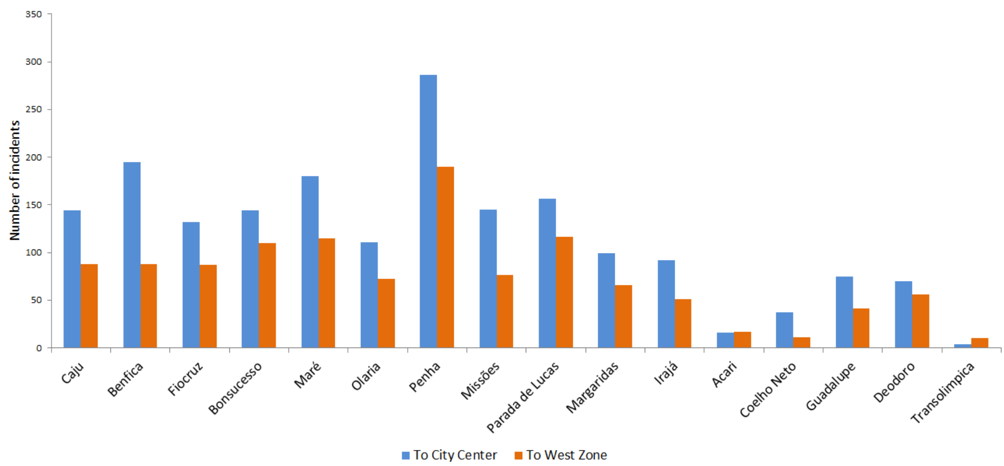

Figure 8 – Number of incidents per section and per way occurred in Avenida Brasil - March to June of 2018.

Source: the authors.

4.2 Proposed scenarios

Table 3 shows the vehicles serviced by each type of tow truck. According to FHWA (2010), an incident is considered intermediate when it affects a traffic lane between 30 minutes and 2 hours. This parameter was used to define the average and the maximum time of permanence of the incident in the system for each type of tow truck.

Table 3 – Types of vehicle serviced by each type of tow truck.

Type of tow truck	Type of vehicle
Light	Cars and motorcycles
Heavy	Buses and small trucks
Super-heavy	Big trucks

Based on the information provided, seven scenarios were created related to the average service times and the tow truck types, three of them for light tow trucks, two for heavy tow trucks and two for super-heavy tow trucks. In the analysis, at first we considered one tow truck available in each scenario, and after two trucks available in each scenario.

Nevertheless, the results of applying the mathematical model described in Section 3 to the two scenarios related to super-heavy tow trucks were extremely similar to the results for heavy tow trucks. As a result, we decided to keep only the first five scenarios related to light and heavy tow trucks, as shown in Table 4.

Table 4 – The first five scenarios of the Case Study - related to average service times.

Scenario	Tow truck type	Average service time
Scenario 1	Light	30 minutes
Scenario 2	Light	40 minutes
Scenario 3	Light	50 minutes
Scenario 4	Heavy	40 minutes
Scenario 5	Heavy	50 minutes

As shown in Section 3, when we use a maximum service time instead of an average service time, we apply Constrains (9) in the mathematical model instead of Constrains (6). This gave rise to seven more scenarios related to the maximum service times and the tow truck types, three of them for light tow trucks, two for heavy tow trucks and two for super-heavy tow trucks. We used $\alpha = 90\%$ as the probability for any incident to be serviced in the maximum time considered in each case.

Also in this case, the results of the two scenarios related to super-heavy tow trucks were extremely similar to the results for heavy tow trucks. Once again, we decided to keep only the first five scenarios related to light and heavy tow trucks, this time numbered from 6 to 10, as shown in Table 5.

When analyzing the two objectives as just one, an optimal solution is obtained without weighing the importance of the distinct objectives existing in the problem, in the present model: maximize the number of incidents response and reduce the congestion queues generated by it. However, when pondering these objectives, it is possible to have a new understanding of the solutions obtained.

Table 5 – The second five scenarios of the Case Study - related to maximum service times.

Scenario	Tow truck type	Maximum service time
Scenario 6	Light	60 minutes
Scenario 7	Light	90 minutes
Scenario 8	Light	100 minutes
Scenario 9	Heavy	90 minutes
Scenario 10	Heavy	100 minutes

Considering the three types of tow trucks involved in incidents responses, the light tow truck is the one that has the highest number of occurrences, so this type of tow truck was selected for the sensitivity analysis considering the objective functions of the model. The maximum average times of 30 and 40 minutes were analyzed.

4.3 Results and discussion

The model was coded using FICO® Xpress-IVE 1.24.24 64 bits on a personal desktop computer Intel® Core™ i7-4510U CPU 2.0GHz 2.6GHz with 8 GB RAM under the operational system Windows 10 Home 64 bits.

In order to justify the need of the three terms of the Objective Function (2), we conduct a gradual analysis of the inclusion of these terms. When we consider only the first term, related to the maximization of the responded incidents, the mathematical model prioritizes the attendance of the sections with the highest speed. This makes sense, since in these sections it is possible to perform a larger number of services in a shorter time.

When we consider the first two terms of Objective Function (2), we are maximizing the number of serviced incidents and minimizing the queue of congestion generated by these incidents at the same time. In this case, the mathematical model leaves some critical sections of the expressway without response, such as sections located at the extremities. As a result, these sections become even more critical, since a longer service delay can have disastrous consequences for traffic. The third term of Objective Function (2), which minimizes the queue of congestion generated by not serviced incidents, is inserted precisely to mitigate this effect.

Fig. 9 shows the number of incidents in Avenida Brasil, within the time window of the case study, per type of tow truck and per neighborhood. The blue line corresponds to the light tow and the orange one to the heavy ones. The section with the highest number of incidents attended by both light and heavy tow trucks is Penha, with the highest peak of incidents related to light tow trucks. The need of using the heavy tow trucks is well perceived mainly between the sections Caju and Parada de Lucas.

Table 6 shows the tow trucks location as a result of the application of the proposed mathematical model in Scenarios 1 to 10 with one tow truck available in each scenario (third column of

Table 6), and with two tow trucks (fourth column of Table 6). The presented solutions considers the same weight for each objective function.

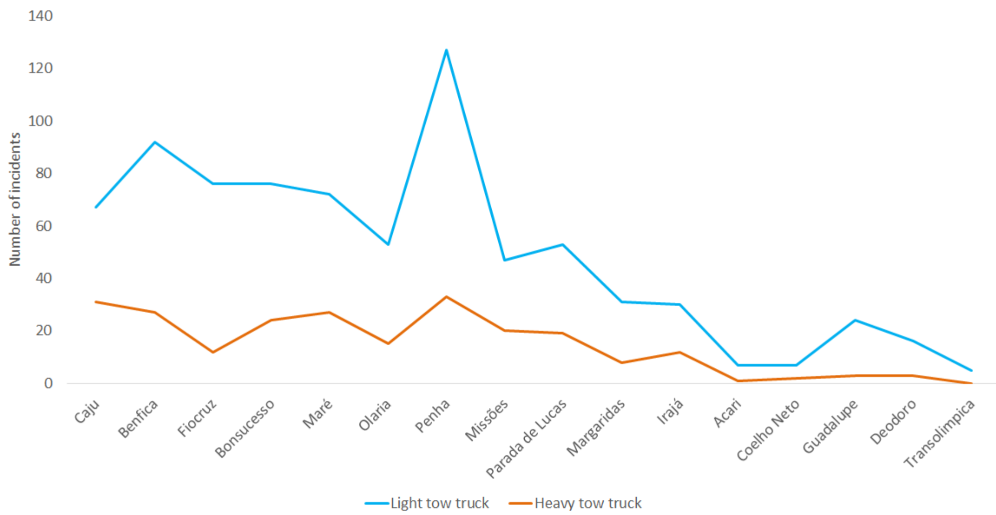

Figure 9 – Number of incidents per section and per type of tow truck occurred in Avenida Brasil - March to June of 2018.

Source: the authors.

Table 6 – Tows trucks location using one or two tows in the proposed mathematical model (1)-(7) for Scenarios 1 to 10.

Type	Scenario	One tow truck	Two tow trucks
	1	Fiocruz to West Zone	Fiocruz and Olaria to West Zone
Light	2	Olaria to West Zone	Acari to city center and Olaria to West Zone
	3	Olaria to West Zone	Olaria and Coelho Neto to West Zone
Heavy	4	Olaria to West Zone	Missões to city center and Maré to West Zone
	5	Olaria to West Zone	Maré and Coelho Neto to West Zone
	6	Fiocruz to West Zone	Fiocruz to city center and to West Zone
Light	7	Olaria to West Zone	Olaria and Margaridas to West Zone
	8	Olaria to West Zone	Olaria and Coelho Neto to West Zone
Heavy	9	Olaria to West Zone	Missões to city center and Maré to West Zone
	10	Olaria to West Zone	Maré to West Zone and Missões to city center

When we have only one tow truck per scenario, we see a clear dominance of position Olaria to West Zone. This can be explained by the strategic positioning of this point in relation to the center of the city that concentrates the largest number of incidents and the highest flow of vehicles (see Fig. 5 and Fig. 9). At the same time, this position is not too far from the other sections located to the right of Olaria in Fig. 9. Scenarios 1 and 6, the only ones to locate the tow truck in Fiocruz to West Zone, are exactly the scenarios that have the smallest time to respond to the incidents. In these cases, the model prioritizes even more the area closest to the center of the city, where the flow and the number of incidents are higher.

When we have two tow trucks per scenario, the locations of the tow trucks proposed by the mathematical model change, depending on the variables involved and the maximum service time used as constraint, as shown in the fourth column of Table 6. Fig. 10 shows the location of the tow trucks chosen by the mathematical model for Scenarios 1, 2 and 3, when two tow trucks are available (the first three lines of the fourth column of Table 6). Each position chosen by the model is associated with a color: blue to Fiocruz, yellow to Olaria, green to Acari and red to Coelho Neto. In addition, the table that comes with the map in Fig 10 shows, in its lines, the 32 sections (16 sections considered in both ways) and the colors correspond to the location of the tow trucks that will make that attendance according to the available scenario time (last three columns of the table). From this table it is possible to observe that, as the time available for service increases, the tow trucks are located more and more distant from the most critical part of Avenida Brazil, being able to respond more quickly to the whole avenue. It is important to point out that despite the allocation of two tow trucks, the incidents sections are served by only one truck.

Today there are three tow trucks to service Avenida Brasil: one light and two heavy trucks. Therefore, in this last study, we apply the proposed mathematical to this situation considering just one light tow truck available for Scenarios 1, 2, 3, 6, 7 and 8; and two heavy tow trucks available for Scenarios 4, 5, 9 and 10. Table 7 shows the number of responded incidents with the number of tow trucks currently available to service Avenida Brasil in each scenario. We believe the incidents' average duration time τ is the main influence on the number of responded incidents. Table 7 shows that in Scenario 6, where a maximum response time of 60 minutes is considered, we have the lowest percentage of incidents' response, showing that the considerable duration time τ is a strong restriction of the problem. When we increase the maximum response time to 90 minutes (Scenario 7), the number of response incidents also increases.

Table 7 – Percentage of responded incidents per scenario: one light tow truck for Scenarios 1, 2, 3, 6, 7 and 8, and two heavy tow trucks for Scenarios 4, 5, 9 and 10.

Type	Scenarios	Percentage of responded incidents (%)
One Light	1	24.78
	2	95.66
	3	100.00
Two Heavy	4	95.36
	5	100.00
One Light	6	8.81
	7	92.98
	8	100.00
Two Heavy	9	94.51
	10	100.00

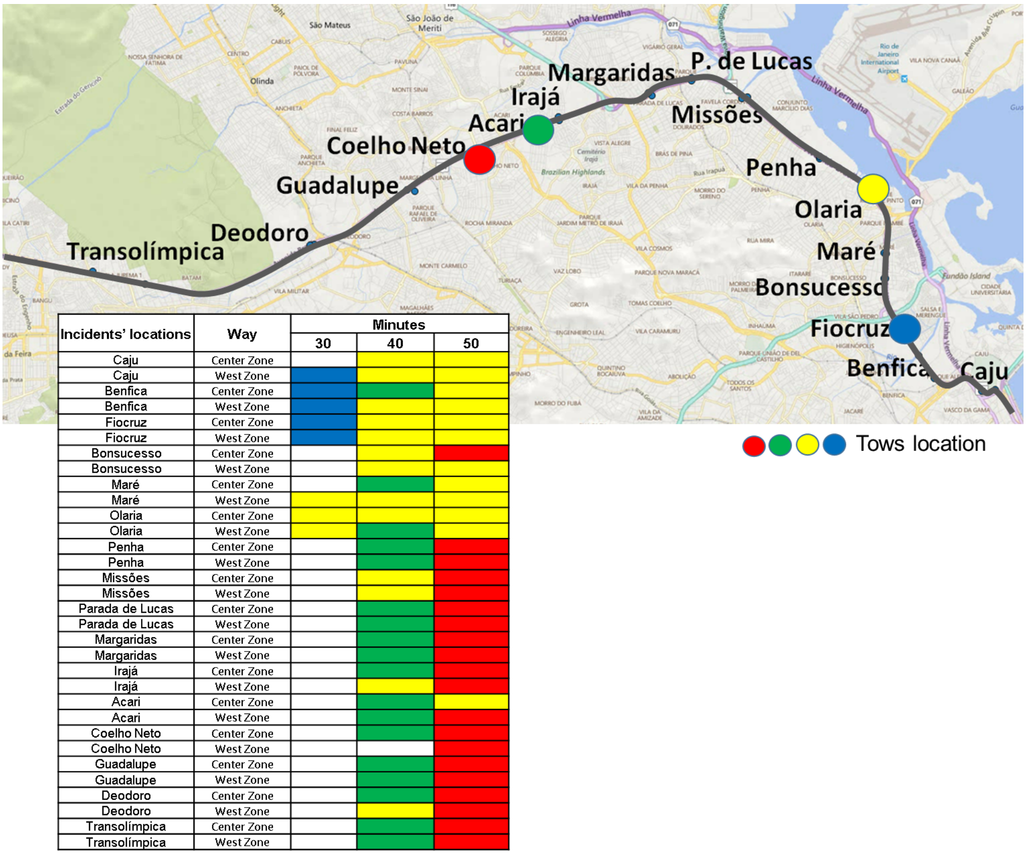

Figure 10 – Location of the tow trucks chosen by the mathematical model for Scenarios 1, 2 and 3, when two tow trucks are available.

Source: the authors.

Table 8 shows the increase in the percentage of responded incidents when we add one extra tow truck to the current available number of tow trucks in each scenario. We can conclude that, regardless of the type of tow truck, as large as the acceptable average or the maximum time of incident duration (in other words, a lower level of service), the impact on the percentage of responded incidents is less than when increasing tow truck numbers.

Fig. 11 compares the percentage of response for Scenarios 3, 6 and 8 when we use the current available light tow truck (in purple) with the percentage of response when we double that number of light tow trucks (in pink). When we compare Scenarios 3 and 8, we see that in both cases 100% of the incidents are responded, however we have 50 minutes of average service time in Scenario 3 and 100 minutes of maximum response time in Scenario 8. Therefore, this shows that if we consider the maximum response time instead of the average time, in the case of light tow trucks, it is necessary to consider twice the time to have 100% of the incidents serviced. We can note that this is even more when we compare Scenarios 6 and 8, since the percentage

Table 8 – Increase in the percentage of responded incidents per scenario when we add one extra tow truck to the current available number of tow trucks in each scenario.

Type	Scenarios	Increase in the percentage of responded incidents (%)
Light	1	42.27
	2	4.41
	3	0.00
Heavy	4	2.21
	5	0.00
Light	6	55.07
	7	5.49
	8	0.00
Heavy	9	2.64
	10	0.00

of response drops ten times when the maximum service time is 60 minutes (Scenario 6) instead of 100 minutes (Scenario 8) and we have only one available light tow truck; and that percentage increases 55.07% (see Table 8) when we make one more light tow truck available.

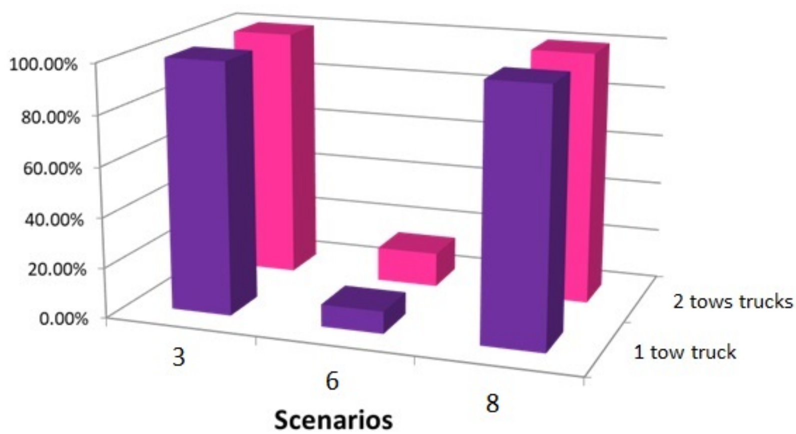

Figure 11 – Comparison between the percentages of responded incidents for Scenarios 3, 6 and 8 when we used 1 or 2 light tow trucks.

Source: the authors.

Fig. 12 compares the percentage of response incidents for Scenarios 1, 2 and 3 when we use the current available light tow truck (in purple) with the percentage of response when we double that number of light tow trucks (in pink). This figure shows that, by doubling the number of light tow trucks available for use, only Scenario 1 (the one with the smallest average service time) has the percentage of serviced incidents substantially increased (in 42,27% - see Table 8), while Scenarios 2 and 3 present only superficial changes. Therefore, this analysis shows that not always when doubling the number of tow trucks we can improve substantially the level of response.

Finally, we analyze the response of the proposed mathematical model when the average incident rate is twice as high. In this case, the greatest decrease in the percentages of responded incidents occurred in Scenarios 1, 2 and 5, the largest reduction is 8% in Scenario 1 when two light tow trucks are available. This analysis is very important because it demonstrates the robustness of the proposed mathematical model and of the data collected for the case study.

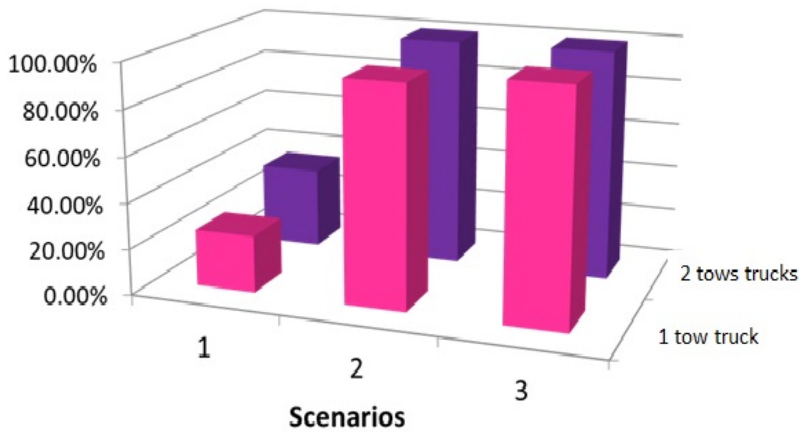

Figure 12 – Comparison between the percentages of serviced incidents for scenarios 1, 2 and 3 when we used 1 light tow truck and 2 light tow trucks.

Source: the authors.

When pondering the objective functions, it is noted that the result does not change independently of the value of α with only one tow truck in Scenario 1, see Fig. 13. Adding one more tow truck, considering only the maximization of incident response, the total number of services increases by 48%. If only the minimization of the congestion queues is considered, the two tow trucks response to 218 incidents and the congestion queue reduces by 30% in relation to the scenario with just one tow truck. Considering the model as bi-objective with two tow trucks, regardless of the value of α, the result is the same: 276 incidents and a congestion queue of 32,515.4m, see Fig. 13.

With three tow trucks, there is more variation in the results, depending on the value of α. When the value varies between 0.2 and 0.4, the results obtained are the same. It also happens when the variation is between 0.5 and 0.8. When considering only the maximization of responses, the model located all tow trucks close to the region where more incidents occur. When considering only the minimization of the congestion queue, the model distributed the tow trucks more spread

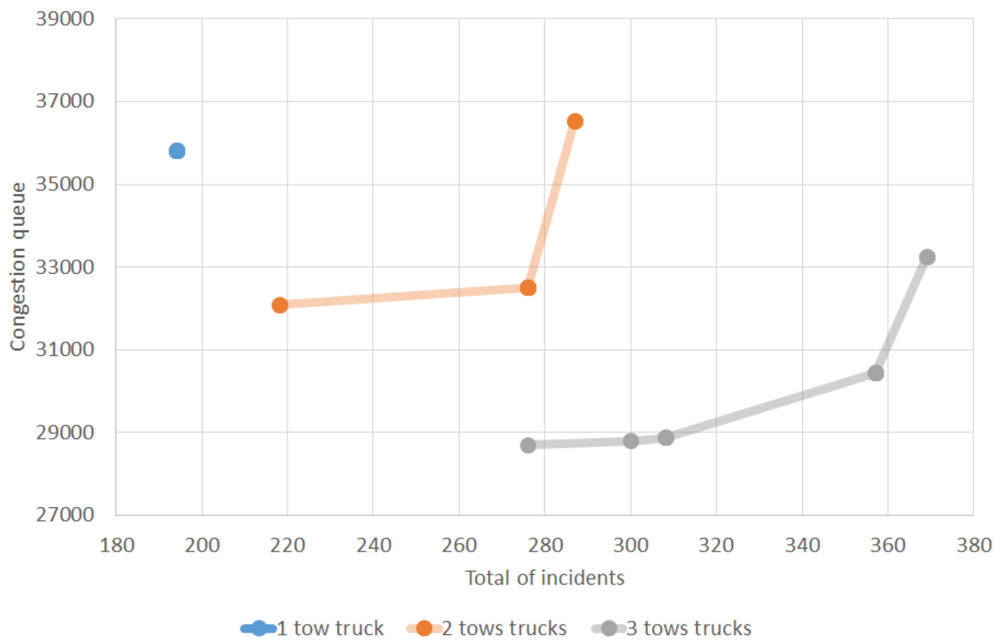

Figure 13 – Total of incidents and congestion queue with 30 minutes.

Source: the authors.

along Avenida Brasil, in order to respond the incidents closest to the city center, in Caju, and one of the tow trucks in a section far from the most critical incidents area, at Transolímpica.

In Scenario 2, shown in Fig. 14, with a time of 40 minutes, one of the tow trucks was always located in the same location, Olaria to west zone, probably due to the central position of this section in relation to the most critical section of the road. Considering the longer time, it was observed that the addition of tow trucks had more impact on the number of incident responses than on the reduction of the queue size. With three tow trucks, it is possible to note that independently of the weight given to each one of the objective functions, the total of incidents does not change, since the tow truck is able to serve all sections. However, when the minimization of the congestion queue was not considered, the model allocates the tow truck in a point where the generated congestion queues will be longer.

5 CONCLUSIONS AND FUTURE WORK

Reducing total incident time in expressways is essential to reduce the anti-economic issues generated by them in big cities. In this scenario, tow trucks have an essential function to reestablish the total road capacity in the shortest time possible. Which is closer to the places with the highest flow and high number of incidents.

Figure 14 – Total of incidents and congestion queue with 40 minutes.

Source: the authors.

In this paper, differently from the ones published in the literature, we proposed a mathematical model which include queueing theory to reduce the negative consequences produced by the incidents taking into account the capacity reduction of the expressway and the places with larger queues of vehicles. Besides, the mathematical model maximizes the quantity of incidents responded providing a medium time to clear them, considering an average or maximum response time, and, at the same time, minimizes the non-recurrent congestion generated by the incidents.

The City of Rio de Janeiro suffers from very high traffic of vehicles on its high-capacity expressways, especially at peak times. In addition, the large number of incidents directly impacts daily urban mobility. In this context, the mathematical model proposed in this work turned out to be very interesting to study alternative scenarios for the tow trucks location. The results of the scenarios analyzed showed that the number of tow trucks is important, but the time for response is the most important variable in the system. It is important to have a large number of tow trucks when we need a low response time.

Besides, the model showed that the incident response operation at Avenida Brasil is not adequately sized due to its low service level. When we require a lower response time, we need a large number of tow trucks to ensure the same amount of attended incidents. Moreover, when we can have a longer response time, the number of tow trucks to maintain the same level of service decreases, but not in a linear way. This is an important conclusion for the public sector, since it is its duty to define the acceptable average time for incident response and subsequently to evaluate the number of tow trucks necessary to achieve a pre-established service level, for example, 100%

or 95% of the demand. The mathematical model proposed in this work can be used as a decision tool to help the expressways operators to define the best public policies according to the present scenarios.

As future research, we suggest the insertion of the issue of public safety in the model, since this is a critical point to the City of Rio de Janeiro. Therefore, in addition to considering the sections with the highest traffic flow as priorities, priority must also be given to the most unsafe zones. It may be also interesting to obtain specific data related to the peak times in the day or to the days with greater vehicular flow in order to suggest studies involving the relocation of tow trucks throughout the day to response, for example, to morning and afternoon peaks, per direction. Lastly, we also suggest considering the possibility that a section can be served by more than one tow truck.

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References

[1] ADLER N, HAKKERT A, KORNBLUTH J, RAVIV T & SHER M. 2014. Location-allocation models for traffic police patrol vehicles on an interurban network. *Annals of Operations Research*, **221**: 9–31.

[2] AKDOGAN M, BAYINDIR Z & IYIGUN C. 2017. Location analysis of emergency vehicles using an approximate queueing model. *Transportation Research Procedia*, **22**: 430–439.

[3] AMSTERDAN SMART CITY. 2018. City alerts: An Incident Message Exchange for emergency services. Available at: https://amsterdamsmartcity.com/projects/city-alerts. Online; accessed 20 December 2019.

[4] CET-RIO - TRAFFIC ENGINEERING COMPANY OF RIO DE JANEIRO. 2018. Dados sobre incidentes no ano de 2018, Rio de Janeiro - Brasil. Available at: https://www.arcgis.com/home/item.html?id=360955161c5843c7856b2668fa87786d. Online; accessed 2 December 2019.

[5] DEB K. 2001. *Multi-objective optimization using evolutionary algorithms*. 1 ed.. New York: Springer.

[6] DELGADO-ANTEQUERA L, LAGUNA M, J P & CABALLERO R. 2020. A bi-objective solution approach to a real-world waste collection problem. *Journal of the Operational Research Society*, **71**: 1–11.

[7] DUNNETT S, LEIGH J & JACKSON L. 2019. Optimising police dispatch for incident response in real time. *Journal of the Operational Research Society*, **70**: 269–279.

[8] FARAHANI RZ, STEADIESEIFI M & ASGARI N. 2010. Multiple criteria facility location problems: A survey. *Applied Mathematical Modelling*, **34(7)**: 1689—1709.

[9] FHWA. 2010. Traffic incident management handbook. Available at: https://ops.fhwa.dot.gov/eto_tim_pse/publications/timhandbook/. Online; accessed 10 December 2019.

[10] FIRJAN. 2014. Os custos da (i)mobilidade nas regiões metropolitanas do Rio de Janeiro e São Paulo. Available at: http://www.firjan.com.br/lumis/portal/file/fileDownload.jsp?fileId=2C908A8F4EBC426A014EC051E736421F. Online; accessed 10 October 2020.

[11] FUKUDA EH & DRUMMOND LMG. 2014. A survey on multiobjective descent methods. *Pesquisa Operacional*, **34-3**: 585–620.

[12] GEROLIMINIS N, KARLAFTIS M & SKABARDONIS A. 2009. A spatial queuing model for the emergency vehicle districting and location problem. *Transportation Research Part B*, **43**: 798–811.

[13] HAJIPOUR V & PASANDIDEH S. 2011. A New Multi-Objective Facility Location Model within Batch Arrival Queuing Framework. *World Academy of Science, Engineering and Technology*, **78**: 1665–1673.

[14] HAKIMI S. 1965. Optimum distribution of switching centers in a communication network and some related graph theoretic problems. *Operations Research*, **13**: 462–475.

[15] HOJATI A, FERREIRA L, WASHINGTON S, CHALES P & SHOBEIRINEJAD A. 2016. Reprint of: Modelling the impact of traffic incidents on travel time reliability. *Transportation Research Part C*, **70**: 86–97.

[16] HUANG J, FADEL G, BLOUIN V & GRUJICIC M. 2002. Bi-objective optimization design of functionally gradient materials. *Materials and Design*, **23(7)**: 657–666.

[17] JEONG Y, CASTRO M & HAN MKJD. 2011. A wavelet-based freeway incident detection algorithm with adapting threshold parameters. *Transportation Research Part C*, **19**: 1–19.

[18] KIM I & WECK O. 2005. Adaptive weighted-sum method for bi-objective optimization: Pareto front generation. *Structural and Multidisciplinary Optimization*, **29**: 149–158.

[19] LI P & WALTON J. 2013. Evaluating Freeway Service Patrols in Low-Traffic Areas. *Journal Of Transportation Engineering*, **139**: 1095–1104.

[20] LOU Y, YIN Y & LAWPHONGPANICH S. 2011. Freeway service patrol deployment planning for incident management and congestion mitigation. *Transportation Research Part C*, **19**: 283–295.

[21] MA X, DING C, LUAN S, WANG Y & WANG Y. 2017. Prioritizing Influential Factors for Freeway Incident Clearance Time Prediction Using the Gradient Boosting Decision Trees Method. *IEEE Transactions on Intelligent Transportation Systems*, **18**: 2303–2310.

[22] MARIANOV V & SERRA D. 1998. Probabilistic,maximal covering location- allocation model for congested systems. *Journal of Regional Science*, **38**: 401–424.

[23] NITSCHE P, JOLSTAM, TAYLOR N, REINTHALER M, WPONWEISER, BERNHARDSSON V, MONACU I, UITTENBOGAARD J & DAM EV. 2016. Pro-active management of traffic incidents using novel technologies. *Transportation Research Procedia*, **14**: 3360–3369.

[24] OBAL TM, VOLPI NMP & MILOCA SA. 2013. Multiobjective approach in plans for treatment of cancer by radiotherapy. *Pesquisa Operacional*, **33-2**: 269–282.

[25] OZBAY K, IYIGUN C, BAYKAL-GURSOY M & XIAO W. 2013. Probabilistic programming models for traffic incident management operations planning. *Annals of Operations Research*, **203**: 389–406.

[26] PAL R & BOSE I. 2009. An optimization based approach for deployment of roadway incident response vehicles with reliability constraints. *European Journal of Operational Research*, **198(2)**: 452–463.

[27] REVELLE C, MARKS D & LIEDMAN J. 1970. An Analysis of Private and Public Sector Location Models. *Management Science*, **16**(11): 692–707.

[28] STEENBRUGGEN J, BORZACCHIELLO M, NIJKAMP P & SCHOLTEN H. 2013. Data from telecommunication networks for incident management: An exploratory review on transport safety and security. *Transport Policy*, **28**: 86–102.

[29] TRANSPORTATION RESEARCH BOARD. 2010. *Highway Capacity Manual*. vol. 1. 5 ed.. Washington D. C.: Transportation Research Board.

[30] WU W, SHEN L, JI X & JIN W. 2014. Analysis of freeway service patrol with discrete event-based. *Simulation Modelling Practice and Theory*, **47**: 141–151.

[31] XIAO Y, COULOMBEL N & PALMA AD. 2017. The valuation of travel time reliability: does congestion matter? *Transportation Research Part B*, **97**: 113–141.

[32] YIN Y. 2006. Optimal Fleet Allocation of Freeway Service Patrols. *Networks and Spatial Economics*, **6**: 221–234.

[33] ZHU S, KIM W, CHANG G, ASCE M & ROCHON S. 2018. Design and Evaluation of Operational Strategies for Deploying Emergency Response Teams: Dispatching or Patrolling. *Journal of Transportation Engineering*, **140**: 04014021.

[34] ZOU Y, YE X, HENRICKSON K, TANG J & WANG Y. 2006. Jointly analyzing freeway traffic incident clearance and response time using a copula-based approach. *Transportation Research Part C*, **86**: 221–234.

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