

We Sold a Million Units – The Role of Advertising Past-Sales*

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Summary: 1. Introduction; 2. The model; 3. The first-period optimal decision; 4. The basic case: Second-generation buyers are uninformed; 5. Extension: Second-generation consumers are better informed; 6. Conclusions.

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In a market where past-sales embody information about consumers' tastes, we analyze a seller's incentives to invest in a costly advertising campaign to report past-sales. If consumers are poorly informed, a *pooling equilibrium with past-sales advertising* obtains. Information revelation only occurs when the seller benefits from the consumers' herding behavior brought about by the advertising campaign. If consumers are better informed, a *separating equilibrium with past-sales advertising* arises. Information revelation always happens, either through prices or through costly advertisements.

Em um mercado no qual as vendas passadas transmitem informação a respeito dos gostos dos consumidores, nós analisamos os incentivos de uma firma a investir em propaganda que divulga o total vendido no passado. Se os consumidores não tem informação própria, o preço de equilíbrio independe da qualidade do produto. Se os consumidores tem informação própria, mesmo que imprecisa, há um equilíbrio separador no qual a firma investe em propaganda.

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1. Introduction

This paper presents a model to examine the role of past-sales advertising. Sales-data advertising is a marketing strategy commonly observed: book publishers often insert ads into newspapers and magazines to promote best-selling titles; pharmaceutical firms often distribute advertisements to report the percentage of doctors or dentists who use certain treatments and health products; automobile companies frequently invest in publicity to stress that certain model has been the most sold during the previous month or year; advertising of music records usually emphasizes the number of units sold in other countries; managers of theater plays or movies commonly produce advertisements reporting the proceeds obtained, or the number of weeks that their plays or films have been performing or on screen; amusement parks and tourism managers repeatedly report the number of tourists who acquire their services; finally, Internet sites often advertise the number of visitors per day, week or month.

Our economic explanation for why these types of advertisements are observed is that monopolistic firms operating in environments where the matching between product characteristics and consumer tastes is uncertain may have an incentive to invest in a costly advertising campaign in an attempt to influence good's future demand in its own interest. The reason is that sales embody aggregate information about the extent to which product's features match consumer tastes. When buyers from distinct generations exhibit similar preferences, the advertisements enable future buyers to make more accurate decisions, and then consumers' *herding* arises naturally.¹ The seller may want to exploit or, contrarily, prevent from buyers' herding in its own interest. In this type of environment we study the occurrence of past-sales advertising.

These issues are studied below in a simple two period model which shares features with Judd and Riordan (1994). A single uninformed seller offers a good to two successive generations of consumers with similar tastes. Before the market opens, Nature selects the quality of the matching between consumers preferences and product features. Consumers privately receive an imperfectly informative signal about the true taste parameter. In the first market opening, the seller, under complete ignorance, sets an initial price and buyers make their demands. First-period sales, which are privately observed by the monopolist, constitute an aggregate indicator (or a summary statistic) of consumers' tastes. In the subse-

¹For early work on consumers' herding and informational cascades see Banerjee (1992) and Bikhchandani et al. (1992). Taylor (1999) studies a model where herding also arises naturally. In his model buyers' herding is always detrimental for the firm which consequently intends to deter herding by strategic pricing.

quent period, the seller sets a price as well as decides whether or not to initiate an advertising campaign to report past-sales.² Finally second-generation consumers make their purchases based upon their private signals (if they receive), the observed price, and the seen advertisement (if advertising occurs). We assume that advertising is costly, truthful and that it reaches all consumers.³ Also, we assume that the price history is observable.⁴

The analysis is carried out under two different informational scenarios. In the *first* scenario, second-period consumers are completely uninformed about the unknown taste parameter. In the *second* scenario we extend the analysis to allow for better, but not completely, informed buyers.

We start with a consideration of an information setting where second-generation buyers do not receive information. In this setting, prices are incapable of transmitting the private information possessed by the seller. In other words, signalling is impossible. The equilibrium we derive in this case is referred to as a *price-pooling equilibrium with past-sales advertising*. It consists of (i) a partition of the set of possible sales observations into an advertising subset and a no-advertising subset, and (ii) a pricing function for each subset. If observed sales-data fall into the advertising subset, sales advertising bringing about buyers herding is worth. By contrast, if sales-data fall into the no-advertising subset, it is optimal for the seller not to invest in advertising (thus preventing from consumers' herding) and charge a pooling (uninformative) price. Of course, consumers are rational and in equilibrium infer the set of sales-data that are not advertised correctly. Therefore, when they do not observe an ad reporting firm's past-sales information, they form the appropriate inferences: "no news" means "bad news." The equilibrium advertising set is therefore greater than expected due to the seller's intention to

²We note that in our setting past-sales advertising conveys 'hard' information, and thus advertising does not function as a signal here (Milgrom and Roberts, 1986). Moreover, consumers know the existence of the product and thus there is no role for advertising other than reporting past-sales.

³We rule out the possibility of misrepresentation. In our context false claims are unlikely to occur because past-sales are verifiable ex-post. In many western European countries advertisers have formed associations which monitor and prevent from misrepresentations, false claims etc., having relegated those cases as mere anecdotal. It is often argued that in general no seller would advertise falsely, since the loss in future gains, or in other markets current gains, or due to fines and penalties, would be considerably larger than the short-run gains from fooling buyers.

⁴We note that posted prices are commonly observable in real world markets. This assumption frees our model from signal jamming possibilities (in contrast to Caminal and Vives (1996) and Taylor (1999)). The assumption allows us to concentrate on the second-period advertising decisions. The first-period helps to understand where the information past-sales provide comes from, how consumers herding arises naturally, and how and why the seller may want to exploit such information in its own interest.

avoid that consumers form pessimistic inferences. In equilibrium, partial disclosure through advertising results.⁵ Moreover, the lower advertising costs the larger is the advertising set.

In the second part of the paper, we turn to the consideration of an informational regime where second-generation buyers are better informed. In particular, second period consumers receive noisy signals. The picture changes dramatically because under this informational assumption prices are capable of signalling the private information possessed by the seller. In this case a *price-signalling equilibrium with past-sales advertising* obtains. The distinguishing feature of this equilibrium is that complete disclosure occurs, either through prices or through costly advertisements, and consequently buyers herding always arises. For promising observations of sales, the seller finds it optimal to initiate an advertising campaign to report past-sales, and quote the price that would be charged if seller and buyers had symmetric information. For unpromising sales-data, the seller uses a price-signalling strategy. As above, the lower the advertising costs, the smaller is the no-advertising set.

A key result of our model is that the market outcome exhibits consumers herding inevitably. Since individual signals are less accurate than the summary statistic which is embodied in the past-sales, consumers rationally employ the information released through the advertisements, or through the prices (if signalling occurs). As a result, it may very well happen that consumers purchase an unsatisfactory good simply because first-generation consumers observed good (but biased) signals. This “path-dependence” effect is caused by the fact that buyers do not observe the *ex-post* utilities of previous customers, but their decisions, which are based on the observed random signals. The seller, by deciding whether to advertise his sales or not and choosing prices appropriately, exploits buyers’ herding behavior in its own interest. We notice that the firm’s ability to take advantage of the rational behavior of the consumers is undermined when second-generation buyers have corroborating information, as in the second informational scenario of our paper.⁶

The literature has identified other reasons for which this type of costly advertisements may be observed. In an imperfectly competitive environment, Jin

⁵Partial disclosure also obtains in Jovanovic (1982). However, in contrast to our model, in his work disclosure is a social waste and sellers profits are totally unrelated to observed signals.

⁶Bikhchandani et al. (1992) report the success and failure of new products apparently due to no objective reason. The idiosyncrasy of such outcomes is observed, for instance, in the cinema industry: even though the public sometimes flood the cinemas’ entrances to get tickets of new films, occasionally the buyers’ ex-post satisfaction is rather low. In the context of our paper, this would be a case where the true value q is bad whereas buyers signals are good.

(1994) argues that such information releases are means to sharing information about common demand uncertainties for the firms, before they compete in the product market. The finance literature agrees in that firms often report their sales or earnings' forecasts to influence investors' perception about the firm's value (Grossman and Hart, 1980, Dye, 1985).⁷ Finally, in an environment where network externalities are present (as in Becker (1991)), sellers' sales releases enable buyers to learn the critical mass of "adopters", which may induce them to acquire the good in question.

The remainder of the paper is organized as follows. Next section describes the model and sets up the problem. The first period optimal decision is discussed in section 3. The results for the case where second-generation buyers do not receive signals is presented in section 4. We extend the analysis to allow better informed consumers in section 5. Section 6 concludes.

2. The Model

We consider a two period model where there is two-sided uncertainty. A single firm sells a good to two successive generations of consumers. The value of the *experience* good is uncertain for the seller and the buyers (Nelson, 1970, 1974). Consumers, however, know the existence of the good and thus there is no role for product advertising here. The uncertain parameter q measures the extent to which product's features and consumer tastes match. We will refer to q as the 'quality' of the good but, in this work, we follow Judd and Riordan (1994) and consider parameter q as a taste index rather than as a parameter of technical superiority. This perspective allows for the abstraction from the dependency of quality and costs. We suppose, for simplicity, that production is costless.

A new cohort of n customers enters the demand side of the market each period. Each consumer has the increasing and strictly concave utility function $U(x) = -qe^{-x}$. The consumer surplus is $-qe^{-x} - px, x \geq 0$. Thus product demand is $x = \log\left(\frac{q}{p}\right)$. For analytical convenience we allow for negative consumption. However since prices are always positive if total demand is negative the firm will not produce. Consumers are short-lived, which implies that they buy only once and that they cannot postpone their purchasing decisions. When the market opens, buyers within a cohort may differ in their information, but they are all identical ex-ante.

The market evolves as follows. At the beginning, before the market opens,

⁷Empirical evidence is found in Lev and Penman (1990) and Clarkson et al. (1994).

q is drawn by Nature and remains fixed thereafter. Thus consumers of different generations have equal tastes. Neither firm nor customers receive full information about it prior to purchasing.⁸ Once Nature has chosen the taste index, all first-generation customers receive a private signal $s_i = q\epsilon_{i1}$, $i = 1, \dots, n$, which conveys (noisy) information about q . Then, under complete ignorance, the seller sets his first-period price and first-generation consumers make their purchases. Once the seller has observed first-period sales, he decides on his marketing strategy for the second-period, he chooses the price to be charged to second-generation consumers and decides whether or not to invest in a *costly* advertising campaign to report first-period sales. Consumers in the second period also receive imperfectly informative signals $t_i = q\epsilon_{i2}$, $i = 1, 2, \dots, n$. Finally, second-generation buyers make their purchases. We suppose that ϵ_{i1} , ϵ_{i2} and q are independent random variables with a log-normal distribution. To fix notation $\log q$ is normal (μ_q, σ_q^2) and $\log(\epsilon_{ik})$ is normal (μ_k, σ_k^2) , $k = 1, 2, i = 1, \dots, n$. Without loss of generality we suppose that $E[q] = 1$. Thus $\mu_q = -\sigma_q^2/2$. Throughout, it is assumed that buyers do not pool their private information, neither within nor between cohorts. Moreover, we assume that customers observe the history of prices charged, but do not observe quantities sold in the past.

We now make some necessary remarks. The *first* observation gives the basis to the problem we analyze. Notice that first-generation consumers will make demand decisions basing upon their private information. As a result, realized first-period sales embed a *summary statistic* of consumers' tastes. This summary statistic is private information to the seller, and contains information (ex-ante) valuable for all the agents in the marketplace. The fact that consumers would be able to make wiser decisions if they knew the sales-data observed by the monopolist raises the question of whether or not the seller is interested in spending resources in advertising sales.

The *second* observation refers to our assumption that the history of prices is observable. Since second-generation consumers observe first-period prices, the seller cannot signal-jam buyers' inferences about the uncertain parameter q by quoting a particular first-period price. Therefore, our model is free of signal jamming. This implies that the intertemporal feature of the monopolist's problem does not affect its first-period price decision. Consequently, the monopolist decision for each period can be computed separately. Next we solve for the seller's first-period decision. Sections 4 and 5 are the core of our analysis, where we study the seller second-period marketing strategy.

⁸More precisely, in our model seller's uncertainty is never fully resolved and consumers' uncertainty is resolved ex-post, after consumption occurs.

3. The First-period Optimal Decision

We first calculate the first-period demand. First-generation consumer i 's information is just his signal s_i . Thus his demand, conditional upon the privately observed signal is:

$$x_1^i = \log(E[q | s_i]) - \log(p)$$

Note that even though consumers observe the price charged by the firm in period 1, p , this price is not informative at all since the seller does not have any private information on q at that stage. We can sum up consumers' demands to obtain the aggregate demand:

$$X = \sum_{i=1}^n \log(E[q|s_1^i]) - n \log(p) = n(Q - \log(p)) \quad \text{where } Q = \frac{1}{n} \sum_{i=1}^n \log(E[q|s_1^i]) \quad (1)$$

Note that, since consumers' signals are private and consumers do not exchange information, realized first-period sales are private information to the seller. The monopolist will condition its second-period marketing strategy on its observation of Q . We now find the first period optimal price. The monopolist sets p to maximize profits:

$$E[pX] = n(pE[Q] - p \log(p))$$

Thus

$$E[Q] - 1 = \log(p^*)$$

Remark 1 Sales will then be $S = n(Q - E[Q] + 1)$. Therefore announcing sales is equivalent to announce Q .

The first-period of our model is thus a period where the seller gathers information about consumer tastes. This information is embedded in the statistic Q . This period is important because it helps understand the content incorporated in an advertisement reporting past-sales. Moreover, it helps explain that buyers' herding arises naturally in markets where consumers are uncertain about their tastes.

We solve the monopolist's second-period problem in what follows. We use the notion of perfect Bayesian equilibrium. This requires consumers' decisions to

be optimal given the seller's strategy and their beliefs about q , and the seller's strategy to be a best-reply to consumers' actions. Besides, all agents' beliefs must conform to Bayes' rule whenever it applies.

4. The Basic Case: Second-generation Buyers are Uninformed

We first examine the case where second-generation consumers do not possess any valuable external information. Consequently, since buyers do not observe first-period sales, they are completely uninformed before the seller sets its marketing strategy. This information setting illustrates a situation where buyers know the existence of the product but are completely uninformed about its features. In section 5, we extend the analysis to allow for better informed second-period buyers.

Advertising Equilibrium

In this section we define an advertising equilibrium and prove its existence when second period consumers do not have signals of their own. Although the most interesting case is when second period consumers have signals of their own as the first period consumers, the no-signals case is easier to understand. Throughout, we assume that the advertising campaign necessary to report past-sales involves a cost $c > 0$. If there is no advertising consumers may try to infer first period sales using the price. This however will not be possible. Even though buyers may try to infer the firm's past-sales S basing upon the observed price p_2 , we show, in what follows, that if the inference rule is Bayesian, then the price is incapable of transmitting such information. In other words, the optimal price will be uncorrelated with S because no inference rule can be part of an equilibrium. The intuition is that if buyers inferred seller's private information from the price, then any "type" of monopolist, by means of his pricing behavior, would have the same incentives to induce an incorrect belief on the part of consumers with the intention to make higher profits. Consumers understand the incentives for misrepresentation that any "type" of seller has, and therefore anticipate that if they made purchases according to an expectations rule such as 'price is positively correlated with quality', they would be dissatisfied after consuming the good almost surely. As a result, rational buyers should expect any quality after observing a price. In equilibrium, consumers will disregard any information conveyed through the price so that their posterior belief equals their prior belief, for all p_2 .

The pooling nature of this equilibrium (price does not depend on S , or equivalently does not depend on Q) stems from the fact that consumers do not have

extra sources of information. In a related paper, Judd and Riordan (1994) demonstrate that when consumers have information of their own, then signalling occurs in equilibrium. In section 5 we extend our analysis to allow for this possibility.⁹

In what follows we characterize a *pooling-price equilibrium with past-sales advertising*. In words, it consists of a partition of the set of past-sales observations into two subsets: *one* for which the monopolist finds it optimal to invest in an advertising campaign to report past-sales, and *another* where the seller conceals its private information. For each subset, the monopolist employs different pricing rules. Consumers decide optimally and their conjectures are correct.

First, we formally define an advertising policy. Then, we define the equilibrium with past-sales advertising and characterize it.

Definition 1 An advertising policy is an interval $A = [q_A, \infty)$

Definition 2 An advertising equilibrium is an advertising policy A and a pricing function $p(Q)$ such that:

- (a) $p(Q)$ is optimal for the monopolist given the advertising policy A .
- (b) Consumers conjectures about the advertising policy A are correct.

Proposition 1 There exists an advertising equilibrium. In this equilibrium prices are uninformative.

Proof Define $A = [x, \infty)$. Let us suppose this is the consumers conjecture about the advertising policy and let us find the firm best response. If the firm advertises demand is $X_2 = n \log(E[q|Q]) - n \log(p)$ and its expected profit is

$$E[pX_2|Q] - c = pn \log(E[q|Q]) - np \log(p) - c$$

⁹The Industrial Organization literature has produced models where prices charged by *fully informed* sellers signal qualities (Wolinsky, 1983). In the basic case of our model, apart from the reason already mentioned, the absence of both cost asymmetries and repeated purchases impede signalling to emerge. In a single-period model, Bagwell and Riordan (1991) show that separation is achieved in equilibrium when higher quality firms have higher costs of production. But even when cost asymmetries are negligible, a high quality type may distinguish himself from a low quality one whenever repeated purchases play an important role in the market (Milgrom and Roberts, 1986).

Thus the optimal price is $\log(p^*) = \log(E[q|Q]) - 1$ and profit is $np^* - c$. If the firm does not advertise consumers infer that $Q < x$ and demand $X_2 = n \log(E[q|Q < x]) - n \log(p)$. The expected profit is therefore

$$E[pn(\log(E[q|Q < x]) - \log(p)) | Q] = pn(\log(E[q|Q < x]) - \log(p))$$

The optimal price is therefore $\log(p^{**}) = \log(E[q|Q < x]) - 1$ and profit is np^{**} . The firm will advertise if and only if

$$\frac{nE[q|Q]}{e} - c \geq n \frac{E[q|Q < x]}{e}$$

Equivalently if and only if

$$E[q|Q = x] - E[q|Q < x] = e^{ux} - E[q|Q < x] \geq \frac{ec}{n}$$

Consider the function $\phi(x) = e^{ux} - E[q|Q < x]$. If x is large enough then $\phi(x) > \frac{ec}{n}$. By continuity there exists a \tilde{x} such that $\phi(\tilde{x}) = \frac{ec}{n}$. It is now immediate that $[\tilde{x}, \infty)$, $p^*(\tilde{x})$ and $p^{**}(\tilde{x})$ is the advertising equilibrium.

Observe that in general the set of events for which advertising occurs shrinks as c increases.

The result above holds also for quadratic utility functions and arbitrary distribution functions (Monteiro and Moraga-Gonzalez, 1999).

A key feature of our equilibrium is that “herding behavior” on the part of the consumers unavoidably occurs when past-sales advertising arises in equilibrium. No individual consumer disregards the information released by the advertisements because it allows him to compute better estimates of q . As a consequence, the idiosyncrasies of the signals observed by predecessors will be inevitably passed on to the successors when the seller advertises its past-sales. As it is common in models of herding behavior, the equilibrium outcome will also exhibit path-dependence here. It is worth noting that even though it is ex-ante optimal for the buyers to use the information revealed through the advertisements, buyers may be dissatisfied ex-post consumption. This may happen when the drawn q is low but the realized noise is biased towards high values. First-generation consumers will demand much and so will second-generation buyers if advertising arises in equilibrium. However, their decisions will turn to be wrong ex-post consumption. It is well known that the success or failure of new products may very well depend

on non-controllable market forces such as consumers external signals.¹⁰ In our pooling-price equilibrium with past-sales advertising, the seller has the ability to exploit the consumer herding behavior in its own interest, by allowing it to happen or not.

5. Extension: Second-generation Consumers are Better Informed

In this section we suppose that consumers in the second period also receive informative signals. This is very natural given the basic model outlined above. In the earlier section prices were incapable of transmitting any information because buyers were totally ignorant before the seller set his marketing strategy. Here, in contrast, we shall see that prices can signal past-sales information.

Assume that second-period consumers exogenously receive extra information about the unknown quality parameter through the signals $t_i = q\epsilon_{i2}$, $i = 1, \dots, n$. As Judd and Riordan (1994) show, a separating equilibrium exists when consumers have an extra piece of information. Signalling emerges due to the fact that buyers have corroborating information of their own. If prices can transmit all the information, the natural question that arises is whether there is a role for past-sales advertising. Our analysis reveals that, for certain sales-data, price signalling is more costly than past-sales advertising, which gives a role for advertising. In what follows we derive a *price-separating equilibrium with past-sales advertising*.¹¹

Proposition 2 Suppose second-generation consumers exogenously receive informative signals. Then there is an advertising equilibrium. This equilibrium involves price signalling.

Proof We first find the optimal price when the firm advertises. If the firm advertises consumer demand is

$$x_i = \log(E[q|Q, t_i]) - \log p$$

Thus from Proposition 3(e) in the appendix it follows that

$$E[x_i|Q] = \hat{\alpha}Q + \hat{\gamma} - \log p$$

¹⁰A restaurant may be crowded certain day while another restaurant located just around the corner may be relatively empty. The next day, however, the opposite may very well happen apparently without any objective reason (Bikhchandani et al., 1992) for other examples.

¹¹We note that in a separating equilibrium consumers correctly infer seller's past-sales from the price. This considerably simplifies the computation of our equilibrium.

The per consumer profit is therefore $p(\hat{\alpha}Q + \hat{\gamma} - \log p)$. Thus the optimal price satisfies

$$\log p^* = \hat{\alpha}Q + \hat{\gamma} - 1 \quad (2)$$

and per consumer equilibrium profit is $e^{\hat{\alpha}Q + \hat{\gamma} - 1} - \frac{c}{n}$. If the firm does not advertise and consumers conjecture the pricing function to be $\phi(Q)$ with inverse $\psi = \phi^{-1}$, consumer demand is

$$\begin{aligned} x_i &= \log E[q|\phi(Q) = p, t_i] - \log p = \log E[q|Q = \\ &\psi(p), t_i] - \log p = \alpha\psi(p) + \beta \log t_i + \gamma - \log p \end{aligned}$$

Since $E[\log t_i|Q] = \frac{\sigma_q^2 + \sigma_1^2}{\sigma_q^2 + \sigma_1^2/n}Q + \mu_2$ (see equation (9) in the appendix) we have that

$$E[x_i|Q] = \alpha\psi(p) + \beta \frac{\sigma_q^2 + \sigma_1^2}{\sigma_q^2 + \sigma_1^2/n}Q + \hat{\gamma} - \log p = \alpha\psi(p) + \hat{\beta}Q + \hat{\gamma} - \log p$$

Here we define $\hat{\beta} = \hat{\alpha} - \alpha$. The monopolist's per consumer expected profit is therefore $p(\alpha\psi(p) + \hat{\beta}Q + \hat{\gamma} - \log p)$. The first order condition is

$$\alpha\psi(p) + \hat{\beta}Q + \hat{\gamma} - \log p + p\alpha\psi'(p) - 1 = 0$$

In equilibrium $p = \phi(Q)$. Thus using that $\psi'(\phi(p)) = \frac{1}{\phi'(p)}$, we have the first order differential equation:

$$\alpha Q + \hat{\beta}Q + \hat{\gamma} - \log \phi(Q) + \frac{\phi(Q)\alpha}{\phi'(Q)} - 1 = 0$$

Rewriting we obtain

$$\phi' = \frac{\alpha\phi(Q)}{1 + \log \phi(Q) - \hat{\gamma} - \hat{\alpha}Q} \quad (3)$$

Define $l(Q) = 1 + \log(\phi(Q)) - \hat{\gamma} - \hat{\alpha}Q$. Then from $l' = \frac{\phi'}{\phi} - \hat{\alpha}$ we obtain the equivalent differential equation

$$l' = \frac{\alpha}{l} - \hat{\alpha}$$

This is a separable variables equation with solution

$$\frac{l(Q)}{\hat{\alpha}} + \frac{\alpha}{\hat{\alpha}^2} \log \left(l(Q) - \frac{\alpha}{\hat{\alpha}} \right) = -Q + C \tag{4}$$

where C is a constant. From examination of the above equation we see that $l(-\infty) = \infty, l(\infty) = \frac{\alpha}{\hat{\alpha}}$ and that $l(Q)$ is strictly decreasing. We recover the pricing function by $\phi(Q) = \exp[l(Q) + \hat{\gamma} - 1 + \hat{\alpha}Q]$. In case of absence of advertising, per consumer profit is $e^{l(Q)+\hat{\gamma}-1+\hat{\alpha}Q}(1-l(Q))$, with the firm not producing if $l(Q) > 1$. We now prove that ϕ is actually the optimal price when there is no advertising. We begin by noting that from (4), $\lim_{Q \rightarrow -\infty} \phi(Q) = 0$. Thus $\phi(\mathbb{R}) = (0, \infty)$. The monopolist per consumer expected profit is

$$p(\alpha\psi(p) + \hat{\beta}Q + \hat{\gamma} - \log p)$$

Making the change of variable $k = \psi(p)$ the monopolist problem is equivalent to maximize

$$h(k) := \phi(k)(\alpha k + \hat{\beta}Q + \hat{\gamma} - \log \phi(k))$$

The derivative of h is

$$\begin{aligned} h'(k) &= \phi'(k)(\alpha k + \hat{\beta}Q + \hat{\gamma} - \log \phi(k)) + \phi(k)\left(\alpha - \frac{\phi'(k)}{\phi(k)}\right) = \\ \phi'(k)(\alpha k &+ \hat{\beta}Q + \hat{\gamma} - 1 - \log \phi) + \alpha\phi(k) = \phi'(k)(\alpha k + \hat{\beta}Q + \\ \hat{\gamma} - 1 - \log \phi) &+ \phi'(k)(1 + \log \phi - \hat{\gamma} - \hat{\alpha}k) = \phi'(k)\hat{\beta}(Q - k) \end{aligned}$$

Thus $k = Q$ and hence $p = \phi(Q)$ maximizes profit.

To find an advertising equilibrium we now must choose C . Define q_C by $l(q_C) = 1$. Choose C such that $e^{\hat{\alpha}q_C+\hat{\gamma}-1} < \frac{c}{n}$. The firm will advertise if and only if its profit is positive and greater than the no-advertising profit:

$$e^{\hat{\alpha}Q+\hat{\gamma}-1} - \frac{c}{n} \geq e^{l(Q)+\hat{\gamma}-1+\hat{\alpha}Q}(-l(Q) + 1) \text{ and } e^{\hat{\alpha}Q+\hat{\gamma}-1} - \frac{c}{n} \geq 0$$

Consider the function

$$h(x) = e^{\hat{\alpha}x+\hat{\gamma}-1} - \frac{c}{n} - e^{l(x)+\hat{\gamma}-1+\hat{\alpha}x}(1-l(x)) = e^{\hat{\alpha}x+\hat{\gamma}-1} \left(1 - e^{l(x)}(1-l(x)) \right) - \frac{c}{n}$$

Let us find h' :

$$\begin{aligned}
h'(x) &= \hat{\alpha}e^{\hat{\alpha}x+\hat{\gamma}-1} - e^{l(x)+\hat{\gamma}-1+\hat{\alpha}x} [(l' + \hat{\alpha})(1 - l(x)) - l'] = \\
&\hat{\alpha}e^{\hat{\alpha}x+\hat{\gamma}-1} - e^{l(x)+\hat{\gamma}-1+\hat{\alpha}x} [-\alpha + \hat{\alpha}] = e^{\hat{\alpha}x+\hat{\gamma}-1} (\hat{\alpha} - e^{l(x)}(-\alpha + \hat{\alpha})) \quad (5)
\end{aligned}$$

If $x = q_C$ we have $h(q_C) = e^{\hat{\alpha}q_C+\hat{\gamma}-1} - \frac{c}{n} < 0$. And we see from (5) that h is first decreasing then increasing. Now choose q_A by $h(q_A) = 0$. Thus since $h(q_A)$ is greater than $h(q_C)$, h is increasing for $x \geq q_A$. We conclude that the pair $([q_A, \infty), \phi(Q))$ is an advertising equilibrium with price signalling.

Remark 2 Note that the optimal price when the firm does not advertise is always higher than the price with advertising $e^{\hat{\gamma}-1+\hat{\alpha}Q}$. This highlights the implicit cost that signalling involves.

Remark 3 In this last remark we analyze the behavior of our solution when the second period signal is less and less informative¹²(when $\sigma_2 \rightarrow \infty$.) If σ_2 goes to infinity then $\hat{\alpha} \rightarrow \alpha$. Also $\hat{\gamma}$ converges to γ . The definition of $l(Q)$ (see 4) now implies that $l(Q) \geq 1$ in the limit. Thus the firm will never produce when there is no advertising. However it will advertise if and only if $e^{\alpha Q+\gamma-1} \geq \frac{c}{n}$.

Again, in our signalling equilibrium with past-sales advertising, consumers' herding behavior necessarily occurs. Notice also that, in contrast to the pooling-price equilibrium with past-sales advertising of section 4, here herding happens for all past-sales observations. A final observation is that since here buyers have corroborating information of their own, the effects of herding are however moderate as compared to the basic case in section 4.

6. Conclusions

In this paper we have studied an interesting practise: past-sales advertising. In a market where a producer sells a commodity of uncertain value to different generations of consumers who exhibit similar tastes, past-sales typically contain (noisy) information about buyers preferences (or equivalently, in the context of our analysis, product attractiveness). In such environments, we have shown that a monopolist has often an incentive to invest in costly advertising activities to report its private sales-data. This type of advertising conveys 'hard' (but noisy)

¹²We thank the referee for this suggestion.

information and thus does not function as a signalling device. By advertising, the supplier enables consumers to estimate good's properties better, and hence make wiser decisions. Often, but not always, the seller benefits from cleverer decisions on the part of the consumers. The equilibrium we derive presents this feature: advertising only occurs for promising past-sales observations. When past-sales advertising arises in equilibrium, the price charged equals the price that would be quoted if there was symmetric information in the market. When advertising does not occur in equilibrium, either separating prices or pooling prices are charged, depending on whether or not consumers have external information of their own.

The information released through the publicity campaign, when advertising arises in equilibrium, is ex-ante useful for the consumers and therefore never disregarded. This fact causes that consumers' herding behavior arises inevitably when advertising is optimal (or when price-signalling occurs in equilibrium). The seller exploits the potential for consumers herding in its own interest by choosing the appropriate advertising-pricing policy. The fact that past-sales advertising embodies a summary statistic of the actions taken by the predecessors – therefore not conveying any information about actual utilities derived by the consumers – implies that consumers may be dissatisfied ex-post consumption.

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Appendix: Conditional Distribution of Log-normals

In this appendix we collect some results on the conditional distribution of log-normal random variable. We fix three independent random variables, X_1, X_2, X_3 , such that $X_i \sim (\mu_i, \sigma_i^2)$, $\sigma_i > 0$, X_i is normally distributed with mean μ_i and variance σ_i^2 .

Lemma 1

(i) The conditional density of X_1 given $X_1 + X_2$ is

$$f_{X_1|X_1+X_2}(a, b) = \frac{r}{\sqrt{2\pi}} e^{-\frac{(ra+s)^2}{2}}, \text{ where } r = \frac{\sqrt{\sigma_1^2 + \sigma_2^2}}{\sigma_1\sigma_2}, rs = -\frac{\mu_1}{\sigma_1^2} - \frac{b - \mu_2}{\sigma_2^2} \quad (6)$$

(ii) The conditional distribution $E[e^{X_1}|X_1 + X_2 = b]$ is given by:

$$E[e^{X_1}|X_1 + X_2 = b] = \exp\left[\frac{\sigma_1^2\sigma_2^2}{(\sigma_1^2 + \sigma_2^2)}\left(\frac{1}{2} + \frac{\mu_1}{\sigma_1^2} + \frac{b - \mu_2}{\sigma_2^2}\right)\right] \quad (7)$$

(iii) The conditional expectation of $E[e^{X_1}|X_1 + X_2, X_1 + X_3]$ is given by

$$E[e^{X_1}|X_1 + X_2 = b, X_1 + X_3 = c] = \exp\left[\frac{\left(\frac{\mu_1}{\sigma_1} + \frac{b - \mu_2}{\sigma_2} + \frac{c - \mu_3}{\sigma_3}\right) + \frac{1}{2}}{r^2}\right], r = \sqrt{\frac{1}{\sigma_1^2} + \frac{1}{\sigma_2^2} + \frac{1}{\sigma_3^2}}$$

We omit the proof.

Proposition 3 (Potpourri of conditional expectations)

a) $\log(E[q|s_i]) = \frac{\sigma_q^2}{\sigma_q^2 + \sigma_1^2} (\log(s_i) - \mu_1)$;

b) $\frac{Q}{t} = \log(q) + \tilde{\varepsilon}$ where $t = \frac{\sigma_q^2}{\sigma_q^2 + \sigma_1^2}$ and $\tilde{\varepsilon}$ is normally distributed with parameters $(0, \frac{\sigma_1^2}{n})$;

$$\text{c) } E[q|Q] = e^{uQ} \text{ where } u = \frac{\sigma_q^2 + \sigma_1^2}{\sigma_q^2 + \sigma_1^2/n};$$

$$\text{d) } \log(E[q|Q, t_i]) = \alpha Q + \beta \log t_i + \gamma, \quad \alpha = \frac{\sqrt{n}}{t\sigma_1 r^2}, \quad \beta = \frac{1}{\sigma_2 r^2}, \quad \gamma = \frac{\frac{\mu_q - \mu_2}{\sigma_q} \frac{\sigma_2}{r^2} + \frac{1}{2}}{r^2},$$

$$r^2 = \frac{1}{\sigma_q^2} + \frac{n}{\sigma_1^2} + \frac{1}{\sigma_2^2};$$

$$\text{e) } E[\log(E[q|Q, t_i])|Q] = \hat{\alpha}Q + \hat{\gamma}, \quad \hat{\alpha} = \alpha + \beta \frac{\sigma_q^2 + \sigma_1^2}{\sigma_q^2 + \frac{\sigma_1^2}{n}}, \quad \hat{\gamma} = \gamma + \beta \mu_2.$$

Proof (a) From lemma 1(ii) above and using that $\mu_q = -\frac{\sigma_q^2}{2}$ we have that

$$\begin{aligned} \log(E[q|s_i]) &= \log\left(E\left[e^{\log(q)}|\log(q) + \log(\varepsilon_{i1}) = \log(s_i)\right]\right) \\ &= \frac{\sigma_q^2(\log(s_i) - \mu_1)}{\sigma_q^2 + \sigma_1^2} \end{aligned}$$

(b) From (a) we have that

$$\begin{aligned} Q &= \frac{1}{n} \sum_{i=1}^n \log(E[q|s_1^i]) = \frac{1}{n} \sum_{i=1}^n \frac{\sigma_q^2}{\sigma_q^2 + \sigma_1^2} (\log(s_i) - \mu_1) \quad (8) \\ &= \frac{1}{n} \sum_{i=1}^n \frac{\sigma_q^2}{\sigma_q^2 + \sigma_1^2} (\log q + \log(\varepsilon_{i1}) - \mu_1) \\ &= \frac{\sigma_q^2}{\sigma_q^2 + \sigma_1^2} \log q + \frac{\sigma_q^2}{\sigma_q^2 + \sigma_1^2} \frac{\sum_{i=1}^n (\log(\varepsilon_{i1}) - \mu_1)}{n} \end{aligned}$$

Therefore $\frac{Q}{t} = \log q + \tilde{\varepsilon}$ where $\tilde{\varepsilon} = \frac{\sum_{i=1}^n (\log(\varepsilon_{i1}) - \mu_1)}{n}$.

(c) Using Lemma 1(ii) and recalling that $\frac{1}{2} + \frac{\mu_q}{\sigma_q^2} = 0$ we have that

$$\begin{aligned} E[q|Q = a] &= E\left[e^{\log q}|\log q + \tilde{\varepsilon} = \frac{a}{t}\right] = \\ \exp\left[\frac{\sigma_q^2 \sigma_1^2/n}{\sigma_q^2 + \sigma_1^2/n} \left(\frac{1}{2} + \frac{\mu_q}{\sigma_q^2} + \frac{a/t}{\sigma_1^2/n}\right)\right] &= \exp\left[\frac{\sigma_q^2}{\sigma_q^2 + \sigma_1^2/n} \frac{a}{t}\right] = \\ \exp\left[\frac{\sigma_q^2 + \sigma_1^2}{\sigma_q^2 + \sigma_1^2/n} a\right] & \end{aligned}$$

We conclude that $E[q|Q] = e^{uQ}$ where $u = \frac{\sigma_q^2 + \sigma_1^2}{\sigma_q^2 + \sigma_1^2/n}$.

(d) Define $r = \sqrt{\frac{1}{\sigma_q^2} + \frac{n}{\sigma_1^2} + \frac{1}{\sigma_2^2}}$. Then from lemma 1(iii) above:

$$E[q|Q = a, t_i] = E \left[e^{\log q} | \log q + \tilde{\epsilon} = \frac{a}{t}, t_i \right] = \exp \left[\frac{\frac{\mu_q}{\sigma_q} + \frac{a}{\sigma_1 \sqrt{n}} + \frac{\log t_i - \mu_2}{\sigma_2} + \frac{1}{2}}{r^2} \right]$$

Thus

$$\log(E[q|Q, t_i]) = \frac{\frac{\mu_q}{\sigma_q} + \frac{Q\sqrt{n}}{t\sigma_1} + \frac{\log t_i - \mu_2}{\sigma_2} + \frac{1}{2}}{r^2}$$

(e) From the last item (d) we have that

$$E[\log(E[q|Q, t_i]) | Q] = E[\alpha Q + \beta \log t_i + \gamma | Q] = \alpha Q + \beta E[\log t_i | Q] + \gamma.$$

Since $\log t_i = \log q + \log \epsilon_{i2}$,

$$E[\log t_i | Q] = E[\log q | \log q + \tilde{\epsilon}] + \mu_2 = \frac{\sigma_q^2}{\sigma_q^2 + \sigma_1^2/n} \frac{Q}{t} + \mu_2 = \frac{\sigma_q^2 + \sigma_1^2}{\sigma_q^2 + \sigma_1^2/n} Q + \mu_2 \tag{9}$$

This ends the proof.