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Speed of Reversion of Deviations of the Purchasing Power Parity for Brazilian Cities

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Contents: 1. Introduction; 2. Literature review; 3. Methodology; 4. Analysis of the results; 5. Concluding remarks; Appendix.

Keywords: Purchasing Power Parity, Speed of Reversion, Nickell Bias, Temporal Aggregation Bias.

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This study estimates the rate of reversion of deviations from the PPP for Brazilian cities considering three possible sources of bias: i) Nickell, ii) the heterogeneity of the autoregressive coefficients, and iii) generated by the temporal aggregation of price indices. The values of the estimated half-lives were approximately 4.41 and 3.18 years when considering the price index of Brazil and the average of the indices of cities as references, respectively. When using the price index for each city as the numeraire, the median half-life is 3.13 years.

O presente estudo estima a velocidade de reversão dos desvios da PPC para as cidades brasileiras considerando três possíveis fontes de viés: i) de Nickell, ii) de heterogeneidade dos coeficientes autorregressivos, e iii) o gerado pela agregação temporal dos índices de preços. Os valores das meias-vidas estimados foram aproximadamente 4,41 e 3,18 anos ao se considerar o índice de preço do Brasil e a média dos índices das cidades como referência, respectivamente. Ao se utilizar o índice de preço de cada cidade brasileira como numerário, a meia-vida mediana é de 3,13 anos.

1. INTRODUCTION

According to the theory of Purchasing Power Parity (PPP), first investigated by Cassel (1921, 1922), in economies whose markets operate in perfect competition there is an equalization of prices, and their real exchange rates converge in the long run to a common stationary value. This hypothesis has become standard in many international macroeconomic models, and due to its importance, several empirical studies have been performed to test its validity. However, despite the extensive and growing literature, PPP is still an important area of research, for its relevance to theoretical models, as well as the difficulty

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empirical studies face in providing convincing evidence of its validity/regularity and in explaining the slow reversal speeds found (Culver & Papell, 1999).

This incomplete adjustment of the level of international relative prices can be justified by issues such as trade barriers; barriers arising from bureaucratic issues in the construction of the distribution system of goods sold; failures in the adjustment of the real exchange rate to shocks in relative prices; market imperfections, such as the presence of firms exercising monopoly power with different prices in segmented markets; transport costs associated with the transportation of goods from one region to another and, lastly, for possible differences in the price indices between countries (Rogoff, 1996; Taylor, 2004).

Therefore, the need to understand the persistence in deviations from the PPP for international data and the existence of large economic regions with a single currency, encouraged a large number of papers to investigate whether countries with continental dimensions, with great regional diversity, satisfy the conditions of PPP regularity and if the speed of reversion of deviations is lesser than shown in the international literature. Papers using data from within the same national borders, with common currency and trade, are to discuss the issues featured above.

Furthermore, this approach is relevant since excessive variations in relative prices, and hence on inflation differentials, lead to the inefficient allocation of resources among economic sectors and determine the differences in real wages and real interest rates that, in turn, influence the flows of labor and capital. Therefore, the movement of relative prices involves substantial loss of welfare to society, besides being useful in the investigation of the degree of integration and regional growth (Nath & Vargas-Silva, 2012; Hegwood & Nath, 2013). Also, with the use of intra data, it is possible to extract a better understanding of sources of persistent deviations from the PPPs present in studies using country information (Cecchetti, Mark, & Sonora, 2002).

In terms of the speed of reversion of deviations from the PPP, Rogoff (1996) finds as the standard in literature a range of 3 to 5 years for the half-life[1] of reversion of deviations from the PPP. However, considering intra information, where it is expected that deviations from the PPP would dissipate quickly, there is great variability in estimated half-lives, and these have been very sensitive to the choice of currency and the methodology used. For American cities, for example, there are estimations of half-lives which vary in the range of 3.82 (Culver & Papell, 1999) to 9.7 (Cecchetti et al., 2002) years.

Even though investigating the PPP and the speed of reversion from its deviations is widespread in international literature, in papers concerning American, Canadian, Mexican, Australian, Japanese and European cities,[2] one can observe in practical terms the absence of papers on the regional evolution of relative prices, with evidence on the speed of reversion of deviations from the PPP for Brazilian cities.

This study aims to fill that gap in literature by providing non-biased estimates of the speed of reversion of deviations from the PPP, denoted by the half-life of convergence for 11 Metropolitan Regions (MRs) in Brazil, for the 1991–2013 period considering Brazil as a whole, the average price level in Brazilian cities (average cross-section) and each of the MRs as the numeraire. To this end, we use the methodology featured in Choi, Mark, & Sul (2006), which proposes a panel estimation method with corrections for three possible sources of bias, namely: inappropriate grouping of biased cross-section units with heterogeneous autoregressive coefficients, Nickell bias (Nickell, 1981), or small samples bias, and bias derived from the temporal aggregation bias in price indices. It is noteworthy that the use of panel data in this kind of approach is desirable, because to combine units with cross-sections of time series considerably expands the number of observations, potentially increasing the accuracy of the estimated half-lives.

[1] Half the time required for a shock on the relative price level to dissipate.

[2] Engel & Rogers (1996); Culver & Papell (1999); Nenna (2001); (Cecchetti et al., 2002); L. L. Chen & Devereux (2003); Carrion-i-Silvestre, Barrio, & López-Bazo (2004); Nath & Sarkar (2009); Faber & Stokman (2009).

Besides this introduction, this work has four more sections. Below is presented a survey of the literature on price convergence, as well as recent papers relating to Brazil. The third section presents the data used and the econometric methodology. Afterwards, the results and concluding remarks of this work are featured.

2. LITERATURE REVIEW

After Cassel's initial papers (1921; 1922) on discussing the purchasing power parity (PPP), various papers, particularly from the 1990s onwards, have tried to better understand the relationship between prices and exchange rates among countries and even within certain countries.

Frankel & Rose (1995) use a set of 150 countries over the period of 1948 to 1992 to explore the cross-section variability provided by the longitudinal structure of the data, and estimate a half-life of about 4 years. Using data from the Organization for Economic Cooperation and Development (OECD) in the period from 1973 to 1986, Wei & Parsley (1995) estimate the half-life for the countries belonging to the European Monetary System (EMS) at 4.25 years and others this value increases to 4.75 years.

As mentioned before, several papers have studied not only the difference in prices among countries, but also within national borders of a given location. In a pioneering work regarding cities, Engel & Rogers (1996), analyze the nature of deviations from the PPP using data from the Consumer Price Index (CPI) of the 14 categories of consumer goods in 14 U.S. cities and 9 cities in Canada. Among their key findings are the positive effect of distance between cities on the volatility of relative prices and the greater variability of prices between equidistant cities in different countries. The authors also emphasize that nominal price rigidity seems to be one of the determinants of the "border effect"[3] on price volatility related factors.

Parsley & Wei (1996) find an upper limit for the speed of convergence of deviations from the PPP using panel data with quarterly prices of 51 goods and services in 48 U.S. cities over the period 1975:1 to 1992:4. Dividing into groups of tradable (perishable and non-perishable) and non-tradable goods, and calculating the half-lives from the median of the autoregressive coefficients of each group, the authors report reversals of approximately 5, 4 and 15 trimesters for non-perishable, perishable and non-tradable goods, respectively. They also find evidence of non-linearity in the convergence rates, since convergence occurs more rapidly when there are large gaps in prices to begin with.

Culver & Papell (1999), using data from the post-Bretton Woods era, find weak evidence of the validity of the PPP with intra-data referring to American and Canadian cities—compared to those obtained for European countries. Reversal speed of deviations from the PPP proved slower in the U.S. than those found for Canada and European countries. That is, even without the problems arising from trade barriers, exchange rate volatility, asymmetries in monetary policy and other factors that restrict arbitrage in the goods market, the authors find a slow process of price convergence in the United States.

The analysis of the sources of persistence in deviations from the PPP is also the main issue in Nenna (2001), which uses monthly data from Italy's main cities in the period from 1947 to 2000, and calculated a half-life of reversal of deviations from the PPP of 23.6 months. Also, the author finds evidence supporting the presence of the Harrod–Balassa–Samuelson effect,[4] and transportation costs as being determinants of the slow adjustment of relative prices.

Cecchetti et al. (2002) study the dynamics of price indices for 19 U.S. cities over the 1918–1995 period using panel data models. The authors find a slow speed reversal of deviations from the PPP,

[3]By controlling the distance—a proxy for transport costs—it is expected that cities from different countries exhibit higher volatility in exchange rates when compared to cities with the same distance, but located under the same border. We define this phenomenon as border effect.

[4]This hypothesis concerns the differential growth in productivity of domestic tradable goods sectors and non-tradable as generators of inflation differentials, changing the structure of internal prices.

with a half-life of approximately 9 years. The authors try to explain this slow adjustment of prices by a combination of the presence of transport costs, differential speeds of adjustment to large and small shocks and the inclusion of non-tradable goods in the computation of the general price index.

Using time series for 19 U.S. cities during the 1918 to 2000 period, L. L. Chen & Devereux (2003) find strong evidence that the prices of these cities converge over time, and that the dispersion of price levels is smaller for these cities than for OECD countries. The authors also claim that the nonstationarity of the real exchange rate is not evidence against the validity of the PPP when there is price convergence, since the real exchange rate[5] of the cities cannot return to a fixed average. Thus, this evidence is consistent with a broader version of the PPP that allows the transport costs and greater market integration reductions.

Carrion-i-Silvestre et al. (2004), in a study of 50 Spanish cities in the 1937–1992 period, find evidence in favor of the PPP, rejecting the null hypothesis of the presence of a unit root in 3 of 4 tests applied with an average half-life of 3.6 years.

Using data for 35 Mexican cities in the 1982–2000 period, Sonora (2005) examines the convergence of the price level for a country with moderately high inflation noting that in economies in such condition, in general, converge more quickly to the relative price of equilibrium. The results of unit root tests do not reject the hypothesis of PPP and display an estimated half-life of between 2 and 3 years. In addition, to examine the PPP in homogeneous areas in terms of productivity and preferences, the author divides the sample in regional areas, obtaining half-lives in the range of 1.7 to 7 years.

Choi & Matsubara (2007) use relative prices in Japanese cities for different types of goods and find that, regardless of the persistence measure used, the average half-lives are shorter than 2 years for most price indices considered. The authors note the existence of heterogeneity in the persistence within the categories of tradable and non-tradable goods and between cities. Thus, the authors assume that the extent of the heterogeneity among the CPI items is related to the degree of tradability and market structure, while physical distance and the relative size of cities can affect the heterogeneity between cities.

Choi et al. (2006) emphasize the existence of three potential sources of bias introduced by the structure of the panel data estimation of the half-life of deviations from the PPP. Such biases are introduced by the inappropriate aggregation of cross-section units with heterogeneous autoregressive coefficients estimates in small samples with constant term, and the presence of a lagged dependent variable (Nickell bias), and the aggregation of prices over time. The authors note, using data for 21 OECD countries, that the heterogeneous cross-section of the convergence rate for the PPP does not seem to be a quantitatively important source of bias. While simultaneously controlling for other sources of bias, the estimates produce an average half-life of 3 years.

Nath & Sarkar (2009) find no evidence of heterogeneity bias using annual data from the consumer price index (CPI) for 17 U.S. cities during the 1918–2006 period. As Choi et al. (2006), this work applies the Nickell bias correction method, as well as the time aggregation bias, producing a half-life of 7.5 years, lower than estimates from previous studies, although still quite slow.

Das & Bhattacharya (2008) use unit root tests for panels which are robust to cross-section dependence for Indian cities in 1995:1–2004:6 period. The authors estimate half-lives of 8.14 and 22.89 months for shocks on the common and idiosyncratic component respectively.

With a database comprising almost the entire period of the European integration, from 1960 to 2003, Faber & Stokman (2009) observe that there is strong evidence of the convergence of price levels in Europe for much of the past 40 to 50 years, levels that have been common over time in the United

[5]It is noteworthy that, for intranational data, relative prices and the real exchange rate are identical, since the nominal exchange rate in this case is equal to 1 ($E = 1$); i.e., the real exchange between cities will be $\theta = EP_i/P_j = P_i/P_j$, where P_i is the price level in city i, P_j is the price level in city j, $i \neq j$. However, henceforth the term relative price will be used in order to standardize the terminology.

States. Among the determinants of the dispersion of European price levels, the authors stress that indirect taxes, convergence of costs of tradable and non-tradable inputs, have contributed to different extents and degrees of variation on the time of convergence of price levels.

Nagayasu & Inakura (2009) use aggregated and disaggregated consumer price indices of Japanese cities in 1990–2003 to verify the convergence of relative prices using Tokyo as a benchmark. The authors find evidence in favor of the PPP, as evidence suggests stationary relative prices in Japan and a half-life of approximately 2 years.

To examine whether the choice of numeraire impacts on the dynamic behavior of relative prices in American cities, Chmelarova & Nath (2010) model the relative price between cities as being composed by two components: a common factor to all cross-sections and an idiosyncratic factor, which varies between cross-sections. The results suggest that the dynamic behavior of relative prices depends on the data of the chosen city as numeraire. With an estimated half-life varying from 7.60 to 18.11 years, the authors also point out that when correcting the estimates for the Nickell bias and temporal aggregation bias, as suggested by Choi et al. (2006), the half-lives obtained are smaller than those presented in a number of previous studies. Table 1 summarizes the main papers in literature with a summary of the data used, the methods and the main results on the reversal speed of deviations from the PPP.

In a general equilibrium approach, Carvalho & Nechio (2011) present a multisector, two-country, sticky-price model. They introduce the heterogeneity in the frequency of price changes across sectors, generating heterogeneous sectoral real exchange rate dynamics. In this framework, they find a half-life of 3.25 years for the PPP deviations. On the other hand, the counterfactual one-sector world economy produces a half-life just above 1 year. So, they conclude that the heterogeneity is an important determinant of the slow speed of reversion of the PPP deviations. Moreover, they point out that the papers which usually find a small role for heterogeneity and aggregation take into account just the aggregation effect—the total heterogeneity effect can be decomposed into an aggregation effect and a counterfactuality effect. However, according to them, the total heterogeneity effect on the half-life is primarily determined by the counterfactuality effect.

Thus, as can be seen, despite the extensive literature on the speed of reversion of deviations from the PPP in papers regarding cities, one sees almost no work in that line about the speed of reversion of deviations from the PPP for Brazilian cities. This work aims to contribute to this issue by providing empirical evidence for a sample of 11 Brazilian cities between 1991 and 2013, through the estimation method proposed by Choi, Mark, & Sul in 2006.

3. METHODOLOGY

3.1. Description and analysis of data

The data used in this work, obtained from the Brazilian Institute of Geography and Statistics (IBGE), contains monthly information on the Consumer Price Index (IPCA) for 11 Brazilian cities,[6] covering the period from January 1991[7] to September 2013.

For purposes of estimating the half-life of the reversal of deviations from the PPP, the information was aggregated from a simple average, taking 2005 as the base year, so that the new data has annual frequency. This aggregation is necessary, since it uses the estimated coefficient of an AR(1) for calculating the speed of reversion and, as emphasized by Choi et al. (2006), this type of analysis is the most appropriate when the frequency of the data is annual. The authors point out that such a specification is appropriate to avoid complications such as the non-uniqueness of the half-life and setting the order of the autoregressive process, which for data on monthly prices requires a higher order.

[6] Belém, Belo Horizonte, Brasília, Curitiba, Fortaleza, Goiânia, Porto Alegre, Recife, Rio de Janeiro, Salvador and São Paulo.

[7] August 1991 was obtained by geometric mean from the values observed in the months of July and September.

Table 1. Half-life results obtained in empirical research.

Author(s)	Index	Period	Cities/Countries	Numeraire	Methodology	$H(\hat{p})$ years
Parsley & Wei (1996)	51 Prices of Commodities	1975:01–1992:04	USA Cities	New Orleans and New York	Levin e Lin (LL) test	1.25, 1.00 and 3.75[vii]
Culver & Papell (1999)	Consumer Price Index	USA (1978:05–1997:04) CAN (1978:09–1997:06)	USA and Canada Cities	All Cities	Feasible GLS	USA: 3.82 CAN: 1.83[i]
Nenna (2001)	Consumer Price Index	1947–2000	Italian Cities	Rome and Cross-section Average	LL test	1.97
Cecchetti et al. (2002)**	Consumer Price Index	1918–1995	USA Cities	Cross-section Average	From LL and Im, Pesaran e Shin (IPS) unit root tests with adjustments for the Nickell bias	8.50–9.70[iii]
L. L. Chen & Devereux (2003)**	Absolute Price Level built from the CPI	1918–2000	USA Cities	USA	Augmented Dickey-Fuller (ADF) test	4.98[ii]
Carrion-i-Silvestre et al. (2004)	Consumer Price Index	1939:07–1992:12	Spanish Cities	Spain	From unit root tests	3.60[ii]
Sonora (2005)	Consumer Price Index	1982:01–2000.12	Mexican Cities	México DF	Mean of $\hat{\rho}$ ADF with bias adjustment estimated through Kendall (1954)	2.00–3.00[iv]
Choi & Matsubara (2007)	36 Disaggregated CPI Items	1970:01–2002:12	Japanese Cities	All Cities	Sum of autoregressive coefficients (SARC), Impulse Response Function (IRF), RGLS and non-linear	< 2.00[v]
Das & Bhattacharya (2008)	Consumer Price Indices for Industrial Workers	1995:01–2004:06	Indian Regions	Cross-section Average and Nagpur	Moon-Perron (MP) test, direct Dickey-Fuller (DDF) and Robust developed by Breitung & Das (2008) with adjustment to the Nickel bias	0.68/1.91[vi]
Nath & Sarkar (2009)*	Consumer Price Index	1918–2006	USA Cities	Cross-section Average	GLS with FE with bias correction for Nickell and temporal aggregation	7.50
Nagayasu & Inakura (2009)	CPI Aggregated and Disaggregated	1990–2003	Japanese Cities	Tokyo	Moon & Perron (2004) unit root test	≈ 2.00
Chmelarova & Nath (2010)*	Consumer Price Index	1918–2007	USA Cities	All Cities	GLS with FE and Nickel bias correction	9.54[i]
Mohsin & Gilbert (2010)	Consumer Price Index	2001:07–2008:06	Pakistani Cities	Karachi and Lahori	Spatial GLS	< 0.42
Frankel & Rose (1995)	Consumer Price Index	1948–1992	150 países	USA	OLS with Standard errors corrected through Huber/White	≈ 4.00
Wei & Parsley (1995)	Sector Price Indices	1973–1986	14 OECD countries	–	–	4.25 and 4.75[viii]
Culver & Papell (1999)	Consumer Price Index	1978:01–1997:02	European Union	All Countries	Feasible GLS	2.19[i]
Choi et al. (2006)	Consumer Price Index	1973–1998	21 OECD coutries	All Countries	GLS with FE with bias correction for Nickell and temporal aggregation	3.00[ii]
Carvalho & Nechio (2011)	Artificial Data Generated by the Model	–	US Economy and Rest of the World	–	Computed directly from the impulse response functions implied by the solution of the models	3.25 and 1.17[ix]

Notes: *Used the same cities. **Used the same cities. (–) Unavailable. [i] Mean. [ii] Median. [iii] Interval tests LL (inferior) and IPS (superior). [iv] Minimum and Maximum intervals. [v] Inferior median 2 years for most numeraires. [vi] For common component and the idiosyncratic factor, respectively. [vii] For non-perishable, perishable and non-tradable goods, respectively. [viii] For countries belonging and not belonging to the European Monetary System, respectively. [ix] Multisector, two-country, sticky-price model and counterfactual one-sector world economy, respectively.

Table 2 reports the volatility of relative prices, measured in terms of their standard deviations, using each of the 11 metropolitan areas as numeraire, as well as the cross-section average and the IPCA in Brazil, aiming to examine the dynamic behavior of prices on the cities in the period. Thus, for each year there is the standard deviation of relative prices compared to its average value for each numeraire. The last row of the table refers to the average annual change in the volatility of relative prices.

It can be observed that the dispersal of relative prices has been reduced on average by 2.28% per year, which reflects a process of convergence and greater market integration among Brazilian cities. It is noteworthy that in terms of relative price, considering the cities of Belém and Recife as references, those two municipalities were the ones with the highest and lowest annual average reduction in volatility, 3.33% and 1.38% annually, respectively

3.2. Methodology

To quantify the half-life of reversal of deviations from the PPP for each numeraire considered, first we estimate the following first-order autoregressive process, AR(1), with fixed effects and possible heterogeneity among cross-sections units:

$$r_{it} = \alpha_i + \rho_i r_{it-1} + \varepsilon_{it}, \qquad i = 1,\dots,N \quad \text{and} \quad t = 1,\dots,T, \tag{1}$$

in which

$$r_{it} = \left(\ln P_{it} - \ln P^*_{jt}\right) \times 100, \tag{2}$$

where r_{it}, is the natural logarithm of the relative price for city i in year t, and r_{it-1} is the first lag of r_{it}.[8] In equation (2), P_{it} is the IPCA for city i in year t, and P^*_{jt} is the IPCA of the numeraire j chosen in year t.[9] From the estimated coefficients, $\hat{\rho}_i$, one obtains the estimated half-lives for each numeraire, defined as the period of time necessary for deviations from the PPP to be dissipated in half, through the following equation:

$$H(\hat{\rho}_i) = -\frac{\ln 2}{\ln \hat{\rho}_i}. \tag{3}$$

However, the speed of reversion of deviations from the PPP is non-linear and extremely sensitive to the value of $\hat{\rho}$. Therefore, a cautious estimation and strong statistical rigor is necessary to obtain the autoregressive coefficient ($\hat{\rho}$) so that there is greater precision in the calculation of $H(\hat{\rho})$.

In that sense, Choi et al. (2006) warn of the possible presence of three important biases in the estimation of the speed of reversion of deviations from the PPP using panel data: the bias generated by the inappropriate aggregation of heterogeneous autoregressive coefficients, the bias originated from the estimation with small samples which occurs when a dynamic regression includes an intercept (Nickell bias), and the bias which occurs because of the time aggregation of data.[10]

The heterogeneity bias arises when different rates of convergence of the PPP are considered identical in the estimation of panel data. Additionally, as the IPCA is constructed from the prices of several individual goods that have different speeds of adjustment, the data is also subjected, as highlighted Imbs, Mumtaz, Ravn, & Rey (2005), to the bias arising from sectorial heterogeneity. However, S.-S. Chen & Engel (2005) shows that sectoral heterogeneity is not a significant source of bias and other empirical papers also do not find evidence in favor of heterogeneity (Choi et al., 2006; Nath & Sarkar,

[8]Note that the log-linear equation of the real exchange rate (2) does not depend on the nominal exchange rate. This happens because these are observations for cities located within the same national boundary, therefore, with the same currency, implying that the nominal exchange $E_{it} = 1$, and, hence $e_{it} = \ln E_{it} = 0$.

[9]The subscript j refers to the aggregate index of Brazil, the average cross-section; or each of the metropolitan regions considered.

[10]For a complete derivation of all sources of bias and the estimation procedure reported in this work, see Choi et al. (2006) and Phillips & Sul (2007).

Table 2. Relative price dispersal.

Year	Numeraire											
	Belém	Belo Horizonte	Brasília	Curitiba	Fortaleza	Goiânia	Rio de Janeiro	Salvador	São Paulo	Porto Alegre	Recife	Brazil	Mean
1991	0.050	0.046	0.051	0.050	0.051	0.051	0.046	0.049	0.050	0.052	0.041	0.049	0.049
1992	0.042	0.040	0.042	0.042	0.037	0.039	0.036	0.037	0.042	0.042	0.038	0.040	0.040
1993	0.033	0.032	0.034	0.034	0.026	0.034	0.030	0.032	0.034	0.034	0.033	0.033	0.033
1994	0.033	0.033	0.034	0.035	0.027	0.035	0.029	0.034	0.035	0.034	0.035	0.033	0.033
1995	0.033	0.037	0.037	0.038	0.034	0.037	0.032	0.037	0.037	0.037	0.038	0.036	0.036
1996	0.033	0.035	0.033	0.035	0.033	0.033	0.031	0.034	0.029	0.032	0.034	0.033	0.033
1997	0.033	0.033	0.032	0.034	0.033	0.032	0.032	0.033	0.021	0.031	0.033	0.032	0.032
1998	0.031	0.031	0.029	0.031	0.030	0.029	0.031	0.031	0.021	0.029	0.031	0.030	0.030
1999	0.025	0.026	0.025	0.026	0.025	0.023	0.026	0.026	0.016	0.025	0.025	0.025	0.025
2000	0.021	0.022	0.021	0.021	0.021	0.018	0.021	0.021	0.013	0.021	0.022	0.020	0.020
2001	0.017	0.017	0.017	0.016	0.017	0.015	0.017	0.017	0.011	0.017	0.017	0.016	0.016
2002	0.015	0.014	0.014	0.014	0.015	0.013	0.015	0.014	0.012	0.015	0.015	0.014	0.014
2003	0.013	0.011	0.013	0.013	0.013	0.012	0.013	0.011	0.013	0.012	0.013	0.013	0.013
2004	0.008	0.008	0.009	0.009	0.009	0.008	0.009	0.008	0.009	0.008	0.009	0.009	0.009
2005	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
2006	0.009	0.007	0.008	0.008	0.008	0.009	0.009	0.008	0.007	0.008	0.008	0.008	0.008
2007	0.014	0.012	0.014	0.013	0.014	0.014	0.014	0.014	0.014	0.013	0.014	0.013	0.013
2008	0.017	0.019	0.021	0.019	0.020	0.020	0.020	0.020	0.019	0.020	0.019	0.020	0.020
2009	0.017	0.022	0.023	0.021	0.023	0.023	0.023	0.023	0.022	0.022	0.022	0.022	0.022
2010	0.018	0.024	0.025	0.024	0.025	0.024	0.025	0.025	0.024	0.024	0.025	0.024	0.024
2011	0.017	0.021	0.022	0.022	0.022	0.021	0.023	0.022	0.021	0.020	0.022	0.021	0.021
2012	0.020	0.024	0.026	0.025	0.026	0.023	0.026	0.026	0.024	0.024	0.025	0.024	0.024
2013	0.023	0.029	0.031	0.030	0.031	0.028	0.031	0.031	0.028	0.028	0.030	0.029	0.029
Δ% per year	-3.33	-2.01	-2.25	-2.26	-2.23	-2.62	-1.78	-2.04	-2.48	-2.66	-1.38	-2.27	-2.27

Notes: Δ% per year refers to the average growth rate of volatility in relative prices between 1991 and 2013, defined as: $[(\ln(t_f) - \ln(t_0))/T] \times 100$, in which t_0 and t_f indicate the volatility in relative prices in the first and last period of the sample, respectively, and $T = 23$.

2009; Chmelarova & Nath, 2010). To verify the assumption of homogeneity of these autoregressive parameters, we use the method proposed by Pesaran & Yamagata (2008), which is appropriate for dynamic panel data models of the AR(1) kind in which $T \geq N$. In case the null hypothesis of homogeneity $H_0\colon \rho_i = \rho,\ \forall i$, is rejected, an estimation procedure based on the Recursive Mean Adjusted Seemingly Unrelated Regression (RSUR)[11] can be applied. This test is based on the dispersion of individual slope estimates from a suitable estimator. Let τ_T be a $T \times 1$ vector of ones, and k the number of regressors. The test statistics[12] used here is defined by

$$\widetilde{\Delta} = \sqrt{N}\left(\frac{N^{-1}\tilde{S} - k}{\sqrt{2k}}\right), \tag{4}$$

$$\tilde{S} = \sum_{i=i}^{N}\left(\widehat{\beta}_i - \widetilde{\beta}_{\text{WFE}}\right)' \frac{X_i' M_\tau X_i}{\widetilde{\sigma}_i^2}\left(\widehat{\beta}_i - \widetilde{\beta}_{\text{WFE}}\right), \tag{5}$$

$$\widetilde{\beta}_{\text{WFE}} = \left(\sum_{i=1}^{N} \frac{X_i' M_\tau X_i}{\widetilde{\sigma}_i^2}\right)^{-1} \sum_{i=i}^{N} \frac{X_i' M_\tau y_i}{\widetilde{\sigma}_i^2}, \tag{6}$$

$$M_i = I_T - Z_i\left(Z_i' Z_i\right)^{-1} Z_i' \quad \therefore \quad Z_i = (\tau_T, X_i). \tag{7}$$

As for the Nickell bias, it occurs because the error of the estimated regression with the variables as deviations from the sample mean is correlated with current and future values of the dependent variable, and as these future values make up the sample mean which is now incorporated into the explanatory variable, the errors are also correlated with the explanatory variable. Under these conditions, by including the constant term, the least squares estimator underestimates the autoregressive coefficient, and, because of the use of panel data, even estimating by least squares with dummy variables (LSDV) does not eliminate this bias.

To correct the Nickell bias in the estimates when this is the only source of bias in the dynamic panel, the inverse of the bias presented in Choi et al. (2006) is applied to the coefficient estimated by Feasible Generalized Least Squares with fixed effects (FGLS) to obtain a non-biased estimator (MUE), which is $\hat{\rho}_{\text{MUE}} = m^{-1}\left(\hat{\rho}_{\text{LSDV}}\right)$.

The third source of bias considered in this work may occur because of the temporal aggregation of data from the mean, which could introduce a moving average MA(1) structure in the regression error. Information that has been collected on a daily, weekly or monthly frequency and is transformed into annual data, and which does not consider this type of problem, tends to overestimate the true value of ρ and, hence, the half-life.

Thus, data which was aggregated is indexed by an interval $t = 1, \ldots, T$, and within each interval there are M subintervals that depend on the frequency with which the data was collected. The estimations carried out in this work were corrected for this problem considering $M = 12, 30$ and 365.[13] To circumvent this problem, the inverse of the temporal aggregation bias in function of M is applied.

Lastly, given the existence of both sources of bias—Nickell and temporal aggregation—there still may be a combined effect of the interaction between them generating an additional bias.[14] The combined net effect of this bias depends on the true value of ρ.

Choi et al. (2006) show that in the neighborhood of $\rho = 0.9$, the biases annul each other; if the true value of $\rho < 0.9$, there is an overestimation of the autoregressive coefficient, and underestimation if the

[11] For a detailed description, see Choi et al. (2004).

[12] Pesaran & Yamagata (2008) show that $\widetilde{\Delta} \to N(0,1)$ as $(N,T) \to \infty$.

[13] We chose to display the estimation for $M = 12$. Estimates for $M = 30$ and $M = 365$ show no significant differences compared to what has been presented here and are available upon request.

[14] In the absence of heterogeneity of the autoregressive coefficient.

contrary occurs. The procedure to correct the combined bias consists in estimating ρ through FGLS, and afterwards applying the inverse function of the combined bias calculated in Choi et al. (2006), which is $\hat{\rho}_{\text{GNTAU}} = B^{-1}(\hat{\rho}_{\text{FGLS}}, M, T)$,[15] in which $B^{-1}(\cdot)$ is the inverse of the Nickell and temporal aggregation biases combined, M is the number of adopted subintervals, and T is the number of periods.

In short, the econometric strategy adopted in this work follows the following steps: first, the homogeneity test proposed by Pesaran & Yamagata (2008) is used to verify if heterogeneity is a relevant source of bias in the data used. Afterwards, in case the autoregressive coefficients are homogenous, we estimate (1) under $\rho_i = \rho$, $\forall i$, through FGLS with fixed effects, $\hat{\rho}_{\text{FGLS}}$. Then, the Nickell bias correction is applied to $\hat{\rho}_{\text{FGLS}}$, obtaining thus $\hat{\rho}_{\text{GMUE}} = m^{-1}(\hat{\rho}_{\text{FGLS}})$. Also, the inverse of the temporal aggregation bias is applied to calculate $\hat{\rho}_{\text{GTAU}}$. Lastly, we correct $\hat{\rho}_{\text{FGLS}}$ considering the combined Nickell and time aggregation biases, to generate $\hat{\rho}_{\text{GNTAU}}$. In all those steps, the half-lives are calculated as defined in equation (3).

4. ANALYSIS OF THE RESULTS

To estimate the dynamic behavior of relative prices among Brazilian cities according to the methodology presented in the previous section, first we verify the validity of the hypothesis of homogeneity of the autoregressive parameters as proposed by the Pesaran & Yamagata (2008) test. Table 1 in the Appendix shows the results of this test conducted for each city used as a numeraire. The hypothesis of homogeneity is not rejected at the 1% level only when Belo Horizonte is the numeraire city.[16] Therefore, cross-section heterogeneity is not shown to be a significant source of bias when estimating the rate of reversion of deviations from the PPP for Brazilian cities. Thus, the estimation procedure adopted here follows the one proposed by Choi et al. (2006).

Table 3 summarizes the results of the steps described in the previous section: (i) estimation without correction, (ii) only correcting the Nickell bias, (iii) only correcting the temporal aggregation bias, (iv) with the correction for the combined Nickell bias and time aggregation.

In general, the evidence, as expected, indicates that the half-lives estimated with correction for both biases are, for all the numeraires considered, within the range of estimates that correct only for the Nickell bias or temporal aggregation bias alone. Furthermore, there is evidence that the effect of the time aggregation bias overlaps the Nickell bias, since, with the exception of estimates having Belém and Porto Alegre as numeraires, the other half lives without corrections were superior to those with correction of both biases.

Although the literature indicates that the half-lives when using data for cities are shown to be quite sensitive to the choice of numeraire, it can be stated that, within the limits of 2.55 to 3.84 years, the choice of numeraire does not exert great influence on the speed of reversion of deviations from the PPP for Brazilian cities.[17] This range observed for the half-lives is reasonably greater than the range obtained by Sonora (2005), which was 2–3 years for Mexican cities. It is noteworthy that, for American cities, these vary between 3.82 (Culver & Papell, 1999) and 9.70 (Cecchetti et al., 2002) years.

With a median half-life of 3.13 years, the results obtained are consistent with some recent studies for American cities such as L. L. Chen & Devereux (2003), with reversal speed of 4.98 years deviations from the PPP; for Spanish cities, Carrion-i-Silvestre et al. (2004) obtained a half-life of 3.60 years.

[15] GNTAU refers to the estimation of FGLS combined with correction of bias Nickell and Temporal Aggregation.

[16] Choi et al. (2006) use a sample with 21 industrial countries and do not reject the null hypothesis of homogeneity only when Germany is the numeraire country. They conclude that evidence against homogeneity is weak in their data set and that pooling is appropriated.

[17] Table A-2, in Appendix, reports the results for aggregate regions. There is no significant difference.

Table 3. Feasible GLS panel estimation.

Numeraire	No Bias Corrections		Nickell Bias Corrected		Time Aggregation Bias Corrected		Nickell and Time Aggregation Bias Corrected	
	$\hat{\rho}_{\text{FGLS}}$	$H(\hat{\rho}_{\text{FGLS}})$	$\hat{\rho}_{\text{GMUE}}$	$H(\hat{\rho}_{\text{GMUE}})$	$\hat{\rho}_{\text{GTAU}}$	$H(\hat{\rho}_{\text{GTAU}})$	$\hat{\rho}_{\text{GNTAU}}$	$H(\hat{\rho}_{\text{GNTAU}})$
Belém	0.829	3.70	0.921	8.42	0.722	2.13	0.830	3.72
Belo Horizonte	0.836	3.87	0.886	5.73	0.681	1.80	0.785	2.86
Brasília	0.826	3.63	0.868	4.90	0.661	1.67	0.762	2.55
Curitiba	0.836	3.87	0.925	8.89	0.726	2.16	0.835	3.84
Fortaleza	0.835	3.84	0.876	5.24	0.670	1.73	0.772	2.68
Goiânia	0.817	3.43	0.899	6.51	0.697	1.92	0.802	3.14
Rio de Janeiro	0.846	4.14	0.894	6.19	0.691	1.88	0.795	3.02
Salvador	0.817	3.43	0.908	7.18	0.708	2.01	0.814	3.37
Porto Alegre	0.814	3.37	0.919	8.21	0.720	2.11	0.828	3.68
Recife	0.849	4.23	0.898	6.44	0.696	1.91	0.801	3.13
São Paulo	0.845	4.12	0.878	5.33	0.672	1.74	0.774	2.71
Brasil	0.939	11.01	0.94	11.2	0.744	2.34	0.855	4.41
Mean	0.832	3.77	0.901	6.65	0.699	1.94	0.804	3.18
Minimum	0.814	3.37	0.868	4.90	0.661	1.67	0.762	2.55
Maximum	0.849	4.23	0.925	8.89	0.726	2.16	0.835	3.84
Mean	0.832	3.76	0.897	6.41	0.695	1.90	0.800	3.15
Median	0.835	3.84	0.898	6.44	0.696	1.91	0.801	3.13

Note: Estimates using the IPCA for Brazil and the mean as numeraires were not considered in the calculation of minimum, maximum, mean, and median.

Additionally, as expected, the evidence of reversal speed of deviations from the PPP for countries,[18] except when compared to those observed in Culver & Papell (1999), were greater than the median half-life of 3.13 years found for Brazilian cities. This reinforces the idea that, for cities located within the same national borders and common currency and the absence of asymmetries in monetary policy, there is greater integration of markets and lower persistence of deviations from the PPP, although the speed of adjustment still appears to be slow.

Table 4 reports the percentage of the estimated half-lives in this work that are below, within or above the range indicated by Rogoff's consensus (1996). The results show that 66.67% of the estimated half-lives are in that range, and that none surpassed such interval.

Table 4. Proportion of half-lives according to Rogoff's (1996) interval consensus.

$H(\hat{\rho}_{\text{GNTAU}}) < 3$	$3 \leq H(\hat{\rho}_{\text{GNTAU}}) \leq 5$	$H(\hat{\rho}_{\text{GNTAU}}) > 5$
33.33%	66.67%	0.00%

Note: Half-lives refer to estimates of both kinds of bias correction. Estimations using the aggregated IPCA for Brazil and the Mean as numeraires were not considered.

[18] Frankel & Rose (1995); Wei & Parsley (1995). Frankel (1986)—4.6 years—and other papers concerning countries which are not discussed here showed a greater half-life than the average result of this work.

It was also observed that 33.33% of estimated reversal speeds are below the stipulated range. This result can be explained by the recent range of the data used, since in recent years the cost of transport and information for consumers has decreased,[19] reducing the time reversal of deviations from the PPP. In addition, some products previously considered to be non-tradable, such as education and financial services, became tradable with the reduction of information costs contributing to the reduction of the half-life of deviations.

5. CONCLUDING REMARKS

This work estimates the rate of reversion of deviations from the PPP for 11 Brazilian cities through the methodology proposed by Choi et al. (2006), which can correct for three possible biases in the estimation of half-lives using panel data, which are: the bias generated by the inappropriate aggregation of heterogeneous coefficients, the Nickell bias, and the bias from the temporal aggregation of data indices.

The estimated half-lives when using the price index of Brazil and the average of price indices of cities as references are 3.18 and 4.41 years, respectively. When using the price index for each city as numeraire, a median half-life of 3.13 years is estimated. It is noteworthy that 33.33% of the half-lives obtained here was inferior to the consensus range suggested by Rogoff (1996) of 3–5 years, and none surpassed such interval.

The results confirm the prediction that speeds of reversion of deviations from the PPP when using data for cities should be smaller than those observed when using data from countries. This finding is explained in terms of greater market integration, more homogeneous areas regarding preferences and productivity, the absence of asymmetries in monetary policy, lower trade barriers and bottlenecks in the distribution system of goods sold, reduced transport costs and, lastly, the more homogeneous composition of price indices between cities within the same country (Sonora, 2005; Carrion-i-Silvestre et al., 2004).

Finally Carvalho & Nechio (2011) reports a half-life of 3.25 years for the PPP deviations, which is close to our results. Nevertheless, this estimate is obtained in the presence of heterogeneity in the frequency of price setting across sectors, and without it, this value is smaller. They show that cross-section heterogeneity always causes a positive bias, and one possibility is that the procedure we used to test the null hypothesis of homogeneity does not capture the type of heterogeneity presented by these authors. Therefore, it will be interesting in future work to investigate the effects of this type of heterogeneity in the estimates of half-live among Brazilian cities.

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[19] With the advent of the internet, for example, relatively simple search tools make comparing prices of different goods (tradables) in different markets an easy task for consumers in general.

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APPENDIX.

Table A-1. Homogeneity test from Pesaran & Yamagata (2008).

Numeraire	$\tilde{\Delta}$	p-value
Belém	-0.255	0.799
Belo Horizonte	**2.65**	**0.008**
Brasília	1.279	0.201
Curitiba	1.753	0.080
Fortaleza	-1.041	0.298
Goiânia	-0.678	0.498
Rio de Janeiro	1.883	0.06
Salvador	0.036	0.971
Porto Alegre	-0.645	0.519
Recife	1.573	0.116
São Paulo	-1.784	0.074

Table A-2. Feasible GLS panel estimation for aggregate regions.

	No Bias Corrections		Nickell Bias Corrected		Time Aggregation Bias Corrected		Nickell and Time Aggregation Bias Corrected	
Numeraire	$\hat{\rho}_{\text{FGLS}}$	$H(\hat{\rho}_{\text{FGLS}})$	$\hat{\rho}_{\text{GMUE}}$	$H(\hat{\rho}_{\text{GMUE}})$	$\hat{\rho}_{\text{GTAU}}$	$H(\hat{\rho}_{\text{GTAU}})$	$\hat{\rho}_{\text{GNTAU}}$	$H(\hat{\rho}_{\text{GNTAU}})$
Nordeste	0.796	3.04	0.915	7.80	0.716	2.07	0.823	3.55
Norte	0.783	2.83	0.856	4.46	0.646	1.59	0.746	2.36
Sul	0.764	2.57	0.859	4.56	0.650	1.61	0.750	2.41
Sudeste	0.795	3.02	0.909	7.26	0.708	2.01	0.814	3.38
Centro-Oeste	0.768	2.63	0.919	8.21	0.720	2.11	0.828	3.67
Brasil	0.798	3.07	0.896	6.31	0.693	1.89	0.798	3.07
Minimum	0.764	2.57	0.856	4.46	0.646	1.59	0.746	2.36
Maximum	0.796	3.04	0.919	8.21	0.720	2.11	0.828	3.67
Mean	0.781	2.82	0.892	6.46	0.688	1.88	0.792	3.07
Median	0.783	2.83	0.909	7.26	0.708	2.01	0.814	3.38

Note: Estimates using the IPCA for Brazil as numeraire was not considered in the calculation of minimum, maximum, mean, and median.