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Comparison of evolutionary algorithms applied to optimal design of water distribution networks

Comparação de algoritmos evolucionários aplicados ao projeto ótimo de redes de distribuição de água

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ABSTRACT

The pursuit of efficient water distribution network (WDN) projects that reflect the complexities of real systems has spurred the development and application of various optimization techniques. Among these, multi and many-objective optimization hold particular significance due to the intrinsic interplay between variables within water distribution networks. Within this domain, evolutionary algorithms have emerged as a promising optimization option, offering a range of methodologies documented in the literature. To systematically evaluate these approaches, a methodology was devised to compare six evolutionary algorithms in the context of water distribution networks optimization: NSGA-II, NSGA-III, U-NSGA-III, R-NSGA-III, MOEA/D, and RVEA, using two distinct objective functions. The comparative analysis utilized as key metrics the efficiency criteria (E), cumulative distribution function (CDF), error statistics and algorithm complexity. The findings revealed that while most algorithms successfully converged to the known global optimum of the employed case study, NSGA-II and NSGA-III exhibited superior performance, notably in minimizing costs. These results demonstrate the efficacy of these algorithms in tackling the complexities inherent in water distribution networks optimization, positioning them as leading contenders in this field.

Keywords: Water distribution networks; Optimization; Multi-objective; Many-objective; Evolutionary algorithms.

RESUMO

A busca por projetos eficientes de redes de distribuição de água (RDA) que refletem as complexidades dos sistemas reais tem estimulado o desenvolvimento e a aplicação de diversas técnicas de otimização. Entre estes, a otimização multi e muitos objetivos possui particular importância devido à intrínseca interação entre variáveis dentro das redes de distribuição de água. Dentro deste domínio, os algoritmos evolutivos surgiram como uma via de otimização promissora, oferecendo uma gama de metodologias documentadas na literatura. Para avaliar sistematicamente essas abordagens, foi desenvolvida uma metodologia para comparar seis algoritmos evolutivos no contexto de otimização de redes de distribuição de água: NSGA-II, NSGA-III, U-NSGA-III, R-NSGA-III, MOEA/D e RVEA, utilizando duas funções objetivo distintas. A análise comparativa utilizou como métricas principais o critério de eficiência (E), a função de distribuição acumulada (FDA), análises estatísticas de erro e complexidade de algoritmos. As descobertas revelaram que, embora a maioria dos algoritmos tenha convergido com sucesso para o ótimo global conhecido do estudo de caso empregado, o NSGA-II e o NSGA-III exibiram desempenho superior, principalmente na minimização de custos. Estes resultados demonstram a eficácia destes algoritmos em lidar com as complexidades inerentes à otimização de redes de distribuição de água, posicionando-os como competidores líderes neste campo.

Palavras-chave: Redes de distribuição de água; Otimização; Multi-objetivo; Muitos objetivos; Algoritmos evolucionários.

INTRODUCTION

The variability in water demand poses a persistent infrastructure challenge for water utilities, whether they're rehabilitating existing networks or expanding new ones and their components. The integration of artificial intelligence into water distribution network (WDN) problems has transformed project planning and operation, shifting the focus from mere cost reduction to a multitude of objectives. These include minimizing hydraulic failures, reducing losses from leaks (Creaco & Pezzinga, 2015), and enhancing water quality (Farmani et al., 2006).

By incorporating multiple objectives into WDN design, the analysis of water distribution systems (WDS) can delve deeper, better reflecting the vast variability and heterogeneity of real-world systems in pursuit of optimal projects. Works such as those by Kapelan et al. (2003), Fu et al. (2013), and Xiong et al. (2018) employed from two to six objectives for the calibration of network models with optimal sensor placement for flow and pressure, as well as optimal design of water distribution networks, varying the objectives from operational cost data to greenhouse gas emissions. Rather than aiming for a single best outcome, the focus shifts to identifying the optimal set of solutions, or Pareto fronts. These fronts are scrutinized and analyzed through the trade-offs between the objectives. It is worth noting that the increasing number of objectives employed hinders the finding and the analysis of the Pareto front.

Various multi and many objective methods have been developed to seek Pareto fronts, with genetic algorithms, particle swarms, and ant colonies among the emphasized approaches. Within WDS optimization, evolutionary algorithms stand out as one of the most utilized and well-established metaheuristics, grounded in Darwinian principles of natural evolution (Maier et al., 2014; Garzón et al., 2022). However, the unique characteristics of WDN, including conflicting objectives, hydraulic constraints, and discrete variables, present optimization challenges. These challenges encompass convergence towards the Pareto front as well as inherent traits of evolutionary algorithms such as fitness function, population diversity, and elitism.

This study undertakes a comparison of six evolutionary algorithms applied to a WDN case study. The performance of NSGA-II, NSGA-III, U-NSGA-III, R-NSGA-III, MOEA/D, and RVEA is examined by varying their initial parameters through a series of simulations and analyzing computational costs in terms of time and efficiency. Additionally, statistical analysis is conducted to assess the variability of the final results obtained and the complexity of algorithms is investigated to understand their equation. The objective of this article is to evaluate the six evolutionary algorithms, highlighting their strengths and weaknesses when applied to WDN, listing those that obtained the best results.

The subsequent sections are structured as follows: Section 2 provides background information and discusses related work, while Section 3 describes the optimization methods employed. The comparison proposal, including scenarios, the case study, and metrics used, is detailed in Section 4. Section 5 presents the results obtained along with their analysis, and finally, Section 6 concludes the study and outlines potential paths for future research.

BACKGROUND AND RELATED WORK

Background

Optimal WDN sizing involves the pursuit of optimal solutions within a vast solution space, guided by one or more objective functions and their associated constraints. The search for solutions that mirror real-world systems implies considering their inherent complexity, demanding a greater number of objectives to be analyzed. Hence, recent studies tend to base their solutions on multi-objective and even many-objective models, recognizing the richer insights derived from considering a spectrum of objectives tied to WDN, as opposed to a singular objective.

One of the challenges associated with the number of objectives is the exponential growth in the number of non-dominated solutions required to approximate the entire Pareto front. As the number of objectives grows, visualizing the front in its entirety becomes increasingly difficult. Fu et al. (2013) demonstrated this complexity by visualizing six objectives in optimal water distribution network designs, highlighting the challenges posed by high dimensionality. Nevertheless, incorporating more objectives enhances stakeholders' decision-making capabilities, providing a more comprehensive understanding of optimal network designs.

Several methods were used for optimal sizing in WDN, such as simulated annealing (Marques et al., 2018), particle swarm (Suribabu & Neelakantan, 2006), genetic algorithms (Walker & Craven, 2020 and Johns et al., 2020) and ant colony (Shokoohi et al, 2017). Despite this array of techniques, evolutionary algorithms enjoy considerable popularity due to their capacity to tackle intricate mathematical challenges, exploring both local and global optima, and seamlessly integrate with simulation models, thereby obviating the need for problem simplification (Maier et al., 2014).

Evolutionary algorithms are a population-based metaheuristic inspired by the processes of biological evolution. Firstly, an initial population is made up of several possible solutions, which are recombined through crossover and mutation operators, giving rise to new generations. While the crossover operator generates a new solution from two or more initial solutions, mutation causes random changes in the generated solutions, introducing a variety component. In this way, new populations are created that evolve towards the set of optimal solutions for the given problem. Notably, the literature features various types of evolutionary algorithms, including genetic algorithms, particle swarm optimization, MOEA/D, RVEA, among others, each with distinct methodologies and contributions, as elaborated in the subsequent subsections.

Related work

Evolutionary algorithms have been comprehensively applied in WDN for decades (Savic & Walters, 1997, Kapelan et al., 2003, Farmani et al., 2006, Sharma et al., 2022). Johns et al. (2020) employed genetic algorithms NSGA-II (Deb et al., 2002), MOALCO-GA and MOPS-GA incorporating the identification and elimination of hydraulic bottlenecks in networks and the smoothing of diameters in connected networks for WDN design. Among these, MOPS-GA demonstrated superior performance over NSGA-II in terms of both optimal solutions and convergence time.

Similarly, Yazdi (2016) explored NSGA-II, SPEAD2 (Zitzler et al., 2001) and MOEA/D for network sizing in large-scale systems, with MOEA/D emerging as the top performer in terms of non-dominance and solution diversity. Shokoohi et al. (2017) used a water quality indicator based on chlorine residual and water age applied to the ant colony algorithm for optimal WDN design. The results obtained presented a lower final cost than the original project.

NSGA-II has been widely used to solve problems related to WDN, such as cost minimization (Fu et al., 2013), loss reduction through the optimal location of pressure control valves (Creaco & Pezzinga, 2015; Creaco & Haidar, 2019) and calibration of network models using an optimal location of flow sensors (Zheng et al., 2016). Its reliability and efficacy in seeking optimal solutions position NSGA-II as a benchmark for comparative performance evaluations with other algorithms. Additionally, Sharma et al. (2022) utilized NSGA-III, an improvement of NSGA-II to handle many objective problems, to optimize measurement and control district boundaries, aiming to manage pressures and monitor water losses.

The great diversity of evolutionary algorithms in the literature, combined with the diverse possibilities of application in different areas, arises a necessity to benchmark their performance. El-Ghandour & Elbeltagi (2018) undertook such a comparison, evaluating five algorithms: genetic algorithms, particle swarm, ant colony, memetic algorithm and a modification of the shuffled frog leaping algorithm. The particle swarm was the top performer across various metrics, including solution quality, efficiency, evaluations of objective functions, and convergence speed. The authors did not use specific algorithms found in the literature, instead, they implemented them in order to investigate the performance of the most common parameters involved in the optimization process. Unlike El-Ghandour & Elbeltagi (2018), this study employs improved evolutionary algorithms with established convergence and efficacy in optimization tasks.

Zhao et al. (2019) conducted a comprehensive analysis and comparison of multi-objective algorithms, categorizing them based on decomposition, dominance, indicators and objective reduction. NSGA-III, based on dominance and classification improvement, and RVEA, based on decomposition, emerged as standout performers across two case studies. In the first case, utilizing the Deb–Thiele–Laumanns–Zitzler (DTLZ) (Deb et al., 2006), RVEA showcased superior convergence and solution diversity and in the second case, the Walking-Fish-Group (WFG) (Huband et al., 2006), NSGA-III stood out for its performance. Although the work of Zhao et al. (2019) evaluated six algorithms, with emphasis on NSGA-III and RVEA, the algorithms considered in our study differ, and our case study entails unique objective functions and constraints specific to the area of WDN.

Given this context, this research aims to compare six evolutionary algorithms within the context of a specific water distribution case study, namely the Alperovitz and Shamir system. By doing so, we aim to provide insights into the relative efficacy and applicability of these algorithms in addressing the challenges inherent to water distribution optimization.

OPTIMIZATION METHODS

In this study, six state-of-the-art multi- and many-objective problem-solving algorithms were chosen. These algorithms include NSGA-II, NSGA-III, U-NSGA-III, R-NSGA-III, MOEA/D, and RVEA. In this study, it has been provided a brief overview of each algorithm's characteristics and methodologies.

NSGA-II

The Non-Dominated Sorting Genetic Algorithm II (NSGA-II), introduced by Deb et al. (2002), employs a quick non-dominance ordering procedure. Initially, this approach classifies the population into dominance fronts, separating dominated from non-dominated individuals. Subsequently, the process iterates, progressively assigning new dominance levels until all individuals are appropriately ranked based on their dominance status.

The clustering distance estimation procedure consists of ordering the population according to each objective function's value, arranged in ascending order of magnitude. For each objective function, the frontier solutions are identified with an infinite distance value, while intermediate solutions are marked with distance values equal to the normalized absolute difference in function values between adjacent solutions. This calculation is applied iteratively across all objective functions. Ultimately, NSGA-II normalizes objective functions and computes the total aggregation distance by summing distances corresponding to each objective (Deb et al., 2002).

This association of non-dominance ordering strategies and algorithmic distance calculation makes NSGA-II an efficient algorithm in tackling optimization problems characterized by conflicting objectives, thereby preserving the best solutions while maintaining diversity.

NSGA-III

The Non-Dominated Sorting Genetic Algorithm III (NSGA-III) algorithm, developed by Deb & Jain (2014) and Jain & Deb (2014), offers a novel approach to tackling many-objective problems, akin to NSGA-II. A key departure lies in its selection operator. NSGA-III initially requires reference points, which, in conjunction with the origin, delineate directions in the solution space. The generated solutions are normalized and then associated with a direction using the orthogonal distance between them. As the algorithm converges, non-dominated solutions exhibit minimal distances.

NSGA-III's strategy of decomposing many-objective problems into a sequence of single-objective subproblems, coupled with its adept selection operator facilitating Pareto front exploration, renders it an invaluable tool for navigating complex multi-objective problem optimization.

U-NSGA-III

The Unified Non-Dominated Sorting Genetic Algorithm III (U-NSGA-III), devised by Seada & Deb (2016), presents a unified approach for addressing single, multi, and many-objective problems. It introduces a selection operator grounded in reference direction niches, which dynamically adjusts solution selection based on the problem's dimensionality. This adaptation stems from employing a larger population size relative to the reference directions and a selection operator that fosters randomness, choice, and the identification of optimal solutions.

U-NSGA-III's proficiency in handling one-to-many problems, yielding Pareto optimal solutions contingent upon the number of objective functions employed, positions it as a valuable tool for resolving optimization challenges and dissecting trade-offs comprehensively.

R-NSGA-III

Vesikar et al. (2018) introduced the Reference Point Based Non-Dominated Sorting Genetic Algorithm III (R-NSGA-III) algorithm, leveraging multiple reference points to search in specific segments of the Pareto front and their corresponding populations. This targeted approach allows for the integration of user preferences regarding the many objectives under scrutiny, facilitating the discovery of desired solutions within the algorithmic framework.

The incorporation of reference points enhances the search process across diverse regions of the Pareto front, providing a rich array of solutions. Moreover, focusing the search within predetermined regions of the Pareto front theoretically boosts computational efficiency, rendering R-NSGA-III an effective tool for tackling optimization challenges.

MOEA/D

The Decomposition-Based Multi-objective Evolutionary Algorithm (MOEA/D) (Zhang & Li, 2007) decomposes a multi-objective optimization problem into N subproblems. Such subproblems are solved simultaneously through the evolution and consequent optimization of the population of solutions. Notably, the optimal solutions for adjacent subproblems exhibit close proximity, with each subproblem benefiting from the optimal solution of its neighboring counterpart. In essence, the algorithm effectively explores the neighborhood relationships inherent among its subproblems.

MOEA/D's straightforward approach to handling multi-objective problems, coupled with its adept exploration of solutions along the Pareto front via decomposition, renders it applicable to a wide array of optimization challenges

RVEA

The Reference Vector guided Evolutionary Algorithm (RVEA) developed by Cheng et al. (2016) employs a scalarization approach, known as penalized angular distance, to dynamically balance the convergence and diversity of solutions according to the number of objectives and generations. Convergence is evaluated by measuring the distance between the candidate solutions and the ideal point, tailored to the problem type, for example, maximization problems feature the maximum value of each objective function as their ideal point. Diversity is quantified by the acute angle between the candidate solutions and the reference vectors.

The integration of reference vectors enhances for diverse and well-distributed solutions along the Pareto front, positioning RVEA as a viable choice for effectively tackling multi-objective problems.

While the algorithms discussed enjoy a solid foundation in the literature, it's crucial to delineate the similarities and distinctions among the six specific algorithms scrutinized in the study. Table 1 highlights the key characteristics of each algorithm, including whether they are designed for single, multi-objective or many-objective optimization, the optimization strategy adopted, the initial parameters, as well as the main advantages and disadvantages of each.This comparative analysis act as a guiding study, revealing the unique traits and shared features inherent in each approach, providing deeper comprehension of their respective strengths and limitations.

COMPARISON PROPOSAL

The comparison of the six algorithms considered different scenarios, presented in the subsection 4.1. Here the formulations of the objective functions, imposed restrictions, simulation input variables and adopted case studies are discussed. Following this, subsection 4.2 explores the metrics employed to compare the performance of the algorithms.

Scenarios

WDN optimization problems are commonly related to the design of network diameters in order to achieve specified objectives. In this context, additional information such as the topology of the networks, the minimum pressure to be met, the demands on the nodes and the operation of the reservoirs are available.

The hydraulic simulation necessary for optimization was carried out using EPANET Software v 2.0 (Rossman, 2000) with an extended simulation period. The multi-objective optimization problem was tackled using the Python programming language.

The optimal design of the present study considers two objective functions: maximizing the Modified Resilience Index (MRI) while minimizing costs.

Objective 1: Maximizing MRI

The Modified Resilience Index was proposed by Jayaram & Srinivasan (2008) and represents the percentage of excess load on the nodes in relation to the required load on the nodes, given by:

$$Max\,IRM = \frac{\sum_{j=}^{n} q_j \left(ha_j - hr_j \right)}{\sum_{j=}^{n} q_j \cdot hr_j} \cdot 100 \tag{1}$$

where q_j is the demand at node *j*, ha_j is the pressure head available at node *j* and hr_j is the pressure head required at node *j*.

Objective 2: Minimizing costs

The cost function is determined by the product of the diameter's price and the length of the section:

$$Min\,Cost = \sum_{t=1}^{np} C_i \left(D_i^k \right) \cdot L_i \tag{2}$$

where np is the total number of pipes; C_i the cost per unit length of pipe i related to diameter D_i, L_i the length of the pipe and k is the regression coefficient.

The objective functions have restrictions related to the laws of conservation of mass in nodes and conservation of energy in loops, given by Equations 3 and 4, respectively.

$$\sum_{i \in NP_{in,n}} Q_i - \sum_{j \in NP_{out,n}} Q_j = S_n, \forall n \in NN \tag{3}$$

with Q being the flow, $np_{in,\,n}$ the set of pipes entering node n, $np_{out,n}$ the set of pipes leaving node n, S the demand and NN the set of nodes.

$$\sum_{k}^{loop\ p} \Delta H_k = 0, \forall\, p \in NL \tag{4}$$

where ΔH is the head loss and NL the loop.

Table 1. Characterization of algorithms.

Algorithm	Type	Optimization strategy	Initial parameters	Advantages	Disadvantages
NSGA-II (Deb et al., 2002)	Multi	Non-dominated Sorting and Crowding Distance	Initial Population Mutation Rate Crossover Rate	It is a proven algorithm that uses Pareto dominance	Applicable to problems with up to 3 objectives due to the non-dominated solution selection mechanism
NSGA-III (Deb & Jain, 2014)	Many	Similar to NSGA-II, but uses reference points for the selection of non-dominated solutions	Initial Population Mutation Rate Crossover Rate Reference points	Adaptively updates a well-distributed set of predefined reference points, effectively solving many-objective optimization problems	The quality of the solutions depends on the selection of reference points, which is typically provided based on the knowledge of the specific problem
U-NSGA-III (Seada & Deb, 2016)	Single, multi and many	Reference function niche selection operator that adapts to the problem's dimensionality	Initial Population Mutation Rate Crossover Rate Reference directions	Proposed as a single algorithm for any number of objectives	The need for adaptive updating and maintenance of reference directionscan introduce additional computational overhead, which might be a concern for very large or complex problems
R-NSGA-III (Vesikar et al., 2018)	Many	Selection of reference points for optimal solution search	Initial Population Mutation Rate Crossover Rate Reference points	Flexibility in handling various types of preference information, making it adaptable to different types of multi-objective optimization problems.	Highly dependent on the quality and accuracy of the points preference information provided by the user
MOEA/D (Zhang & Li, 2007)	Many	Decomposition into subproblems that are solved simultaneously	Mutation Rate Crossover Rate Reference Directions Number of Neighbors Probability of Combining Neighbors	Decomposition of the multi-objective optimization problem into a set of single-objective problems. Introduces neighborhood relationships to share evolutionary information	Lack of diversity and slow convergence speed in the later-stage evolution species
RVEA (Cheng et al., 2016)	Many	Angular distance provides convergence and diversity to the solutions	Initial Population Mutation Rate Crossover Rate Reference directions	The reference vectors can not only be used to decompose the original multiobjective optimization problem into a number of single objective sub-problems, but also to elucidate user preferences to target a preferred subset of the whole Pareto front	Difficulty in adapting the reference vectors to the distribution of candidate solutions according to the estimated geometric characteristics of the Pareto front

The pressure loss is the difference between the loads of two nodes in a pipe, calculated using the Hazen-Willians formula:

$$\Delta H_k = H_{1,k} - H_{2,k} = w \frac{L_k}{C_k^{\beta} D_k^{\gamma}} Q_k Q_k^{\beta-1}, \forall k \in NP \tag{5}$$

where w is a numerical constant that depends on the units used, C is the material roughness coefficient and β and γ are the regression coefficients.

The other restrictions of the problem are related to the minimum pressures in the networks, given by Equation 6, the minimum diameter and available commercial diameters, related to Equations 7 and 8.

$$H_n \geq H_{n\,min}, \forall n \in NN \tag{6}$$

where $H_{n\,min}$ is the minimum pressure in each node.

$$D_k = \geq D_{min}, \forall n \in NN \tag{7}$$

where D_{min} is the minimum diameter.

$$D_k \in D, \forall k \in NP \tag{8}$$

with D being the set of commercial diameters.

In order to evaluate the performance of the algorithms given the complexity of the WDN, the simulations were carried out on Alperovits and Shamir system (Alperovits & Shamir, 1977).

Case study: Alperovitz and Shamir

The case study proposed by Alperovits and Shamir has 6 nodes, 8 network sections of 1000 m and 1 reservoir of fixed height of 210m. The minimum pressure at all nodes is 30m, the demand and quota information at the nodes can be viewed in Figure 1 and the unit costs of the diameters are presented in Table 2.

The simulation scenario was designed to facilitate a comparison of the six algorithms, with variable parameters including the initial population size and the probabilities of recombination and mutation. The number of generations was set at 1000, and each algorithm utilized the same initial random seed in every simulation. Nonetheless, six simulations were conducted with random seed variations to assess their impact on the results. The number of partitions and reference points, consistent across all algorithms except NSGA-II, remained fixed.

4.2 Comparison metrics

The first comparison metric employed in this study was the Efficiency Criterion (E), formulated by Mora-Melia et al. (2015).

Table 2. Unit costs per diameter.

Diameter (mm)	Cost (unit)
25.4	2.0
50.8	5.0
76.20	8.0
101.6	11.0
152.4	16.0

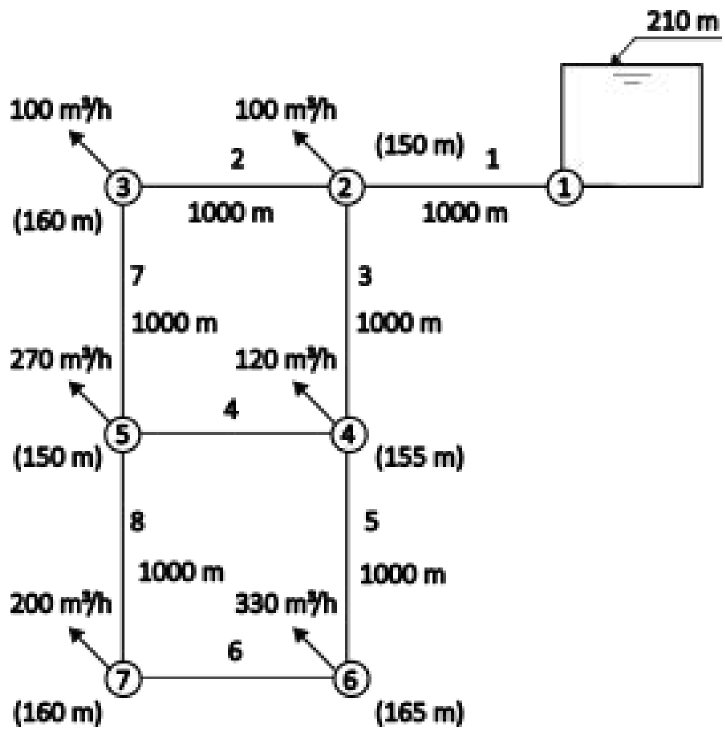

Figure 1. Alperovits and Shamir network.

This metric assesses an algorithm's performance by balancing the quality of solutions attained with the computational effort expended. The calculation of E is defined by Equation 9.

$$E = \frac{n_q}{n_c} \tag{9}$$

where n_c represents the convergence speed of the algorithm, quantified by the total number of objective function evaluations conducted until reaching the final solution and the quality of the solution, n_q, correlates with the proportion of successful high-quality solutions relative to the total number of solutions acquired.

Good solutions are those that maintain a predetermined threshold value, indicating the number of results falling below the user-specified lower limit in minimization problems. The calculation of n_q is determined by Equation 10.

$$n_q = \frac{n_s}{n_{sim}} \tag{10}$$

where n_s is the number of good solutions and n_{sim} is the total number of solutions obtained.

The second metric employed was the cumulative distribution function (CDF), which assesses the probability of a variable being less than or equal to a specific value. It was utilized to scrutinize the variability of diameters concerning costs. This method facilitates examining the diameters utilized to attain the minimum costs in the simulations, while also enabling an analysis of the variability among the resulting diameters.

RESULTS AND DISCUSSIONS

In each simulation, the initial population sizes considered were 10 and 100, with mutation probabilities of 0.01, 0.05 and 0.1 and crossover probabilities of 0.1, 0.5 and 0.9. All six algorithms shared the same initial random seed, resulting in six simulations utilizing six different seeds and six seeds. All analyzes were categorized based on the same population-cross-mutation trio of results from the simulations, resulting in a total of 18 analyzes per algorithm.

Figure 2 depicts the Pareto front of cost versus MRI for the six algorithms. Only the results from the second simulation were used, segregated into populations of 10 and 100 in order to emphasize variability, convergence, and result quantity for both populations. Notably, R-NSGA-III produced fewer results for both populations, with 18 results each. Although MOEA/D also appears to have fewer solutions, it yielded the same number as the other algorithms (around 1000), with its solutions concentrated at the extremes of the graph, either minimizing costs or maximizing MRI. Conversely, other algorithms demonstrated similar behavior, yielding a more diverse array of solutions, particularly outside the extremities.

The purpose of the Pareto front is to identify trade-offs among different solutions, thereby identifying those offering the best cost-benefit ratio for decision-making. This is crucial because converging towards lower-cost solutions inherently entails reducing MRI, an indicator of network quality. A lower MRI signifies increased vulnerability to demand variations or unforeseen events within the supply system, potentially compromising the system's ability to maintain the minimum pressure required in the networks. Consequently, algorithms showcasing a greater diversity of results

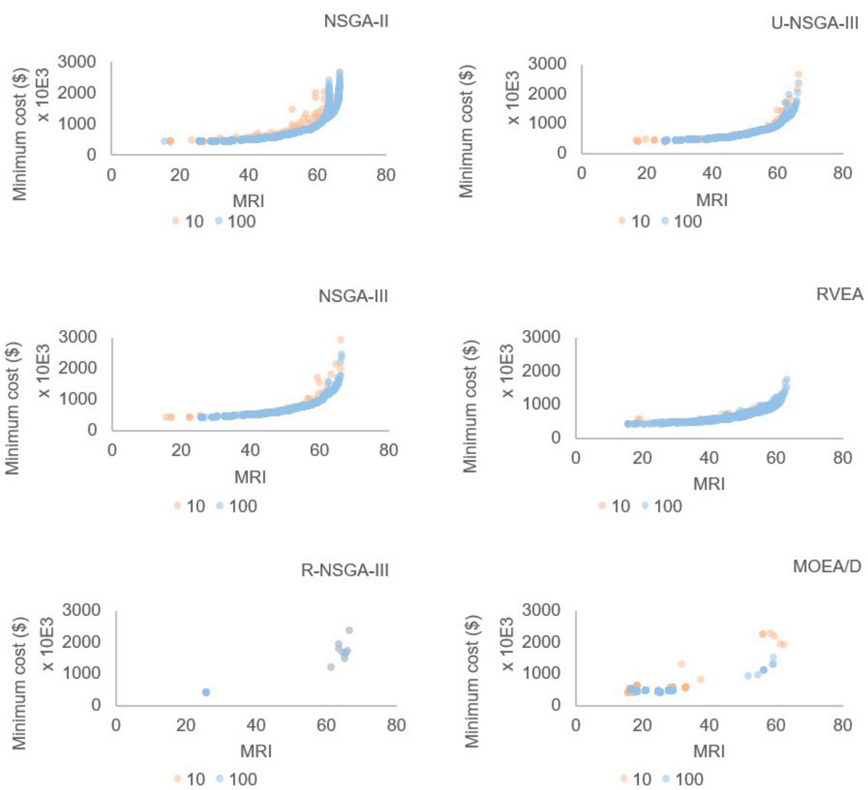

Figure 2. Pareto front.

are deemed preferable compared to those offering fewer results or solely converging on lower-cost solutions or maximal MRI. In light of these criteria, NSGA-II, NSGA-III, U-NSGA-III and RVEA have exhibited superior performance over R-NSGA-III and MOEA/D.

While emphasizing the Pareto front is crucial for revealing trade-offs, the sheer quantity of solutions – around 1000 for most of the algorithms – poses graphical representation challenges. Hence, a strategy was adopted wherein, for every set of population-crossover-mutation parameters, only the minimum cost values and their corresponding MRI were selected. This approach enables us to scrutinize algorithmic behavior in achieving minimal costs amidst parameter variations. Given potential duplications of costs, a bubble graph format was adopted to ensure clear visualization, accounting for frequency distributions as well.

In Figure 3, the convergence of NSGA-II, R-NSGA-III, and U-NSGA-III becomes apparent as costs hover around 420000 regardless of the initial parameters employed. Notably, these three algorithms yielded only two pairs of results. In contrast, NSGA-III, RVEA, and particularly MOEA/D exhibited a wider diversity in minimum costs, although occurrences near 420000 were more frequent compared to other values.

This simplified Pareto front serves as a visual sensitivity analysis depicting how algorithms respond to variations in initial population parameters, crossover, and mutation rates. NSGA-III, RVEA, and MOEA/D emerge as the most responsive, showcasing a broad spectrum of outcomes. This underscores the importance of meticulously selecting initial parameters when employing these algorithms.

Figure 4 shows the Efficiency (E) obtained for the six algorithms considering all simulations, that is, the parameters for the number of successes, simulations and objective function evaluations were added for each population-crossover-mutation grouping. The highest E were obtained by MOEA/D, RVEA and R-NSGA-III algorithms. In the case of MOEA/D, the E peaks are the result of the high number of successes achieved, that is, costs less than or equal to 430000. For RVEA, the E peak is related to the low number of simulations and consequently of evaluations of objective functions. R-NSGA-III, on the other hand, presented the same number of successes, simulations and evaluations of the objective functions for all clusters, resulting in the constant value of E. The remaining algorithms presented very similar behavior, especially for the initial population of 100.

Student's T test was used to evaluate deviations from minimum cost values obtained through sampling error. The sampling error represents the variability of minimum costs in relation to their respective averages, visually depicted in graphical form. In this way, the average of the minimum costs in each simulation for each group was computed alongside the sampling error. The requisite number of samples for conducting the test equaled the number of seeds used.

Figures 5 and 6 illustrate the results categorized by populations of 10 and 100, respectively. In instances where the hydraulic simulation failed to attain the minimum pressure of 30 mH2O, resulting in the application of a cost penalty of 100000, such instances were excluded from the standard deviation calculation.

All algorithms exhibit higher error values for the population of 10 in comparison to the population of 100, indicating greater

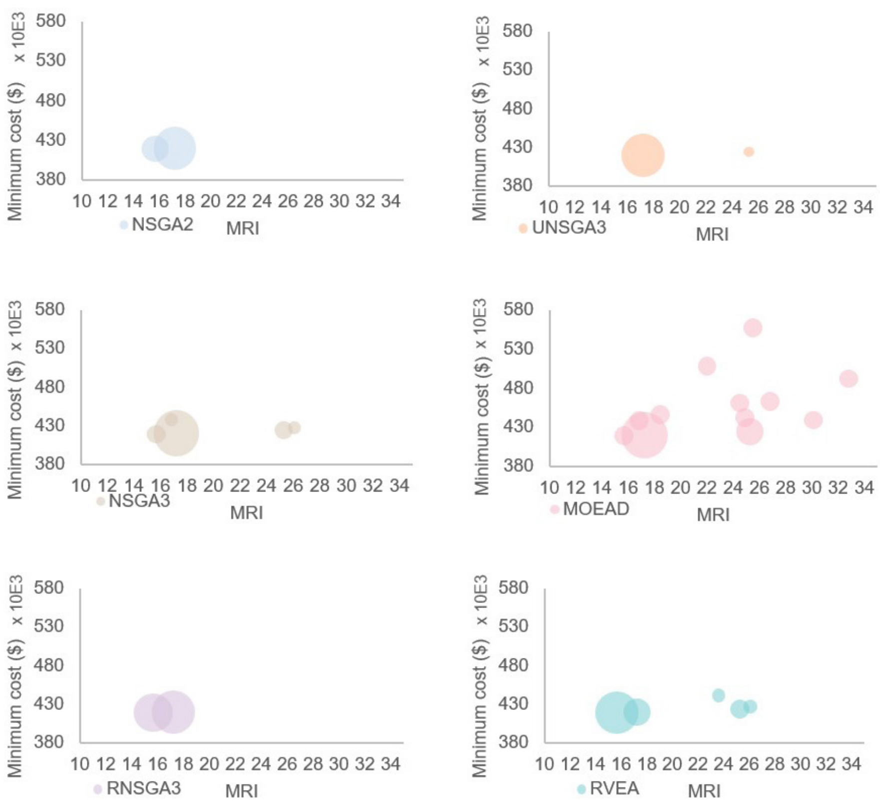

Figure 3. Miminum costs of Pareto Front.

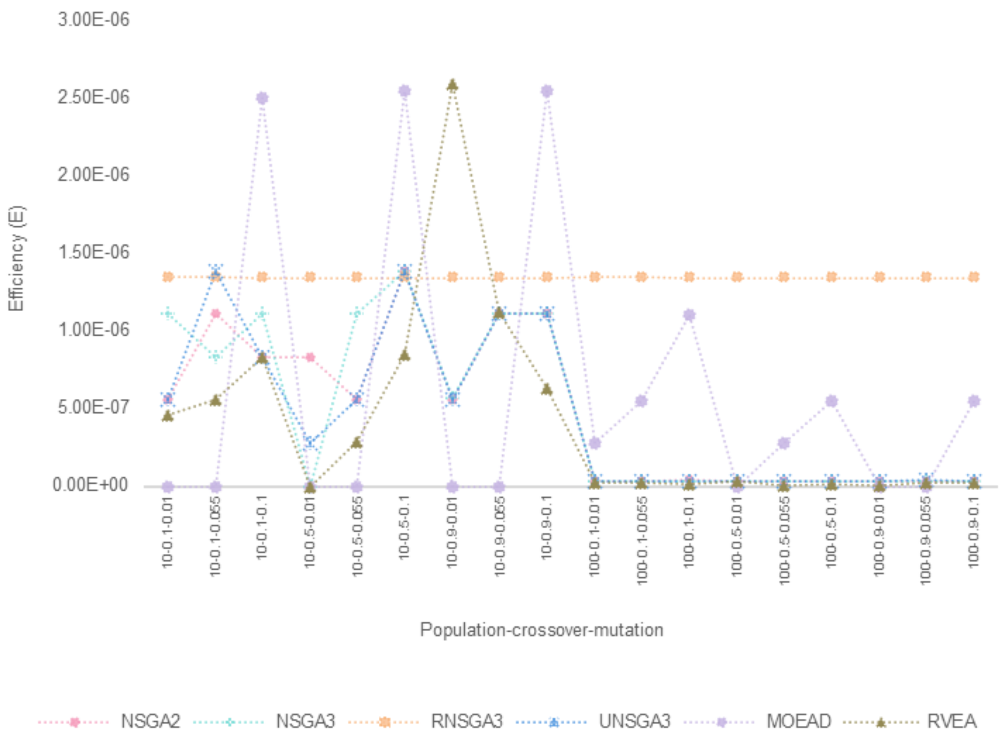

Figure 4. Efficiency per algorithm.

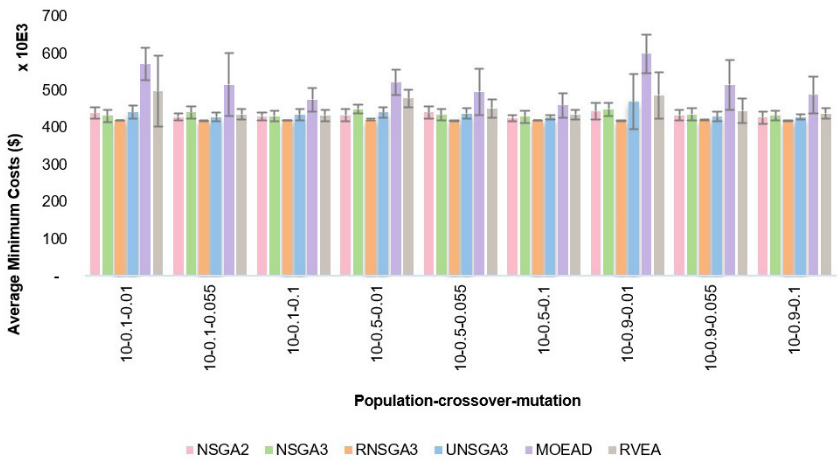

Figure 5. Errors associated with average minimum costs for initial population of 10.

sensitivity of this parameter in achieving lower cost values. Another factor that corroborates this evidence is that the average minimum costs were lower for the population of 100 compared to 10. Variations in the mutation and crossover rate parameters were not significant in terms of reducing costs or errors.

Despite MOEA/D and RVEA algorithms displaying the highest E values, their errors were greater in comparison to other algorithms, signaling increased variability in results across both populations. This suggests that while these algorithms frequently achieve minimum cost values, they also exhibit a notable degree of error in the process.

Table 3 presents the configurations and results for achieving the minimum cost with each algorithm. As all simulations were considered, the *time* results, *ns*, *nsim*, *nc*, *nq* and *E*, represent their cumulative values across the six simulations. U-NSGA-III was the only one that didn't achieve the minimum cost of 419000, reaching 420000 instead. All algorithms used a crossover rate of 0.1 to reach the minimum value, except NSGA-II, which required a crossover rate of 0.9.

In terms of processing time, NSGA-II, NSGA-III, U-NSGA-III and RVEA exhibited identical values of 23 minutes, while MOEA/D required 44 minutes and R-NSGA- III 154 minutes.

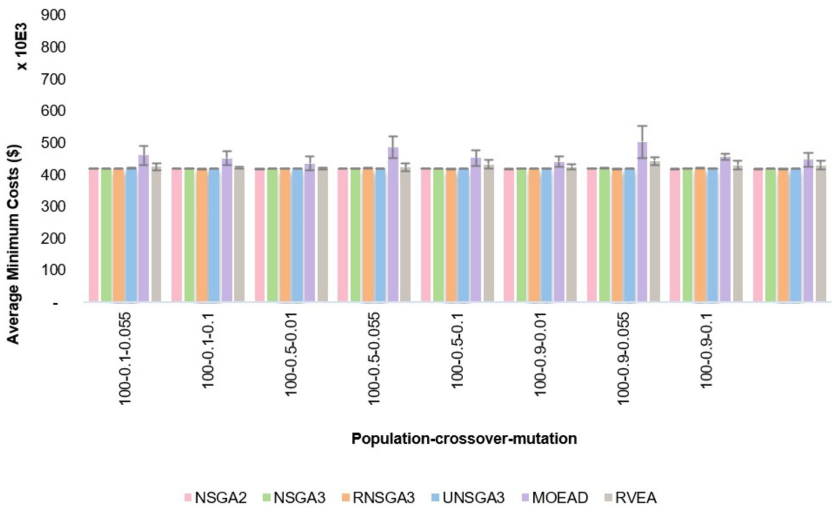

Figure 6. Errors associated with average minimum costs for initial population of 100.

Table 3. Simulation results for the lowest cost.

			NSGA-II							
Population	**Crossover**	**Mutation**	**Minimum cost**	**MRI**	**Time (min)**	n_s	n_{sim}	n_c	n_q	**E**
10	0.9	0.01	419000	15.68	23	2	60	60000	3.3%	5.56E-07
			NSGA-III							
Population	**Crossover**	**Mutation**	**Minimum cost**	**MRI**	**Time (min)**	n_s	n_{sim}	n_c	n_q	**E**
10	0.1	0.01	419000	15.68	23	4	60	59993	6.7%	1.11E-06
			U-NSGA-III							
Population	**Crossover**	**Mutation**	**Minimum cost**	**MRI**	**Time (min)**	n_s	n_{sim}	n_c	n_q	**E**
10	0.1	0.01	420000	17.22	23	2	60	59996	3.3%	5.56E-07
			R-NSGA-III							
Population	**Crossover**	**Mutation**	**Minimum cost**	**MRI**	**Time (min)**	n_s	n_{sim}	n_c	n_q	**E**
10	0.1	0.055	419000	15.68	154	6	12	371996	50.0%	1.34E-06
			MOEA/D							
Population	**Crossover**	**Mutation**	**Minimum cost**	**MRI**	**Time (min)**	n_s	n_{sim}	n_c	n_q	**E**
10	0.1	0.1	419000	15.68	44	9	60	60000	15.0%	2.50E-06
			RVEA							
Population	**Crossover**	**Mutation**	**Minimum cost**	**MRI**	**Time (min)**	n_s	n_{sim}	n_c	n_q	**E**
10	0.1	0.055	419000	15.68	23	2	60	59518	3.3%	5.60E-07

Despite the high processing time, R-NSGA-III presented the highest success rate (*nq*) due to the smaller number of simulations carried out and the second highest Efficiency (E), being surpassed solely by MOEA/D in this regard and considering this specific population-crossover-mutation configuration. Table 4 displays the optimal diameters computed for each section of the system alongside the corresponding costs and MRI values for all algorithms.

Since the processing time in Table 3 represents the duration of the six simulations under those configurations, Figure 7 illustrates the average time required to execute the 6 complete simulations, categorized by population size. Additionally, the sampling error of the average time was calculated using the Student's T test, similar to Figures 5 and 6.

MOEA/D exhibited significantly higher average processing time compared to other algorithms for the population of 100, while R-NSGA-III did so for the population of 10. Regarding the average error depicted by the bars, it was generally higher for the population of 100 across all algorithms, except for R-NSGA-III, which exhibited a larger error for the population of 10 compared to the population of 100.

The CDF was computed using the results obtained from the parameters outlined in Table 2 to scrutinize the variability of the listed diameters. Particularly, the minimum cost of 419000 was achieved in all algorithms except for U-NSGA-III.

The CDF values for NSGA-II, NSGA-III and U-NSGA-III are very close, suggesting that similar diameters were obtained in the final result. The RVEA's CDF, despite also being similar to the aforementioned algorithms, diverges from the others at the beginning of the graph due to a higher utilization of the 25.4mm diameter (approximately 14% of the total). Aditionally, MOEA/D is the only one that significantly uses the diameters 254 and 304.8mm, evidenced by the pronounced slope of its graph at these points. The R-NSGA-III predominantly favors larger diameters, 355.6mm and mainly 558.8mm, at the expense of smaller diameters.

Table 4. Optimal diameters.

Pipe	NSGA-II, NSGA-III, R-NSGA-III, RVEA and MOEA/D	U-NSGA-III
	Diameters (mm)	
1	457.2	508
2	254	254
3	406.4	406.4
4	101.6	25.4
5	406.4	355.6
6	254	254
7	254	254
8	25.4	25.4
Cost (units)	419000	420000
MRI	15.68	17.22

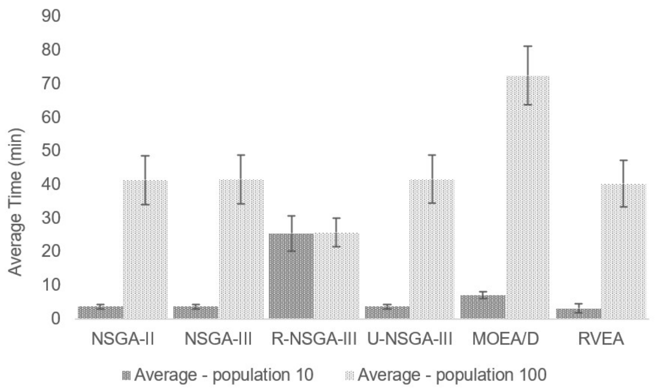

Figure 7. Errors associated with average time processing.

The wider range of diameters used in sizing WDN not only leads to increased logistical and operational costs for sanitation companies but also requires additional resources for managing and maintaining diverse parts and accessories. This includes the need for stocking spare parts tailored to each diameter size and allocating storage space accordingly. Moreover, the efficiency of networks, particularly in terms of minimizing pressure loss, is significantly enhanced when diameters are more uniform throughout the system.

In this regard, the R-NSGA-III algorithm stands out for its tendency towards less variation in diameters. In fact, diameters of 50.8mm, 76.2mm, and 304.8mm were not utilized in any section of the network by this algorithm. This uniformity suggests a more streamlined and cost-effective approach to infrastructure management.

Figure 8 provides a visual representation of the CDF, offering insights into the distribution and utilization of diameters across different sections of the network.

Another significant aspect analyzed in this study was the complexity of algorithms, which relates to the mathematical formulation of each algorithm rather than the specific results they produce. Complexity analysis quantifies the computational effort required by an algorithm as a function of the size of the input data. This analysis offers valuable insights into comparing and selecting the most suitable algorithms based on factors such as input data size, available computational resources, and desired performance, for example.

Conventionally, the notation *O* is employed to denote time complexity, representing time as a function of the size of the

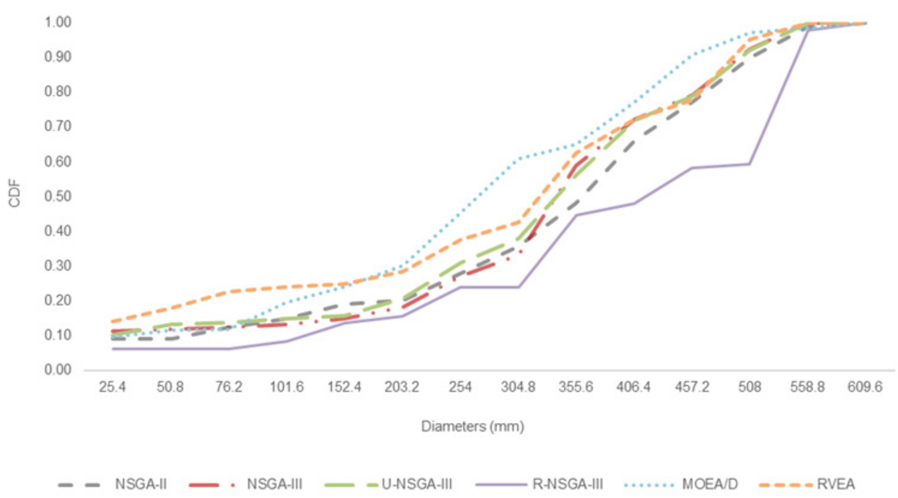

Figure 8. CDF.

Table 5. Complexity of algorithms.

	Algorithm					
	NSGA-II	**NSGA-III**	**R-NSGA-III**	**U-NSGA-III**	**MOEA/D**	**RVEA**
Complexity	$O(m \cdot N^2)$	$O(m \cdot N^2)$ ou $O(N^2 \cdot \log N^{(M-2)})$	$O(m \cdot N^2)$ ou $O(N^2 \cdot \log N^{(M-2)})$	$O(m \cdot N^2)$	$O(m \cdot N \cdot T)$	$O(m \cdot N^2)$

input data. Table 5 outlines the complexity of the six algorithms, where *m* denotes the number of objective functions, *N* is the size of the initial population, and *T* represents the number of solutions. NSGA-II, U-NSGA-III, and RVEA exhibit identical complexity, expressed in terms of *m* and a quadratic relationship with *N*. Conversely, MOEA/D demonstrates a linear relationship between *m*, *N*, and *T*, while NSGA-III and R-NSGA-III share the same complexity, featuring a quadratic relationship with *N* and a logarithmic relationship with *M-2*.

Considering the average time depicted in Figure 5, it becomes apparent that algorithms with similar complexity, such as NSGA-II, U-NSGA-III, and RVEA, exhibit comparable average times and errors. However, NSGA-III and R-NSGA-III showcase markedly different times and errors across the two populations. One potential explanation for these disparities could be attributed to the reference points used in R-NSGA-III, which necessitate preprocessing for the starting point. This preprocessing incurs additional computational overhead, particularly evident for smaller populations like 10. However, for larger populations exceeding 100, preprocessing becomes more efficient, rendering R-NSGA-III the fastest algorithm among all.

Another factor contributing to algorithmic delays could be the allocation of reference points in suboptimal regions of the solution space, resulting in inefficient processing. Vesikar et al. (2018) discuss this issue and suggest relocating points to regions where non-dominated solutions are already discovered as an enhancement for algorithmic efficiency.

MOEA/D was the only algorithm that exhibited both linear complexity and dependence on the variable *T*, relative to the number of solutions. Although linearity presupposes a faster algorithm, its relationship with *T* positions MOEA/D as the poorest performer in terms of time efficiency. This observation raises the hypothesis that there may be a dominance of *T* to the detriment of other variables in the algorithm's performance.

CONCLUSIONS

Among the various optimization techniques applied to WDN projects, evolutionary algorithms stand out as versatile optimization techniques, each with its unique characteristics and methodologies. The proliferation of these algorithms has necessitated comparative studies to discern their respective strengths and weaknesses, thereby facilitating informed decision-making regarding their application in diverse studies.

In this study, six evolutionary algorithms—NSGA-II, NSGA-III, R-NSGA-III, U-NSGA-III, MOEA/D, and RVEA—were subjected to rigorous comparison using a variety of metrics. These metrics included Efficiency, Cumulative Distribution Function (CDF), Pareto front analysis, algorithmic complexity, and average errors related to minimum cost and execution time. The case study centered on the Alperovitz and Shamir network, providing a context for evaluation.

Based on all the metrics used, it can be seen that, with the exception of U-NSGA-III, all algorithms reached the minimum cost of $419,0000, demonstrating their successful convergence to the minimum found in other studies that applied the same WDN. Furthermore, it was found that only the initial population variable brought significant changes to minimize average costs and errors.

The comprehensive investigation of these metrics yielded insightful conclusions regarding the performance of the algorithms. It was noted that no single algorithm emerged as universally superior across all metrics; rather, each algorithm exhibited superiority in some metrics to the detriment of others. This nuanced understanding emphasizes the importance of employing diverse analyses to obtain a comprehensive overview of the algorithm performance.

Moreover, the exploration of algorithmic complexity highlighted the underlying mathematical formulations governing each algorithm. Interestingly, algorithms with similar complexity levels yielded divergent time results due to variations in initial parameters and search methodologies for non-dominated solutions within the solution space. Additionally, the algorithm with linear complexity, conventionally expected to execute faster due to its simpler mathematical structure, turned out to be the least efficient in terms of time expenditure.

In the overall assessment, NSGA-II and NSGA-III emerged as superior algorithms, demonstrating higher performance in various metrics. They exhibited satisfactory Efficiency peaks, achieved low minimum costs, and maintained efficient processing times, all while minimizing average errors.

On the other hand, U-NSGA-III failed to meet the minimum cost threshold, R-NSGA-III presented the longest processing time for the population of 10, and MOEA/D and RVEA exhibited high average costs and errors.

In conclusion, the comparative analysis conducted in this study offers valuable insights into the performance of evolutionary algorithms in the context of WDN optimization. By systematically evaluating various metrics and considering algorithmic complexities, decision makers can acquire a nuanced understanding of each algorithm's strengths and weaknesses. With this knowledge, they can make informed decisions when selecting algorithms for specific optimization tasks, thereby maximizing the likelihood of achieving desired outcomes within resource constraints. Future research could expand this analysis by comparing the results obtained using the two objective functions simultaneously with those generated by simulations that consider each objective separately. This comparative study would allow for an assessment of the computational effort required by each algorithm, providing a more detailed view of the efficiency and effectiveness of multi-objective approaches compared to single-objective approaches. Additionally, this analysis could identify which algorithms are better suited for scenarios with varying levels of complexity, offering valuable insights for selecting methodologies in optimization problems.

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Alexandre Kepler Soares*:* Conceptualization, methodology, theoretical framework, analytical methods, reviewing, and supervising.

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