Civil Engineering
Engenharia Civil

Risk assessment in a rock slope stability analysis using a Mamdani fuzzy controller

http://dx.doi.org/10.1590/0370-44672022760061

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Abstract

The shearing behavior of discontinuities is one of the factors with major influence on rock slope stability analysis, which is predictable when adopting different analytical models. However, these methodologies, generally of a purely deterministic nature, do not allow an assessment of the influence of the variability of the input parameters of the models on the shear behavior of unfilled rock discontinuities and, consequently, on the risk involved in the rock slope stability analyses. The purpose of this article is to present a methodology for rock slope risk assessment based on the development of a model to predict the shear behavior of unfilled rock discontinuity considering the variability of its input parameters, using a Mamdani fuzzy controller. The input variables of the model are the boundary normal stiffness and initial normal stress acting on the discontinuity, its roughness, the uniaxial compressive strength, the basic angle of friction of the intact rock and the shear displacement imposed on the discontinuity. The model outputs are the membership functions for the shear strength and dilation of the unfilled rock discontinuity, and from which the membership function can be defined for the factor of safety of the rock slope considering the failure mechanism governed by discontinuity. The results reveal the use and importance of fuzzy logic and fuzzy number operations in assessing the risk in rock slopes and may be used even in situations where there are major uncertainties on the existing information of the characteristics of the unfilled discontinuities.

Keywords: unfilled joints, rock slope stability analysis, risk assessment, Mamdani.

1. Introduction

Usually, fractured rock slope stability may be analyzed using the classic methods based on the limit equilibrium condition of the rock mass. Meanwhile, the presence of fractures in the rock mass is one of the key factors influencing the slope failure mechanism, making the shear behavior of discontinuities to have a strong influence on the stability of the fractured rock mass.

Several studies have been done on the main mechanisms governing the shear behavior of rock discontinuities (Barton, 2013; Benmokrane & Ballivy 1989; Skinas *et al.*, 1990; Papaliangas *et al.*, 1993). Oliveira (2009) listed some of the key factors influencing the shear strength of the rock joints as follows: (i) roughness of their walls; (ii) strength and deformability of their asperities; (iii) the mechanical characteristics of the intact rock; (iv) presence or not of infill and its thickness; (v) external boundary conditions; and (vi) the conditions of drainage.

For unfilled rock joints, there are several methodologies to predict their shear behavior. Some traditional models, such as Barton & Choubey (1977), were developed based on the results of direct shear tests performed under constant normal loading conditions, and allow only to estimate the peak shear strength, while others, such as those proposed by Barton & Bandis (1990), provide a full representation of the shear stress x shear displacement of the unfilled rock discontinuities.

Indraratna & Haque (2000) present an analytical model in which the estimated shear strength of unfilled rock discontinuities is based on their external boundary conditions, roughness, and basic friction angle. This model could be considered the latest model developed to predict the shear strength of unfilled rock joints and offers as benefits in relation to other more traditional models the fact that it provides a full prediction of the shear stress x shear displacement behavior for both constant normal loading (CNL) and constant normal stiffness (CNS) conditions. However, applying this model is somewhat laborious and its main difficulty lies in the need to know the variation of the dilation of the discontinuity during its shearing, which can only be obtained by performing large-scale direct shear tests, not always available or representative of the external boundary conditions that the discontinuity undergoes.

In order to more simplify the study of the shearing behavior of the unfilled rock discontinuities and to be less dependent on the availability of large-scale direct shear test results, Dantas Neto *et al.* (2017) proposed a prediction model developed using artificial neural networks. The results obtained by Dantas Neto *et al.* (2017) indicated that the neural model can express the influence of the input variables in the shearing behavior of the joints, and their results fit the experimental data better than the analytical model of Indraratna & Haque (2000) for a wide range of rocks (soft to very hard) and joint conditions.

Despite the good performance of some of previously mentioned models, it is found that they all address the prediction process of shear behavior of the unfilled rock discontinuities by using a deterministic methodology; that is, they do not consider the variability of the input parameters in the models' response, thereby leading to uncertainties in the obtained results. Such uncertainties arise from many factors, namely the sampling process to obtain test samples to be used in the laboratory tests required to define the values of the input parameters, the formation process and weathering of the rock and discontinuity, among others. One example of this is the use of the JRC value, which in the traditional deterministic approaches is normally assumed to be the only one throughout the discontinuity, this fact being seldom observed in the field. In this scenario, it is known that the probability theory and fuzzy set theory (Zadeh, 1965) are useful tools to describe complex and nonlinear multivariable problems, and whose input parameters bring a high degree of variability, or uncertainty, as in the case of the rock slope stability analyses.

Regarding slope stability analysis, several assessment techniques have been developed which include a range of simple evaluations, planar failure, limit state criteria, limit equilibrium analysis, empirical techniques, numerical methods, hybrid and high-order approaches, and implementation in both two and three dimensions (Azarafza *et al.*, 2022). Among these techniques, intelligent systems have shown to be a promising alternative to improve slope stability analysis. Table 1 presents some research that confirmed the great potential of this systems for application in slope stability analysis. Although these models are powerful tools, their outputs are still deterministic and are not able to assess the risk.

In this context, Mamdani fuzzy controllers may be useful for quantifying risk in slope stability analysis, especially when the number of available data is scarce to characterize the uncertainty through probability theory, since they are intelligent systems whose input and output variables are fuzzy sets, thus enabling to obtain the safety factor as a fuzzy number. Applications of Mamdani fuzzy inference systems in rock mechanics were made by several researchers. Some of them focused on determining the properties of rock masses (Kayabasi *et al.*, 2003; Sonmez *et al.*, 2003; Hamidi *et al.*, 2010; Daftaribesheli *et al.*, 2011; Asadi, 2016; Sari, 2016), while others investigated the intact rock parameters (Gokceoglu, 2002; Gokceoglu & Zorlu, 2004; Monjezi & Rezaei, 2011; Asadi, 2016). None of them studied the shear behavior of rock joints.

Therefore, the aim of this paper is to present a methodology which incorporates fuzzy logic and fuzzy number operations to evaluate the risk in rock slopes when failure mechanism is governed by their joints. Thus, the proposed methodology involves (i) the development of a Mamdani fuzzy controller for the prediction of clean rock joints shear behavior, (ii) the definition of the safety factor used in the rock stability analysis as a fuzzy number and finally (iii) the risk assessment for a rock slope whose failure is controlled by a hypothetical unfilled joint. This methodology may be an alternative to the existing analytical models and probabilistic methods which sometimes require several large-scale laboratory tests and are not always available to characterize the uncertainty during a rock slope stability analysis.

Table 1 – Intelligent systems for slope stability analysis.

Authors	Methodology	Main conclusions
Dantas Neto *et al.* (2017)	Proposed a model to predict the shear behaviour of clean rock joints developed by using a multilayer perceptron. A rock slope stability problem was also used as an example for applying the model in practice.	The developed neural model was able to express the influence of the input variables in the shearing behavior of joints and may be an alternative to the existing analytical models which sometimes require certain parameters obtained from large-scale laboratory tests which are not always available.
Matos *et al.* (2019a, 2019b)	Developed Takagi-Sugeno fuzzy systems for predicting shear strength of clean rock joints incorporating uncertainties in the variables that govern their shear behavior.	Despite the proposed models present the advantage of considering the uncertainties of their input variables, their responses are still deterministic.
Azarafza *et al.* (2020)	Presented a fuzzy logical decision-making algorithm based on block theory to determine discontinuous rock slope reliability by classifying the slope into expressive classes such as stable or unstable.	The proposed algorithm is relatively simple, requires low computational cost, has excellent compatibility, provides intuitive and experimental satisfaction and can be used by students and less experienced people.
Ahangari Nanehkaran *et al.* (2022)	Investigated the performance of multilayer perceptron, support vector machines, k-nearest neighbors, decision tree and random forest models to predict the soil slope safety factors (FS).	Slope height, total slope angle, dry density, cohesion and internal friction angle were considered to predict the FS for 70 slopes. Among the five machine learning models, the multilayer perceptron was found to be the most reliable for predicting FS.
Nanehkaran *et al.* (2023)	Provided a comparative analysis between multilayer perceptron, decision tree, support vector machines and random forest learning algorithms to predict soil slope safety factors, using a dataset of 100 records of slopes.	The paper tried to fill the gap in traditional analysis procedures based on advanced methods in slope stability assessments. The multilayer perceptron achieved the highest accuracy and precision in predicting the FS.

2. Materials and methods

The risk assessment methodology on rock slope analyses using Mamdani fuzzy controllers (Mamdani, 1973) involves three general stages: (i) the development of the fuzzy model to predict the shear behavior of unfilled rock discontinuities governing the rock slope failure; (ii) obtaining the factor of safety of the rock slope as a fuzzy number; and (iii) establishing a risk factor that takes into account the variation of the parameters governing the shear behavior of the unfilled rock joint and its influence on the results obtained for the factor of safety of the analyzed rock slope.

The models were built based on a dataset of 44 direct shear tests presented by Skinas *et al.* (1990), Papaliangas *et al.* (1993), Indraratna & Haque (2000), and Indraratna *et al.* (2010), performed on different types of joints and different boundary conditions. The models were developed using 673 examples of shear strength-dilation-shear displacement (15 examples for each direct shear test on average) that were used to define the rules of inference for the fuzzy systems, while considering as input variables the main factors governing the shear behavior of clean rock joint: the normal boundary stiffness (k_n), initial normal stress (σ_{no}) acting on the discontinuity, joint roughness coefficient (JRC), uniaxial compressive strength of the intact rock (σ_c), the basic friction angle (ϕ_b), and shear displacement (δ_h) having as its response the shear strength of the discontinuity (τ_h) or the dilation (δ_v). It is noteworthy that the Mamdani fuzzy systems structure presented herein were used as a basis for Matos *et al.* (2019a, 2019b) models' development, which used another type of inference process.

Figure 1 shows the configuration to be used for the risk analysis involved in the stability of a rock slope undergoing an overload F, at height H, gradient α_s and with the potential failure surface defined by an unfilled rock discontinuity at angle α_j. The presence of the force applied by a rock bolt T defines the boundary stiffness of the problem (CNS).

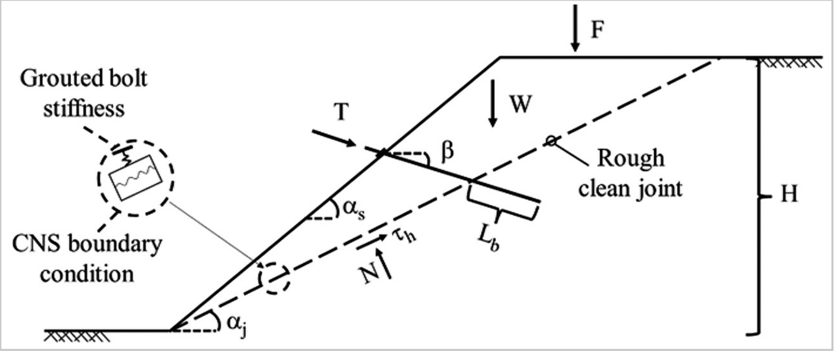


Figure 1 – Stability analysis of a rock slope.

The wedge weight (W) defined by the discontinuity in the rock considering its specific weight (γ) can be determined according to Equation 1.

$$W = 0.5\gamma H^2(\cot \alpha_j - \cot \alpha_s) \tag{1}$$

The T value can be calculated by Equation 2 in function of the deformability modulus (E_b), cross-section area (A_b), length (L_b) and inclination β of the rock bolts, and dilation of the discontinuity (δ_v), which could occur during shearing.

$$T = \frac{E_b A_b}{L_b} \frac{\delta_v}{\sin(\alpha_j + \beta)} \tag{2}$$

Assuming n rock bolts are installed with horizontal spacing s_h, the factor of safety (FS) is obtained by the ratio between the resistant forces acting on the wedge and the forces favorable to their failure along the discontinuity according to the expression in Equation 3.

$$FS = \frac{\tau_h \left(\frac{H}{\sin \alpha_j}\right) + \left(\frac{n}{s_h}\right) T\cos(\alpha_j + \beta)}{(W + F)\sin\alpha_j} \tag{3}$$

From Equation 3, it is noticeable that the factor of safety is a function of the shear strength (τ_h) and dilation of the joint (δ_v) considered in the calculation of the burden of rock bolt T as shown in Equation 2. Since these two parameters are fuzzy variables ($\tilde{\tau}_h$ and $\tilde{\delta}_v$), they bear within their input parameters their own uncertainties, whereupon the factor of safety may also be considered as a fuzzy number ($\widetilde{FS}$), as shown in Equation 4.

$$\widetilde{FS} = \frac{\tilde{\tau}_h \left(\frac{H}{\sin \alpha_j}\right) + \left(\frac{n}{s_h}\right) \tilde{T}\cos(\alpha_j + \beta)}{(W + F)\sin\alpha_j} \tag{4}$$

Equation 5 shows the expression for the load applied by the rock bolts in the form of the fuzzy number in Equation 4.

$$\tilde{T} = \frac{E_b A_b}{L_b} \frac{\tilde{\delta}_v}{\sin(\alpha_j + \beta)} \tag{5}$$

The factor of safety in the form of the fuzzy number in Equation 4 can be obtained for a certain membership degree h using the rules of interval arithmetic according to Equation 6.

$$\overline{FS}(h) = [\, FS_i(h), FS_f(h)] \tag{6}$$

Considering the fuzzy numbers ($\tilde{\tau}_h$ and $\tilde{\delta}_v$) obtained from the Mamdani fuzzy controller, the minimum and maximum values for the interval $\overline{FS}(h)$ can be obtained from Equations 7, 8, 9 and 10, which allow the definition of the membership function for the fuzzy number ($\widetilde{FS}$).

$$FS_i(h) = \frac{\tau_{hi}(h) \left(\frac{H}{\sin \alpha_j}\right) + \left(\frac{n}{s_h}\right) T_i(h)\cos(\alpha_j + \beta)}{(W + F)\sin\alpha_j} \tag{7}$$

$$FS_f(h) = \frac{\tau_{hf}(h) \left(\frac{H}{\sin \alpha_j}\right) + \left(\frac{n}{s_h}\right) T_f(h)\cos(\alpha_j + \beta)}{(W + F)\sin\alpha_j} \tag{8}$$

$$T_i = \frac{E_b A_b}{L_b} \frac{\delta_{vi}(h)}{\sin(\alpha_j + \beta)} \tag{9}$$

$$T_f = \frac{E_b A_b}{L_b} \frac{\delta_{vf}(h)}{\sin(\alpha_j + \beta)} \tag{10}$$

The concept of risk factor (R_f) presented by Ganoulis (1994) was used to assess the risk involved in a rock slope stability analysis in this study. The value of R_f is obtained by dividing the area under the membership function for the fuzzy number ($\widetilde{FS}$) until a given value of the factor of safety (M) by its total area, as represented in Equation 11. This allows to assess the risk of the factor of safety is less than 1, for example, even when in a deterministic analysis is obtained a satisfactory value for the factor of safety which was obtained not considering the uncertainties of the input parameters.

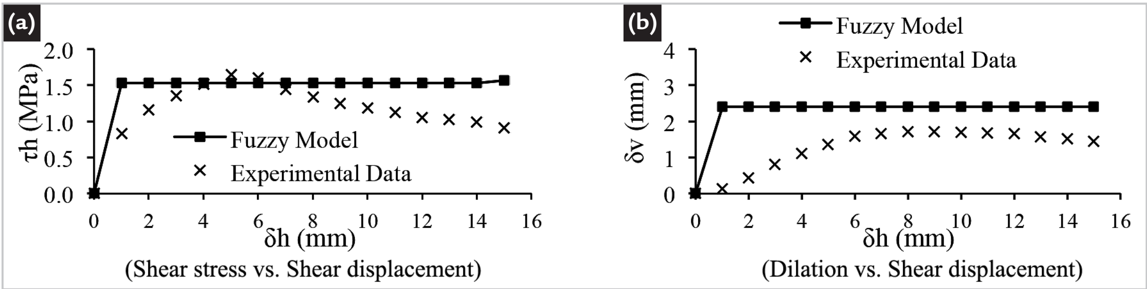
$$R_f = \frac{\int_{-\infty}^{M} \mu(FS)dFS}{\int_{-\infty}^{+\infty} \mu(FS)dFS} \tag{11}$$

3. Results and discussion

In order to apply the fuzzy model when assessing the risk in the rock slope stability analysis shown in Figure 1, the following characteristics were adopted for the slope and the unfilled rock discontinuity: H = 30.5 m, α_s = 80°, and α_j = 50°, γ = 27.5 kN/m³ and F = 25,000 kN. The rock bolts had a diameter of 63.5 mm, L_b = 1.0 m, β = 15°, and s_h = 1.4 m. Taking the module E_b = 200 GPa and n = 30 for the number of bolts in the cross-section, the initial normal stress acting on the discontinuity and its normal boundary stiffness are equal to 540 kPa and 380 kPa/mm, respectively. The joint properties used as input for predicting the fuzzy models are: S_c = 12 MPa, ϕ_b = 37.5° and JRC = 12.

Figure 2 provides a comparison between the experimental results of Indraratna & Haque (2000) for an unfilled rock discontinuity undergoing initial normal stress of de 560 kPa, normal boundary stiffness of 453 kPa/mm, uniaxial compressive strength of intact rock of 12 MPa, basic friction angle of 37.5° and JRC of 13, and the predictions by using the fuzzy models for shear stress and dilation developed with the Mamdani fuzzy controller. These results were obtained by applying the centroid method at the defuzzification stage after obtaining the membership function for both output variables by the fuzzy model. It is worth pointing out that the results of Indraratna & Haque (2000) were used to compare them with those supplied by the fuzzy model because the boundary conditions given in terms of initial normal stress and normal boundary stiffness were closer to the values defined for the unfilled rock discontinuity, according to its boundary conditions and configuration of rock bolts adopted in this study.

The results in Figure 2 show that the Mamdani fuzzy controller has some difficulty in predicting the shear behavior of the unfilled rock discontinuity at low levels of shear displacement when compared to the experimental results provided by Indraratna &Haque (2000). However, the results of the fuzzy models when predicting the peak shear stress and its corresponding dilation which are used in the evaluation of the rock slope stability shown in Figure 1 can be considered satisfactory.



(Shear stress vs. Shear displacement)

(Dilation vs. Shear displacement)

Figure 2 – Comparison between experimental results and fuzzy models' predictions.

Figure 3 illustrates the fuzzy sets for the peak shear stress and corresponding dilation after the Mamdani controller's aggregation stage, representing the influence of uncertainties of the input parameters governing the shear behavior of the unfilled rock discontinuity. These results show the variation of obtained shear stress and dilation due the variability of the considered input variables. These results show clearly how useful the Fuzzy Set Theory can be to represent the uncertainties of the input parameters governing the shear behavior of the unfilled rock discontinuities. Using the centroid method for defuzzification, the values of 1.41 MPa and 2.42 mm were obtained for the shear stress and dilation, respectively, which can be used to perform the deterministic rock slope stability analysis.

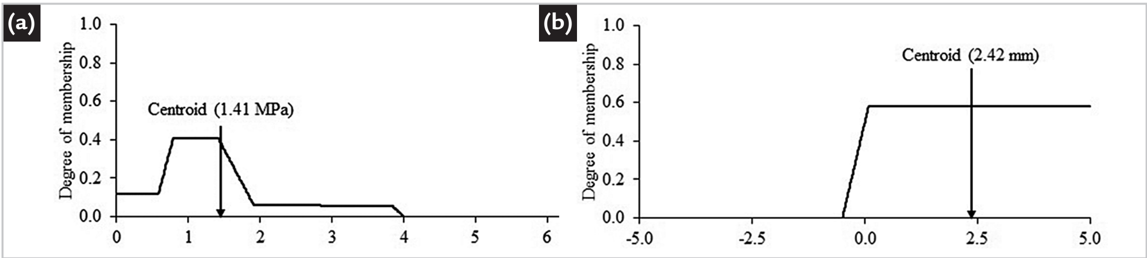


Figure 3 – Membership functions at peak shear stress.

Figure 4 shows the variation in the factor of safety with the shear displacement of the unfilled rock discontinuity from the results of the fuzzy models for the shear stress and dilation of the experimental data of Indraratna & Haque (2000) and considering the geometric characteristics and burdens for the rock mass presented previously in Figure 1. The results indicate that the fuzzy model developed with the Mamdani controller showed quite a satisfactory result for the assessment of the factor of safety (FS = 2.8), considering the level of shear displacement corresponding to peak shear stress, when compared to the values obtained using the experimental data for the joint's shearing behavior. The difficulty for representing the increase of the factor of safety is due the limitation of the fuzzy model used for predicting the shear stress and dilation at low levels of shear displacement as discussed previously.

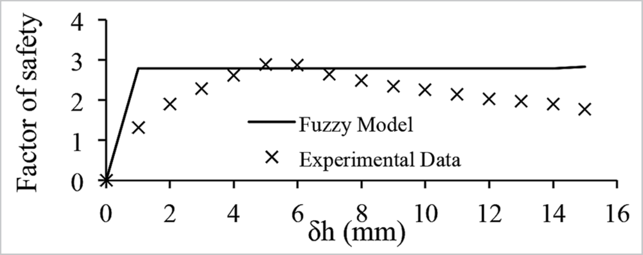


Figure 4 – Safety factor versus shear displacement in the rock slope stability analysis.

With the membership functions for the fuzzy numbers representing the peak shear stress (Figure 3a), and corresponding dilation (Figure 3b), and applying the interval arithmetic described previously, the safety factor is obtained as a fuzzy number whose membership function is given in Figure 5a. According to the results, the rock slope has a factor of safety of 2.8, but a risk factor in the failure (FS < 1) obtained by dividing the hatched area by the total area under the membership function, according to Equation 11, is equal to 10%. The risk assessment of the safety factor being less than any specific value can be obtained in a similar way to be able to know the risk curve shown in Figure 5b.

These results show that by applying a methodology based on the concepts of the fuzzy sets in assessing the risk of failure in rock slopes, whose failure mechanism is governed by an unfilled rock joint, can be quite useful, since it allows consideration of the uncertainties existing in the characterization of the different elements and structures of the rock mass and the discontinuity. Evidently, the proposed methodology is still in its early days and its results must be compared to other risk assessment proposals and criteria based on probabilistic methods (Fell, 1994; USACE, 1999). However, the application when analyzing rock slope stability confirmed the great potential of fuzzy controllers to assess the risk of failures in problems involving complex scenarios and uncertain parameters, such as those found in designed and implemented geotechnical projects in rock, especially in situations with few available data, in which it would be difficult to apply the probabilistic methods.

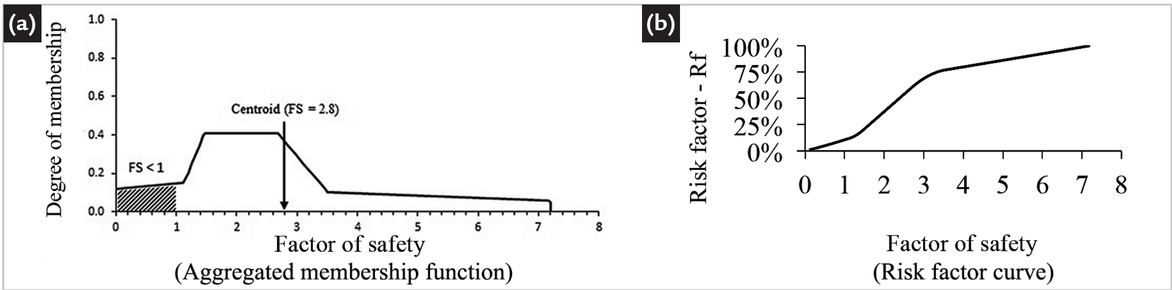


Figure 5 – Risk assessment of the safety factor.

3.1 Methodology limitations

Although the proposed fuzzy methodology may be a promising alternative for rock slope risk assessment, there are certain restrictions regarding its application. The first is related to the Mamdani fuzzy models. Despite delivering good results, they have some limitations, most of which are concerned with the distribution of input variables within the construction of their rules of inference. Figure 6 show histograms and cumulative frequencies curves for the boundary

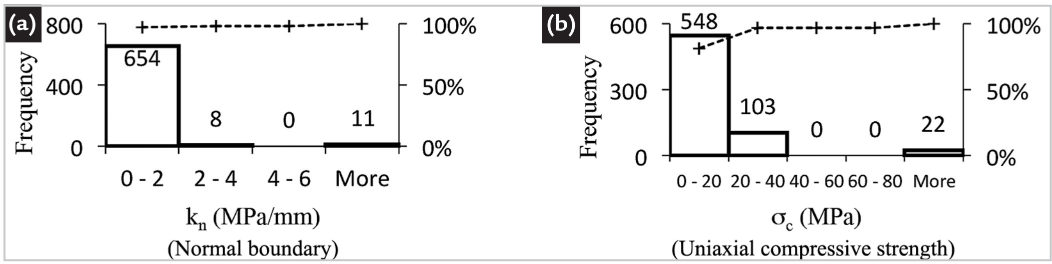


Figure 6 – Histograms and cumulative frequencies curves.

normal stiffness and uniaxial compressive strength which exhibited the most lack of values within the database that were used while developing the fuzzy models. This input lack of values affects the performance of the fuzzy systems as it does not allow to establish a greater number of rules of inference decreasing the generalization in their predictions.

4. Conclusions

This article proposed a novel methodology using Mamdani fuzzy controllers for rock slope risk assessment based on the prediction of the shear behavior of unfilled rock discontinuities considering the uncertainties inherent in their governing parameters. Considering the fuzzy sets defined for all input parameters and the configuration adopted for the slope, for the acting discontinuity and burden, the models enabled the estimation of a safety factor of 2.8 and obtained the variation in the risk factor for the safety factor's fuzzy set at any rock joint's shear displacement level.

It should be emphasized that application of the methodology addressed herein for failure risk assessment of rock slopes should be compared to those obtained using other risk assessment methodologies based on the probability theory. However, the results confirmed that it is possible to define and easily adopt the risk when analyzing rock slope stability using a Mamdani fuzzy controller, because rock slopes represent complex and nonlinear multivariable problems, and their failure mechanism is ruled by input parameters bearing a major uncertainty due to the formation of the rocks and their discontinuities.

Finally, it is worth mentioning that the developed fuzzy models do not consider scale effects, weathering, pore water pressure and drainage conditions on the shear behavior of clean rock joints. Also, they are limited by the areas of their input variables that were defined during their construction, and thus do not permit in their input data the inclusion of measurements outside their predefined interval of action. Nevertheless, these areas may be adjusted as new sets of data become available.

Acknowledgments

The authors would like to thank the Fundação Cearense de Apoio ao Desenvolvimento Científico e Tecnológico (Funcap), the Coordenação de Aperfeiçoamento de Pessoal de Nível Superior (CAPES) and the Universidade Federal do Ceará (UFC) for logistical and financial support.

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Received: 9 October 2022 - Accepted: 1 May 2023.