Qualification exams carried out in the professional master's program in mathematics in the national network*

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Abstract

This article analyzes the nature of the knowledge, especially that of a specialized nature, evoked in the solution of the questions that made up the National Qualification Exams (NQE), developed within the scope of the Professional Master's Program in Mathematics in the National Network (PROFMAT), between the years 2012 and 2019. The focus of this program was based on the development of a solid education in Mathematics, which contemplates the needs arising from the daily lives of teachers in the school space and their broad needs for development and professional appreciation. Therefore, a qualitative study was developed in which, from the interpretative content analysis, a total of 62 questions were classified in the light of the conceptual-analytical model *Mathematics Teachers' Specialized Knowledge* (MTSK), which composed the NQE applied in the first semester of the 8 years analyzed. The analysis carried out showed that: a) more than 70% of the questions focus exclusively on knowledge that appears in only one subdomain of *Mathematical Knowledge* (MK), the Knowledge of Topics, while only 5% of them address knowledge that appears in all subdomains of MK; b) Absence, in all questions, of knowledge associated with the *Pedagogical Content Knowledge* (PCK) domain. Thus, it is concluded that the evaluation developed through the NQE privileges knowledge that appears only in one of the six subdomains of knowledge that really support the teacher in his daily work with mathematics, which is opposed to the results of many relevant studies that emphasize the importance and impact of knowledge associated with the PCK domain in teaching.

* English version by Vincent Field. The authors take full responsibility for the translation of the text, including titles of books/articles and the quotations originally published in Portuguese.

† When this article was edited, Professor Marlova Estela Caldatto and José Carrillo Yanez had died.

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2- Universidade de Huelva, Huelva, Andalucia, Spain. When this article was accepted, Professor Carrilo had died. The aforementioned professor, an expressive member of the European Society for Mathematics Education, is the main author of Mathematics Teachers' Specialized Knowledge, among many works and research published in various journals of international relevance.

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Keywords

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Introduction

The research presented here is part of an investigative project whose main object is the *Professional Master's Program in Mathematics in the National Network* (PROFMAT), a postgraduate program in Mathematics coordinated by the Brazilian Society of Mathematics (SBM), supported by the National Institute of Pure and Applied Mathematics (IMPA) and recognized, evaluated and promoted by the *Coordination for Improvement of Higher Education Personnel* (CAPES).

The focus on PROFMAT was given because, according to its bylaws, it aims to "provide in-depth and relevant mathematical education for teaching in Basic Education, aiming to give graduates a certified qualification for the exercise of the Mathematics teaching profession" (SBM, 2016, p. 01).

In addition, PROFMAT became a reference in the establishment of a "national policy for the continuous training of teachers in the postgraduate modality [professional masters in a network], given the subsequent emergence of other professional master's programs in a network" (CALDATTO; FIORENTINI; PAVANELLO, 2018). In 2019, in addition to PROFMAT, under PROEB[3], 11 more master's programs of the same nature are being offered, namely[4]: ProFis, PROFCIAMB, ProfLetras, ProfArtes, ProfHistória, PROFBIO, PROFQUI, PROFilo, PROFSOCIO, PROFEDFÍSICA and ProfÁgua.

Offered through a network of Higher Education Institutions (HEIs) and currently present in the 27 units of the federation, PROFMAT is apparently consolidated, as it has expanded its reach since its implementation in 2011. At the year of its inception, the network consisted of 48 HEIs (54 centers), in 2013 it had 58 HEIs (71 centers), in 2015 there were 66 institutions (80 centers) in the network, in 2017 it had 67 HEIs (79 centers) and in 2019 it consisted of 75 HEIs (100 centers)[5].

3- ProEB (Professional Master's Programs for the Qualification of Teachers of the Public Basic Education Network) aims at the stricto sensu continuous training of teachers working in the public basic education network, in accordance with the policy of the Ministry of Education (MEC), through support for higher education institutions or a network of associated institutions in the country responsible for the implementation and execution of courses with areas of concentration and themes directly linked to the improvement of Basic Education. CAPES support will be given through the granting of scholarships and promotion of ProEB's Professional Master's courses, in the face-to-face and distance modalities, within the scope of the Open University of Brazil System - UAB, (for more information, see: http://www.capes.gov.br/educacao-a-distancia/proeb?fbclid=IwAR3Qyj6T1j6KLT1H1rQXXTpu7bouKJlgIuSijlrMWKGbw4NNOBtq6Mxl_Sl).

4- Respectively, Professional Master's in: Physics Teaching; Environmental Sciences Teaching for Basic Education Teachers; Language Teaching; Art; History Teaching; Biology Teaching; Chemistry; Philosophy; Sociology; Physical Education; Water Resources Management.

5- Avaible in: http://www.profmat-sbm.org.br/instituicoes-associadas/, access in: 05/01/2019.

In this scenario, different elements that make up and interfere in the implementation and execution of PROFMAT have been the object of study, as is the case with: the phases that make up the implementation of its curriculum (CALDATTO; PAVANELLO; FIORENTINI, 2016); changes in its curriculum (FARIAS; VILELA, 2017); the academic project approved by CAPES, which authorized its opening (CALDATTO; FIORENTINI; PAVANELLO, 2018); and textbooks that guide teaching activities in subjects offered in its scope (CALDATTO, 2015; CALDATTO; RIBEIRO, 2019). And it is in this same context that this research is presented, although focusing on an element not yet investigated: the National Qualification Exams (NQE) applied under PROFMAT.

Thus, this research will seek to answer the following question: what knowledge is mobilized by the questions that composed the National Qualification Exams of PROFMAT, during the years 2012 and 2019, given that this program is aimed at the professional development of teachers in relation to to the teaching of mathematics in basic education?

In this context, the following are presented: the context and the methodological options adopted for the development of this research; the conceptual bases of Mathematics Teachers' Specialized Knowledge (MTSK), a conceptual-analytical model aimed at discussing the specialized knowledge of the Mathematics Teacher (MT) and which was used as a theory that structured and outlined the analyses developed; emerging data and analyses; final considerations.

Objective, object, methodology and context of the investigation

The objective of this article was to investigate and analyze the nature of theknowledge, especially that of a specialized nature, evoked in the solution of the questions that made up the Qualification Exams, developed within the scope of PROFMAT, between the years 2012 and 2019. This was done taking into account that this program aims to develop a training process based on a "solid education in Mathematics, which addresses the needs arising both from the daily work of teachers in the school space and from their broad needs for professional development and appreciation" (BRASIL, 2010, p. 9).

As a result of this objective, a qualitative study was developed in which, from the interpretative content analysis (MARCONI; LAKATOS, 2017), the questions that made up the National Qualification Exams were classified in the light of the MTSK conceptual-analytical model (CARRILLO *et al.*, 2018), within the scope of PROFMAT, in the first semester of the years 2012 to 2019. It is worth noting that, for the purposes of analysis, the knowledge considered was that which relates to the solution of each of the questions, as provided in the answers published on the website of the aforementioned program[6]. This is because they present the parameters to be followed in the correction process of the exams applied throughout Brazil and, therefore, they also indicate the knowledge considered relevant in the training process developed.

Thus, for presentation purposes, these questions were named by an expression associated with the acronym EAQN, in which the letters refer to: the word Exam (E), the year in which the exam was administered (A=12; 13; ...; 19), the word Question (Q) and

6- Available at: http://www.profmat-sbm.org.br/exame-nacional-de-qualificacao/, accessed on: 20/04/2019.

the number (N=1; 2; ...; 8) of the question through the sequence that composed each test. Thus, for example, the expression E12Q1 refers to Question 1 of the NQE applied in the first semester of 2012.

The adoption of Mathematics Teachers' Specialized Knowledge (CARRILLO *et al.*, 2018) as a conceptual-analytical model occurred because, in addition to conceptualizing the knowledge mobilized by the mathematics teacher, it is also configured as an analytical tool for investigative contexts linked to to the professional practice of MTs, especially in basic education. As detailed below, MTSK is composed of the Mathematical Knowledge (MK) and Pedagogical Content Knowledge (PCK) domains, which, exclusively for analytical purposes, are organized into subdomains and categories of knowledge. This favors the process of classifying and analyzing data from different contexts (teaching situations, training situations, interviews, documents, etc.) related to MTs in teaching mathematics.

The option to focus on NQE was given because obtaining a Master's degree in Mathematics by PROFMAT, which traditionally occurs in postgraduate programs in the area of Mathematics, is conditioned, among four other factors[7], on the approval of academics in the examination. This is a written assessment, offered twice a year (one per semester), composed of discursive questions that cover the contents of the basic subjects of the program's curriculum (namely: Real Numbers and Functions; Discrete Mathematics; Geometry; Arithmetic). This exam is prepared and corrected by the National Commission for the Evaluation of PROFMAT Students, designated by ordinance of the coordination of the National Academic Commission, and applied to academics simultaneously in all PROFMAT centers (same date and time). In addition, each academic "has only two consecutive opportunities to pass the National Qualification Exam," which must be taken "immediately after having passed the four basic disciplines and within the completion period of the course" (SBM, 2016, p. 05).

In this way, the PROFMAT student who fails the NQE, the approval of which requires at least 50% of the test being correct (obtaining a grade equal to or greater than 5, on a scale from 0 to 10), will have the opportunity to take the exam in the following semester and, if receiving the scholarship (from CAPES), the scholarship will be cancelled. In the event of a second failure of the NQE, the student will be dismissed from the program and, if they are a scholarship holder, they must refund the "amount spent on the scholarship, according to the legislation in effect" (SBM, 2018, p. 03).

In this scenario, the NQE becomes an important component of the training process developed, since it is configured as one of the main assessment instruments (if not the main one) of PROFMAT, since the failure of students causes them a series of consequences such as, for example, their withdrawal from the program and the refund of the amount received as a scholarship. Thus, research focused on this element of PROFMAT is relevant,

7- I Have been approved in at least 9 (nine) subjects, including all the compulsory diciplines defined in the Dicipline Catalog; II- Have passed the NQE; III- Have been approved in the presentation of the final conclusion work of PROFMAT; IV- Have the final version of the formally-written text inserted in the Academic Control System and in the Sucupira Platform by the Institutional Academic Coordination; V- Satisfy all legal requirements of their Associated Institution (SBM, 2016, p. 06).

especially if we consider the objectives of the program and the fact that it involves the use of public money in its conception and execution.

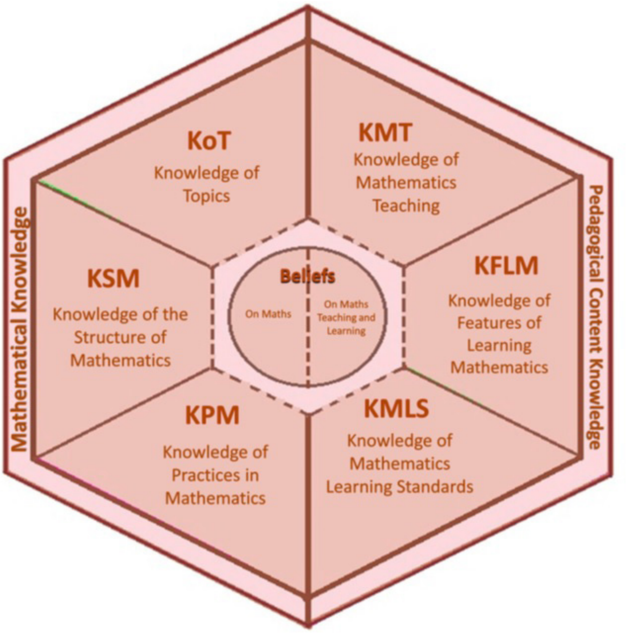
The conceptual-analytical model *Mathematics Teachers' Specialized Knowledge*

Teachers are constantly on the agendas, both political-educational and academic, since their knowledge is considered among the controllable and intervening factors in the teaching process, which has the most repercussions on student learning and results (DARLING-HAMMOND, 2000). Commonly associated with the conceptualizations of Content Knowledge (CK) and Pedagogical Content Knowledge (PCK) developed by Shulman (1986), especially at an international level, are studies that discuss "a knowledge base for teaching that does not depend only on the mastery of content, personal style or good teaching communication" (BORN; PRADO; FELIPPE, 2019, p. 03). From these studies emerged different conceptual and analytical models aimed at the investigation of knowledge that directly impacts the teacher's action when teaching each of the various areas of knowledge [physics (ETKINA, 2010), chemistry (ALVARADO; CAÑADA; GARRITZ, 2015), biology (JÜTTNER; NEUHAUS, 2013) etc.].

With regard to mathematics, research has particularly focused on the analysis of the dimensions of knowledge that directly impact the teaching of mathematics (BAUMERT *et al.*, 2010; SCHEINER *et al.*, 2017). In this regard, several works can be highlighted, such as those of Rowland and Turner (2007) in conceiving the Knowledge Quartet; Ball and collaborators (BALL; THAMES; PHELPS, 2008) in developing Mathematical Knowledge Teaching (MKT); Baumert and collaborators (KLEICKMANN *et al.*, 2015) in investigating within the scope of the Professional Competence of Teachers, Cognitively Activating, Instruction, and the Development of Students' Mathematical Literacy (COACTIV); Stylianides and Stylianides (2014) in forming the Mathematics for Teaching; and Carrillo and collaborators (CARRILLO *et al.*, 2018) in developing Mathematics Teachers' Specialized Knowledge.

This last model, when turning to the specialization of the knowledge of the MT, assumes that the practice of this professional needs exclusive and intrinsic knowledge of the teaching of mathematics, so that it does not allude and compare this knowledge with that demanded by teachers of other areas – such as COACTIV, for example (KRAUSS; BAUMERT; BLUM, 2008) – or other professionals – such as MKT, for example (BALL; THAMES; PHELPS, 2008). Thus, MTSK assumes that the MT needs knowledge composed of the dimensions Beliefs, Mathematical Knowledge (MK) and Pedagogical Content Knowledge (PCK) (and their sub-dimensions, as shown in Figure 1), whose theoretical division occurs exclusively for analytical purposes (FLORES *et al.*, 2016), since they are constantly interrelated and feed back when the teacher teaches mathematics. In light of this framework, the adoption of MTSK was chosen as an analytical model for the discussion of the PROFMAT NQE, since both have the same focus: mathematical knowledge exclusively evoked by the teacher when teaching mathematics.

Figure 1 – MTSK domains and subdomains



Source: Carrillo *et al.* (2018).

As shown in Figure 1, MTSK is composed of domains and subdomains, such that MK is defined and structured from the Knowledge of Topics (KoT), Knowledge of the Structure of Mathematics (KSM) and Knowledge of the Practice of Mathematics (KPM), while PCK is defined and structured from the Knowledge of Mathematics Teaching (KMT), Knowledge of Features of Learning Mathematics (KFLM) and Knowledge of Mathematics Learning Standards (KMLS). The domain focused on the teacher's conceptions and beliefs (Beliefs) is related to both mathematics and mathematics teaching and, therefore, is located at the center of the model.

The KoT subdomain includes knowledge directly related to the themes/topics that the teacher needs to teach, such as procedures (seeking to discuss how it is done, when it can be done, why it is done this way, and the characteristics of the results obtained from the aforementioned mathematical procedures), phenomenological and application aspects of topics/themes, definitions, properties and their foundations, as well as knowledge about the different representation records of specific themes (CARRILLO *et al.*, 2018). It is worth noting that in the KoT, there is mathematical knowledge that complements the approach to the topic in question in the curricular documents.

KSM refers to the teacher's knowledge that allows them to identify and establish relationships between mathematical themes and topics in the course of their teaching. Thus, instead of the concepts being seen as isolated elements, the appropriation of KSM enables their integration in a system of connections, which allows the teacher to understand and approach certain advanced concepts from an elementary perspective (simplification), to develop certain elementary concepts from an advanced perspective (complexification), to relate different topics based on common characteristics (cross-cutting) or to aim at the development of one of the topics from the other (auxiliaries) (CARRILLO *et al.*, 2017).

KPM encompasses aspects related to mathematical thought and practice, such as knowledge about the different ways of defining, arguing, demonstrating and validating mathematics, as well as knowledge of mathematical syntax (FLORES *et al.*, 2016).

The action of teaching involves the knowledge of how this teaching can and should be developed. Thus, KMT includes knowledge of different teaching theories, strategies, techniques, tasks, examples and teaching resources (material and virtual) that allow the teacher to foster the development of students' procedural and conceptual mathematical skills. Likewise, this subdomain predicts that the teacher needs to know examples that awaken in the student the intuition about certain concepts, as well as resources that allow them to induce their students to learn, through manipulation, certain mathematical concepts.

The knowledge linked to how students interact with mathematical content is a highly significant area of knowledge for the authors (FLORES *et al.*, 2016; CARRILLO *et al.*, 2017; CARRILLO *et al.*, 2018) and, therefore, makes up the MTSK through KFLM. Therefore, KFLM encompasses the knowledge of the characteristics of the process of understanding the different contents by the students, the errors, difficulties, aptitudes and obstacles associated with the learning of each concept, in addition to the language used by the students in relation to the concept worked in the classroom and learning theories that appear in different literature.

KMLS particularly refers to mathematics learning standards, such as learning expectations in each educational cycle/phase and the expected level of conceptual and/or procedural development, in addition to the sequence of knowledge at each level. Thus, as an example, KMLS includes knowledge related to the curriculum adopted by the institution at all stages/levels of education, in addition to information present in productions originating from research in the area of mathematics education, with information provided by experienced teachers about the expected learning at each step.

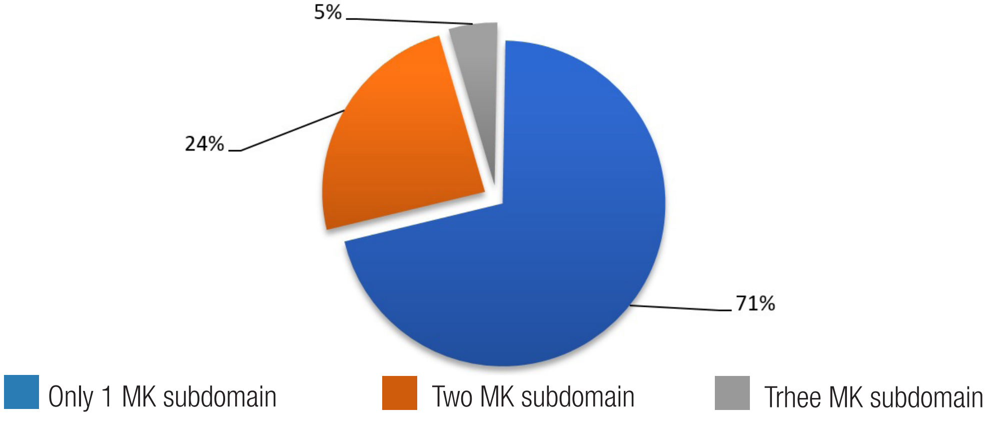
Presentation and discussion of data: the NQE applied between 2012 and 2019

In view of the objective of this research, the content of the questions that composed 8 NQEs applied within the PROFMAT scope will be analyzed, in particular, those that were applied in the first semester of the years 2012, 2013, 2014, 2015, 2016, 2017, 2018 and 2019. In view of this scenario, the corpus of analysis will consist of 62 discursive questions, since 7 NQEs were composed of 8 questions and 1 NQE (applied in 2012, the first exam applied in PROFMAT) was composed of 6 questions.

A first feasible assessment of the analyses carried out relates to the absence of knowledge that can be associated with the PCK domain of MTSK. That is, none of the questions, and therefore none of the NQEs analyzed, address mathematics from a perspective of its teaching and learning in the final years of Elementary and High School (CARRILLO *et al.*, 2017; 2018; FLORES *et al.*, 2016). In the course of the text, we will present and discuss a portion of these questions, in which such absence will be crystallized.

With regard to the MK domain of MTSK, the first categorization produced was embodied in Graph 1.

Graph 1 – Categorization of questions in relation to the approach of knowledge of the subdomains of MK



Source: The authors.

According to graph 1, of the total number of questions under analysis, 5% of them covered knowledge belonging to the three MK subdomains, 24% simultaneously addressed knowledge from two subdomains and 71% of them involved knowledge that appears in only one subdomain.

With regard to the 5% of the questions that simultaneously covered knowledge that appears in the KoT, KSM and KPM domains, they originated a cluster (1) of analysis composed of questions E12Q2, E13Q3 and E18Q8. The solutions of these questions, which were included in the answer sheet and therefore considered correct by the exam graders, involved the knowledge detailed in Table 1.

Table 1 - Questions that simultaneously address knowledge of KoT, KSM and KPM

KoT, KSM and KPM Categories	Associated Questions
Procedures – KoT	E12Q2, E13Q3 and E18Q8
Definitions, properties and their foundations – KoT	
Phenomenology and Applications – KoT	
Auxiliary Connections – KSM	
Forms of Validation and Demonstration – KPM	

Source: Research data.

According to table 1, the 3 questions address the knowledge related to procedures, definitions, properties and their foundations, and phenomenology and applications (which, among others, make up the KoT) in the same way, which fostered the development of auxiliary connections between distinct topics of mathematics (KSM) and Forms of Validation and Demonstration (KPM) (CARRILLO *et al.*, 2017; 2018; FLORES *et al.*, 2016).

For the purposes of exemplification and discussion, let us consider question E13Q3, composed of the following statement:

Figure 2 - Question statement E13Q3

The objective of this question is to demonstrate that the function $f(x) = \cos\sqrt{x}$, $x\ 0$, is not periodic, that is, there is no positive real number T such that $\cos\sqrt{x+T} = \cos\sqrt{x}$ for all $x\ 0$.
a) Find all values of $T\ 0$ for which $f(T) = f(0)$ and then find all values of $T\ 0$ for which $f(T) = f(2T)$.
b) Use item a) to show that $f(x)$ is not periodic.

Source: Research data.

In fact, the solution of the question E13Q3 published on the program's website raises the knowledge of the procedures involved in the study of the cosine function, the knowledge of the definition of periodic function and application of the concept of function periodicity (KoT). Furthermore, it also encompasses the knowledge about the irrationality of $\sqrt{2}$, evidencing the realization of an auxiliary connection with a content that is not the object of the question (KSM) and the demonstration by absurdity (item b of the question) (KPM).

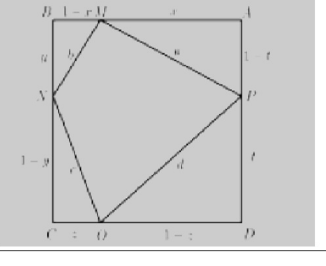
Regarding the 24% of the questions that simultaneously addressed knowledge of two subdomains, which total 15 questions (cluster 2), 13 of them are associated with KoT and KPM, while 2 of them are associated with KoT and KSM.

The solution of the two questions associated with KoT and KSM, E14Q3 and E15Q2, raise knowledge associated with procedures, definitions, properties and their foundations (KoT), in addition to promoting connections between the central topic of the question and themes that are not the direct object of discussion (auxiliary connections, KSM) (CARRILLO *et al.*, 2017; 2018; FLORES *et al.*, 2016). For the purposes of appreciation, let us consider question E14Q3, which presents the following statement:

Figure 3 - Question statement E14Q3

A quadrilateral has its vertices on each side of a square, whose side has measurement 1. Knowing that the measurements of the sides of this quadrilateral are a, b, c, and d, prove that

2 [illegible].



Source: Research data.

Indeed, the solution of question E14Q3 published on the PROFMAT website covers knowledge about procedures inherent to the manipulation of numerical/algebraic expressions of inequalities, in addition to knowledge of definitions, properties and procedures involving geometric figures (particularly, quadrilaterals) (KoT). It also evokes knowledge associated with the definition and properties of a quadratic function in the course of solving a geometric problem, characterizing an auxiliary connection (KSM).

Regarding the 13 questions associated with KoT and KPM, the solution of three of them, E12Q3, E13Q6 and E16Q4, involves knowledge about procedures, definitions, properties (KoT), in addition to exploring forms of validation and demonstration and processes associated with the solution of problems as a way of producing mathematics (KPM). Furthermore, question E12Q3, unlike E13Q6 and E16Q4, also involves phenomenology and applications of knowledge linked to the topic at hand (KoT) (CARRILLO *et al*., 2017; 2018; FLORES *et al*., 2016), as discussed below.

Figure 4 - Question statement E12Q3

A fair coin is tossed successively.
a) If the coin is tossed 4 times, what is the probability that the observed number of heads is odd? What if the coin is tossed 5 times?
b) Observing the result of item a), formulate a conjecture about the probability of seeing an odd number of heads in (n) tosses of the coin.
(c) Demonstrate, using finite induction, the conjecture of item b).

Source: Research data.

Indeed, the solution of question E12Q3 involves the definition, properties and procedures associated with the calculation of probability, in addition to the application of this knowledge (KoT) and forms of validation and demonstration in mathematics, as is the case of the solution of item c that involves the finite induction demonstration technique (KPM). Finally, note that the student is led to produce a conjecture in item b, which must be proved in item c. This conjecture production process (production of a mathematical result) is a characteristic of mathematical practice in the problem solving process (KPM).

Of these 13 questions, 2 of them, E18Q6 and E18Q7, involve knowledge related to procedures, definitions, properties and their foundations, and representation records (KoT), and forms of validation and demonstration (KPM) in their solution (CARRILLO *et al*., 2017; 2018; FLORES *et al*., 2016) (Table 2).

Table 2 – Questions that simultaneously address knowledge of KoT and KPM

KoT and KPM Categories	Associated Questions
Procedures – KoT	E18Q6 and E18Q7
Definitions, properties and their foundations – KoT	
Representation Records – KoT	
Forms of Validation and Demonstration – KPM	

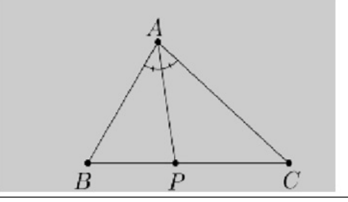
Source: The authors.

As an example of the analysis process that culminated in table 2, let us consider the statement of question E18Q7:

Figure 5 – Question statement E18Q7

Let ABC be a triangle. If P is the foot of the internal bisector relative to the side BC, prove the **Internal Angle Bisector Theorem**, that is, that

$$\frac{\overline{BP}}{\overline{PC}} = \frac{\overline{BA}}{\overline{AC}}$$



Source: Research data.

According to the answer sheet, the knowledge involved in solving question E18Q7 involves procedures associated with alternate interior angles, in addition to the knowledge of various geometric definitions, such as isosceles triangles. Still, the objective of the question is to elaborate a proof for the Internal Angle Bisector Theorem, which is accompanied by the graphic representation of the theorem.

Of the 13 questions, 8 simultaneously involve KoT and KPM. However, as shown in Table 3, they involved only two categories of KoT (procedures and definitions, properties and their foundations) and one of KPM (forms of validation and demonstration) (CARRILLO *et al.*, 2017; 2018; FLORES *et al.*, 2016).

Table 3 - Questions that simultaneously address knowledge of KoT and KPM

KoT and KPM Categories	Associated Questions
Procedures – KoT	E13Q4, E14Q5, E14Q7, E14Q8, E15Q1, E15Q6, E17Q6, E19Q2
Definitions, properties and their foundations - KoT	
Forms of validation and demonstration - KPM	

Source: The authors

Regarding the discussion and exemplification of the discussions addressed in table 3, let us consider question E17Q6, composed of the following statement:

Figure 6 - Question statement E17Q6

(a) Prove Stifel's relation for all (n) and (p) positive integers with $n \geq p$

$$C_{n+1}^{p+1} = C_n^{p+1} + C_n^p$$

(b) Consider the sequence of integers

$$\begin{cases} a_1 = C_2^2, \\ a_n = C_2^2 + \cdots + C_{n+1}^2, n \geq 2. \end{cases}$$

Show that $a_n = C_{n+2}^3$

Source: Research data.

Certainly, the knowledge evoked in the course of the solution presented in the answer of question E17Q6 is about procedures and definitions and properties of combinations (KoT). In addition, the final objective of the question is the elaboration of proofs (validation and demonstration forms) for two mathematical sentences involving the topic combination (KPM), in the case demonstration using finite induction to prove item b.

Regarding the portion of questions whose solutions address knowledge that appears only in one MTSK domain (cluster 3), 71% of the total under analysis (44 questions), these cover knowledge linked to KoT, 17 of which are associated with the categories procedures and definitions, properties and their foundations (CARRILLO *et al.*, 2017; 2018; FLORES *et al.*, 2016), as shown in Table 4.

Table 4 - Questions that address only two categories of KoT knowledge

KoT Categories	Associated Questions
Procedures	E13Q8, E14Q1, E14Q2, E15Q5, E16Q1, E16Q6, E17Q1, E17Q7, E17Q8, E18Q2, E18Q4, E18Q5, E19Q1, E19Q3, E19Q4, E19Q6 and E19Q8
Definitions, properties and their foundations.	

Source: The authors.

For the purposes of discussion and presentation of the analyses that were embodied in table 4, let us consider question E19Q8, composed of the following statement (Figure 7):

Figure 7 - Question statement E19Q8

a) Determine the smallest natural number *c* for which the equation

$$5X + 7Y = C$$

Has exactly 4 solutions in [illegible].

b) Determine, explicitly, the 4 solutions obtained in item a).

Source: Research data.

In fact, the knowledge that appears in the answer sheet for question E19Q8 is associated with the definition and properties of a Diophantine Equation, as well as the knowledge of the standard algorithm associated with solving this type of equation (KoT).

Of these 44 questions, 6 are associated with the categories procedures, definitions, properties and their foundations, and representation records (CARRILLO *et al.*, 2017; 2018; FLORES *et al.*, 2016), as shown in table 5.

Table 5 - Questions Addressing Three Categories of KoT Knowledge

KOT Categories	Associated Questions
Procedures	
Definitions, properties and their foundations.	E14Q6, E16Q7, E17Q2, E17Q5, E18Q3, E19Q7
Representation records	

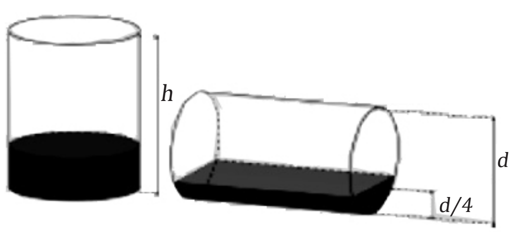
Source: The authors.

Indeed, taking into account question E19Q7 (figure 8) in a scenario of elucidation of the analyses crystallized in table 5, its solution involves the procedure of calculating the volume of a cylinder, in addition to the definition of this geometric figure and its properties, such as generatrix, for example, in addition to the graphic representation of the geometric object in question.

Figure 8 - Question statement E19Q7

A cylindrical container with a circular base is partially filled with water. When "lying down", that is, leaning on one of its generatrixes, the water level reaches a quarter of the diameter d of the base of the cylinder, as shown in the figure. When the cylinder is "standing", that is, supported on one of its bases, the water level reaches what fraction of the height h of the cylinder?

Note - Consider the thickness of the walls and bases of the container to be negligible.



Source: Research data.

Of the 44 questions strictly associated with KoT, 16 are linked to the categories procedures, definitions, properties and their foundations, and phenomenology and applications (CARRILLO *et al.*, 2017; 2018; FLORES *et al.*, 2016), as shown in table 6.

Table 6 - Questions Addressing Three Categories of KoT Knowledge

KoT Categories	Associated Questions
Procedures	E12Q1, E12Q4, E12Q6, E13Q5, E13Q7, E14Q4, E15Q3, E15Q7, E15Q8, E16Q2, E16Q3, E16Q5, E17Q3, E17Q4, E18Q1, E19Q5
Definitions, properties and their foundations.	
Phenomenology and Applications.	

Source: The authors.

Indeed, in a context of illustrating the analyses that generated table 5, let us consider question E18Q1 presented in Figure 9:

Figure 9 - Question statement E18Q1

Radioactive isotopes of a chemical element are subject to a process of radioactive decay. Over time, a sample of such isotopes disintegrates, that is, emitting radiation and transforming into a sample of more stable atoms.
It is known that this decay is exponential, that is, denoted by *m(t)* the mass of a certain radioactive isotope at time *t*, we have

$$m(t) = m_0 \,.\, b^t,$$

for some $0 < b < 1$, being $m_0 > 0$ the initial mass. The half-life of this isotope, denoted *T*, is the time required for the mass *m* to reduce to half its initial value.
a) Determine *b* as a function of *T*.
b) Determine, as a function of *T*, the time required for *m* to reduce to one-third of its initial value.

Source: Research data.

When we turn to the solution that appears in the answer sheet of the aforementioned question, we find that it involves the definition and properties of an exponential function, in addition to evoking the manipulation of procedures associated with such a function, from the application of this knowledge in solving a problem which involves calculating the half-life of radioactive isotopes.

Finally, still in relation to the questions that make up cluster 3, five of them evoke knowledge that appears in the categories of KoT procedures, definitions, properties and their foundations, representation records, and phenomenology and applications (CARRILLO *et al.*, 2017; 2018; FLORES *et al.*, 2016), as shown in table 7.

Table 7 - Questions covering four categories of KoT knowledge

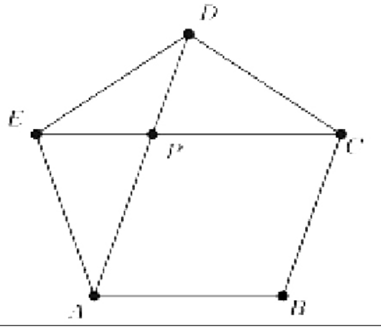
<table>
<tr><th>KOT Categories</th><th>Associated Questions</th></tr>
<tr><td>Procedures</td><td rowspan="4">E12Q5, E13Q1, E13Q2, E15Q4, E16Q8</td></tr>
<tr><td>Definitions, properties and their foundations.</td></tr>
<tr><td>Representation records</td></tr>
<tr><td>Phenomenology and Applications</td></tr>
</table>

Source: The authors.

For the purposes of exemplification and discussion of the analyses that originated table 7, let us consider question E15Q4, whose statement is presented in figure 10.

Figure 10 - Question statement E15Q4

The diagonals AD and CE of the regular pentagon ABCDE with sides of length a, intersect at point P. Determine $\overline{AP}$ and $\overline{PD}$ as a function of a.



Source: Research data.

The solution process presented in the answer sheet for this question involves knowledge of procedures, definitions, properties and foundations of plane geometric figures, especially the pentagon. In particular, the question raises the establishment of the relationship (through an algebraic expression) between the measure of the length of the side of a pentagon (a), and the segments originating from the intersection between two of its diagonals, namely $\overline{AP}$ and $\overline{PD}$ (property). This expression can be configured as an aid in the process of determining (procedure) the length of the segments $\overline{AP}$ and $\overline{PD}$ from the value of the length of the side of the pentagon. This question is also accompanied by a graphic representation of the statement, in addition to being an application of the geometric concepts of congruence and similarity of triangles during the solution.

Final Considerations

The analysis presented showed that the assessment process developed within the scope of PROFMAT through the National Qualification Exam is aimed at the exploration of knowledge that, according to the conceptual-analytical model Mathematics Teachers' Specialized Knowledge (CARRILLO *et al.*, 2017, 2018; FLORES *et al.*, 2016), is exclusively in the Mathematical Knowledge domain. Furthermore and predominantly, the knowledge that appears in the questions that made up the aforementioned exam, throughout all the years in which it was implemented (and that the program has been in progress), is associated with the Knowledge of Topics (KoT) subdomain. The knowledge associated with the Knowledge of the Structure of Mathematics (KSM) and Knowledge of the Practice of Mathematics (KPM) subdomains appear with significantly less intensity, since more than 70% of the questions focus exclusively on knowledge that appears in KoT, while only 5% of them address knowledge that appears in the 3 subdomains.

Thus, from a perspective of discussion of the articulation between the (decisive) evaluation process that takes place in PROFMAT and the objectives proposed by this policy of continuing teacher education at the professional master's level, it is possible to verify that: I) Taking into consideration that the practice of MTs teaching mathematics, when analyzed in the light of Mathematics Teachers' Specialized Knowledge, raises the mobilization of knowledge linked to the subdomains KoT, KSM and KPM in an interrelated and articulated way, it is possible to affirm that the assessment developed in PROFMAT privileges only one of the subdomains of mathematical knowledge that really supports teachers in their daily work with mathematics; II) The assessment process developed by the National Qualification Exam does not address at any time knowledge associated with the Pedagogical Content Knowledge (PCK) domain.

The lack of knowledge linked to Pedagogical Content Knowledge conflicts with the results obtained by research that discuss the influence of the mathematical education of MTs on their practice of teaching mathematics, since, according to the conceptualization of MTSK and several conceptual-analytical models (some even linked to educational policies) – such as, for example, the works of Rowland and Turner (2007); Ball and collaborators (BALL; THAMES; PHELPS, 2008), Baumert and collaborators (KLEICKMANN *et al.*, 2015) and Stylianides and Stylianides (2014) – the teaching practice is dependent on this type of knowledge (PCK) and it is also closely related to the mathematical dimension of MT training.

The analyses carried out of the PROFMAT NQEs converge with those developed by other research (CALDATTO, 2015; CALDATTO; PAVANELLO; FIORENTINI, 2016; FARIAS; VILELA, 2017; CALDATTO; FIORENTINI; PAVANELLO, 2018; CALDATTO; RIBEIRO, 2019), which concludes that the elements that make up the training process developed within the scope of this postgraduate program are considerably removed from the demands related to the practice of MTs in teaching mathematics.

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