# FORMAL SEMANTICS FOR PROPOSITIONAL ATTITUDES

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**Abstract:** Contemporary logic is confined to a few paradigmatic attitudes such as belief, knowledge, desire and intention. My purpose is to present a general model-theoretical semantics of propositional attitudes of any cognitive or volitive mode. In my view, one can recursively define the set of all psychological modes of attitudes. As Descartes anticipated, the two primitive modes are those of belief and desire. Complex modes are obtained by adding to primitive modes special cognitive and volitive ways or special propositional content or preparatory conditions. According to standard logic of attitudes (Hintikka), human agents are either perfectly rational or totally irrational. I will proceed to a finer analysis of propositional attitudes that accounts for our imperfect but minimal rationality. For that purpose I will use a non standard predicative logic according to which propositions with the same truth conditions can have different cognitive values and I will explicate subjective in addition to objective possibilities. Next I will enumerate valid laws of my general logic of propositional attitudes. At the end I will state principles according to which minimally rational agents dynamically revise attitudes of any mode.

**Keywords:** Philosophical logic. Propositional attitudes. Psychological modes. Minimal rationality.

## 1. ANALYSIS OF PROPOSITIONAL ATTITUDES

Let me first explain my componential analysis of psychological modes and the principles of my logic of propositional attitudes. (For more explanations see my paper Vanderveken (2008)). By nature attitudes, which are directed at objects and facts of the world, have logically related conditions of possession and of satisfaction. Whoever

possesses an attitude is in principle able to determine what has to happen in the world in order that his or her attitude is satisfied. Beliefs are satisfied whenever they are true, desires whenever they are realized and intentions whenever they are executed. Propositional attitudes consist of a *psychological mode M* with a *propositional content P.* They are the simplest kinds of individual attitudes directed at facts. Many philosophers tend to reduce all propositional attitudes to sums of beliefs and desires. However, our intentions are much more than a desire to do something with a belief that we are able to do it. Of course, all cognitive attitudes (e.g. conviction, faith, confidence, knowledge, certainty, presumption, pride, arrogance, surprise, amazement, stupefaction, prevision, anticipation and expectation) are beliefs and all volitive attitudes (e.g. wish, will, intention, ambition, project, hope, aspiration, satisfaction, pleasure, enjoyment, delight, gladness, joy, elation, amusement, fear, regret, sadness, sorrow, grief, remorse, terror) are desires. But psychological modes divide into other components than the *basic categories of cognition* and *volition.* In my view, complex modes have a *proper way* of believing or desiring, proper *conditions on their propositional content* or proper *preparatory conditions.* We feel our beliefs and desires in a lot of ways. Many modes require a special *cognitive or volitive way* of believing or desiring. Thus, *knowledge* is a belief based on strong evidence that gives confidence and guarantees truth. Whoever has an *intention* feels such a strong desire that he or she is disposed to act in order to satisfy that desire. From a logical point of view, a *cognitive or volitive way* is a function $f_{\bar{\omega}}$ which restricts basic psychological categories. Like illocutionary forces, modes also have *propositional content* and *preparatory conditions. Previsions* and *anticipations* are directed towards the future. Intentions are desires to carry out a present or future action. From a logical point of view, a *condition on the propositional content* is a function $f_{\theta}$ that associates which each agent and moment a set of propositions. Any agent of an

attitude or of an illocution *presupposes* certain propositions. His or her attitude and illocution would be *defective* if these propositions were then false. Thus *promises* and *intentions* have the preparatory condition that the agent is then able to do the action represented by their propositional content. No agent can lie to himself. Whoever has an attitude both believes and presupposes that its preparatory conditions are fulfilled. A preparatory condition is a function $f_\Sigma$ associating with each agent, moment and propositional content a set of propositions that the agent would presuppose and believe if he had then an attitude with that preparatory condition and propositional content. The sets of cognitive and volitive ways, of propositional content and of preparatory conditions are *Boolean algebras.* They contain a *neutral* mode, preparatory or propositional condition and they are closed under the operations of *union* and intersection.

On the basis of my analysis, one can formally distinguish different modes of attitudes like fear, regret and sadness which apparently reduce to the same sums of beliefs and desires. Identical psychological modes have the same components. Possession conditions of propositional attitudes are entirely determined by components of their mode. By definition, an agent *a possesses a cognitive (or volitive) attitude of the form M(P) at a moment m* when he or she then *believes* (or *desires*) the propositional content $P$, he or she feels that belief or desire that *P in the cognitive or volitive way* $\bar\omega_M$ proper to psychological mode $M$, the *proposition P then satisfies propositional content conditions* $\Theta_M(a, m)$ and finally that *agent then presupposes and believes all* propositions determined by *preparatory conditions* $\Sigma_M(a, m, P)$ of mode $M$ with respect to the content $P$. Thus an agent *intends that P* at a moment when proposition $P$ then represents a present or future action of that agent, he or she desires so much that action that he or she is committed to carrying it out and moreover that agent then presupposes and believes to be able to carry it out. An *attitude strongly commits an*

*agent to another at a moment* when he or she could not then have that
attitude without having the second. Thus whoever believes that it will
rain tomorrow then foresees rain tomorrow. Some attitudes strongly
commit the agent to another at particular moments. Whoever believes
now that it will rain tomorrow foresees rain tomorrow. The day af-
ter tomorrow the same belief won't be a prevision. It will be a belief
about the past. An attitude *contains another* when it strongly com-
mits any agent to that other attitude at any moment. There are *strong
and weak psychological commitments* just as there are strong and weak
illocutionary commitments (See Searle and Vanderveken (1985)). One
must distinguish between the overt possession of an attitude and a
simple psychological commitment to that attitude. Whoever believes
that every man is mortal is weakly committed to believing that Ne-
buchadnezzar is mortal, even if he has not Nebuchadnezzar's concept
in mind and if he or she does not then possess the second belief. No
one could simultaneously believe the first universal proposition and the
negation of the second.

   Psychological modes are not a simple sequence of a basic psycho-
logical category, a cognitive or volitive way, a propositional content
condition and a preparatory condition. For their components are not
logically independent. Certain components *determine* others of the
same or of another kind. Thus the volitive way of the mode of *inten-
tion* determines the propositional content condition that it represents
a present or future action of the agent and the preparatory condition
that that agent is then able to carry out that action. The two primitive
modes of *belief* and *desire* are the simplest cognitive and volitive modes.
They have no special cognitive or volitive way, no special propositio-
nal content or preparatory condition. Complex modes are obtained by
adding to primitive modes special cognitive or volitive ways, proposi-
tional content conditions or preparatory conditions. Thus the mode
of *prevision* $M_{foresee}$ is obtained by adding to the mode of belief the

propositional content condition $\Theta_{future}$ that associates with each agent and moment the set of propositions that are future with respect to that moment. $M_{foresee} = [\Theta_{future}]Belief.$ The mode of *expectation* is obtained from that of prevision by adding the special cognitive way that the agent is then in a state of expectation. $M_{expect} = [\bar{\omega}_{expecation}]M_{foresee}.$ The mode of *hope* is obtained from that of *desire* by adding the special cognitive way that the agent is then uncertain as regards the existence and the inexistence of the represented fact and the preparatory condition that that fact is then possible. The mode of *satisfaction* is obtained from that of *desire* by adding the *preparatory condition* that the desired fact exists. The mode of *pleasure* has, in addition, the *volitive way* that the satisfaction of the desire puts the agent in a state of pleasure and the preparatory condition that it is good for the agent. Because all operations on modes add new components, they generate stronger modes. Attitudes $M(P)$ with a complex mode *contains* attitudes $M'(P)$ whose modes have less components. A lexical analysis of terms for attitudes based on my componential analysis explains which name stronger psychological modes. One can show comparative strength by drawing tableaux. See the appendix.

From a logical point of view, propositional attitudes are not pairs of a mode and a propositional content. Whoever possesses a propositional attitude of the form $M(P)$ *applies* the mode $M$ to the content $P$ so as to determine under which conditions it is satisfied. Identical propositional attitudes can have a different psychological mode. What matters is that their propositional content is the same and that whoever possesses one attitude also possesses the other. Attitudes with the same content and possession conditions fulfill the same function in psychological life. Thus the *belief* that it will always be the case that $2 + 2 = 4$ is also a *prevision*.

Propositions with the same truth conditions are not the contents of the same attitudes. Moreover agents ignore the necessary truth of

many propositions that they understand. They have to learn a lot of essential properties of objects. By *essential property* of an object I mean a property that it *really* possesses in any possible circumstance. An essential property of each agent is to have certain parents. Some do not know their parents. Others are wrong about their identity. According to standard logic of attitudes (cf. Hintikka (1971)), relations of psychological compatibility with the truth of beliefs and the realization of desires are modal relations of accessibility between agents and moments, on one hand, and possible circumstances, on the other hand. Possible circumstances are compatible with agents' beliefs at each moment of time. To each agent $a$ and moment $m$ there corresponds a unique set $Belief(a, m)$ of possible circumstances that are compatible with the truth of all beliefs of that agent at that moment. In Hintikka's logic, an agent *believes a proposition at a moment* when that proposition is true in all possible circumstances that are compatible with what that agent then believes. Given this approach, human agents are logically omniscient. They believe necessarily true propositions and their beliefs are closed under strict implication. Moreover, they are either *perfectly rational or totally irrational.* Agents are perfectly rational when at least one circumstance is compatible with what they believe. Otherwise, they are totally irrational. Whoever believes a necessary falsehood believes everything.

**In my approach, all** *circumstances* **remain** *possible.* So objects keep their essential properties (each of us keeps his real parents) and necessarily true propositions remain true in all circumstances. In order to account for our human inconsistency and minimal rationality, I advocate a *predicative* non classical propositional logic which takes into account acts of predication and reference that we make in apprehending propositions.

**In my view, each proposition has a finite** *structure of constituents.* It predicates *attributes* (properties or relations) of *objects subsu-*
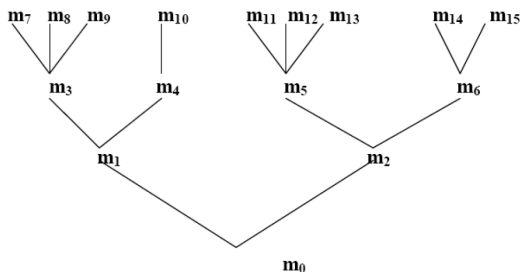
*med under concepts.* We understand a proposition when we understand which attributes our objects of reference must possess in a possible circumstance in order that this proposition be true in that circumstance.

We understand most propositions without knowing in which circumstances they are true, because we ignore *real denotations* of their attributes and concepts in many circumstances. One can refer to a colleague's mother without knowing who she is. However we can in principle think of persons who could be that mother. So in any possible use of language, **there are a lot of** *possible denotation assignments to attributes and concepts* **in addition to the standard** *real denotation assignment* which associates with each propositional constituent its actual denotation in every possible circumstance. They are functions of the same type that associate with each individual concept a unique individual or no individual at all in every possible circumstance. According to the real denotation assignment, my colleague's mother is the woman who gave birth to him. According to other possible denotation assignments, his mother is another person. However, **all possible denotation assignments respect** *meaning postulates.* According to any, a mother is a female parent. We ignore the real denotation of most concepts and attributes in many circumstances. But we can think of denotations that they could have. When we have in mind concepts and attributes only some possible denotation assignments to them *are then compatible with* our beliefs. Suppose that according to you my mother is Belgian. In that case, possible denotation assignments according to which she is not Belgian are incompatible with your beliefs. **Possible denotation assignments rather than possible circumstances are compatible with agents' beliefs**. So logic can account for subjective possibilities. Because my truth definition is relative to both circumstances and denotation assignments, most propositions have *a lot of possible truth conditions.* Of course, in order to be *true in a circumstance* propositions have to be *true in that*

*circumstance according to the real denotation assignment.* So among all possible truth conditions of a proposition, its *real Carnapian truth conditions* determine the set of possible circumstances where it is true according to the real denotation assignment.

In my view, *identical* propositions make the same predications and they are true in the same circumstances according to the same possible denotation assignments. Necessarily true propositions with a different structure of constituents are therefore different. So are propositions that are not true according to the same possible denotation assignments to their constituents. We do not understand them to be true in the same possible circumstances. Few necessarily true propositions are *pure tautologies* that we know *a priori*. A proposition is *necessarily true* when it is true in every possible circumstance according to the real denotation assignment. In order to be *tautologically true*, a proposition has to be true in every circumstance according to every possible denotation assignment. Unlike the proposition that my mother is a mother, the necessarily true proposition that my mother is Gabrielle Louise Albertine Charron is not a pure tautology. It is false according to possible denotation assignments. A proposition is *subjectively possible* when it is true in a circumstance according to a denotation assignment. In order to be *objectively possible* it must be true in a circumstance according to the real denotation assignment. The logic of attitudes requires a ramified conception of time compatible with indeterminism. Agents live and persist in an indeterminist world. First of all their attitudes and illocutions are not determined. They could have thought or spoken differently. Secondly, their future is open. In branching time, a *moment* is a complete possible state of the actual world at a certain instant and the *temporal relation* of *anteriority* between moments is partial. There is a single causal route to the past. However, there are multiple future routes. Consequently, the set of moments of time is a *tree-like frame*:

$m_7$  $m_8$  $m_9$      $m_{10}$      $m_{11}$ $m_{12}$ $m_{13}$      $m_{14}$   $m_{15}$

$m_3$        $m_4$        $m_5$        $m_6$

$m_1$                              $m_2$

$m_0$

A maximal chain $h$ of moments of time is called a *history*. It represents a *possible course of history of our world.* As Belnap, Perloff and Xu (2001) pointed out, each *possible circumstance* is a pair of a moment $m$ and of a history $h$ to which that moment belongs. Thanks to histories temporal logic can analyze important modal notions like settled truth and historic necessity. Certain propositions are true at a moment according to all histories. Their truth is then *settled at that moment* no matter how the world continues. So are propositions which attribute attitudes to agents. Whoever desires something at a moment then desires that thing no matter what happens later. Contrary to the past, the future is open. The world can continue in various ways after indeterminist moments. Thus the truth of future propositions is not settled at such moments. It depends on which historical continuation of that moment is under consideration. When a moment can continue in different possible ways, its actual future continuation is not then determined. However, as Occam (see Prior (1967)) pointed out, if the world continues after a moment, it will continue in one way. The actual historic continuation of any moment is unique even if it is still undetermined at that moment. Indeterminism cannot prevent that unicity. So each moment $m$ has a *proper history $h_m$* in each model. A proposition is *true at a moment $m$* according to a denotation assignment when it is true according to that assignment at that moment in its proper history $h_m$. Coinstantaneous moments which belong to the same instant are

on the same horizontal line in each tree-like frame. Logic analyzes *historic necessity* by quantifying over coinstantaneous moments.[1]

Like elementary illocutions propositional attitudes are directed towards facts of the world represented by their propositional content. Most often agents establish a correspondence between their ideas and things to which they refer. According to them their ideas correspond to represented things in the world or represented things have to correspond to their ideas. Their attitudes and illocutions have for that reason *satisfaction conditions.* At each moment where agents think and act they ignore how the world will continue. However, their attitudes and actions are directed toward the real historic continuation of that moment. In order that a present desire directed at the future is realized, it is not enough that things will be at a posterior moment as the agent now desires. They must be so later in the real future. So the *satisfaction* of propositional attitudes and elementary illocutions of an agent at an indeterminist moment requires the *truth* of their propositional content *at that moment* in its real historic continuation. The notion of *satisfaction* is a generalization of the notion of *actual truth* that covers attitudes and elementary illocutions with a non empty direction of fit. Just as a *belief* at a moment is *satisfied* when it is *then true*, a *wish* and a *desire* are *satisfied* when they are *then realized*; a *prevision*, an *expectation*, a *hope* and an *aspiration* when they are *fulfilled*; an *intention*, a *project* and a *plan* when they are *then executed*; a *fear* and a *fright* when the feared thing does not *then* happen.

Like assertive illocutions, *cognitive attitudes* have the *mind-to-things direction of fit.* They are *satisfied* when their propositional content is true at the moment under consideration. The agent's ideas correspond to things as they are then in the world. In the cognitive case, when the

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[1]The proposition that $P$ is then necessary (in symbols $\Box P$) is true at a moment when $P$ is true at all coinstantaneous moments according to all histories.

agent realizes that there is no correspondence, he immediately changes his beliefs. This is why *satisfaction* amounts to *truth* in the case of *cognitive* attitudes. However, this is not the case for *volitive* attitudes whose direction of fit goes from things to ideas. For the world and not the agent is at fault in the case of dissatisfaction of volitive attitudes. The agent can keep his desires and remains dissatisfied. Most often agents having a *volitive attitude desire the fact* represented by the propositional content *no matter how that fact turns to be existent in the world.* So most volitive attitudes are *satisfied* when their content is then true, no matter for which reason. Things are then such as the agent desires them to be, no matter what is the cause of their existence. However *volitive attitudes* like *will, intentions, pretensions* and *ambitions* have a special volitive way which requires that things fit the agent's ideas because he or she *wants* them in that way. Such wanting attitudes as well as illocutions like orders, commands, pledges and promises that express the speaker's will have *self-referential satisfaction conditions.* Their satisfaction requires more than the actual truth of their propositional content. It requires that the represented fact turns to be existent in order to satisfy the agent's attitude. In order to execute a prior intention, an agent must later carry out the intended action because of that previous intention. If the agent is later obliged to act, he or she does not then execute the prior intention. My logic of attitudes explains such cases of self-referential satisfaction by relying on the notion of *intentional causation.* The agent's attitude must then be a *practical reason* why the represented fact turns to be existent.

*Volitive* modes like *joy, gladness, pride, pleasure regret, sadness, sorrow,* and *shame* have the *empty direction of fit.* Agents of such attitudes do not want to establish a correspondence between their ideas and things in the world. They just take for granted either correspondence or lack of correspondence. In the case of *joy, gladness, pride* and *pleasure,* the agent believes that the desired fact exists. In the case of

*regret, sorrow* and *shame*, he believes the contrary. The first attitudes have the preparatory condition that their propositional content is then true, the second attitudes have the opposite preparatory condition that their content is then false. Volitive attitudes with such special *preparatory condition* have the *empty direction of fit* and no proper *satisfaction conditions*. Instead of being satisfied or dissatisfied, they are just *appropriate* or *inappropriate*.

## 1.1. My new explication of the basic categories of cognition and volition

In my approach, attitudes of agents are about objects that they represent under concepts. No agent can have a propositional attitude without having in mind consciously or potentially all attributes and concepts of its content. Whoever has a conscious propositional attitude has in mind consciously its attributes and concepts. Whoever has an unconscious attitude can in principle express these senses thanks to his language. Otherwise, he would be unable to determine under which conditions his attitude is satisfied. Secondly, possible denotation assignments to propositional constituents rather than possible circumstances are compatible with the satisfaction of agents' attitudes. So there corresponds to each agent $a$ and moment $m$ a unique set $Belief(a, m)$ of possible denotation assignments to attributes and concepts that are compatible with the truth of beliefs of that agent at that moment. $Belief(a, m)$ is the set $Val$ of all possible denotation assignments when agent $a$ has no attribute or concept in mind at moment $m$. That agent has then no attitudes. Otherwise, $Belief(a, m)$ is a *non empty proper* subset of $Val$. Any agent who has in mind propositional constituents respects meaning postulates governing them in determining truth conditions of propositions. Otherwise he could not express them. Because meaning postulates are entirely determined by meaning, agents necessarily internalize such postulates in learning their language.

Consequently there always are possible denotation assignments compatible with what an agent believes. In my view, an agent *a believes a proposition* at a moment $m$ when he has then in mind all its concepts and attributes and that proposition is true at that moment according to all possible denotation assignments belonging to $Belief(a, m)$. We all now have beliefs directed at the future. Such beliefs are true when things will be as we believe in *the real future continuation* of the present moment.

Similarly, to each agent $a$ and moment $m$ there corresponds a unique non empty set $Desire(a, m)$ of possible denotation assignments to attributes and concepts that are compatible with the realization of all desires of that agent at that moment. There is however the following important difference between desire and belief. We often believe, but we cannot desire, that objects are so and so without believing that they could be otherwise. For any desire contains a *preference*. Whoever feels a desire distinguishes two different ways in which represented objects could be in the actual world. In a first preferred way, objects are in the world as the agent desires, in a second way, they are not. The agent's desire is realized in the first case, it is unrealized in the second case. In order that an agent *a desires the fact represented by a proposition* at a moment $m$, it is not enough that he has then in mind all its constituent senses and that the proposition is true at that moment according to all possible denotation assignments of $Desire(a, m)$. That proposition must moreover be false in at least one circumstance according to that agent.

My explication of belief and desire is compatible with philosophy of mind (See my paper Vanderveken (2009)). It distinguishes conscious and unconscious attitudes and accounts for the fact that *human agents are not perfectly rational*. We do not have in mind all concepts and attributes. So we ignore logical as well as necessary truths. Our knowledge is limited: we ignore which objects possess many properties in

a lot of circumstances. In that case assignments associating different denotations to these properties in these circumstances are then compatible with our beliefs. We have false beliefs and unsatisfied desires. Possible denotation assignments compatible with our beliefs and desires need not assign real denotations to attributes that we have in mind. They can even violate essential properties of objects. In that case we have necessarily false beliefs and insatisfiable desires. My analysis solves traditional paradoxes. However human agents cannot be totally irrational. On the contrary, they are *minimally rational*. First of all, agents are paraconsistent in Newton da Costa's sense (see da Costa, Béziau and Bueno (2005)). They cannot believe or desire everything since some possible denotation assignments are always compatible with the satisfaction of their beliefs and desires. Moreover, their beliefs and desires commit them to others. For possible denotation assignments respect meaning postulates. Human agents are therefore *minimally logically omniscient*: they cannot have in mind what I called a pure tautology without knowing for certain that it is necessarily true. Represented objects could not be in another way according to them. Similarly, *pure contradictions* (negations of pure tautologies) are false in every possible circumstance according to any agent. We can neither believe nor desire contradictory things. Some hope that arithmetic is complete (a necessarily false proposition if Gödel's proof is right). But we could never believe nor desire both the completeness and the incompleteness of arithmetic (a pure contradiction). There is a new *strong* propositional *implication* much finer than Lewis' strict implication that is important for the analysis of psychological and illocutionary commitments. A proposition *strongly implies* another when whoever expresses that proposition is able to express the other and it cannot be true in a circumstance according to a possible denotation assignment unless the other proposition is also true in that circumstance according to that

assignment. Strong implication is finite, tautological, paraconsistent, decidable and *a priori known.*

## 2. THE IDEAL OBJECT LANGUAGE

The ideal language $\mathscr{L}$ of my logic of propositional attitudes contains logical constants expressing material and formal universal features of psychological modes. Its formulas can express the logical form and determine felicity conditions of all kinds of propositional attitudes. Like natural languages, $\mathscr{L}$ contains indexical expressions whose sense is context-dependent.

**The primitive types of $\mathscr{L}$ are:** $\#e, r, p, \bar{\omega}, \theta, \Sigma$ and $M$. $\#e$ is the type of *concepts* of individual objects, $r$ is the type of *attributes* of or *relations* between individuals, $p$ is the type of *propositions*; $\bar{\omega}$ is the type of *ways of believing or desiring*; $\theta$ the type of *propositional content conditions*; $\Sigma$ the type of *preparatory conditions*; and $M$ the type of *psychological modes.*

The vocabulary of $\mathscr{L}$ contains:

- **individual constants** of type $e$ expressing *concepts of individual objects* and *agents*; for each natural number $n$, **predicates of degree** $n$ expressing $n$-ary *relations* between individuals including the binary identity predicate $=_2$ and

- the **logical constants:** $Bel$, $Des$, $K_{\bar{\omega}}$, $Want_{\bar{\omega}}$ and $Int_{\bar{\omega}}$ that are of type $\bar{\omega}$, the constant $Int_{\theta}$ of type $\theta$, the constant $Truth_{\Sigma}$ of type $\Sigma$ and a constant $1_{\zeta}$ for each type $\zeta \in \{\bar{\omega}, \theta, \Sigma\}$. $Bel$ and $Des$ name respectively the general *categories of cognition and volition.* Constants of type $\bar{\omega}$ name *ways* of believing and desiring. $K_{\bar{\omega}}$ names the *special cognitive way of knowledge* and $Want_{\bar{\omega}}$ and $Int_{\bar{\omega}}$ the special *volitive ways of will* and *intention.* Constants of type $\theta$ name *propositional content conditions* and constants of type $\Sigma$ *preparatory conditions.* In particular, $Past_{\theta}$ names the condition that the propositional content

is past and $Future_\theta$ that it is future. $Int_\theta$ names the *special proposi-tional content condition of intentions.* $Truth_\Sigma$ names the preparatory condition that the propositional content is then true. $1_{\bar{\omega}}$ names the neutral way of having cognition and volition, $1_\theta$ the neutral propositio-nal content condition and $1_\Sigma$ the neutral preparatory condition.

The **syncategorematic symbols** of $\mathscr{L}$ are: $\neg, \wedge, \cap, \cup, \}, \square, \diamond,$ *Was, Will, Settled, Actually, Tautological*, $\rho$, [ , ( , ] and ).

## 2.1. Rules of formation of $\mathscr{L}$

### Terms for components of psychological modes

(i) Constants of type $\bar{\omega}, \theta$ and $\Sigma$ are terms of $\mathscr{L}$ naming *components of modes.*

(ii) If $A$ and $B$ are component terms of L of the same type so are new terms $(A \cup B)$ and $(A \cap B)$. They name respectively the *intersection* and the *union* of components named by $A$ and $B$.

### Terms for psychological modes

Expressions of the form $[A_{\bar{\omega}}, A_\theta, A_\Sigma, Bel]$ and $[A_{\bar{\omega}}, A_\theta, A_\Sigma, Des]$ are *terms* of type $M$ of $\mathscr{L}$. The first term names the weakest cognitive mode and the second term the weakest volitive mode which have the way $A_{\bar{\omega}}$, the propositional content condition $A_\theta$ and the preparatory condition $A_\Sigma$.

### Propositional formulas

(i) If $R_n$ is a predicate of degree $n$ and $t_1, \ldots, t_n$ is a sequence of $n$ individual constants, then $[R_n t_1 \ldots t_n]$ is a propositional formula of $\mathscr{L}$ of type $p$ expressing the *elementary proposition* that predicates the attribute expressed by $R_n$ of the $n$ individuals under concepts expressed by $t_1, \ldots, t_n$ in that order.

(ii) If $A_p$ and $B_p$ are propositional formulas and $a$ an individual constant, $\neg A_p, Settled A_p, \square Ap, Was A_p, Will A_p, Actually A_p, Tautolo-gical A_p, (A_p \wedge B_p), (A_p \} B_p), [Bel a A_{\bar{\omega}} A_p], \diamond a A_p, [Des a A_{\bar{\omega}} A_p], [\rho A_p B_p], (A_\theta a A_p), (A_\Sigma a A_p)$ and $[a >> A_p]$ are new propositional formulas.

$\neg A_p$ expresses the *negation* of proposition $A_p$. $Will A_p$ expresses the *future* proposition that *it will be the case* that $A_p$. $Was A_p$ expresses the *past* proposition that *it has been the case* that $A_p$. $Settled A_p$ expresses the modal proposition that *it is settled* that $A_p$. $\Box A_p$ means that *it is historically necessary* that $A_p$, in the sense that it could not then have been otherwise than $A_p$. $Actually A_p$ expresses in each context the indexical proposition that *it is then actual* that $A_p$, that is to say that $A_p$ is true at the moment and in the history of that context[2]. $Tautological A_p$ means that proposition $A_p$ is a pure tautology. $(A_p \wedge B_p)$ expresses the *conjunction* of the propositions expressed by $A_p$ and $B_p$ and $(A_p \} B_p)$ means that proposition $A_p$ contains all elementary propositions of $B_p$. $[Bela A_{\bar{\omega}} A_p)]$ and $[Desa A_{\bar{\omega}} A_p)]$ respectively mean that agent a believes and desires that $A_p$ with the mode named by $A_{\bar{\omega}}$. $\Diamond a A_p$ means that $A_p$ could then be true given all attitudes of agent $a$ [3]. $[\rho A_p B_p]$ means that $A_p$ is true *because of* $B_p$.

$(A_\theta a A_p)$ means that proposition $A_p$ *satisfies the propositional content condition* $A_\theta$ as regards agent $a$[4]. $(A_\Sigma a A_p)$ expresses in each context a proposition that is true in a circumstance $m/h$ when all the preparatory conditions $A_\Sigma$ are fulfilled as regards the agent $a$, the propositional content $A_p$ and the moment $m$[5]. $[a >> A_p]$ means that agent $a$ presupposes $A_p$.

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[2]The interpretation of the actuality connective makes a reference to the proper history of the moment of utterance.

[3]$\Diamond a A_p$ is true in circumstance $m/h$ when $A_p$ is true in a coinstantaneous circumstance where agent a has all attitudes that he or she has at moment $m$.

[4]Thus $(Int_\theta a A_p)$ means that proposition $A_p$ represents a present or future action of agent $a$.

[5]If $Int_\Sigma$ represents preparatory conditions of intention. $(Int_\Sigma a A_p)$ means that agent is capable to carry out action $A_p$.

## 2.2. Rules of abbreviation

I will use ordinary rules of abbreviation for the elimination of parentheses and type symbols and for the introduction of truth, modal and temporal connectives.

*Historical possibility:* $\Diamond A =_{def} \neg \Box \neg A$

*Universal Necessity:* $\blacksquare A =_{def} Always \Box A$

*Strong implication:* $(A_p \mapsto B_p) =_{def} (\Box A_p \} B_p) \wedge Tautological(A_p \to B_p)$

*Same structure of constituents:* $A_p \} \{ B_p =_{def} (A_p \} B_p) \wedge (B_p \} A_p))$

*Propositional identity:* $(A_p = B_p) =_{def} (A_p \mapsto B_p) \wedge (B_p \mapsto A_p)$

*Identity of individual concepts:* $(a = b) =_{def} [a =_2 b] = [a =_2 a]$

*Identical attributes:* $R_n = R'_n =_{def} [R_n t_1 \ldots t_n] \} [R'_n t_1 \ldots t_n]$

$Present_\theta a A_p =_{def} \neg(Past_\theta a A_p \vee Future_\theta a A_p)$

$Falsehood_\Sigma a A_p =_{def} Truth_\Sigma a \neg A_p$

$\Diamond_\Sigma a A_p =_{def} Truth_\Sigma a \Diamond A_p$ and $\neg \Box_\Sigma a A_p =_{def} Truth_\Sigma a \neg \Box A_p$

*The primitive mode of belief:* $Belief =_{def} [1_{\bar{\omega}}, 1_\theta, 1_\Sigma, Bel]$

*The primitive mode of desire:* $Desire =_{def} [1_{\bar{\omega}}, 1_\theta, 1_\Sigma, Des]$

*The operation of imposing a cognitive or volitive way to a psychological mode:* $[B_{\bar{\omega}}][A_{\bar{\omega}}, A_\theta, A_\Sigma, \vartheta] =_{def} [(B_{\bar{\omega}} \cap A_{\bar{\omega}}), A_\theta, A_\Sigma, \vartheta]$ *where $\vartheta$ is $Bel$ or $Des$.*

*The operation of adding a new propositional content condition to a psychological mode:* $[B_\theta][A_{\bar{\omega}}, A_\theta, A_\Sigma, \vartheta] =_{def} [A_{\bar{\omega}}, (B_\theta \cap A_\theta), A_\Sigma, \vartheta]$

*The operation of adding a new preparatory condition to a psychological mode:* $[B_\Sigma][A_{\bar{\omega}}, A_\theta, A_\Sigma, \vartheta] =_{def} [A_{\bar{\omega}}, A_\theta, (B_\Sigma \cup A_\Sigma), \vartheta]$

*The cognitive mode of knowledge:* $Knowledge =_{def} [K_{\bar{\omega}}, 1_\theta, Truth_\Sigma, Bel]$

*The volitive mode of will:* $Will =_{def} [(Want_{\bar{\omega}}, Present_\theta \cup Future_\theta, \neg \Box_\Sigma \cup \Diamond_\Sigma, Des]$

*The volitive mode of intention:* $Intention =_{def} [Int_{\bar{\omega}}][Int_\theta]Will$

*The volitive mode of prior intention:* $Priorintention =_{def} [Future_\theta] Intention$

*Components of a psychological mode* $A_M(A_\varsigma) =_{def} [A_\varsigma]A_M = A_M$

$A_M(A_\varsigma)$ means that $A_\varsigma$ is a component of mode $A_M$.

*The null direction of fit:* $\varnothing(A_M) =_{def} A_M(Des) \wedge ((A_M(Truth_\Sigma) \vee (A_M(Falsehood_\Sigma)))$

*Belief attribution:* $[BelaA_p] =_{def} [aBel1_{\bar{\omega}}A_p)]$

*Desire attribution:* $DesaA_p =_{def} [aDes1_{\bar{\omega}}A_p)]$

$[BelaA_p]$ expresses the proposition that agent a believes $A_p$ and $DesaA_p$ that a desires $A_p$.

*Conditions of possession of propositional attitudes*

$[aHas([A_{\bar{\omega}}, A_\theta, A_\Sigma, Bel]A_p)] =_{def} ([BelaA_{\bar{\omega}}A_p] \wedge (A_\theta aA_p) \wedge [BelaA_\Sigma a A_p] \wedge [>> A_\Sigma aA_p]$

$[aHas([A_{\bar{\omega}}, A_\theta, A_\Sigma, Des]A_p)] =_{def} ([DesaA_{\bar{\omega}}A_p] \wedge (A_\theta aA_p) \wedge [BelaA_\Sigma a A_p] \wedge [>> aA_\Sigma aA_p] \wedge [Bela\neg TautologicalA_p])$

$[aHas(A_M A_p)]$ means that agent $a$ has the attitude with the mode $A_M$ and the content $A_p$.

*Intention:* $aIntendsA_p =_{def} [aHas(IntentionA_p)]$

*Knowledge:* $KaA_p =_{def} [aHas(KnowledgeA_p)]$

*Strong psychological commitment for an agent:* $\{(A_M A_p), (B_M B_p)\} \blacktriangleright a(C_M C_p) =_{def} \Box(([aHas(A_M A_p)] \wedge [aHas(B_M B_p)]) \Rightarrow [aHas(C_M C_p)])$

*Strong psychological commitment for all agents:* $\{(A_M A_p), (B_M B_p) \blacktriangleright (C_M C_p) =_{def} Tautological(A_M A_p), (B_M B_p) \blacktriangleright a(C_M C_p)$

*Psychological necessity* $\boxdot aA_p =_{def} \neg \diamondsuit a\neg A_p$

*Weak psychological commitment for an agent:* $a \triangleright (A_M A_p) =_{def} \boxdot a \diamondsuit a [aHas(A_M A_p)]$

$(A_M A_p) \triangleright a(B_M B_p) =_{def} \Box((aHasA_M A_p) \Rightarrow a \triangleright (B_\tau B_p))$

*Weak psychological commitment for all:* $(A_M A_p) \triangleright (B_M B_p) =_{def} Tautological(A_M A_p) \triangleright a(B_M B_p)$

*Identical attitudes* $A_M A_p = B_M B_p =_{def} (A_p = B_p) \wedge (A_M A_p \blacktriangleright B_M B_p) \wedge (B_M B_p \blacktriangleright A_M A_p)$

*Satisfaction conditions of propositional attitudes:*

$Satisfied(A_M aA_p) =_{def} [aHasA_M A_p]] \wedge \neg\varnothing(A_M) \wedge ActuallyA_p \wedge (A_M$

$(Want_{\bar{\omega}}) \Rightarrow [\rho(ActuallyA_p)[aHasA_MA_p]])$. $Satisfied(A_MaA_p)$ means that the attitude of mode $A_M$ and propositional content $A_p$ of agent $a$ is (or will be) satisfied.

## 3. THE STRUCTURE OF A SEMANTIC INTERPRETATION

On the basis of previous philosophical considerations on the nature and felicity conditions of propositional attitudes, I have developed the following model-theoretical semantics in order to state valid laws of my logic. A *standard possible interpretation* or *model* for $\mathscr{L}$ is a structure $\mathcal{M} = < Time, Individuals, Agents, Concepts, Attributes, Val, Context,$ $\mathcal{U}, Belief, Knowledge, Desire, Intention, Granted, Goal, Reason, \|\|\| >$ where $Time, Individuals, Agents, Concepts, Attributes, Val$ and *Context* are disjoint non-empty sets and $Belief, Desire, Intention, Want,$ $Reason, Val, \mathcal{U}, Granted, Goal$ and $\|\|\|$ are functions which satisfy the following meaning postulates.

(1) $Time$ is a non-empty set of moments $m, m', m'', \ldots$ which represent *complete possible states* of the world at certain instants. There is a partial order $\leq$ on the set $Time$ representing the temporal relation of *anteriority* or *identity*. When $m < m'$, moment $m$ is in the past of moment $m'$ and moment $m'$ is in the future of possibilities of $m$. A moment is *final* when no moment is posterior to it. Because of indeterminism, the future can be open. But the past is unique. Thus there is *historical connection* and there is *no backward branching*. Consequently, the set $Time$ of all moments of time is a tree-like frame. A maximal chain h of moments of $Time$ is called a *history*. It represents a *possible course of history* of our world. Let $History$ be the set of all histories. Among all histories to which belongs any non final moment there is one that will be *the continuation* of that moment. Any moment $m$ has then a *proper history* $h_m$ in each model. When $m$ is the last moment

of a single history, that history is $h_m$. Since the proper history of any moment is unique, when moment $m$ continues, all moments $m' \in h_m$ have the same real historic continuation: $h_{m'} = h_m$. Two histories $h$ and $h'$ are *undivided* at a moment $m$ when they have the same present and past at these moments. *Alternative* moments have the same past. *Instant* is a partition of the set $Time$ that satisfies *unique intersection* and *order preservation*. Its members represent *instants*. Two moments of time $m$ and $m'$ are *coinstantaneous* (in symbols: $m \oplus m'$) when they belong to the same instant.

(2) *Circumstance* is the subset of the Cartesian product $Time \times History$ that contains all pairs of the form $m/h$ such that $m \in h$. Its elements are called *possible circumstances*. Circumstances are *coinstantaneous* when their moments are coinstantaneous.

(3) *Individuals* is a set of *individual objects* that contains a non empty subset *Agents* of agents. $Individuals_m$ is the set of individuals existing at moment $m$ according to $\mathcal{M}$.

(4) *Concepts* is the set of *individual concepts* and *Attributes* is the set of *attributes* of individuals under concepts considered in model $\mathcal{M}$. For each number $n$, $Attributes(n)$ is the subset of $Attributes$ containing all *relations* of degree $n$. The set $Attributes$ contains the binary relation of identity $Identity$ and is closed under logical operations. See Bealer (1982) for operations on attributes.

(5) *Context* is an arbitrary non-empty set of *possible contexts of utterance* considered in model $\mathcal{M}$. Each context $c$ has distinctive features: a *speaker* $a_c$, a *hearer* $b_c$ who are agents ($a_c$ and $b_c \in Agent$) and a *moment of utterance* $m_c \in Time$. By definition, the history $h_c$ of context $c$ is the historic continuation $h_m^c$ of its moment $m_c$ and the pair $m_c/h_c$ is the *proper circumstance* of that context. Any speaker exists at the moment of his utterance. Thus $a_c \in Individuals_m^c$.

(6) The set $Val$ is a proper subset of $((Concepts \times Circumstances)$

$\rightarrow (Individuals \cup \{\varnothing\})) \cup \bigcup_{n}((Attributes(n) \times Circumstances)$

$\rightarrow \mathcal{P}(Concepts^n))$. $Val$ contains all possible *denotation assignments to propositional constituents* of the model $\mathcal{M}$. For any individual concept $c_e$ and possible circumstance $m/h$, $val(c_e, m/h) \in Individuals$ when individual concept $c_e$ has a denotation in the circumstance $m/h$ according to assignment $val$. Otherwise $val(c_e, m/h) = \varnothing$. Because many attributes are intensional, attribute denotations are sequences of individual concepts rather than sequences of individuals (see Lewis (1972)). For any attribute of degree $n$, $val(R_n, m/h) \in \mathcal{P}(Concepts^n)$; $val(R_n, m/h)$ is the set of $n$-uples of individual concepts to which the attribute $R_n$ applies in circumstance $m/h$ according to $val$. As usual, an attribute $R_0$ of degree 0 holds in circumstance $m/h$ according to a denotation assignment $val$ when $val(R_n, m/h)$ is the set containing the empty sequence.

The set $Val$ contains a *real valuation* $val\mathcal{M}$ that assigns to concepts and attributes their *actual denotation* in each possible circumstance according to model $\mathcal{M}$. Moreover, there corresponds to each agent $a$, moment $m$ and assignment $val$ a particular set $val(a, m)$ containing all concepts and attributes that the agent *a has in mind* at that moment according to that assignment. Of course, $val(a, m) = \varnothing$ when $a \notin Agents \cap Individuals_m$. Any non empty set $val(a, m)$ contains the rigid individual concept $c_a$ whose denotation is agent $a$ in any possible circumstance for any denotation assignment. For any agent who thinks or acts has a concept of him or herself. Possible denotation assignments respect the nature of the identity relation and of logical operations on attributes like reflexivization and necessitation. By definition, $< c_e^1, c_e^2 > \in val(Identity, m/h)$ iff $val(c_e^1, m/h) = val(c_e^2, m/h)$. Two alternative moments $m$ and $m'$ are *psychologically compatible as regards an agent* according to a denotation assignment $val$ (in symbols $m' \in Compatible_m^a(val)$) when that agent could then possess according

to that assignment all the attitudes that he or she has at both moments. The relation of psychological compatibility between moments is reflexive and symmetric for each agent. In order that a moment $m'$ be compatible with all the attitudes of an agent $a$ at another moment $m$, both must belong to histories with the same past. Every agent persists in the world. What an agent thinks or attempts at each moment depends on how the world has been up to that moment. The possible causes and effects so to speak of the attitudes and actions of an agent at a moment are limited to those which are possible outcomes of the way the world has been up to that moment. This is why the relation of compatibility with attitudes has to satisfy Belnap's *historical relevance condition.*

(7) $Belief, Knowledge, Desire, Intention, Want$, and $Granted$ are functions from $Agents \times Moments \times Val$ into $\mathcal{P}(Val)$ that determine agents' attitudes in model $\mathcal{M}$.

By definition, $Belief_m^a(val)$ is the set of possible denotation assignments that are compatible in model $\mathcal{M}$ with the *truth* of all beliefs that the agent $a$ possesses at the moment $m$ according to $val$. In particular, $Belief_m^a(val\mathcal{M})$ is the set of possible denotation assignments that are compatible with the *truth* of all beliefs that agent $a$ has at moment $m$ according to model $\mathcal{M}$. Similarly, $Desire_m^a(val)$ is the set of possible denotation assignments that are compatible in model $\mathcal{M}$ with the *realization* of all desires that agent $a$ possesses at the moment $m$ according to $val$. An agent has no attitude at a moment when he or she has no attribute or individual concept in mind at that moment. Thus $Belief_m^a(val)$ and $Desire_m^a(val)$ are the whole set $Val$ when $val(a, m) = \varnothing$. Otherwise, they are non empty proper subsets of $Val$. Agents are competent. Whenever an agent has in mind attributes and concepts, *some* possible denotation assignments to these senses are compatible with what that agent then believes or desires. Moreover, that agent has then in mind all these senses in compatible assignments.

If $val'' \in Belief_m^a(val) \cup Desire_m^a$ then $val(a, m) \subseteq val'(a, m)$. Agents can have desires that they do not desire to have. But they cannot have a belief without believing that they have it. The relation of compatibility with the truth of beliefs is then transitive. If $val' \in Belief_m^a(val)$ and $val'' \in Belief_m^a(val')$ then $val'' \in Belief_m^a(val)$.

*Knowledge* represents the *cognitive way proper to knowledge*; *Want* and *Intention* the *volitive ways proper to will and intention. Knowledge* and *Want* are functions from $Agents \times Moments \times Val$ into $\mathcal{P}(Val)$ that restrict respectively *Belief* and *Desire*. Agents can have false beliefs but they can only know true propositions. The cognitive mode of knowledge is then *reflexive: $val \in Knowledge_m^a(val)$*. Whoever intends something wants that thing. Thus $Intention_m^a(val) \subseteq Want_m^a(val)$. Like the relation of cognitive compatibility corresponding to beliefs, characteristic cognitive and volitive ways of knowledge and intention are *transitive*. Agents can have desires that they do not desire to have. But they cannot know something without knowing that they know it. And they cannot execute an intention without intending to execute that intention. Thus if $val' \in Knowledge_m^a(val)$ and $val'' \in Belief_m^a(val')$ then $val'' \in Belief_m^a(val)$. And similarly for $Intention_m^a(val)$. For each agent $a$, moment $m$ and valuation $val$, $Granted_m^a(val)$ is the non empty set of all possible denotation assignments that are compatible with the truth of all propositions that $a$ *presupposes* at moment $m$ according to $val$. $Granted_m^a$ is transitive.

(8) The set *Predications* is a subset of $\mathcal{P}(Attributes \cup Concepts)$ that contains all sets of propositional constituents with which one can make a *predication $\mathscr{L}$*. Such sets are of the form $\{R_n, c_e^1, \ldots, c_e^k\}$; they contain a single attribute $R_n$ of degree $n$ and a positive number $k$ ($k \leq n$) of individual concepts $c_e^1, \ldots, c_e^k$ when $n$ is positive. Unlike truth functions, modal, temporal and attitude propositional operations serve to make predications of new *modal, temporal, epistemic and volitive attributes*. In thinking that it is impossible that God makes mistakes we predicate

of Him the modal property of infallibility. Whoever thinks that God created the world predicates of Him the past property of having created the world. Whoever thinks that the pope believes that God exists predicates of God the epistemic property of being existent according to the pope. Whoever wishes that God forgives him predicates of God the property that he would prefer God's pardon.

The power set $\mathcal{P} Predications$ is then closed in each model under union, a modal and temporal unary operation $^*$ and, for each individual concept $c_e$, a unary psychological and praxeological operation $\otimes c_e$. For any set $\Gamma \subseteq Predications$, $\cup^* \Gamma$ contains all modal and temporal attributes that one can form with attributes of $\cup \Gamma$. Similarly $\cup \otimes c_e \Gamma$ contains epistemic and volitive properties corresponding to beliefs or desires that the agent falling under concept $c_e$ could form with attributes of $\cup \Gamma$. As usual, modal and temporal operation $^*$ and praxeological operations of the form $\otimes c_e$ obey the following postulates: $\Gamma \subseteq^* \Gamma$ and $^* \Gamma \subseteq \otimes c_e \Gamma.^*(\Gamma_1 \cup \Gamma_2) =^* \Gamma_1 \cup^* \Gamma_2$ and $^{**}\Gamma =^* \Gamma$. Similarly, $\otimes c_e(\Gamma_1 \cup \Gamma_2) = \otimes c_e \Gamma_1 \cup \otimes c_e \Gamma_2$ and $\otimes c_e \otimes c_e \Gamma = \otimes_{c^e} \Gamma$. Whoever has in mind an individual concept can attribute attitudes to the individual to which that concept applies. Consequently, when $\cup \Gamma \subseteq val(a, m)$ and $c_e \in val(a, m)$, $\cup \otimes c_e \Gamma \subseteq val(a, m)$.

(9) $\mathcal{U}$ is a function that associates with each type $\alpha$ the set $\mathcal{U}_\alpha$ of all entities that are possible semantic values of formulas of that type in model $\mathcal{M}$. By definition, (i) $U_{\#e} = Concepts$, (ii) $U_r = Attributes$. (iii) $U_p \subseteq \mathcal{P}[Predications] \times (Circumstances \rightarrow \mathcal{P}(Val))$. (iv) $U_{\bar\omega} \subseteq (Agents \times Times \times Val) \rightarrow \mathcal{P} Val$. (v) $U_\theta \subseteq (Agents \times Times) \rightarrow \mathcal{P}(U_p))$ and (vi) $U_\Sigma \subseteq (Agents \times Times \times Up) \rightarrow \mathcal{P} U_p$. The union of all such sets $U_\alpha$ is the *domain* of the possible interpretation $\mathcal{M}$.

$U_p$ is the set of *first order propositions* of model $\mathcal{M}$. When an attribute is predicated of $n$ individuals under concepts in a proposition $P$, the set containing that attribute and these $n$ concepts belongs to the first term $id_1 P$ of that proposition. Thus *the set of all propositional*

*constituents* of a proposition $P$ is the union $\cup id_1 P$. The second term $id_2 P$ of proposition $P$ is the family of all sets of possible denotation assignments that are compatible with its truth in each circumstance. Thus proposition $P$ is *true in circumstance $m/h$ according to a possible denotation assignment val* iff $val \in id_2 P(m/h)$.

(10) *Goal* is a function from $Agent \times Time \times Val$ into $\mathcal{P}U_p$ that serves to determine *propositional content conditions of agents' intentions* in model $\mathcal{M}$. $Goal_m^a(val)$ is the set of propositions representing *objectives* that agent $a$ *has* at moment $m$ according to denotation assignment $val$. Thus $\bigcup_{val \in Val} Goal_m^a(val)$ contains propositions representing *objectives* that agent $a$ *could have* at moment $m$. *Goal* satisfies the following postulates:

- *Any agent intends to carry it out and can express his or her goals.* Thus if $P \in Goal_m^a(val))$ then $\cup id_1 P \subseteq val(a, m)$ and $Intention_m^a(val) \subseteq id_2 P(m/h_m)$.

- *Agents have a free will.* $P \in Goal_m^a(val)$ only if $P \notin Goal_{m'}^a(val)$ for at least one moment $m'$ coinstantaneous with moment $m$.

- *Many individual intentions and attempts are personal.* An agent can only intend *that he or she do something.* So when $a \neq b, Goal_m^a(val) \neq Goal_m^b(val)$.

- *Agents' intentions are directed towards the present or the future.* When $P \in Goal_m^a(val)$ there is no proposition $Q$ such that $id_2 P(m^*/h) = \bigcap id_2 Q(m'/h)$ for moments $m'$ anterior to $m$.

- *Agents always believe that they can reach their objectives.* So they do not intend to do things that they believe to be necessary or impossible. When $P \in Goal_m^a(val)$ and $val' \in Belief_m^a(val)$ for one but not all circumstances $m'/h'$ coinstantaneous with $m, val' \in id_2 P(m'/h')$.

(11) *Reason* is a function from $Times \times U_p \times Val$ into $\mathcal{P}(U_p)$. The set $Reason(m, P, val)$ contain all propositions that represent reasons why proposition $P$ is true at moment $m$ according to assignment

*val*. In my view, any reason for $P$ is *a cause* of the fact that represents. Causes precede or are simultaneous with their effects. There are *natural causes*. Some facts are effects of other facts given laws of nature. There are also *intensional causes*. Some facts turn to be existent because of attitudes or actions of agents. *Agents have attitudes and make attempts for practical reasons*. They have intentions because they have desires and beliefs about how to satisfy these desires. They act in order to keep previous commitments. Sometimes there is a causal over-determination. They think or do something for different reasons. By definition, causes produce their effect: when a proposition $Q \in Reason(m, P, val)$, $val \in id_2P(m/h_m)$ and $val \in id_2Q(m/h_m)$. Moreover the same causes produce the same effects at alternative moments which are psychologically compatible as regards an agent. Consequently when $Q \in Reason(m, P, val)$, at all moments $m' \in Compatible_m^a(val)$) if $val \in id_2Q(m'/h_{m'})$ then $val \in id_2P(m'/h_{m'})$.

(12) Finally, $\|\|$ is a function that associates with each term $A$ of type $\alpha$ and possible context of utterance $c$ the entity $\|A\|_c$ of $U_\alpha$ that $A$ expresses in that context according to the possible interpretation $\mathcal{M}$.

The evaluation function $\|\|$ satisfies the following clauses:

- For any constant $A$ of $\mathscr{L}$ of type $\alpha$ $\|A\|_c \in U_\alpha$. Thus if t is an individual constant, $\|t\|_c \in Concepts$ and for any predicate $R_n$ of degree $n$, $\|R_n\|_c \in Attributes(n)$.

- In particular,$\| =_2 \|_c = Identity$; $\|Bel\|_c = Belief$ and $\|Des\|_c = Desire$.

- $\|Want_{\bar\omega}\|_c = Want$, $\|Int_{\bar\omega}\|_c = Intention$, $\|K_{\bar\omega}\|_c = Knowledge$ and $\|Int_\theta\|_c(a, m) = Goal_m^a$.

- $\|1_{\bar\omega}\|_c(a, m, val) = Belief_m^a(val) \cup Desire_m^a(val)$ and $\|1_\theta\|_c(a, m) = U_p$ and $\|1_\Sigma\|_c(a, m, P) = \varnothing$.

- $\|A_\theta \cap B_\theta\|_c(a, m) = \|A_\theta\|_c(a, m) \cap \|B_\theta\|_c(a, m)$. And similarly for $\cup$.

- $\|A_\Sigma \cup B_\Sigma\|_c(a,m,P) = \|A_\Sigma\|_c(a,m,P) \cup \|B_\Sigma\|_c(a,m,P)$.  And similarly for $\cap$.

- $\|A_{\bar\omega} \cap B_{\bar\omega}\|_c(a,m,val) = \|A_{\bar\omega}\|_c(a,m,val) \cap \|B_{\bar\omega}\|_c(a,m,val)$.  And similarly for $\cup$.

- $\|Past_\theta\|_c(a,m) = \{P \in U_p /$ for some proposition $Q$ and for some moments $m'$ anterior to $m$, $id_2 P(m^*/h) = \bigcap id_2 Q(m'/h)\}$.

- $\|Future_\theta\|_c(a,m) = \{P \in U_p /$ for some proposition $Q$ and for some moments $m'$ posterior to $m, id_2 P(m^*/h) = \bigcap id_2 Q(m'/h)\}$.

- $\|[A_{\bar\omega}, A_\theta, A_\Sigma, Bel]\|_c$ is the function $h$ from $Agent \times Times \times Val$ into $\mathcal{P}U_p$ such that $P \in h(a,m,val)$ iff $(\|A_{\bar\omega}\|_c(a,m,val) \cap Belief_m^a(val)) \subseteq id_2 P(m/h_m)$, $P \in \|A_\theta\|_c(a,m)$, $\cup id_1(P) \subseteq val(a,m)$ and, for all $Q \in \|A_\Sigma\|_c(a,m,P)$, $\cup id_1(Q) \subseteq val(a,m)$ and $Belief_m^a(val) \cup Granted_m^a(val) \subseteq id_2 Q(m/h_m)$. Similarly for $\|[A_\mu, A_\theta, A_\Sigma, Des]\|_c$ with $Desire_m^a(val)$ instead of $Belief_m^a(val)$ and the additional condition that for at least one $val' \in Belief_m^a(val)), val' \notin id_2\|P\|(m/h_m)$.

- $id_1\|[R_n t_1 \ldots t_n]\|_c = \{\{\|R_n\|_c, \|t_1\|_{c,\ldots}, \|t_n\|_c\}\}$ and $val \in id_2\|[R_n t_1 \ldots t_n]\|_c(m/h)$ iff $< \|t_1\|_{c,\ldots}, \|t_n\|_c)) \in val(\|R_n\|_c, (m/h))\}$.

- $id_1\|\neg A_p\|_c = id_1\|A_p\|_c$ and $val \in id_2\|\neg A_p\|_c(m/h)$ iff $val \notin id_2\|A_p\|_c(m/h)$.

- $id_1\|Settled A_p\|_c =^* id_1\|A_p\|_c$ and $id_2\|Settled A_p\|_c(m/h) =$
$\bigcap\limits_{m \in h'} id_2\|A_p\|_c(m/h')$.

- $id_1\|Actually A_p\|_c =^* id_1\|A_p\|_c$ and $id_2\|Actually A_p\|_c(m/h) = id_2\|A_p\|_c(m_c/h_c)$.

- $\|Truth_\Sigma\|_c(a,m,\|A_p\|_c) = \{\|Actually A_p\|_c\}$

- $id_1\|\Box A_p\|_c =^* id_1\|A_p\|_c$ and $id_2\|\Box A_p\|_c^\sigma(m/h) =$
$\bigcap\limits_{m' \approx m} \bigcap\limits_{m' \in h'} id_2\|A_p\|_c(m'/h')$.

- $id_1\|TautologicalA_p\|_c =^* id_1\|A_p\|_c$ and $id_2\|TautologicalA_p\|_c$ $(m/h) = Val$ when $\bigcap\limits_{m'/h'} id_2\|A_p\|_c(m'/h')) = Val$. Otherwise, $id_2\|TautologicalA_p\|_c(m/h) = \varnothing$.

- $id_1\|WillA_p\|_c =^* id_1\|A_p\|_c$ and $id_2\|WillA_p\|_c(m/h) =$ $\bigcup\limits_{m'>m} id_2\|A_p\|_c(m'/h)$.

- $id_1\|WasA_p\|_c =^* id_1\|A_p\|_c$ and $id_2\|WasA_p\|_c(m/h) =$ $\bigcup\limits_{m'<m} id_2\|A_p\|_c(m'/h)$.

- $id_1\|\diamond a A_p\|_c = \otimes_{\|a\|_c} id_1\|A_p\|_c$ and $val \in id_2\|\diamond a A_p\|_c(m/h)$ if and only if, for some $m' \in Compatible_m^a(val)$ and for some $h'$, $val \in id_2\|A_p\|_c(m'/h')$.

- $id_1(\|B_p \wedge C_p\|_c) = id_1(\|B_p\|_c) \cup id_1(\|C_p\|_c); id_2\|B_p \wedge C_p\|_c(m/h) =$ $id_2\|B_p\|_c(m/h) \cap id_2\|C_p\|_c(m/h)$.

- $id_1(\|B_p\}C_p\|_c) =^* (id_1(\|B_p\|_c) \cup id_1(\|C_p\|_c))$ and $id_2\|B_p\}C_p\|_c$ $(m/h) = Val$ when $id_1\|B_p\|_c \supseteq id_1\|C_p\|_c$. Otherwise, $id_2\|B_p\}C_p\|_c$ $(m/h) = \varnothing$.

- $id_1\|[aBelA_{\bar{\omega}}A_p)]\|_c = \otimes_{\|a\|_c} id_1\|A_p\|_c$ and $val \in id_2\|[aBelA_{\bar{\omega}}A_p)]\|_c(m/h)$ iff $\|A_p\|_c \in \|[A_{\bar{\omega}}, 1_\theta, 1_\Sigma, Bel]\|_c(val$ $(\|a\|_c, m_c/h_c), m, val)$. And similarly for $\|[aDesA_{\bar{\omega}}A_p)]\|_c$ with $Des$ instead of $Bel$.

- $id_1\|[\rho B_p C_p]\|_c = (id_1\|B_p\|_c \cup id_1\|C_p\|_c)$ and $val \in id_2\|[\rho B_p C_p]\|_c$ $(m/h)$ iff $\|ActuallyC_p\|_c \in Reason(m, \|B_p\|_c, val)\}$.

- $id_1\|A_\theta a A_p\|_c = \otimes_{\|a\|_c} id_1\|A_p\|_c$ and $val \in id_2\|A_\theta a A_p\|_c(m, h)$ iff $\|A_p\|_c \in \|A_\theta\|(val\|a\|_c(m_c/h_c), m)\}$.

- $id_1\|(A_\Sigma a A_p)\|_c$ is $\otimes_{\|a\|_c} id_1\|A_p\|_c$ and $id_2\|(A_\Sigma a A_p)\|_c(m/h)$ is the intersection of all sets $id_2 P(m/h)$ for all propositions $P \in \|A_\Sigma\|_c(val(\|a\|_c, m_c/h_c), m, \|A_p\|_c)$.

- $id_1\|[>> a A_p]\|_c = \otimes_{\|a\|_c} id_1\|A_p\|_c$ and $val \in id_2\|[>> a A_p]\|_c(m/h)$ iff $\cup id_1(\|A_p\|_c) \subseteq val((val\|a\|_c(m_c/h_c), m)))$ and $Granted_m^a(val) \subseteq$ $id_2\|A_p\|_c(m/h_m)$ where $a = val\|a\|_c(m/h_m)$.

- $\|(A_M A_p)\|_c = < \|A_p\|_c, \{(a, m, val)/\|A_p\|_c \in \|A_M\|_c(a, m, val)\}$.

A propositional formula $A_p$ of $\mathscr{L}$ is *true* in a possible circumstance $m/h$ according to a standard model $\mathcal{M}$ when $\|A_p\|$ is true in $m/h$ according to $val\mathcal{M}$. $A_p$ is *valid* (in symbols: $\models A_p$) when it is true in all possible circumstances according to all standard models.

## 4. VALID LAWS

Here is a list of fundamental valid laws of my logic of propositional attitudes. As one can expect, all the instances in language $\mathscr{L}$ of classical axiom schemas of truth functional logic, $S5$ modal logic for settled truth, historic and universal necessities, Brouwer's modal logic for psychological necessity and branching temporal logic for past and future are valid formulas.

$S5$ **Modal logic for settled truth, historic and logical necessity**

$(S1) \models (Settled A_p \Rightarrow A_p)$
$(S2) \models (Settled(A_p \Rightarrow B_p) \Rightarrow (Settled A_p \Rightarrow Settled B_p))$
$(S3) \models (\neg Settled\neg A_p \Rightarrow Settled\neg Settled\neg A_p)$ and similarly for $\Box$ and $\blacksquare$.

Notice that $\not\models Settled A_p \Rightarrow \Box A_p$ and $\not\models \blacksquare A_p \Rightarrow Tautological A_p$.

**Brouwer's modal logic for psychological necessity**

$(\Diamond 1) \models (A_p \Rightarrow \Diamond a A_p)$ $(\Diamond 2) \models (\Box a(A_p \Rightarrow B_p) \Rightarrow (\Box a A_p \Rightarrow \Box a B_p))$
$(\Diamond 3) \models (A_p \Rightarrow \Box a \Diamond a A_p)$

**Valid schemas for tautologies**

$(T1) \models Tautological A_p \Rightarrow \blacksquare A_p$. However $\not\models \blacksquare A_p \Rightarrow Tautological A_p$.
$(T2) \models Tautological A_p \Rightarrow Tautological Tautological A_p$
$(T3) \models \neg Tautological A_p \Rightarrow Tautological\neg Tautological A_p$.

$(T4) \models Tautological A_p \Rightarrow (Tautological(A_p \Rightarrow B_p) \Rightarrow Tautological B_p)$.

$(T5) \models Tautological B_p \Rightarrow (Tautological(A_p \Rightarrow B_p) \Rightarrow Tautological A_p)$.

$(T6-7) \models (A_p\}B_p) \Rightarrow Tautological(A_p\}B_p) \models \neg(A_p\}B_p) \Rightarrow Tautological\neg(A_p\}B_p)$.

**Valid schemas for propositional identity**

$(I1) \models ([R_n t_1 \ldots t_n] = [R_n^k d_1 \ldots d_n]) \Leftrightarrow (((\hat{R}_n = \hat{R}_n^k)) \wedge ((\hat{t}_1 = \hat{d}_1) \vee \ldots \vee (\hat{t}_1 = \hat{d}_n)) \wedge \ldots \wedge (\hat{t}_n = \hat{d}_1) \vee \ldots \vee (\hat{t}_n = \hat{d}_n))) \wedge \ldots \wedge ((\hat{d}_1 = \hat{t}_1) \vee \ldots \vee (\hat{d}_1 = \hat{t}_n)) \wedge \ldots \wedge (\hat{d}_n = \hat{t}_1) \vee \ldots \vee (\hat{d}_n = \hat{t}_n))) \wedge Tautological((R_n t_1 \ldots t_n) \Leftrightarrow (R_n^k d_1 \ldots d_n))$

$(I2) \models A_p = A_p$

$(I3) \models (A_p = B_p) \Rightarrow (C \Rightarrow C^*)$ where $C^*$ and $C$ are propositional formulas which differ at most by the fact that an occurrence of $B_p$ in $C^*$ replaces an occurrence of $A_p$ in $C$.

$(I4) \models (A_p = B_p) \Rightarrow Tautological(A_p = B_p)$

$(I5) \models \neg(A_p = B_p) \Rightarrow Tautological\neg(A_p = B_p)$

**Valid schemas for propositional composition**

$(C0) \models ([R_n t_1, \ldots, t_n]\}A_p) \Rightarrow (A_p = [R_n t_1, \ldots, t_n])$

$(C1) \models A_p\}A_p$

$(C2) \models (A_p\}B_p) \Rightarrow ((B_p\}C_p) \Rightarrow (A_p\}Cp))$

$(C3) \models (A_p \wedge B_p)\}A_p$

$(C4) \models (A_p \wedge B_p)\}B_p$

$(C5) \models ((C_p\}A_p) \wedge (C_p\}B_p)) \Rightarrow C_p\}(A_p \wedge B_p)$

$(C6) \models A_p\}\{\neg A_p$

$(C7) \models \Box A_p\}\{Tautological A_p$ and similarly for $Will, Was,$ *Actually* and other modal and temporal connectives.

$(C8) \models (A_p\}B_p) \equiv \Box(A_p \wedge B_p)$

$(C9) \models \Box A_p\}A_p$

$(C10) \models [aHas(A_M A_p)]\}\Box A_p$

$(C11) \models \Box\neg A_p\}\{\Box A_p$ and similarly for $[aHas(A_M A_p)]$

$(C12) \models \Box(A_p \wedge B_p)\}\{(\Box A_p \wedge \Box B_p)$ and similarly for $[aBelA_{\bar{\omega}}A_p)]$ and $[aDesA_{\bar{\omega}}A_p)]$.

$(C13) \models \Box\Box A_p\}\{\Box A_p$ and similarly for $[aBelA_{\bar{\omega}}A_p)]$ and $[aDesA_{\bar{\omega}}A_p)]$.

$(C14) \models [aBelA_{\bar{\omega}}A_p)]\}\{[aDesA_{\bar{\omega}}A_p)]$

### Branching time logic

$(TL1) \models (Will-always(A_p \Rightarrow B_p) \Rightarrow (Will-alwaysA_p \Rightarrow Will-alwaysB_p))$ where $Will - alwaysA_p =_{def} \neg Will\neg A_p$.

$(TL2) \models (Was-always(A_p \Rightarrow B_p) \Rightarrow (Was-alwaysA_p \Rightarrow Was-alwaysB_p))$

$(TL3) \models (A_p \Rightarrow Was - alwaysWillA_p)$

$(TL4) \models (A_p \Rightarrow Will - alwaysWasA_p)$

$(TL7) \models (WasA_p \Rightarrow Will - alwaysWasA_p)$

$(TL8) \models (WillA_p \Rightarrow Will - always(WillA_p \vee A_p \vee WasA_p)$

$(TL9) \models (WasA_p \Rightarrow Was - always(WillA_p \vee A_p \vee WasA_p)$

### Historic modality with time

$(MT1) \models (Was((A_p \wedge Will - alwaysB_p) \wedge Was - always\neg(B_p \wedge \Diamond C_p)) \Rightarrow ((Will - alwaysD_p \wedge WasC_p \Rightarrow Was(A_p \wedge (C_p \vee WasC_p) \wedge Will - always(C_p \Rightarrow Will - alwaysD_p)))$

$(MT2) \models (Was-always(A_p \wedge Was-always\neg(B_p \wedge \Diamond C_p) \wedge Will(B_p \wedge A_p \wedge \Diamond D_p)) \wedge Was((E_p \wedge Will - alwaysB_p)) \Rightarrow ((Will - alwaysQ_p \Rightarrow Was(E_p \wedge Will - always(C_p \Rightarrow Will - always(D_p \Rightarrow Will - always Q_p))))^6$

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[6]Axioms (MT1-2) are A. Zanardo [1985]'s axioms of local correspondence.

**Indexical Temporal logic**

$(IT1) \models (Actually(ActuallyA_p \Rightarrow A_p))$

$(IT2) \models (ActuallyA_p \Rightarrow AlwaysActuallyA_p)$

$(IT3) \models (ActuallyA_p \Rightarrow SettledActuallyA_p)$

$(IT4) \models (Actually(A_p \wedge B_p) \Leftrightarrow (ActuallyA_p \wedge ActuallyB_p))$

$(IT5) \models (ActuallyActuallyA_p \Leftrightarrow ActuallyA_p)$

**Valid schemas for belief**

$(B1) \models (BelaA_p \wedge BelaB_p) \Rightarrow Bela(A_p \wedge B_p)$

$(B2) \models TautologicalA_p \Rightarrow \neg Bela\neg A_p$

$(B3) \models TautologicalA_p \Rightarrow (((BelaB_p) \wedge (B_p\}A_p) \Rightarrow$
$BelaTautologicalA_p)$

$(B4) \models BelaA_p \Rightarrow ((A_p \mapsto B_p) \Rightarrow (BelaB_p))$

$(B5) \models BelaA_p \Leftrightarrow (BelaBelaA_p)$

$(B6) \models Satisfied[BelaA_p] \Leftrightarrow ([BelaA_p] \wedge ActuallyA_p)$

Notice that beliefs are detachable: $\models BelaA_p \Rightarrow ((Bela(A_p \Rightarrow B_p)) \Rightarrow (BelaB_p))$.

But agents do not believe all universal necessities. $\not\models \blacksquare A_p \Rightarrow BelaA_p$.

They can even believe necessarily false propositions. $\not\models \neg \blacklozenge A_p \Rightarrow Bela\neg A_p$.

Inconsistent agents are paraconsistent. $\not\models \neg \blacklozenge A_p \Rightarrow (BelaA_p \Rightarrow BelaB_p)$.

Agents are also not logically omniscient. $\not\models TautologicalA_p \Rightarrow BelaA_p$.

In particular, agents do not make all tautological inferences.
$\not\models (Tautological(A_p \Rightarrow B_p)) \Rightarrow (BelaA_p \Rightarrow BelaB_p))$

For conclusions of tautological inferences can contain new predications. $\not\models (Tautological(A_p \Rightarrow B_p)) \Rightarrow (A_p\}B_p))$.

However agents are minimally consistent; they cannot believe strongly incompatible propositions. $\models Tautological(A_p \Rightarrow B_p) \Rightarrow$

$(BelA_p \!\!\!-\!\!\!\in \neg Bela\neg B_p)$

All the classical *Boolean laws of idempotence, commutativity, associativity* and *distributivity* are valid laws of propositional identity:

So $\models BelaA_p = Bela(A_p \wedge A_p)$; $\models Bela(A_p \wedge B_p) = Bela(B_p \wedge A_p)$ and $\models Bela\blacksquare(A_p \wedge B_p) = Bela(\blacksquare A_p \wedge \blacksquare B_p)$.

The classical laws of *reduction* are also valid: $\models \neg\neg A_p = A_p$ and $\models BelaBelaA_p = BelaA_p$. Unlike hyperintensional logic, predicative logic does not require that identical propositions be *intensionally isomorphic*. The order of predication does not always affect truth conditions. Similarly, the order and number of applications of propositional operations does not always affect the logical form. Clearly, $\models Bela(A_p \Leftrightarrow B_p) = Bela(B_p \Leftrightarrow A_p)$. Intensional isomorphism is too strong a criterion of propositional identity.

However, propositional identity requires more than the *co-entailment* advocated in the logic of relevance. As M. Dunn pointed out, it is unfortunate that $A_p$ and $(A_p \wedge (A_p \vee B_p))$ co-entail each other. For most formulas of such forms are not synonymous. Co-entailment is not sufficient for synonymy because it allows for the introduction of new sense. $\not\models A_p \mapsto (A_p \wedge (A_p \vee B_p))$.

For $\not\models BelaA_p \mapsto Bela(A_p \wedge (A_p \vee B_p))$ in my logic.

**Valid schemas for desire**

$(D1) \models (DesaA_p \wedge DesaB_p) \Rightarrow Desa(A_p \wedge B_p)$. But the converse is not valid.

$(D2) \models TautologicalA_p \Rightarrow \neg(DesaA_p \vee Desa\neg RA_p)$

$(D3) \models DesaA_p \Rightarrow (((A_p \mapsto B_p)\wedge\neg TautologicalA_p)) \Rightarrow (DesaB_p)))$

$(D4) \models DesaA_p \Rightarrow Bela\neg TautologicalA_p$

$(D5) \models Satisfied[DesaA_p] \Leftrightarrow ([DesaA_p] \wedge ActuallyA_p)$

Notice that unlike belief, desire is not closed under strong implication. $\not\models (A_p \mapsto B_p) \Rightarrow (DesaA_p \Rightarrow DesaB_p)$. For any desire contains a preference. Thus agents cannot desire tautological things (Axiom D2).

$\models A_p \mapsto (A_p \Rightarrow A_p)$. But $\not\models Desa A_p \Rightarrow Desa(A_p \Rightarrow A_p)$.

Unlike beliefs, desires are not detachable. $\not\models Desa A_p \Rightarrow ((Bela\blacksquare(A_p \Rightarrow B_p)) \Rightarrow (Desa B_p))$.

Agents can desire things that they believe to be impossible.
$\not\models Bela\blacksquare B_p \Rightarrow \neg Desa A_p$.

However $\models Tautological(A_p \Rightarrow B_p) \Rightarrow (Desa A_p \!-\!\!\!\in \neg Desa\neg B_p)$.

### Valid schemas for propositional attitudes

$\models [aHas([(A_{\bar{\omega}}, A_{\theta}, A_{\Sigma}, Bel]A_p)] \Leftrightarrow ([aHas([A_{\bar{\omega}}]Belief A_p)] \wedge (A_{\theta}a A_p) \wedge [Bela A_{\Sigma}a A_p] \wedge (>> a A_{\Sigma}a A_p))$

$\models [aHas([(A_{\bar{\omega}}, A_{\theta}, A_{\Sigma}, Des]A_p)] \Leftrightarrow ([aHas([A_{\bar{\omega}}]Desire A_p)] \wedge (A_{\theta}a A_p) \wedge [Bela A_{\Sigma}a A_p] \wedge (>> a A_{\Sigma}a A_p) \wedge Bela\neg Tautological A_p)$

$\models [aHas((A_M A_p] \Rightarrow Settled[aHas((A_M A_p)] \models ([A_{\varsigma}]A_M A_p) \blacktriangleright (A_M A_p)$

$\models Satisfied(A_M a A_p) \Rightarrow (\neg\varnothing(A_M) \wedge Actually A_p)$

$\models (\neg\varnothing(A_M) \wedge \neg A_M(Want_{\bar{\omega}}) \wedge [aHas(A_M A_p)]) \Rightarrow (Satisfied(A_M a A_p) \Leftrightarrow Actually A_p)$

$\models (\neg\varnothing(A_M) \wedge A_M(Want_{\bar{\omega}})) \Rightarrow (Satisfied(A_M a A_p) \Leftrightarrow [\rho(Actually A_p)[aHas(A_M A_p)]])$

### Valid schemas for knowledge

$(K1) \models [KaA_p] \Rightarrow (A_p \wedge [Bela A_p]$

$(K2) \models [KaA_p] \Rightarrow [KaKaA_p]$

$(K3) \models Tautological A_p \Rightarrow ([aHas(A_M A_p)] \Rightarrow [KaA_p])$. But
$\not\models Tautological A_p \Rightarrow KaA_p$.

$(K4) \models KaA_p \Rightarrow ((A_p \mapsto B_p) \Rightarrow (KaB_p)) \not\models (Tautological(A_p \Rightarrow B_p)) \Rightarrow (KaA_p \Rightarrow KaB_p))$

**Valid schemas for intention**

$(I1) \models Int_\theta aA_p \Rightarrow \neg(A_p = WasB_p)$

$(I2) \models Satisfied(IntaA_p) \Leftrightarrow [\rho(ActuallyA_p)[IntaA_p]]$

$(I3) \models [IntaA_p] \Rightarrow ((Bela\Diamond A_p \wedge Bela\neg\Box A_p) \wedge DesaA_p))$

## 4. PRINCIPLES OF REVISION OF ATTITUDES

Agents keep many attitudes during an interval of time. Whenever they acquire a new propositional attitude they then lose old attitudes that are incompatible with the new attitude and they acquire others whose nature can be determined with the conceptual apparatus of my logic. Minimally rational agents could not have simultaneously attitudes of certain forms at certain moments. One can introduce psychological incompatibility in my ideography as follows: $(A_M A_p) ><_a (B_M B_p) =_{def} \neg\Diamond([aHas(A_M A_p)] \wedge [aHas(B_M B_p)])$. Certain incompatible attitudes are such that no agent could possess them at any moment. In that case, $\models [aHas(A_M A_p)] \mapsto \neg[aHas(B_M B_p)])$.

My first principle of update is a principle of revision: any agent who acquires a new attitude loses *eo ipso* all previous attitudes that are incompatible with that new attitude. There are many reasons of incompatibility between attitudes. Propositional attitudes can be incompatible because they contain incompatible beliefs or desires or incompatible cognitive or volitive ways, propositional content or preparatory conditions. For example agents lose beliefs that they discover to be false and they revoke previous intentions that they discover to be impossible to execute. My logic gives a general account for psychological incompatibility. My second principle of update is an expansion principle: Whenever an agent acquires a new propositional attitude $M(P)$ he or she acquires *eo ipso* many other new attitudes $M'(P)$ with the same propositional content $P$ whose modes $M'$ can be determined given propositional content conditions that $P$ then satisfies and other

attitudes that that agent has at that moment about $P$. Let me give a few examples. Suppose that an agent who sees a lot of heavy low clouds in the sky gets to believe that it will soon rain. Because that new belief is directed towards the future, he or she then *foresees* that it will rain. For the mode of *prevision* is obtained from that of belief by adding the *special propositional content condition that the represented fact is future* with respect to the moment of the attitude. Suppose that that new belief puts then the agent in a state of expectation *(special cognitive way)*, he or she then also expects rain. Suppose moreover that that agent previously *hoped* and still desires that it would not soon rain. Because hope is a desire whose satisfaction is then possible according to the agent (special preparatory condition), the agent then loses the previous hope and feels moreover a *disappointment*. Whoever is *disappointed* with a fact desires its inexistence while believing (special preparatory conditions) that the fact exists and that he or she previously believed that it would not exist.

The expansion principle that I have just stated generates new attitudes having the same propositional content but a new mode, my second expansion principle generates new attitudes having the same mode but a new propositional content. That second expansion principle is based on the fact that most components of psychological modes $M$ are compatible with conjunction. Suppose that an agent $a$ acquiring a new attitude $M(P)$ at a moment also possesses at that moment another attitude $M(Q)$ with propositional content $Q$. Then he or she also acquires the new attitude $M(P \wedge Q)$ whose content is the conjunction of these propositions in case both attitudes $M(P)$ and $M(Q)$ strongly commit that agent to the new attitude $M(P \wedge Q)$ that is to say when $\{(A_M A_p), (A_M B_p)\} \blacktriangleright a(C_M C_p)$. This is always the case when $M$ is a primitive psychological mode. For the categories of cognition and volition are compatible with conjunction. Whoever believes (or desires) two things believes (or desires) both. Moreover many but

not all cognitive and volitive ways and preparatory and propositional content conditions are compatible with conjunction. So are for example the special volitive way of pleasures, the propositional content condition of previsions and the preparatory condition of satisfactions. Suppose that an agent who is satisfied to win a competition is also satisfied with the received price. Then that agent is satisfied with both facts. For the mode of satisfaction is obtained from that of desire by adding the preparatory condition that the desired fact exists. Whoever is pleased with something feels a satisfied desire whose satisfaction puts him or her in a state of pleasure. Clearly no agent can be pleased with one fact and with another without being pleased with both. Similarly two previsions strongly commit the agent to foreseeing their conjunction. For if two propositions are future with respect to a moment so is their conjunction. Notice that the volitive mode of agreements and the propositional content condition of approvals are not compatible with conjunction. Sometimes it is good to do one thing and good to do another thing but it is not good to do both. So one can approve one thing and approve another thing without approving both. A cognitive or volitive way $A_{\bar{\omega}}$ is *compatible with conjunction* when $\models ([aHas([A_{\bar{\omega}}]A_M A_p)] \wedge [aHas([A_{\bar{\omega}}]A_M B_p)]) \Rightarrow [aHas([A_{\bar{\omega}}]A_M(A_p \wedge B_p)])$. A propositional content condition $A_\theta$ is compatible with conjunction when $\models ((A_\theta a A_p) \wedge (A_\theta a B_p)) \Rightarrow A_\theta a(A_p \wedge B_p))$. A preparatory condition $A_\Sigma$ is compatible with conjunction when $\models ((A_\Sigma a A_p) \wedge (A_\Sigma a B_p)) \Rightarrow (A_\Sigma a(A_p \wedge B_p))$. On the basis of previous considerations, one can formulate as follows my *update theory for attitudes* where $[aacquires(A_M A_p)]$ and $[akeeps(A_M A_p)]$ are new formulas.

*The revision principle:* $\models ([aacquires(A_M A_p)] \wedge (A_M A_p) ><_a (B_M B_p)) \Rightarrow \neg[akeeps(A_M A_p)]$

*First expansion principles:* $\models ([aacquires(A_M A_p)] \wedge (A_\theta a A_p)) \Rightarrow [aacquires([A_\theta]A_M A_p)]$;

$\models ([aacquires(A_M A_p)] \wedge [Bel a A_{\bar{\omega}} A_p]) \Rightarrow [aacquires([A_\theta] A_M A_p)]$ when $Bel$ is part of $A_M$ and $\models ([aacquires(A_M A_p)] \wedge [Des a A_{\bar{\omega}} A_p]) \Rightarrow [aacquires([A_\theta] A_M A_p)]$ when $Des$ is part of $A_M$.

$\models ([aacquires(A_M A_p)] \wedge [Bel a A_\Sigma a A_p] \wedge [>> a A_\Sigma a A_p]) \Rightarrow [aacquires([A_\Sigma] A_M A_p)]$

*Second expansion principles:* $\models ([aacquires(A_M A_p)] \wedge [aHas(A_M B_p)]) \Rightarrow [aHas(A_M(A_p \wedge B_p)])$ when $A_M = Bel$ or $Des$.

Suppose that $M(P) \in NewAttitudes_m^a(val)$. If $P \in \theta(a,m)$ then $[\theta]M(P) \in NewAttitudes_m^a(val)$. For $\models [aHas([A_\theta]A_M A_p)] \Leftrightarrow ([aHas(A_M A_p)] \wedge (A_\theta a A_p)$. If moreover $P$ is true in $m$ according to all possible denotation assignments of $\bar{\omega}(a,m,val)$ then $[\bar{\omega}]M(P) \in NewAttitudes_m^a(val)$. Indeed $\models [aHas([A_{\bar{\omega}}]A_M A_p)] \Leftrightarrow ([aHas(A_M A_p)] \wedge [Bel a A_{\bar{\omega}} A_p])$ when $Bel$ is part of $A_M$ and $\models [aHas([A_{\bar{\omega}}]A_M A_p)] \Leftrightarrow ([aHas(A_M A_p)] \wedge [Des a A_{\bar{\omega}} A_p])$ when $Des$ is part of $A_M$. Finally, if every proposition $Q \in \Sigma(a,m,P)$ is true at moment $m$ according to all possible denotation assignments of $Belief_m^a(val) \cap Granted_m^a(val)$ then $[\Sigma]M(P) \in NewAttitudes_m^a(val)$. For $\models [aHas([A_\Sigma]A_M A_p)] \Leftrightarrow ([aHas(A_M A_p)] \wedge [Bel a A_\Sigma a A_p] \wedge [>> a A_\Sigma a A_p]$.

## APPENDIX: SEMANTIC TABLEAUX

My lexical analysis of terms for attitudes explains comparative strength between psychological modes. Here are two semantic tableaux showing relations of comparative strength between cognitive and volitive modes. The initial node of the first tableau is the term "belief" which names the primitive cognitive mode; the initial node of the second is the term "desire" that names the primitive volitive mode. Any immediate successor of a term names a stronger mode obtained by applying one or more operations whose nature is indicating by symbols in the branch between the two terms. See my next book **Speech Acts in Dialogue** for more explanation.
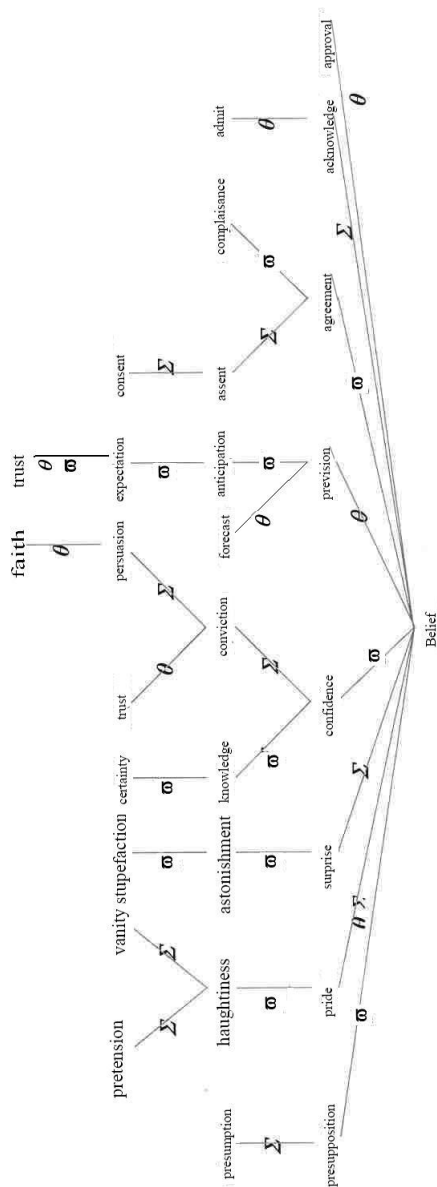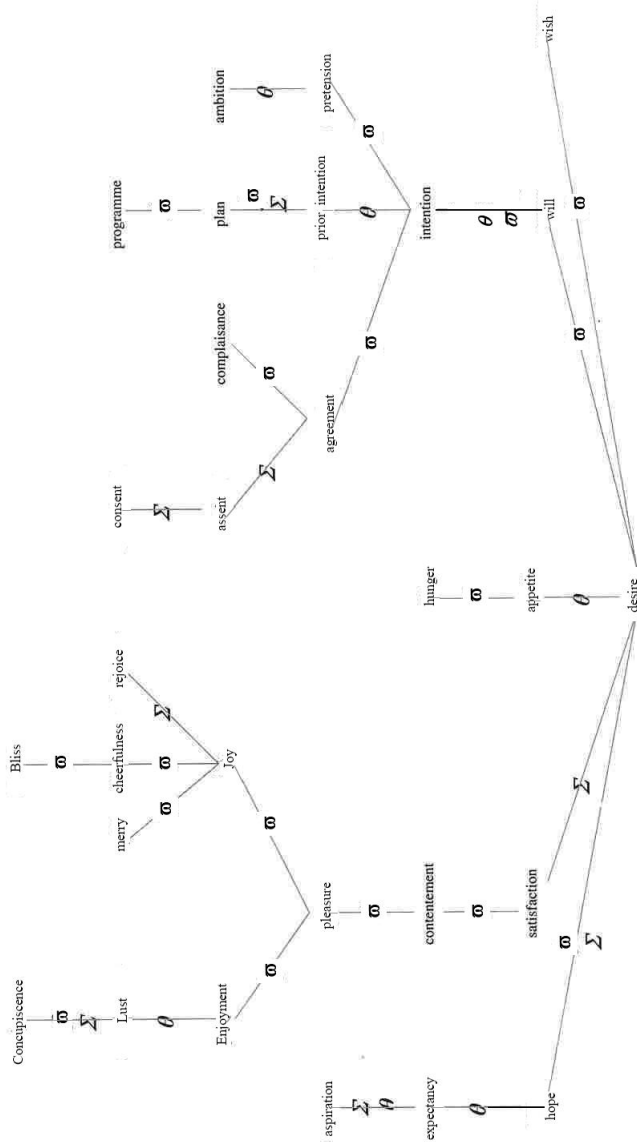
*Figure 1. Cognitive Modes*

Figure 2. Volitive Mode

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