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HYDROLOGICAL REGIONALIZATION OF THE ANNUAL MAXIMUM STREAMFLOWS OF THE UPPER AND MIDDLE PARAOPEBA RIVER – MG USING THE INDEX-FLOOD TECHNIQUE

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KEYWORDS

design floods, L-moments, frequency analysis, return periods.

ABSTRACT

Estimating peak streamflows in a basin is an essential issue in hydrology, especially for projects and management of hydraulic infrastructures such as dams, spillways, and bridges. In this context, this article presents a regionalization of annual maximum streamflows for the upper and middle Paraopeba River region in Minas Gerais, proposing a regional equation to calculate maximum streamflows using the index-flood technique by the generalized extreme value (GEV) distribution. GEV distribution parameters were estimated using the L-moments method, and the adjustment of the statistical model was performed by applying the Anderson-Darling test. The results of the hydrological regionalization were compared with those obtained by the local frequency analysis, performed for the considered fluviometric stations, observing absolute deviations from 11.91 to 58.78%.

INTRODUCTION

The 20th century was characterized by intense changes in land use regarding agricultural practices, urbanization, and forest management, and these changes can cause alterations in hydrological and ecological systems, affecting the rainfall-runoff ratio and contributing to the risk of flooding (O'Brien & Burn, 2014; Markert et al., 2018).

The estimation of the frequency of flooding is fundamental both for an approach to temporal variability (Bocchiola et al., 2003) and for environmental planning, and, according to Formetta et al. (2018), the hydrological records of a place of interest are often not available or are too short to allow reliable statistical analysis. Moreover, the observed data series in some situations, mainly in the Brazilian scenario, are rarely long enough to provide reliable estimates of flood quantiles (Wu et al., 2019a), leading to a large number of uncertainties in the estimates of hydrological variables in engineering projects (Wu et al., 2019b).

Thus, hydrological regionalization has been used as an approach to overcome this limitation, consisting of a technique that allows the transfer of data between points where there is an operational fluviometric station to other points without monitoring (Lelis et al., 2020), provided that this region is considered homogeneous.

In this sense, a large number of models and regionalization methods based on the concept that information on the hydrological response can be transferred between hydrologically homogeneous basins have been proposed in the literature, such as SMAP (Lopes et al., 1982), SWAT (Arnold et al., 1998), traditional method (Eletrobras, 1985), the method proposed by Chaves et al. (2002), differing in terms of the type of information that is transferred, transfer method, and intrinsic properties of the basin used to justify the similarity (Pagliero et al., 2019).

According to Vogel (2005), one of the essential issues in hydrology is the transfer of hydrological data to regions without monitoring and, in this sense, hydrological regionalization can be used to increase the reliability of the estimated quantiles for an already monitored point, as well as for estimating quantiles in locations lacking an efficient hydrometric monitoring network.

In addition, basin dynamics can be incorporated into statistical models to obtain reliable estimates. According to Bocchiolla et al. (2003), the methodology for statistical forecasting of floods by regionalization started with

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Darlymple in the 1960s, with the index-flood method, which requires a two-step procedure. The first step refers to the identification of a homogeneous region where a maximum probability model can accommodate the normalized streamflow, while the second step involves the search for the appropriate index-flood estimator for the specific basin.

Maximum streamflow events can be characterized as a hydrological phenomenon, presenting great temporal and spatial variability due to climate variations on a global and regional scale, as pointed out by Naghettini (2017). This hydrological variable is related to stochastic processes, considered a random variable described by probability distributions and, therefore, allowing statistical hydrology, whose purpose is to provide adequate means to interpret these characteristics of hydrological processes and extract more accurate information about the studied events.

Considering the different options for streamflow regionalization methodologies and aiming to contribute to users, technicians, and managers of water resources in the state of Minas Gerais in the choice of methods for estimating streamflows associated with a return period in the upper and middle Paraopeba River basin, this study presents the regionalization of annual maximum streamflows, proposing a regional equation to calculate these streamflows using the index-flood technique by the generalized extreme values (GEV) distribution, with its parameters estimated by the L-moments method (LMM), as presented by Hosking & Wallis (1997).

MATERIAL AND METHODS

Study area

The Paraopeba River basin is a sub-basin of the São Francisco River, located in the central region of Minas Gerais, in the range of coordinates −20°51′ S and −18°35′ S latitude and −45°11′ W and −43°38′ W longitude, with a drainage area of approximately 13,640 km^2, covering 48 municipalities and over 1,318,000 inhabitants. Its main rivers are the Paraopeba, Águas Claras, Macaúbas, Betim, Camapuã, and Manso. The Paraopeba River rises in the city of Cristiano Otoni and has a total length of 428 km, and its mouth is located at the Três Marias Hydroelectric Power Plant dam (Teixeira et al., 2021).

The upper and middle Paraopeba River region in the present study is considered to be the one upstream of the control section of the Alberto Flores fluviometric station (40740000), with a drainage area of 4,120 km^2. Seven fluviometric stations were used to regionalize the maximum streamflows in the study area, which are shown in Figure 1.

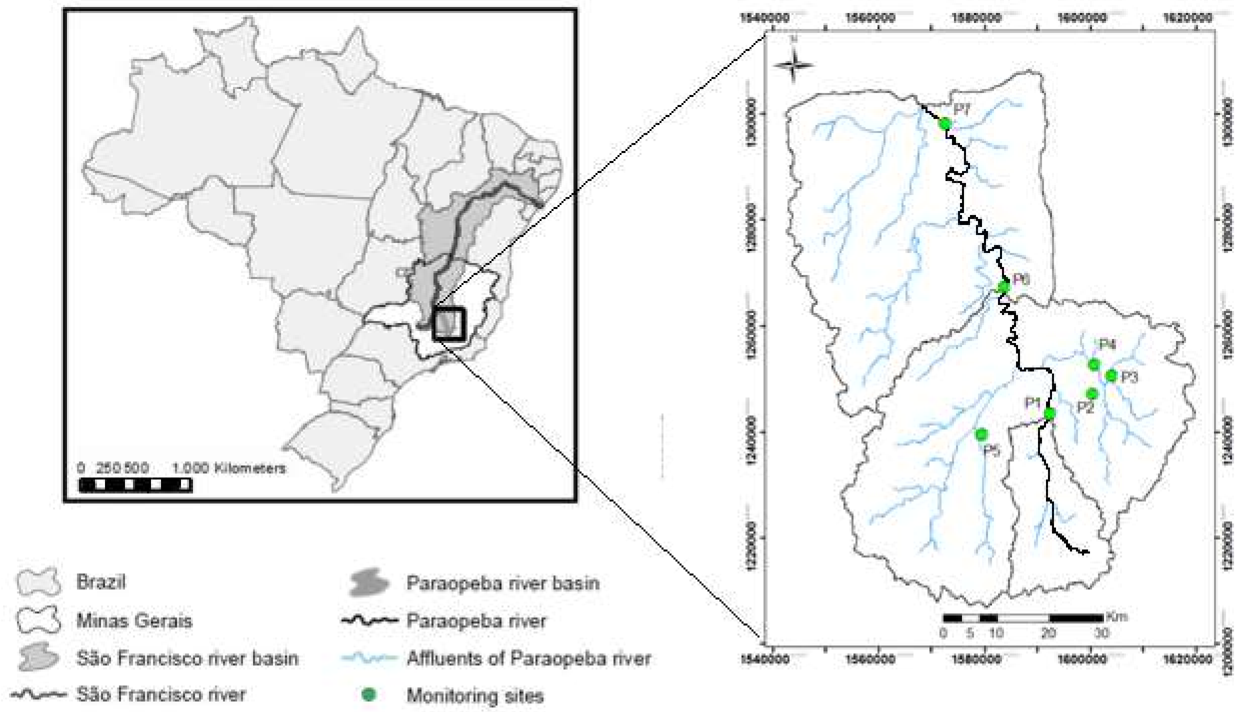

FIGURE 1. Upper and middle Paraopeba river basin and the considered monitoring sites.

The application of hydrological regionalization requires that the region under analysis be considered homogeneous, which is described by Naghettini & Pinto (2017) as those in which there is sufficient evidence that the different samples of a group have the same frequency distribution. In this sense, CPRM (2001) mentions that homogeneous regions can be defined from geographic characteristics and the similarity of the trend of individual frequency curves.

Table 1 shows the fluviometric stations and their main attributes. The stations are operated by the Geological Survey of Brazil – CPRM and are part of the National Hydrometeorology Network managed by the National Water and Sanitation Agency (ANA).

TABLE 1. Main attributes of the fluviometric stations.

Code	Point	Station	Latitude	Longitude	Area (km^2)	Series
40549998	P1	São Brás do Suaçuí Montante	−20.6039	−43.9086	462	1982–2014
40573000	P2	Joaquim Murtinho	−20.5667	−43.8333	284	1945–1965
40577000	P3	Ponte Jubileu	−20.5333	−43.8000	130	1942–1965
40579995	P4	Congonhas Linígrafo	−20.5186	−43.8356	569	1987–2017
40665000	P5	Usina João Ribeiro	−20.6500	−44.0333	293	1938–1985
40710000	P6	Belo Vale	−20.4083	−44.0217	2770	1965–2014
40740000	P7	Alberto Flores	−20.1550	−44.1647	4120	1964–2015

Although Dalrymple (1960) has shown that the used series must contain common periods of data for the application of regionalization, Hosking & Wallis (1997) argue that it is not necessary to use periods in common if the series are homogeneous and representative of the variable under analysis (in this case, annual maximum streamflows), as considered in the present study.

Subsequently, each element X_{ij} of the series, where i is the order number of the element at station j, is nondimensionalized through the relationship between the element and the nondimensionalization factor μ_j of station j, forming the series of dimensionless elements X_{ij}/μ_j.

Definition of the regional frequency curve

The calculation of regional parameters of the distribution adopted for the homogeneous region allows the estimation of the dimensionless curve of regional quantiles. According to Hosking & Wallis (1997), the parameters of the regional curve of dimensionless quantiles, denoted by $x(F;\ \theta_1,\ \ldots,\ \theta_p)$, can be obtained by weighting the local parameters $\hat{\theta}_k^{(j)}$, $k = 1, \ldots, p$, estimated for each station j by the respective sample sizes.

Thus, the regional parameters can be obtained by the means of the local parameters of the distribution adopted for the homogeneous region, weighted by the respective lengths of the series n_j of dimensionless values of each station in the region, according to [eq. (1)]:

$$\hat{\theta}_k^{j} = \frac{\sum_{j=1}^{N} n_j\, \hat{\theta}_k^{j}}{\sum_{j=1}^{N} n_j} \tag{1}$$

The L-moments method (LMM), which consists of equating the first k sample L-moments to their corresponding population moments by applying L-statistics, was applied to estimate the local parameters of the probability distribution, as described by Hosking & Wallis (1997).

The generalized extreme value (GEV) distribution was applied in this study, as this distribution model is derived from the classical theory of extreme values (Gumbel, 1958) and has theoretical justifications for its use for modeling maximum values of hydrological data, as reported by Naghettini & Pinto (2017).

The mathematical equation of the inverse function of the cumulative probability function of the GEV distribution model is given by:

$$x(T) = \beta + \frac{\alpha}{\kappa}\left\{1 - \left[-\ln\left(1 - \frac{1}{T}\right)\right]^{\kappa}\right\} \tag{2}$$

Where:

α, β, and κ are the parameters of scale, position, and shape, respectively, which describe the GEV distribution. The chi-square (χ^2), Kolmogorov-Smirnov (KS), and Anderson-Darling (AD) tests were applied to verify the fit of this distribution model to the sample data, as presented in Sabino et al. (2021).

Regression analysis

According to Costa (2017), a regression analysis aims to explain the spatial variation of the nondimensionalization factor μ_j of each station j based on basin characteristics, such as drainage area, annual precipitation, and channel slope.

There are several regression models, such as potential, exponential, and logarithmic, among others. The drainage area of a basin is known to be correlated with the maximum streamflows observed at that site. According to Singh (1992), a potential model that relates the maximum streamflows Q_{max} and the drainage area of basin A is given by:

$$Q_{\max} = kA^n \tag{3}$$

The k and n coefficients can be estimated through statistical analysis of peak streamflow data (Q_{max}), observed in basins of different drainage areas. The model of [eq. (3)] was adopted in the present study, which was also applied to some homogeneous regions delimited by CPRM (2001).

The coefficients of correlation r and determination r^2 were calculated and the Student's t-test (with a 5% significance level) was applied to verify the linearity relationship between the variables considered in the regression.

Estimation of an event associated with any return period

The dimensionless quantile associated with a given return period $(X/\mu)_T$ is estimated from the regional dimensionless curve. The event X_T is calculated for a given return period T using regression to estimate the nondimensionalization factor μ_j, valid for the homogeneous region, that is:

$$X_T = (X/\mu)_T\, \mu_j \tag{4}$$

According to Pires (1994), the index-flood method, in the context of flood streamflow regionalization, has an advantage over other methods because the regression is performed with a measure of central tendency, as the mean values tend to have less sampling variability.

RESULTS AND DISCUSSION

Verification of regional homogeneity

A regional analysis of data consistency was performed based on usual techniques of data consistency analysis and auxiliary statistics, in which the results indicated that the samples presented no discrepant statistical characteristics. In this sense, the final definition of homogeneity was performed using the routines described by Hosking (1991), allowing considering that the region under analysis is characterized as homogeneous. Moreover, the alignment of dimensionless empirical distributions is a positive indicator that the region is homogeneous (Figure 2). The coefficients of variation of the series of annual maximum streamflows of the stations were calculated to complement this analysis (Table 2).

Importantly, this methodology for the definition of hydrologically homogeneous regions was also used by Moreira & Silva (2014), who analyzed the methods for estimating the minimum reference streamflows in the Paraopeba River basin and identified two homogeneous regions, with the region corresponding to the upper Paraopeba River called Region 1 in that study, and also by Dantas et al. (2014) in the characterization of flood formation in the Una River basin in Pernambuco.

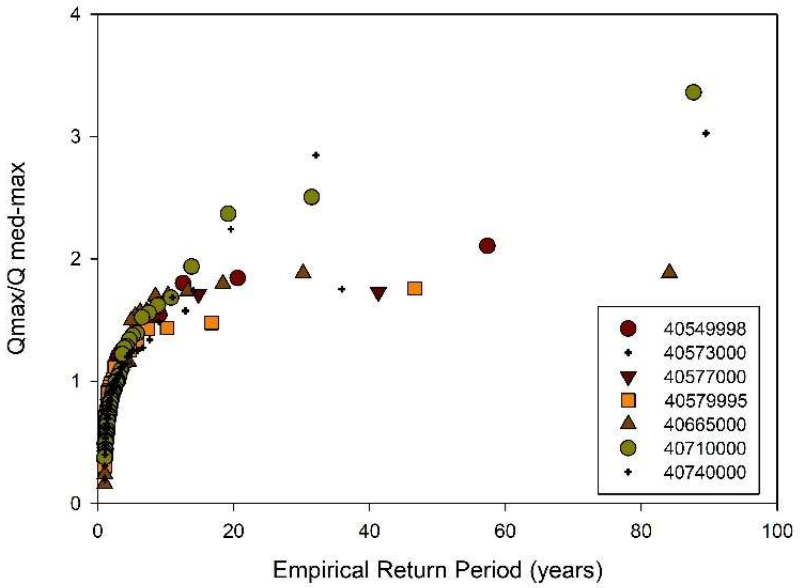

FIGURE 2. Dimensionless frequency curve for the analyzed stations.

TABLE 2. Local statistics of the series of annual maximum streamflows of the stations.

Station	40549998	40573000	40577000	40579995	40665000	40710000	40740000
Mean ($m^3\ s^{-1}$)	63.15	30.26	25.53	70.18	26.08	392.91	363.75
SD ($m^3\ s^{-1}$)	27.63	10.68	9.61	23.29	11.02	236.54	199.54
CV	0.44	0.35	0.38	0.33	0.42	0.60	0.55

Adherence tests were applied after verifying the regional homogeneity to examine the goodness of fit provided by the GEV distribution for the data series at a 5% significance level. The KS, χ^2, and AD tests indicated the applicability of GEV for the selected stations. Among these tests, KS is the least rigorous in rejecting hypotheses of adherence to the series of annual maximums, as its application assumes that the distribution to be tested is previously known (Back & Bonfante, 2021). Still, the KS test is widely used many times as the only adherence test (Ottero et al., 2018) and to select the best distribution (Back & Cadorin, 2020).

As shown by Naghettini & Pinto (2007), the LMM method was used to estimate the local parameters of the selected stations, while the regional parameters were obtained by the means of the local parameters of the distribution adopted for the homogeneous region, weighted by the respective series lengths. The parameters estimated by this method were considered good, following the results by Franco et al. (2014) and Junqueira Júnior et al. (2015), who evaluated the precision of fit of GEV and other probability distributions for Minas Gerais and found that the best fits were obtained by LMM and MLM.

According to Alam et al. (2018), who evaluated probability distributions applied to monthly maximum rainfall events in Bangladesh, the LMM developed by Hosking in 1990 is more robust than conventional methods, in addition to having greater sensitivity in the estimation of parameters.

GEV distribution application in the seven analyzed stations allowed obtaining the dimensionless quantiles for the respective return periods considered for different engineering applications. Morais et al. (2020) analyzed the efficiency of 10 probabilistic models in a regionalization study in the Araguaia River basin and identified GEV as the only one that had a good fit for the maximum streamflow series. This robustness was also observed by Cassalho et al (2018) when evaluating six probability distributions in the Mirim-São Gonçalo basin in Rio Grande do Sul and Rodrigues et al. (2021), who regionalized reference streamflows in the Cerrado biome in Tocantins.

The results obtained for the regional parameters of scale (α), position (β), and shape (κ) were 0.334, 0.784, and −0.046, respectively. The inverse function of the regional GEV distribution was rewritten with the parameters estimated for the homogeneous region, according to [eq. (5)], presented below.

$$\frac{Q_{\max}}{Q_{\text{med}-\max}}(T) = 0{,}784 + \frac{0{,}334}{-0{,}046}\left\{1 - \left[-\ln\left(1 - \frac{1}{T}\right)\right]^{-0{,}046}\right\} \tag{5}$$

Table 3 shows the dimensionless quantiles associated with different return periods.

TABLE 3. Nondimensionalized regional quantiles.

Return period (years)	2	5	10	20	25	50	100
Regional quantile	0.907	1.303	1.577	1.848	1.936	2.214	2.498

Once the inverse function and the nondimensionalized quantiles were obtained, we proceeded with the regression application between the means of the annual maximum streamflows and the drainage areas of the analyzed stations to obtain the estimate of the index-flood. Table 4 shows the values of the drainage areas of the stations and their respective annual maximum mean streamflows, together with the logarithmic transformation of these variables.

TABLE 4. Fluviometric stations and values of areas and means of annual maximum streamflows and logarithmic transformation.

Station	**40549998**	**40573000**	**40577000**	**40579995**	**40665000**	**40710000**	**40740000**
Area (km^2)	462.0	284.0	130.0	569.0	293.0	2770.0	4120.0
$Q_{\text{mean-max}}$ (m^3 s^{-1})	63.15	30.26	25.53	70.18	26.08	392.91	363.75
ln (A)	6.13556	5.64897	4.86753	6.34388	5.68017	7.92660	8.32361
ln ($Q_{\text{mean-max}}$)	4.14549	3.40966	3.24004	4.25103	3.26121	5.97358	5.89647

The linearization of [eq. (3)] allowed obtaining the parameters a = ln (k) and b by applying simple linear regression, as shown in the equation below:

$$\ln(Q) = \ln(k) + b\ln(A) \tag{6}$$

Figure 3 shows the dispersion diagram relating the drainage area and the mean annual streamflows (after logarithmic transformation), where the approximately linear relationship between the two variables can be seen in a log-log graph, suggesting the applicability of a potential model, as shown in [eq. (3)].

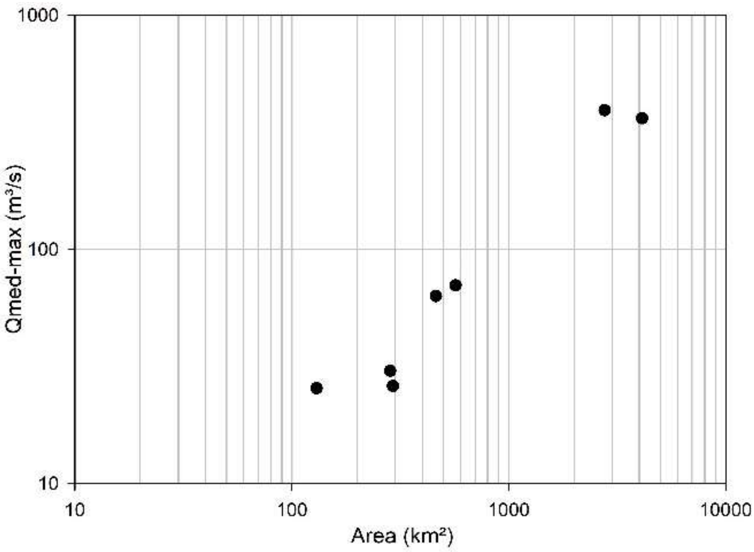

FIGURE 3. Linearity between the means of annual maximum streamflows and the drainage area.

The linear relationship was also evaluated by applying the Student's t-test at a 5% significance level. The relationship between the analyzed variables was considered linear with 95% confidence. Table 5 shows the results of the linear regression application.

TABLE 5. Result of coefficients obtained by linear regression.

a* = ln *(k)	***B***	***r***	***r²***
−1.5526	0.9136	0.975	0.951

Pruski et al. (2016) regionalized the long-term mean streamflow (Q_{ltm}) for the São Francisco River sub-basins and obtained models with r^2 equal to 0.898 and 0.893, using the drainage area and the mean annual precipitation as explanatory variables for each model, respectively. Morais et al. (2020) found a similar result with r^2 ranging from 0.83 to 0.96. In this context, the models proposed for the upper Paraopeba River are parsimonious and can be considered effective and efficient to estimate the flood streamflow.

Therefore, [eq. (6)] can be rewritten as shown below:

$$\ln(Q) = -1{,}5526 + 0{,}9136\ln(A) \tag{7}$$

Table 6 shows the result of calculating the residuals of linear regression after estimating coefficients *a* and *b*, while Figure 4 shows the residuals of the regression carried out around the predicted variable (mean of annual maximum streamflows) by the regression model. The variance can be considered approximately constant, indicating the fulfillment of the hypothesis of homoscedasticity.

TABLE 6. Result of the regression residuals.

Station	**40549998**	**40573000**	**40577000**	**40579995**	**40665000**	**40710000**	**40740000**
$\ln(Q_{\text{mean-max}})$	4.05299	3.60843	2.89448	4.24331	3.63693	5.68932	6.05203
Residual	0.09250	−0.19876	0.34556	0.00772	−0.37572	0.28426	−0.15556

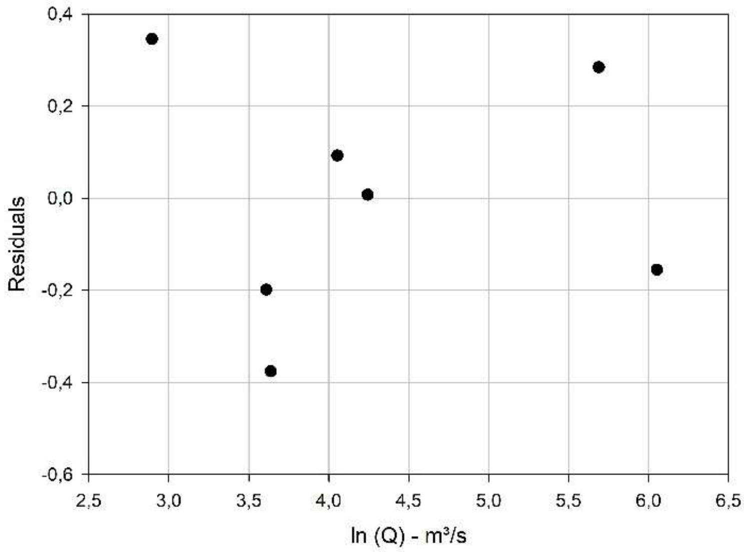

FIGURE 4. Variation of residuals around the mean of the annual maximum streamflows obtained by the regression model.

Equation 7 describes the simple linear regression of the transformed variables, but the equation written in potential form is used to estimate the index-flood, that is:

$$Q_{\text{med-max}} = 0{,}2117 \times A^{0{,}9136} \tag{8}$$

Where:

$Q_{\text{mean-max}}$ is the mean of the annual maximum streamflows in $m^3\ s^{-1}$, and

A is the basin drainage area in km^2.

A comparison was performed between the values of the means of the observed annual maximum streamflows relative to those estimated according to the application of [eq. (8)], as shown in Table 7, together with the calculation of the respective percentage deviations (*PD*), with a mean value of 2.98%.

TABLE 7. Comparison of streamflows estimated by linear regression relative to the mean values of the observed maximum streamflows.

Point	**P1**	**P2**	**P3**	**P4**	**P5**	**P6**	**P7**
$Q_{\text{mean-max observed}}$ ($m^3\ s^{-1}$)	63.1	30.3	25.5	70.2	26.1	392.9	363.8
$Q_{\text{med-max calculated}}$ ($m^3\ s^{-1}$)	57.6	36.9	18.1	69.6	38.0	295.7	425.0
PD (%)	−8.84	21.99	−29.22	−0.77	45.60	−24.74	16.83

Importantly, the flood data quality has to be verified before any application of hydrological analysis because some factors associated with flood data cannot be fully controlled due to their nature. Therefore, the local factors that influence the index-flood may not be adequately represented in the regression models, as pointed out by Muhammad & Lu (2020), and the regionalization equation should be used with caution.

Thus, the calculation of the nondimensionalization factor $Q_{\text{mean-max}}$, obtained by linear regression, can be substituted in [eq. (5)], that is:

$$Q_{\max}(T) = \left(0{,}2117 \times A^{0{,}9136}\right) \times \left\{0{,}784 + \frac{0{,}334}{-0{,}046}\left\{1 - \left[-\ln\left(1 - \frac{1}{T}\right)\right]^{-0{,}046}\right\}\right\} \tag{9}$$

Thus, the estimation of quantiles associated with different return periods for unmonitored locations, located within the considered homogeneous region (upper Paraopeba), can be performed by applying [eq. (9)]. This regional equation for calculating maximum streamflows is valid for drainage areas between 130 and 4,120 km^2.

Figure 5a shows the estimate of maximum streamflow quantiles associated with different return periods, obtained as a function of the application of [eq. (9)], considering the respective drainage areas of the stations used in the study. Additionally, a local frequency analysis was performed for the stations, whose results are shown in Figure 5b. The GEV distribution was also applied with its local parameters estimated by the LMM method to estimate maximum streamflow point quantiles.

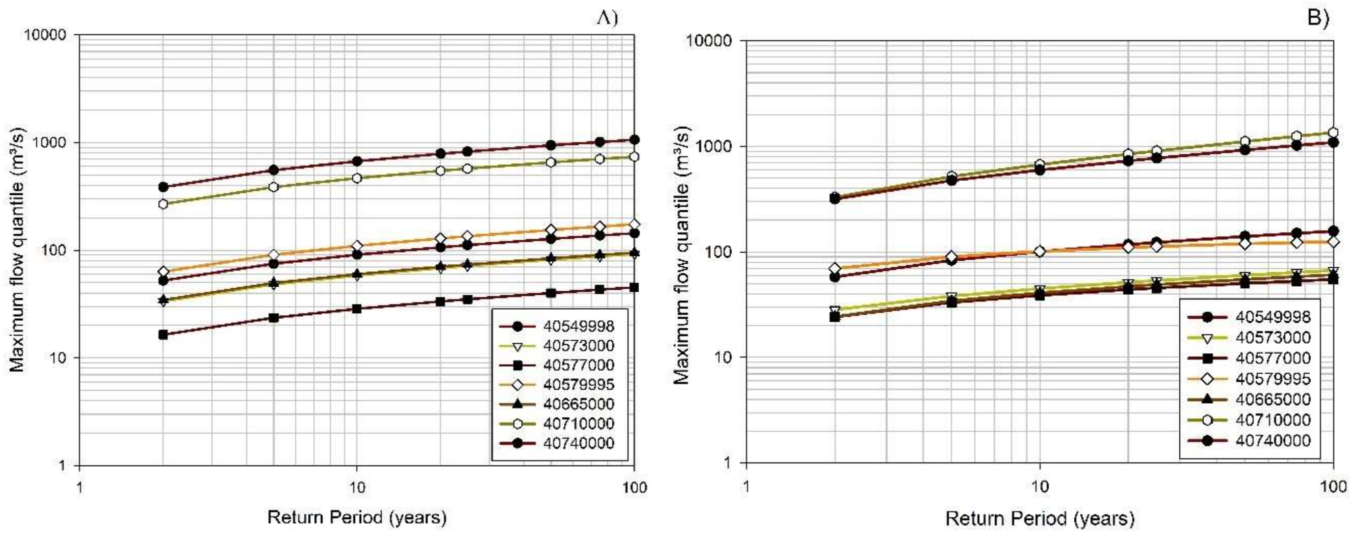

FIGURE 5. Results of quantile regionalization by regional (a) and local (b) frequency analysis for the analyzed fluviometric stations.

Figure 6 shows the percentage deviations between the maximum streamflow quantiles using regional and local frequency analysis.

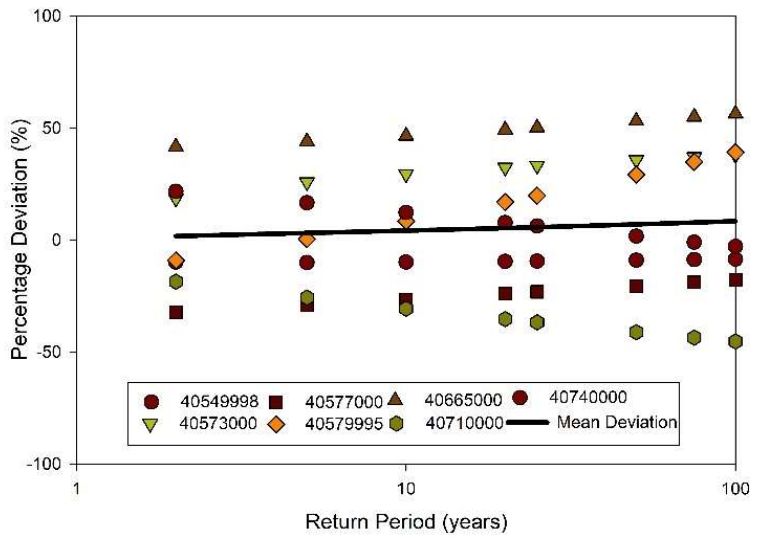

FIGURE 6. Percentage deviations between the quantiles estimated by applying regional and local frequency analysis.

The absolute deviations calculated by estimating the maximum streamflow using the regional equation varied between 11.91% (for T = 2 years) and 58.78% (for T = 100 years) compared to that obtained by the local frequency analysis for each return period. However, the mean of the percentage deviations obtained for all monitoring points (return periods from 2 to 100 years) was 5.40% (mean of the obtained deviations). Zhao et al. (2021) developed a global model for flood estimation and found deviations of up to 70% for a return period equal to 100 years. It demonstrates the challenge of estimating the flood rates for a given location even for locations provided with monitoring.

CONCLUSIONS

This study proposed a regional equation for the estimation of maximum streamflow quantiles in the upper and middle Paraopeba River basin region, in Minas Gerais. The region was considered homogeneous since the dimensionless frequency curves for the fluviometric stations showed the same trend, in addition to similar coefficients of variation.

The linearity of the relationship between the mean of the annual maximum streamflows and the respective drainage areas of the stations was observed, with a correlation coefficient of 0.975. The linear regression performed to obtain the parameters of the potential model of the relationship between the mentioned variables resulted in a coefficient of determination r^2 of 0.951. The application of Equation 9 depends only on the value of the drainage area, which must be between 130 and 4,120 km^2, which are the limits of the contribution area of the stations considered in the regionalization.

The mean deviations between the streamflow quantiles obtained by the application of the regional equation relative to those by the application of the local frequency analysis were calculated for each return period. Absolute deviations between 11.91 and 58.78% were observed, which are in the same range of variation obtained in similar studies of the literature.

An overall mean deviation (mean of the obtained deviations) of 5.40%, considering all monitoring stations and return periods from 2 to 100 years, was observed despite the values mentioned above. The application of other distribution models is recommended for future investigations so that the reduction of these percentage deviations can be verified.

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