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ARTICLES

A FUZZY SCALE APPROACH TO THE THOR ALGORITHM

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ABSTRACT. The use of the Multicriteria Decision Support Hybrid Algorithm for Decision Making Processes with Discrete Alternatives, acronym THOR, requires from the decision maker, during the judgment insertion stage, a significant amount of information that needs to be valued, which may cause the decision maker great cognitive fatigue. Therefore, this article aims to reformulate the THOR algorithm, in the judgment insertion stage, based on the inclusion of a fuzzy measurement scale, allowing the decision maker to express only a single value judgment. This reformulation follows the three steps of a fuzzy system: fuzzification, fuzzy inference and defuzzification. In addition, a comparative analysis is performed between the THOR algorithm and its new version based on the construction of the fuzzy scale. It should be noted that its reformulation does not compromise the methodological efficiency of the THOR algorithm, it only reduces the complexity of decision making.

Keywords: multicriteria method, fuzzy, THOR.

1 INTRODUCTION

The speed of information and the concern for obtaining the best response in a short period of time are outstanding characteristics in the current scenario, directly implicated in decision-making processes dependent on human and behavioral conduct.

The decision-making process, according to Hilletofth et al. (2019), is dependent on the human being and is subject to failures, as well as to other feelings. Guerrero et al. (2021) state that in solving decision problems, it is necessary to deal with many factors of uncertainty whose

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nature manifests itself in different ways, such as randomness, imprecision, indistinguishability, and incompleteness.

In this sense, traditional methods, such as the Analytic Hierarchy Process (AHP), proposed by Saaty (1980), methodology are insufficient since, according to Salih et al. (2020), in this methodology the decision maker cannot determine his preference only among extreme values of the adopted scale since this causes difficulty for the method in selecting alternatives based on the hierarchy built by the method.

In the 70s, researchers perceived that to make a decision that is consistent with reality, it would be necessary to consider, in addition to economic factors, the subjective values inherent to the human decision-making process. For Liao (1996), as judgment is a complex and uncertain task, a solution would be to transform human thinking into linguistic labels, creating a pleasant environment to make judgments. Balusa and Gorai (2019) add that to contain the uncertainty of judgements, multicriterial methods seek to ally themselves to theories that address such adversities, among which one can highlight Fuzzy theory.

According to Kotb et al. (2021), the inclusion of multicriteria methodologies in a fuzzy environment enables decision makers to express their assessments in a more complete manner, since through linguistic terms included in the scales, the results obtained become more sensitive and accurate. Salih et al. (2020) also state that with the inclusion of fuzzy logic, it is possible to define fair and understandable comparisons for the decision maker while preventing inconsistency and reducing inaccuracy and comparison execution time.

Within this context, this study proposes to reformulate data entry for the Hybrid Algorithm for Multicriteria Decision Support for Decision Making Processes with Discrete Alternatives, acronym THOR. With the inclusion of a fuzzy measurement scale, the decision maker shall be allowed to express a single value judgement.

Although the methodology used in the development of the THOR algorithm is well structured, the inaccuracy in the decision-making process presents a disadvantage regarding the data input because it requires four value judgments, requiring a great cognitive effort on the part of the decision maker. In this sense, with the development of a fuzzy scale, by maintaining the quality of the decision-making process, it is possible to make the use of the algorithm by the decision maker more accessible. Building the fuzzy scale involves the steps of fuzzification, inference, and defuzzification.

This paper is structured into five sections. After the introduction, Section 2 presents the description of THOR methodology. In Section 3 the theoretical framework of fuzzy logic is presented with the methods involving the multicriteria approaches of operational research, including the description of the THOR methodology and its merger with fuzzy logic. In the subsequent section, the results obtained by comparing the methodologies are presented. Finally, Section 5 addresses the final considerations.

2 THOR ALGORITHM AND ITS SPECIFICITIES

According to Gomes (2005), the THOR algorithm is grounded on both the American school, which uses the theory of utility and multicriteria, and the French school, which uses preference modelling. Gomes et al. (2021) highlight that the developed algorithm applies utility theory to evaluate the value of alternatives in complex situations and preference modelling to indicate overranking relations as well as the multi-attribute theory to present the dominance and value hierarchy of alternatives. Furthermore, the THOR methodology employs the theory of fuzzy sets, which along with the theory of approximate sets deals with fuzziness, imprecision, and indiscernibility.

Rough set theory, being currently known as *Dominance-Based Rough Set Approach* (DRSA), according to Pawlak (1982), aims to give a treatment to uncertainty and approximate classification. Slowinski et al. (2012) add that DRSA is mainly based on replacing the indiscernibility relation by a dominance relation in the irregular approximation of the decision class. Thus, it is possible to deal with inconsistencies without removing them before the analysis.

Gomes et al. (2002) specify 12 steps for the application of the THOR algorithm as summarized in Table 1.

The first four steps of the algorithm are the same adopted in the classic multicriteria methods.

In Step 5, two scales are utilized by the decision maker. The first is used for the criteria classifications and the second for determining the relevance degree $\mu c_j \in [0,1]$ of the criterion C_j. For each criterion C_j the decision maker must assign a weight wc_j and its respective relevance value μc_j. The calculation of the Hamming distance, measured between the pertinence degrees μc_j individually and globally determines the criteria pertinence index C_j. The relevance index in the decision-making process determines the exclusion of the criterion C_j. A criterion C_j is excluded when its individual pertinence index is greater than the global pertinence index.

In Step 6, two scales are utilized by the decision maker. The first scale is applied to the classifications of alternatives and the second is utilized to determine the degree of pertinence $\mu a_i \in [0,1]$ of the alternative a_i. For each alternative a_i, the decision maker must attribute a weight wa_i and its respective degree of relevance μa_i. The Hamming distance calculation, measured between the pertinence degrees, individually and globally, determines the pertinence index of the alternative a_i in the decision process, which determines the exclusion of the alternative a_i such that an alternative a_i will be excluded when its pertinence index is greater than the global pertinence index.

In Step 7, the disagreement for each criterion is defined, which as per Gomes and Costa (2015) is responsible for controlling the preference intensity, so that it does not exceed a permitted limit. In addition, this variable can only assume values greater than or equal to zero.

In Step 8, for each criterion C_j the decision maker defines the preference limits $p \geq 0$ and indifference thresholds $q \geq 0$. Boundaries classify alternatives into: indifferent, weak preference, and

Table 1 – THOR algorithm steps.

Step 1	Problem identification, formulation, and analysis.
Step 2	Object setting and preferences.
Step 3	Identification of constraints and/or relaxations.
Step 4	Identification of criteria, through cognitive maps, fishbone diagrams, and *brainstorming*.
Step 5	Weightings of the criteria.
Step 6	Weights of the alternatives.
Step 7	Definition of the disagreement for each criterion.
Step 8	Classification of alternatives based on the definition of the limits p and q. For each criterion C_j criteria, the alternatives a and b are classified as: • $-q \leq \lvert g(a) - g(b) \rvert \leq q$, a and b will be considered indifferent; • $q < \lvert g(a) - g(b) \rvert \leq p$, a will have weak preference with respect to b, otherwise b will be weakly preferred with respect to a. • $g(a) - g(b) > p$, a will have strong preference with respect to b. Where: $g(.)$ represents the gain in the criterion C_j for the alternative $(.)$
Step 9	Comparison of the alternatives, following the formalism of the scenarios S_1, S_2, and S_3 presented by equations (1), (2) and (3): $$S_1 : \sum_{j=1}^{n} (w_j \vert aP_jb) > \sum_{j=1}^{n} (w_j \vert aQ_jb + aI_jb + aR_jb + bQ_ja + bP_ja) \quad (1)$$ $$S_2 : \sum_{j=1}^{n} (w_j \vert aP_jb + aQ_jb) > \sum_{j=1}^{n} (w_j \vert aI_jb + aR_jb + bQ_ja + bP_ja) \quad (2)$$ $$S_3 : \sum_{j=1}^{n} (w_j \vert aP_jb + aQ_jb + aI_jb) > \sum_{j=1}^{n} (w_j \vert aR_jb + bQ_ja + bP_ja) \quad (3)$$ where: R: non-comparability P: strong preference I: indifference Q: weak preference
Step 10	Choice of alternatives.
Step 11	Implementation of the alternatives.
Step 12	System feedback.

strict preference. Table 1 shows how the classification of alternatives is conducted in Step 8 of the algorithm.

In Step 9, through the formalisms (1), (2) e (3) we have the scenarios S_1, S_2, and S_3 presented in Table 1, comparisons between alternatives are conducted. Using the overranking relations for each alternative, the additive, non-transitive function compares alternatives. According to Gomes et al. (2008), scenario S_1, equation (1), will score only the attractiveness that presents strong preference over the others. The S_1 scenario is considered the most demanding scenario when compared to the scenarios S_2, equation (2) and S_3, equation (3).

In Steps 10 and 11, the choice of alternative occurs. At this stage the level of imprecision of alternatives and criteria is verified. The decision maker is the one who chooses the best alternative, the method only suggesting an ordering of alternatives. We note that in Steps 5 and 6 of the algorithm, the pertinence index indicates both criteria and alternatives to be excluded from the model as well as their relative ranking.

In Step 12, the feedback of the system occurs, that is, a sensitivity analysis is performed for the values of p, q, and the discordance value for each criterion. Thus, the classifications are updated and scenarios are recalculated.

For a better understanding of the THOR method, refer to Figure 1 follows, which presents the main steps of the method in flowchart format.

With the description of the THOR algorithm, it is important to bear in mind that although it is a very well-founded algorithm, it remains little explored due to the cognitive effort demanded of the decision maker. Of the total of five stages (identification, assignment, classification, comparison, and choice), the decision maker has cognitive responsibility over three stages. The steps are explained briefly below.

At the identification stage, the decision maker and others assist in the formulation and analysis of the problem as well as in the definition of criteria and alternatives.

At the assignment stage, the decision maker informs the weightings for criteria and alternatives as well as their respective pertinences. This means that for each evaluation, the decision maker must be capable of informing the certainty level of his answer. Furthermore, for each criterion the disagreement and limiting values are determined, the latter determining relevant and irrelevant differences for the decision maker.

The following stages are classification and comparison and occur without the intervention of the decision maker. These steps are designed to perform the overranking of alternatives in relation to each of the criteria in a paired manner, and three scenarios are built to yield the attractiveness score.

In the final choice stage, the veracity of the proposed model is verified by means of the pertinence index. At this stage the decision maker, utilizing the pertinence index, has the responsibility of excluding or retaining certain criteria or alternatives that might increase system inaccuracy. Thus, if the decision maker opts for exclusion, then it is necessary to return to the comparison stage, refeeding the system. Otherwise, the hierarchy is established and the THOR algorithm is concluded.

Since the emergence of the THOR algorithm, its applicability has been gaining ground in the most diverse areas, such as: in the management of the ballast water problem (Gomes, 2005), in

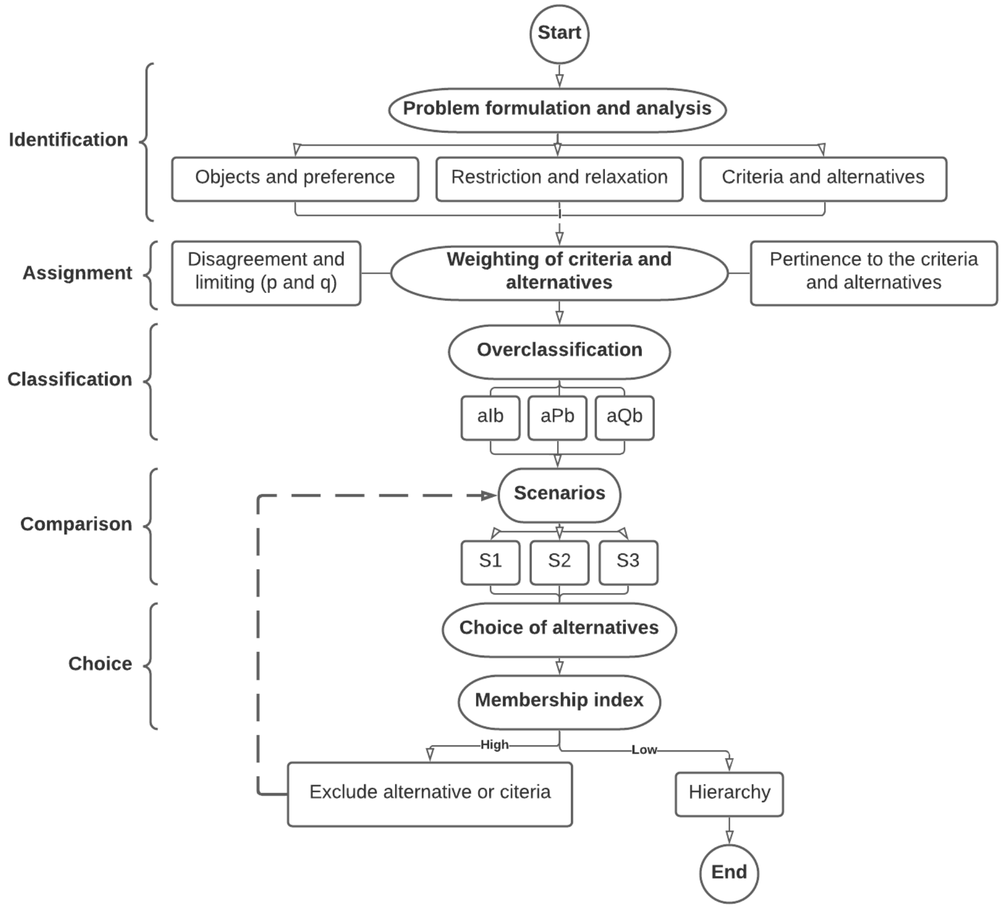

Figure 1 – THOR steps.

Source: Authors.

the choice of credit card technology (Gomes and Costa, 2015), in the selection of a ship for the Brazilian Navy (Tenório et al. 2020), in project management (Santos et al. 2022) among others.

Although Gomes et al. (2002) emphasize that the THOR algorithm has made an important contribution to Multicriteria Decision Support (MDA) models, it still lacks much use on the part of users and researchers. Furthermore, Tenório et al. (2020) affirm that the application of the algorithm is limited to Brazilian researchers, given the discontinuity in the development of the decision support system and improvement of the algorithm in light of the complexity of the adopted methodology.

Gomes et al. (2021), making an axiomatic evolution of the THOR method, develops the THOR 2 method. Furthermore, Tenório et al. (2021), adds that with the THOR 2 algorithm it is possible to avoid the elimination of alternatives or criteria that are not weighted, by assigning a low relevance value. The difference between the two methodologies occurs only in the weighting of the overclassifications.

The overclassification executed in the THOR algorithm, considering two alternatives ***a*** and ***b***, with "***aPb***" strict preference, "***aQb***" weak preference and "***aIb***" indifferent preference, occurs as follows:

- The weight assigned by the decision maker, in a preference situation "***aPb***" situation, will be retained.
- The weight assigned by the decision maker in a preference situation "***aQb***", will suffer a correction. This is calculated through the product of the weight and the fuzzy factor, where the fuzzy factor is defined as the arithmetic mean of the pertinence values of alternatives ***a*** and ***b*** and the corresponding criterion.
- The weight assigned by the decision maker in a situation "***aIb***" will be half the weight of the criterion.

As in THOR algorithm, THOR Algorithm 2 maintains the criterion weighting for strong preference and half of the weighting for indifference preference. However, for the weak preference a new formalism is applied to lower the criterion weight, which is calculated through the proportion between the half of the criterion weight and the total weight. More details are found in Gomes et al. (2021).

It is believed that THOR 2 improved the THOR algorithm so far as overranking weightings are concerned, to the extent that it attributes to these classifications all the uncertainty defined by the decision maker. However, both THOR and the new version, THOR 2, were not improved in relation to Steps 5 and 6 of the algorithm, i.e., they still demand a great cognitive overload from the decision maker.

The cognitive overload occurs when the decision maker chooses to establish a value judgment for each criterion and alternative, including the values for vetoes, making the process exhaustive, as shown in Figure 1 for the step assignment. Due to the success of fuzzy theory, in order to minimize the weakness pointed out in the algorithms with respect to data input, this study proposes an update in THOR algorithm through the inclusion of a fuzzy measurement scale.

3 FUZZY THEORY AND METHODS OF MULTICRITERIA DECISION SUPPORT

Existing sets in the real world do not have precise limits, causing difficulty for the modeling process of the same. Therefore, in order to implement a relaxation of the classical definition of these sets and allow a mathematical treatment for imprecise information, the theory of fuzzy sets was created by Lotfi Asker Zadeh.

Another feature of this theory is that it allows the programming and storage of vague concepts in computers, making it possible to produce calculations with imprecise information, as human beings do. Guerrero et al. (2021) adds that fuzziness allows a more realistic representation of real world information through a simple approach.

In fuzzy theory there is a broader concept with respect to the relevance of an element than in classical theory. To define the belongingness of an element, the fuzzy approach determines the degree to which it belongs to a set, that is, it provides a gradual transition from falsehood to truth. This degree of membership is defined in the interval [0, 1] through the construction of a membership function (μ_A). Details and formal proofs for fuzzy theory can be found in Zadeh (1965).

As per Dhunny et al. (2019), the application of fuzzy theory to modeling a problem involves three important steps: fuzzification, fuzzy inference, and defuzzification.

The first step is fuzzification, which transforms the variables of the problem in fuzzy values. In this step occurs the quantification of imprecision, which is obtained through the definition of fuzzy numbers. The main types of fuzzy numbers are: triangular (Jun Li, 1999), trapezoidal (Parvathi and Malathi, 2012), Gaussian (Barros et al. 2017), pentagonal (Pathinathan and Ponnivalavan, 2014), and diamond (Pathinathan and Ponnivalavan, 2015).

The second stage is fuzzy inference, which is responsible for the rules and operations with the fuzzy values. It is important to note that these operations are not conventional; for more details see Zadeh (1965).

The third step is characterized by the transformation of the fuzzy result into a crisp result; this process is termed defuzzification. For the application of this transformation, the chosen method must be compatible with the fuzzy number defined in the first step.

Multicriteria methods, according to Balusa and Gorai (2019), utilize expert opinion to generate conclusions. However, according to Dong and Zhang (2015), when dealing with the judgements of experts, inconsistencies may occur due to subjectivity and uncertainty, thus invalidating the results obtained.

According to Gligoric and Simeunovic (2010) the adversities mentioned above are difficult to model by traditional mathematical methods, thus necessitating an update in the methods employed. Kafuku et al. (2019) manage uncertainty and subjectivity by applying fuzzy logic, which plays a significant role in understanding such complexities. Karmarkar and Gilke (2018) state that with the use of fuzzy theory it is possible to insert linguistic variables, making the decision process easier.

According to Romeo and Marcianò (2019), integrating traditional methods into a fuzzy environment provides a simpler interpretation for decision making. For Liao (1996), the multicriterial methods that employ fuzzy logic have a successive gain in acceptance because they have the ability to handle imprecision. Gupta et al. (2018) highlight the reduction in computational time over classical approaches. Li et al. (2018) add that Fuzzy logic is also responsible for reducing the complexity of the implemented algorithms.

Kotenko et al. (2019) demonstrate that with the inclusion of fuzzy logic, their results demonstrated the correct functioning of the modeling and the gradual increase of accuracy over classical methods. In addition, Caprioli and Bottero (2021) add that with this combination the methodologies give more importance to the difficulty, imprecision, and uncertainty in the weighting phase. Vahidnia et al. (2009) state that with this association it is possible to leave the decision maker free to choose a range of values that reflect their confidence, enhancing the capacity of these methods.

The table below presents, based on a brief literature review, articles that apply the Fuzzy theory along with multicriterial methods.

Basically, the interest of the studies in Table 1 lies in the resolution of applied problems utilizing the multicriteria methodology. There are still few studies that present the methodologies involved in decision support algorithms. In this sense, this study's greatest contribution specifically consists in the improvement of the Hybrid Algorithm of Multicriteria Decision Support for Decision Making Processes with Discrete Alternatives - THOR for the identification of needs, evolution, and updating.

3.1 Fuzzy theory added to the THOR method

As mentioned, fuzzy theory combined with the multicriteria methods has been gaining increasing attention in operations research and decision making. Tosun and Akyüz (2015) reveal in their work important methodologies applying fuzzy logic, including fuzzy AHP, fuzzy TOPSIS, fuzzy VIKOR, fuzzy DEMATEL.

Accordingly, as per Krohling and De Souza (2012), the Multicriteria Interactive Decision Making method (TODIM) in risk prospecting theory was developed, having a Brazilian origin and universal reputation. However, due to the weakness of the decision matrix not taking information uncertainty into account, the adopted methodology was reformulated. Thus, as an improvement of the classical mathematics of the TODIM method by using the operational peculiarities of fuzzy logic, the F-TODIM method was developed.

The updates to the TODIM and THOR methods use fuzzy theory in order to improve the structure of imprecision and ambiguity. In addition to maintaining the aforementioned characteristics, the present study aims to encourage the application of the THOR algorithm, as it significantly reduces the cognitive effort on the part of the user, as mentioned in section 2.

The proposed update will be through the reformulation of the THOR algorithm's data entry, from a scale integrated to the fuzzy theory, providing the decision maker with a less tiring experience in relation to their value judgments. Another concern in this adaptation was to maintain the efficiency of the THOR algorithm, without the need to adjust it to the fuzzy mathematical formalism.

The THOR method combined with fuzzy theory modifies Steps 5 and 6 of the algorithm described in Section 2 in relation to the data input; these steps are responsible for the allocation

Table 2 – Review of the fuzzy approach.

Authors	Application of fuzzy theory
Su et al. (2012)	Applies fuzzy logic to assist in calculating the temperature of a wireless sensor network monitoring area.
Mukherjee and Dasgupta (2013)	Adopts fuzzy logic to calculate the quality of control satisfaction in control system applications.
Aksoy et al. (2014)	Uses fuzzy logic to solve multi-period dynamic decision making for strategic supplier selection with stochastic demand.
Perera and Lahat (2014)	Applies fuzzy logic for real-time flood forecasting in the Kelatan River Basin in Malaysia.
Sheehan and Gough (2016)	Employs fuzzy logic to evaluate landscapes for conservation and resource planning.
Loh et al. (2017)	They perform an analysis of the supply chain of ports, assessing the probability and severity of threats of disruption of this chain centered on ports, applying an evaluation by the comprehensive fuzzy method.
Karimi et al. (2018)	Fuzzy logic is applied to ANP for the SWOT analysis of a ceramic and tile factory in Iran.
Zhou et al. (2018)	They propose a quantitative human reliability analysis (HRA) model based on fuzzy logic theory, Bayesian networks, and the cognitive reliability and error analysis method (CREAM) for tanker transportation industries.
Rajasekhar et al. (2019)	Applies fuzzy logic to a multi-criteria method to construct groundwater potential mapping in India.
Pandey and Shukla (2019)	Through *fuzzy multicriteria decision making* an evaluation of the factors influencing the human performance of air traffic control in Thailand was conducted.
Tseng and Pilcher (2019)	It uses fuzzy logic together with AHP method to analyze the main factors affecting green port policies, applied to three ports in Taiwan.
Wu et al. (2020)	Applies fuzzy logic to assist in decision making for the selection of navigation strategy in the scheme of separation of inland traffic, being safe navigation or in the process of autonomous navigation.
Zindani et al. (2021)	Applies the fuzzy intuitionist approach along with the TODIM method to assist in choosing the best machine for a particular company based on four criteria: reliability, safety, flexibility, and productivity.

of weights by the decision maker to the criteria and the alternatives, specifying their degrees of relevance. In this reformulation of the THOR algorithm, using fuzzy logic, the decision maker needs only express a value judgment to the criteria. For this, a fuzzy scale was built allowing the pertinence indices to be determined without the need for information about the degree of pertinence from the decision maker.

Thus, as in F-TODIM, the input scale in THOR with Fuzzy logic was defined under a triangular fuzzy number. According to Liu et al. (2014), these numbers have a high performance in modeling the uncertainty associated with the decision process. Junior (2018) adds that these present a linear utility function, simplifying the execution of the calculations. Furthermore, its computational implementation presenting quality results is easy.

According to Van Laarhoven and Pedrycz (1983), the base triangular fuzzy number δ is defined according to formalism (4):

$$(b-\delta, b, b+\delta) \tag{4}$$

where b is the value of the weight stipulated by the decision maker with the highest degree of pertinence.

As per Jun Li (1999), a triangular fuzzy number $A = (a,b,c)$ is a fuzzy R subset with a piecewise linear association function μ_A defined according to formalism (5):

$$\mu_A = \begin{cases} \frac{x-a}{b-a}, a \le x \le b \\ \frac{c-x}{c-b}, b \le x \le c \\ 0, \ otherwise \end{cases} \tag{5}$$

where $a = b - \delta$ and $c = b + \delta$.

The triangular fuzzy number, as shown in Figure 2, has the following geometric representation.

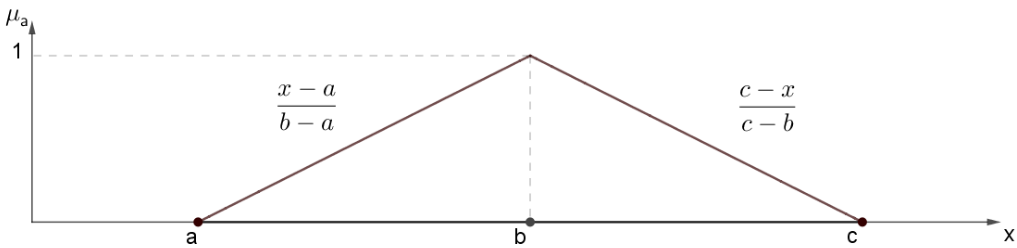

Figure 2 – Triangular Fuzzy number.

Source: Authors.

After the choice and definition of the fuzzy number it is necessary to establish the number of classes on the fuzzy scale. Thus, the review of research that applies the THOR methodology finds that Gomes et al. (2008) and Cardoso et al. (2009) use six cardinal points with a range from 1 to 11; Jun Li (1999) makes a classification using five categories; Rouyendegh and Erol (2012) uses six categories with a weighting from 1 to 9; and Zandi and Roghanian (2013) apply seven categories ranging 0 to 9.

In order to maintain the characteristics of the scales applied in THOR, it was decided that the THOR with fuzzy data input will have six classes, using a weighting beginning at 1 and ending at 11. According to Van Laarhoven e Pedrycz (1983), the construction of the fuzzy scale requires a value for the base δ of the triangular fuzzy number. Thus, through simulation the value $\delta = 2$ was established. The summary of the analysis obtained for the three values $\delta = 0.5$, $\delta = 1.0$, and $\delta = 2.0$ are presented in Tables 3, 4, and 5.

For the construction of the scale, as we see taking the value $\delta = 0.5$ in Figure 3, there are no intersections between the triangular fuzzy numbers, i.e., the classes have no points in common. The cardinal values, 2, 4, 6, 8, and 10 are not represented in any of the classes obtained. Therefore, the decision maker has no choice options for these values.

Table 3 – Summary of the analysis of the base of the triangle for 0.5.

Value	Fuzzy scale application	
0.5	Class 1	$(0.5, 1, 1.5)$
	Class 2	$(2.5, 3, 3.5)$
	Class 3	$(4.5, 5, 5.5)$
	Class 4	$(6.5, 7, 7.5)$
	Class 5	$(8.5, 9, 9.5)$
	Class 6	$(10.5, 11, 11.5)$

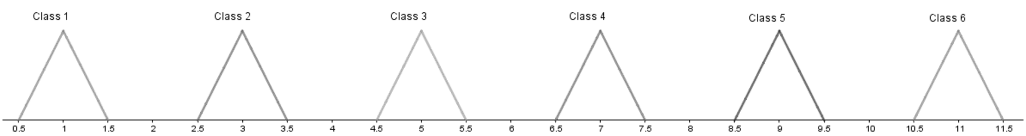

Figure 3 – Fuzzy scale with 0.5 base.

Source: Authors.

For $\delta = 1.0$, we note in Figure 4 the absence of intersections between the values 1, 3, 5, 7, 9, and 11, as there are only intersections for even values. Moreover, Table 4 and Figure 4 show that the number of classes defined is exceeded, because the last class does not contain the cardinal 11. Therefore, to establish $\delta = 1.0$ within the interval from 1 to 11, it would be necessary to insert new classes. However, this insertion would make the decision process more arduous and perhaps more confusing.

Table 4 – Summary of the analysis of the base of the triangle for 1.0.

Value	Fuzzy scale application	
1.0	Class 1	$(1, 1, 2)$
	Class 2	$(1, 2, 3)$
	Class 3	$(2, 3, 4)$
	Class 4	$(3, 4, 5)$
	Class 5	$(4, 5, 6)$
	Class 6	$(5, 6, 7)$

It was also observed that for $\delta = 1.5$, the number of classes was also exceeded.

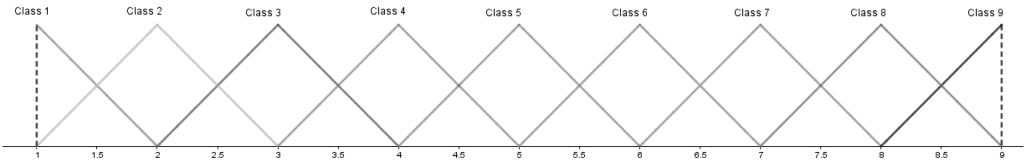

Figure 4 – Fuzzy scale with 1.

Source: Authors.

Thus, from the analysis of the value $\delta = 2.0$ in Table 5 and Figure 5, we note the existence of all the intersections necessary for the construction of the scale. As the even values are included, there is no need for the inclusion of new classes. Therefore, this scale can be used in the fuzzy algorithm, thus implying a better classification by decision makers.

Table 5 – Summary of the analysis of the base of the triangle for 2.0.

Value	Fuzzy scale application	
2.0	Class 1	$(1,1,3)$
	Class 2	$(1,3,5)$
	Class 3	$(3,5,7)$
	Class 4	$(5,7,9)$
	Class 5	$(7,9,11)$
	Class 6	$(9,11,11)$

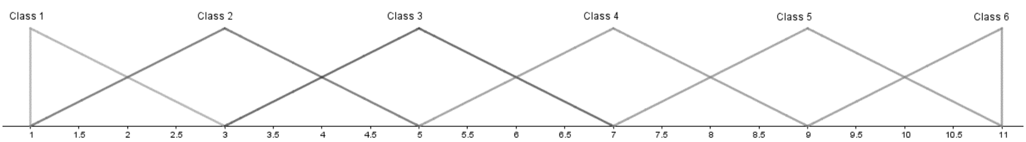

Figure 5 – Fuzzy scale with 2.

Source: Authors.

Thus, in the definition of the fuzzy scale, the decision makers will attribute the weights for value judgements through a parity comparison. Subsequently, it is necessary to determine the relevance of the judgment. Importantly, the determination of the pertinence of the judgment is no longer the responsibility of the decision maker, as in the THOR algorithm. In the modified algorithm, when the decision maker assigns the weights, the pertinence function defined in (5) is determined automatically. In other words, the weight is fuzzified, i.e., is transformed into a triangular fuzzy number. Fuzzy inference is responsible for aggregating the decision matrices of the decision makers through operations with fuzzy numbers. It should also be noted that the modified algorithm allows the decision maker to choose two forms of judgment, quantitative or qualitative.

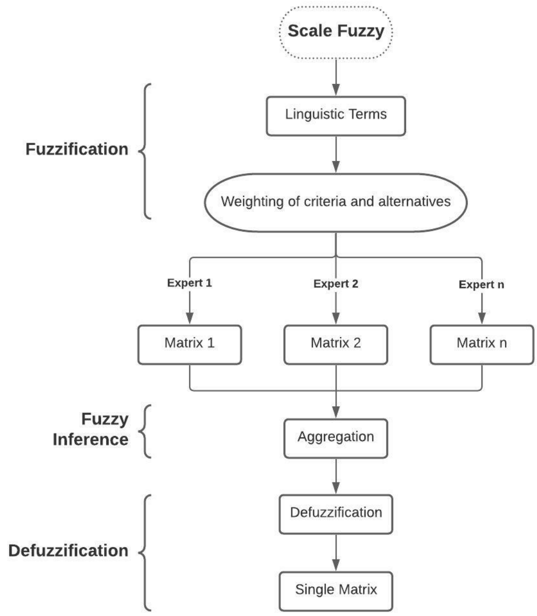

Figure 6 – The fuzzy scale and its processes.

Source: Authors.

As the goal is to update only the data input, the existing mathematical processes in THOR will be maintained without having to adapt them to fuzzy operations, that is, processes such as the construction of scenarios S_1, S_2, and S_3, defined respectively by (1), (2) and (3), are unchanged. Therefore, it is necessary to include a defuzzification step. According to Ghadimi et al. (2018), the results of defuzzification are the weights of the criteria defined in relation to the evaluation of decision makers. Bajestani et al. (2018) add that the results obtained become sharper, due to the conversion of the fuzzy number to a crisp number.

Triverdi et al. (2017) define defuzzification as the new centroid method, through the following formalism (6).

$$D_A = \frac{1}{3}\sqrt{(a+b+c)^2+1} \tag{6}$$

where a, b, and c are the values of the triangular fuzzy number stipulated by the decision maker.

Two important factors were considered when choosing the new centroid method. The first related to Longaray et al. (2019), who demonstrated statistically that the method, when applied to both consistent and inconsistent matrices, presents significant applicability. The second factor is the simplicity of the computational implementation of the method.

To illustrate the inclusion of fuzzy scale in THOR method, Figure 6 presents the steps of the fuzzy modeling process in the method data input.

In addition, the THOR algorithm with fuzzy data input determines the value of disagreement by the maximum total amplitude of judgments and no longer by the value informed by the decision maker, as in the THOR algorithm. Another modification occurs in the definition of the preference (p) and indifference (q), which will be respectively $\frac{2}{3}$ and $\frac{1}{3}$ of the standard deviation of the population of all judgments. The classifications follow THOR methodology, as shown in Section 2.

The other steps follow the same process as THOR, as can be seen in Figure 7, which presents a flowchart of the main steps of the new algorithm.

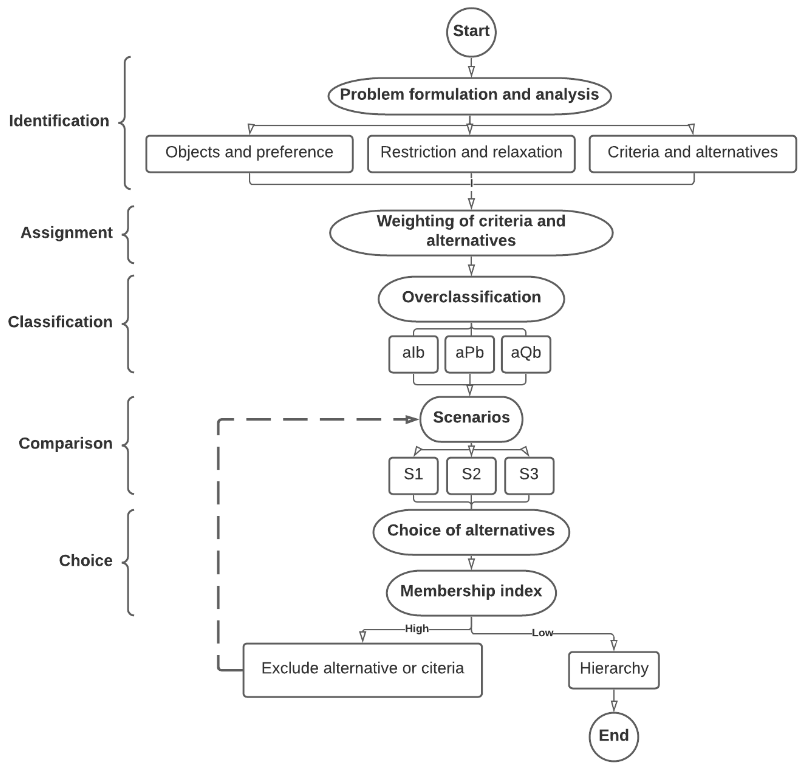

Figure 7 – THOR steps with the fuzzy scale.

Source: Authors.

It is important to emphasize that for the tests with the new method, a spreadsheet was developed that presents all the steps mentioned in this subsection.

The next section is devoted to the results of a comparison of the THOR and THOR Fuzzy algorithms.

4 RESULTS

The spreadsheet was developed to optimize the decision making process without sacrificing quality. For the process to be more accessible, the spreadsheet developed offers the decision maker two options to insert their judgments, which are: qualitative, via linguistic terms, and quantitative, with reference to the terms of each class, as can be seen in Figure 8. It is important to emphasize that the spreadsheet is in to Brazilian Portuguese.

1	Judgment	Abbreviation	Fixed Value	Class	Fuzzy Number
2	Indifferent	I	1	1	1,1,3
3	Reasonably Important	R	3	2	1,3,5
4	Considerably Important	C	5	3	3,5,7
5	Important	IM	7	4	5,7,9
6	Strongly Important	S	9	5	7,9,11
7	Extremely Important	E	11	6	9,11,11

Figure 8 – Data input.

Source: Authors.

Data can also be entered combining both aforementioned ways, as shown in Figure 9.

Alt/Crit	Criterion 1	Criterion 2	Criterion 3	Criterion 4
A1	IM	4	5	R
A2	5	3	5,5	7
A3	IM	4	5	2
Weights	6	5	4	2

Figure 9 – Spreadsheet judgments.

Source: Authors.

The results of the fuzzy algorithm will be compared with those of Gomes et al. (2001). In this work, the authors present an example using four criteria and three alternatives, as shown in Table 6.

Table 6 – Example Data.

Alternative / criterion	C_1	C_2	C_3	C_4
a_1	7	4	5	3
a_2	5	3	5,5	7
a_3	5	4	5	2
Weights	6	5	4	2

Table 6 represents Steps 5 and 6 of the algorithm. Initially, the fuzzification was performed on these data according to the scale defined in Table 5. For each triangular fuzzy number in Table 5, a pertinence value is determined. Figure 1 shows the spreadsheet developed for the calculations.

Delta	2							
	TRIANGULAR FUZZY NUMBER				VALUE		MEMBERSHIP FUNCTION	
1	1	1	3		4,7			0,85
3	1	3	5					
5	3	5	7		CATEGORY			
7	5	7	9		3			
9	7	9	11					
11	9	11	11					

Figure 10 – Spreadsheet.

Source: Authors.

Subsequently, the defuzzification of the judgments of value was performed through equation (6). The results are presented in Table 7.

Table 7 – Defuzzified data for the example.

Alternative / criterion	C_1	C_2	C_3	C_4
a_1	7,007932	4,013865	5,011099	3,018462
a_2	5,011099	3,018462	5,510092	7,007932
a_3	5,011099	4,013865	5,011099	2,027588
Weights	6,00952	5,011099	4,13865	2,02588

In Steps 7 and 8, the discordance values are determined for the limiters p and q. Table 8 shows the results obtained by both algorithms.

Table 8 – Veto values.

Veto	THOR	THOR + Fuzzy Scale
Disagreement	4	4,013865
p	0,9	0,959745
q	0,9	0,959745

Once judgements and vetoes have been established, the classification of alternatives is conducted by building three scenarios, as described in Section 3 of this study. We note that there is no difference between the methods in the ranking of alternatives.

The spreadsheet developed to obtain the classifications was programmed so that the calculation of the difference between the judgments of two different alternatives in relation to the same criterion was determined in two ways: one through the module and the other through any real value. Figure 11 shows the spreadsheet with the classifications obtained thereby.

Scenario S1											
	a1Pa2	>	a1Qa2	+	a1Ia2	+	a2Pa1	+	a2Qa1	+	a1Ra2
	6,00952	>	5,0111	+	4,01387	+	2,02759	+	0	+	0
	a1 does not dominate a2										
Scenario S1											
	a2Pa1	>	a2Qa1	+	a1Ia2	+	a1Pa2	+	a1Qa2	+	a2Ra1
	2,02759	>	0	+	4,01387	+	6,00952	+	5,0111	+	0
	a2 does not dominate a1										
Scenario S1 completion		a1 and a2 will be the same for the decision maker									

Figure 11 – Classification for alternatives a_1 and a_2.

Source: Authors.

The ranking is obtained from the scenarios, as mentioned in Section 2, Step 9. The scenario results S_1, equation (1), with the comparisons of the alternatives a_1 and a_2 is presented in Figure 12.

Scenario S1											
	a1Pa2	>	a1Qa2	+	a1Ia2	+	a2Pa1	+	a2Qa1	+	a1Ra2
	6,00952	>	5,0111	+	4,01387	+	2,02759	+	0	+	0
	a1 does not dominate a2										
Scenario S1											
	a2Pa1	>	a2Qa1	+	a1Ia2	+	a1Pa2	+	a1Qa2	+	a2Ra1
	2,02759	>	0	+	4,01387	+	6,00952	+	5,0111	+	0
	a2 does not dominate a1										
Scenario S1 completion		a1 and a2 will be the same for the decision maker									

Figure 12 – Exemplification of the scenario S_1.

Source: Authors.

Table 9, from the pairwise comparisons between the alternatives, presents the three scenarios that were obtained by the THOR methodology.

Table 10, from the parity comparisons between the alternatives, shows the three scenarios that were obtained by the THOR method with fuzzy data entry.

In tables 9 and 10, it is noted that the parity comparisons between the alternatives, for scenarios S_1 e S_2, expose the supremacy of alternatives considered equal by the decision maker, resulting in a value of 0.5 for the aforementioned positions in the decision matrix. After the sum of the rows for each matrix of scenarios, the result is hegemony in equality, thus guaranteeing an absence of ranking among the alternatives.

In Tables 9 and 10 we note that both the THOR methodology and the THOR Fuzzy methodology present the same rankings, i.e., there are no alternatives with overranking over the others. In the S_3 scenario, the two methodologies result in a ranking that suggests the dominance of the

Table 9 – Decision matrix for scenarios.

S_1	a_1	a_2	a_3	Score
a_1	0	0,5	0,5	1
a_2	0,5	0	0,5	1
a_3	0,5	0,5	0	1
S_2	a_1	a_2	a_3	Score
a_1	0	0,5	0,5	1
a_2	0,5	0	0,5	1
a_3	0,5	0,5	0	1
S_3	a_1	a_2	a_3	Score
a_1	0	0,5	1	1,5
a_2	0,5	0	0,5	1
a_3	0,52	0,5	0	1,02

Table 10 – Decision matrix for scenarios obtained by THOR with fuzzy data input.

S_1	a_1	a_2	a_3	Score
a_1	0	0,5	0,5	1
a_2	0,5	0	0,5	1
a_3	0,5	0,5	0	1
S_2	a_1	a_2	a_3	Score
a_1	0	0,5	0,5	1
a_2	0,5	0	0,5	1
a_3	0,5	0,5	0	1
S_3	a_1	a_2	a_3	Score
a_1	0	0,5	1	1,5
a_2	0,5	0	0,5	1
a_3	0,52895	0,5	0	1,02895

alternative a_1, followed by the alternatives a_3 and a_2. However the differences presented by the fuzzy methodology are greater than by the THOR methodology, in part due to working with fuzzy numbers, which are converted into crisp numbers, taking into account the problems of ambiguity and imprecision for each trial.

Finally, the S_3 scenario was the only one that presented options of choice for the decision maker. Thus, neither method provides an optimal solution, but rather a ranking of alternatives.

5 FINAL CONSIDERATIONS

The vast majority of multicriterial tools developed employ the need for judgement on the part of decision makers who often give up using them due to the complexity of handling. Thus, robust methodologies cease to be utilized given the non-accessibility of the same on the part of managers. In this sense, the work presents an important contribution with respect to the indication of fuzzy logic to make more accessible the use of the THOR tool in the decision making process.

In addition, the work contributes to the construction of a fuzzy scale, where the "parameter under analysis" (base of the triangle), the fuzzy number, needs to have a range of variation such that a number within this range must have a higher possibility of occurrence than in other ranges; the construction of a fuzzy scale with six classes for categorization affords the classification system autonomy for the decision maker and increased accuracy, expressiveness, diversity, and subjectivity, which is invaluable to the decision-making process; the use of the centroid method of defuzzification, which was successfully applied by Kaufmann and Grupta (1988), and Chang (1981), was incorporated by the authors of this work.

Another relevant aspect of updating the THOR algorithm is that in this reformulation the decision maker needs to weigh only one judgment for each criterion and alternative, and the insertion of weighting values and limiting values, in addition to their respective degrees of relevance, is no longer under their responsibility.

However, despite this update bringing significant improvements facilitating the cognitive process, the algorithm has a limitation regarding data entry. Both algorithms, THOR or THOR 2, present good performance when they allow the user not to insert the weights. However, the idea obtained, to develop the THOR algorithm with fuzzy scale, consists in the construction of a fuzzy triangular number from the decision maker's weighting, which is an indispensable presupposition that must be fulfilled. If this assumption is not satisfied, the present algorithm cannot be used.

Regarding the comparison of the algorithms, in the first two orders the results were equivalent, both in the hierarchy and in the weights. The third scenario yielded the same hierarchy; however, the results of the weights showed a slight difference in the third decimal place. This does not invalidate the results, as the difference can be justified by the insertion of the fuzzy scale, as a different way of obtaining the veto values.

Finally, the results obtained through the comparison of the two algorithms are encouraging, suggesting the inclusion of fuzzy logic as an ally in multicriteria methodologies, specifically in the THOR methodology.

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