Pesquisa Operacional (2022) 42(nspe1): e263499 p.1-15
doi: 10.1590/0101-7438.2022.042nspe1.00263499

Printed version ISSN 0101-7438 / Online version ISSN 1678-5142
www.scielo.br/pope
ARTICLES

PROPOSAL OF AN OPTIMAL REDUNDANCY AND RELIABILITY ALLOCATION APPROACH FOR DESIGNING COMPLEX SYSTEMS

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Received April 27, 2022 / Accepted August 24, 2022

ABSTRACT. Problems of optimal reliability and redundancy allocation are not new. However, the literature describes two isolated situations, either when redundancy allocation is the target and the reliability metrics of the components are known, or focusing on the reliability allocation, that demands the metrics to be known in advance and the use of some optimization approach. In the present paper, we consider a combined situation, i.e., when both situations must be considered together. To guide the allocation process, we developed a cost function that considers the costs of acquisition, development and/or improvement as a function of monetary effort, along with the reliability target and expected failure costs. The results considering two classical test problems in the literature demonstrated the efficacy of the proposed approach to deal with similar situations.

Keywords: optimal reliability allocation, optimal redundancy allocation, reliability optimization.

1 INTRODUCTION

Observing the technological advances, the complexity of systems has increased in parallel with the expected performance of these systems and their components. Therefore, problems associated

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with optimizing reliability are increasingly important in the development of engineering designs as stated by Coit & Zio (2019). The design of reliable systems is subject to various challenges due to limitation of resources. Among the ways to improve reliability are: (i) intrinsic reliability allocation in component level; (ii) redundancies allocation (active, standby or mixed); and (iii) allocation of redundancy and intrinsic reliability simultaneously. In standby redundancy, inactive components are allocated in the system and a process of monitoring and switching is used to activate the standby component when the active one fails. In cases of active redundancy, the redundant component has identical conditions as the main one (both operating), while mixed redundancy situations involve a mixture of active and standby redundancy, as presented in Kuo & Zuo (2003) Ardakan & Hamadani (2014) and Ardakan et al. (2016).

Due to the high complexity of modern systems, in terms of configuration and heterogeneity, the optimization of reliability is becoming increasingly difficult. According to Mellal et al. (2020) and Coit & Zio (2019), the evolution of the approaches for reliability optimization can be classified into three eras: (i) that of mathematical programming; (ii) that of pragmatism; and (iii) that of active reliability improvement. In this evolution, the use of metaheuristics has enabled the modeling of practical problems that are ever nearer to reality.

Sakawa (1978) originally proposed a multi-objective approach to deal with problems of allocating redundancy and reliability. More recently, Garg et al. (2014) presented a bi-objective approach called Particle Swarm Optimization (PSO), in which reliability is treated as a triangular fuzzy number in one of the objective functions, while the other objective function reflects the associated costs. Zoulfaghari et al. (2014) proposed a bi-objective approach in which some components were reparable and others were not. Ardakan et al. (2015) suggested a bi-objective approach in which the reliability function considers the mixed approach for redundancy allocation and the second objective function is a cost function that considers the acquisition of components and switches. Qiu et al. (2017) proposed a bi-objective approach based on evolutionary computation to maximize reliability and minimize the costs associated with the redundancy allocation. Ardakan & Rezvan (2018) put forward a multi-objective approach, treated by means of NSGA-II in which reliability is modeled by a continuous Markov process to deal with cases of standby redundancy. Finally, Dobani et al. (2019) proposed a multi-objective approach in which the components allocated for redundancy are not necessarily homogeneous. Their optimization process is instead based on a stochastic fractal search (SFS) algorithm.

In all the cases mentioned above, the authors considered problems with known reliability metrics of the components, and then searched for an optimal redundancy allocation; or problems with unknown reliability metrics, where the optimization process involved searching for a minimum reliability requirement and the number of redundancies to be allocated. To the best of our knowledge, no work in the literature has yet covered a problem in which the reliability metrics of some components are known and the others are unknown, involving application of a mixture of the approaches presented. The type of problem addressed in this study has a complicating factor in the formulation of the cost function, because it is necessary to consider the cost of developing or improving the technology. Another matter not addressed in the articles mentioned above is the failure cost, i.e., the expected cost of the failure of a subsystem.

In light of the above arguments, here we present an approach for optimal allocation of redundancy and reliability in which the system has known components with known reliability metrics, and others that need to be developed or improved, so as to contemplate the expected development costs and the expected cost of failure. Hence, this involves optimal allocation of systems in the conceptual or design phase Blanchard (2008). This is as important approach for system engineering, as in Oil & Gas field development, projects of new autonomous vehicle, and any other complex system that can be developed.

To achieve the intended objectives, this paper is structured in the present introduction section, followed by a theoretical framework in Section 2, the methodology development in Section 3, application cases in Section 4, and the conclusions are presented in the 5 Section.

2 THEORETICAL FRAMEWORK

2.1 Optimal Allocation of Redundancy and Reliability

Among the ways to increase the reliability of systems, the inclusion of redundancies has been a common topic of research into optimized solutions. The main question addressed is: How many components should be allocated for redundancy and what levels of reliability must be attained by the components under development to maximize the system's reliability while minimizing the expected costs of acquisition, development and the expected failure costs? This question has been studied for over six decades. Black & Proschan (1959), Kettelle Jr (1962) and Proschan & Bray (1965) all proposed exact methods for the problem of redundancy allocation involving different types of constraints. Studies in the following two decades began to consider combined problems, i.e., to determine the level of reliability of a given component of a system and the number of redundancies to be allocated. This problem, according to Tillman et al. (1977), can be modeled as a nonlinear mixed programming problem in which the system's reliability must be maximized as a function of the level of reliability of its components and the number of redundancies to be considered in each of its stages, with restrictions of cost and physical characteristics being commonly considered. Traditionally, two types of redundancy have been considered, cold standby and hot standby. The cold standby case assumes that the backup component is not activated until the main component fails, while in the hot standby case the backup component is always activated so it is subject to the same stress as the principal component. According to Kuo & Zuo (2003) there is also an intermediate situation called warm standby, where the backup component's failure rate is deemed to between the failure rates of cold standby and hot standby components. As an evolution of these traditional approaches, Ardakan & Hamadani (2014) and Ardakan et al. (2016) proposed a hybrid approach involving a mixture of hot and cold standby components. From the standpoint of mathematical modeling, equation (1) defines the analysis of active redundancy:

$$R_{Hstb}(t) = \prod_{i=1}^{S}(1-(1-r_i(t)))^{n_{Ai}} \tag{1}$$

where: $r_i(t)$ is the reliability of the components (assumed equal) of the i-th subsystem, n_{Ai} is the number of active redundancies of the i-th subsystem, and S is the number of subsystems connected in series.

Equation (2) applies to the case of subsystems with mixture standby redundancy:

$$R_{Cstb}(t) = \prod_{i=1}^{S}\left[r_i(t) + \theta_i(t)\sum_{j=1}^{n_{si}-1}\int_0^t r_i(t-u) f_i^{(j)}(u)du\right] \tag{2}$$

where: $r_i(t)$ is the reliability of the active component; $\theta_i(t)$ is the switching reliability, $f_i^{(j)}$ is the probability density function associated with the occurrence of the j-th failure of subsystem i (in turn associated with the failures of the active and redundant components, assumed to be equal), and n_{si} is the number of redundant components (with this number considering the active component plus the redundant ones on standby, so that $n_{si}-1$ is the upper limit of the summation).

These models have been used to investigate the problem of joint redundancy and reliability allocation. The optimization method should specify the type of redundancy to be employed (active or standby), and how many will be allocated to each subsystem, besides indicating the reliability level of the components. The possibility also exists of considering a mixture of redundancy types, i.e., some active and other standby as proposed in Coelho (2009), Ardakan & Hamadani (2014), Ardakan et al. (2016), Muhuri & Nath (2019), Juybari et al. (2019), Mellal et al. (2020), and Peiravi et al. (2020). In all the cases mentioned so far, the aim has been to find the optimal reliability of a system subject to some constraints. In general, these restrictions are associated with the cost and physical characteristics (size and weight). The cost function most often applied is that described by Tillman et al. (1977) given by equation (3) below:

$$TotalCost(R,n,\alpha,\beta) = \sum_{i=1}^{S}\alpha_i\left(\frac{-t}{\ln(R_i)}\right)^{\beta_i}\left[n_i + \exp\left(\frac{n_i}{4}\right)\right] \tag{3}$$

In this equation, the total cost has a term associated with the level of reliability, $n_i \times \alpha_i\left(\frac{-t}{\ln(R_i)}\right)^{\beta_i}$, and another associated with the interplay of the redundant components, $\alpha_i\left(\frac{-t}{\ln(R_i)}\right)^{\beta_i} \times \exp\left(\frac{n_i}{4}\right)$. In these terms, α_i and β_i are characteristics of the components, namely scale and shape factors. Other simpler models, associated exclusively with the acquisition costs, can be specified according to equation (4), with a few variations.

$$TotalCost = \sum_{i=1}^{S}(c_{iz_i}n_i + c_{switch,i}) \tag{4}$$

In this equation, c_{iz_i} is the cost of the type z component of the i-th subsystem, n_i is the number of redundancies considered for the i-th subsystem, and $c_{switch,i}$ is the cost of the switching component of the i-th subsystem.

Considering the functional structure of the above mentioned equations, evolutionary computing has been the commonly adopted approach to reach optimized solutions, and genetic algorithms is the base-line to all of them.

2.2 Genetic Algorithms

Genetic Algorithms (GAs) and their variants have been employed to resolve problems of the *NP*-Hard and *NP*-Complete classes since the 1970s with Holland (1975). These algorithms are based on the concept of the evolution of biological species, in which only the fittest are used to create the elements of the next generation. This evaluation of fitness, which reflects the environment in which these "individuals" are inserted, is accomplished by means of objective functions.

The versatility and simplicity of the concepts contained in the GA framework has led to the creation of various algorithms inspired by these concepts. According to Talbi (2009), this type of algorithm should contain at least the following steps: 1) initialization of the population; 2) evaluation of each individual of the population according to the objective(s); 3) generation of new individuals from the elements of the population, through crossover and mutation; and 4) selection of individuals to compose the next generation. Steps 2 to 4 compose an iteration of the method, which continues until a stopping criterion is reached. The population is the set of solutions stored by the algorithm. In the context of GAs, a solution to the problem is called an individual, and the population is composed of multiple individuals.

The creation of new individuals entails the application of genetic operators. The best known are those that accomplish crossover and mutation. In crossover, two or more individuals are chosen and the parts that compose them (genes) are selected and inserted into a new individual, called offspring. In mutation, normally part of an individual is altered seeking to introduce modifications in the population.

One of the variants of GAs with greatest presence in the literature is the Biased Random Key Genetic Algorithm (BRKGA). In this algorithm, unlike traditional GAs, the representation of the solution is composed of random real numbers, a characteristic that allows the crossover operator to produce valid new individuals, which is not always possible using binary or integer encoding. This representation via random numbers is indirect, so it is necessary to convert the random sequence into a solution within the search space. For interpretation of the random values, a decoding function is used, which is responsible for interpreting the random values that compose the individual, thus identifying the solution to the problem. In the crossover operation, the BRKGA normally chooses an individual of the elite set and another from the remainder of the population. Once all the generation solutions have been defined, a strategy based on parameterized uniform crossover is utilized. Mutation takes place by substitution of one or more individuals of the population by a new randomly generated individual, permitting the algorithm to escape from local optima. The next generation is composed of three portions: the first consists of the best chromosomes of the current generation (elite set), the second is composed of the solutions generated by the crossovers, and the third is composed of mutants as in Gonçalves & Resende (2011).

This algorithm is widely used in the literature and has obtained good results in various types of single-objective and multi-objective problems, as is our case here, where a single solution may not be sufficient to represent all the objectives established, so a diversified Pareto optimal set is sought Li et al. (2015), represented by means of a frontier as in Prasetyo et al. (2018) Martínez et al. (2011), Gonçalves & Resende (2011), Deb et al. (2002), Emmerich & H (2018), and Katoch et al. (2021).

3 METODOLOGY

In this article we consider, without loss of generality, that the redundant components will be allocated actively. Thus, we consider the reliability function of a system commonly employed in the literature, equation (1). The cost function is composed of three terms: (i) acquisition cost, (ii) cost of development or improvement, and (iii) expected cost of failure of the subsystem.

$$CT = \sum_{i \in A} C_{aq,i} + \sum_{j \in D} C_{d,j}(R_j(t)) + \sum_{\forall k \in S} CF_k \tag{5}$$

In equation (5), the first term, $C_{aq,i}$, which is known, refers to the acquisition cost of subsystem i, in other words, the number of redundancies envisioned, such that $C_{aq,i} = c_{a,i}m_i$, where m_i is the number of redundant components to be allocated in subsystem i. The second term in equation (5) is referred as the development costs, i.e., is the estimated cost for improvement of an existent technology, or the cost for completely develop a new one. This term is dependent of the reliability target $R_j(t)$, that can be previously stated by the designer (project man), or it can be established by the optimization process. It must be observed that the development, or improvement, is reliability-driven, meaning that the higher the reliability requirement the higher the cost for development or improvement. The third term of equation (5) refers to the expected cost associated with failure of a specific subsystem. Considering the redundancies allocated to it, i.e., for complete subsystem k we have:

$$CF_k = (1 - R_k(t))C_k \tag{6}$$

where: $R_k(t)$ denotes the reliability of subsystem k over mission time t, and C_k is the cost associated with its failure, considering all the redundancies allocated in it. Notice that in the estimated cost C_k we can considers, among any others things, the environmental impacts and lost profit.

The development, or improvement, costs are specially common in Oil & Gas industry, when a new oil field, in different geological environment, must be explored, some new or improved technologies must be considered in upstream projects.

In the second term of equation (5), which refers to the cost of development or improvement of an existent component, consideration is given to the variation of the failure rate, which depends on a monetary effort function, proposed by Yadav & Zhuang (2014), given by the following formulation:

$$\frac{\partial \lambda(t)}{\partial c} = -\rho \lambda(t) \tag{7}$$

where: c is the monetary effort, and ρ is an effort rate. In other words, the higher the effort is, the greater will be the reduction in the failure rate. By resolving this differential equation for the average failure rate, considering an initial condition $(c_0, R_0(t))$ - which is generally obtained from the manufacturer/developer through negotiation – for a representation in a reliability function, the following development or improvement cost function is obtained as:

$$C_{d,j} = C_{0,j} - \rho \ln\left(\frac{\ln(R_j(t))}{\ln(R_{0,j}(t))}\right), R_j(t) \geq R_{0,j}(t) \tag{8}$$

where $C_{d,j}$ is the cost associated to the development or improvement of component j with target reliability $R_j(t)$. $C_{0,j}$ is the cost associated to the initial reliability $R_{0,j}(t)$, and ρ is as previously defined. Initial reliability and its associated cost can be obtained based on existent technology, and ρ can be obtained through previous experience within credited suppliers/developers.

With this, the total cost function's three terms are: (i) the acquisition cost of the known components and the number of redundancies, (ii) the cost of developing/improving the components (considering that the development contracts include the number of components to be supplied), and (iii) the costs of failure of all the subsystems with their respective redundancies. With this, the problem to be resolved is expressed as follows:

$$\{Max\, R_s, Min\, CT \mid 0 \leq R_k(t) \leq 1\} \tag{9}$$

where R_s is the reliability of the entire system, and CT is the total cost.

Because of the large number of articles in the literature that have used the BRKGA in this kind of problem, we decided to use an adaptation of the meta-heuristic to the problem described here. In the implementation, we followed the concepts established in Gonçalves & Resende (2011) and Deb et al. (2002).

The encoding is given by a vector with $2n$ positions containing random numbers in the interval $[0,1)$, with n being the number of subsystems analyzed. For the purpose of decoding, the first n positions denote the reliability of each subsystem. In the subsystems for which this reliability is known, the respective positions are simply filled with the input reliability. In turn, for subsystems in which the reliability needs to be determined, the number inserted by the algorithm is treated directly as reliability. The positions from $n+1$ to $2n$ specify the redundancy. Since one of the input parameters is the maximum number of components permitted in each subsystem, it is possible to calculate ranges within the interval $[0,1)$ to represent whole numbers. For example, considering that a subsystem permits 2 components, the interval $[0,1)$ can be partitioned into two sub-intervals: $[0,\frac{1}{2})$ and $[\frac{1}{2},1)$. Thus, numbers smaller than one-half mean the subsystem has only one component, and if not, the subsystem has two components.

We utilized two objective functions: (i) system reliability, as described by equation (1); and (ii) subsystem cost, as specified by equation (5). The selection of the individuals for crossover is accomplished randomly in two groups as in Gonçalves & Resende (2011): one is chosen from the elite set and the other is chosen from the remaining population. To define which solutions belong to the elite set, we used the concepts of the Non-dominated Sorting Genetic Algorithm II (NSGA-II) proposed by Deb et al. (2002). First, the size of the set is defined as a percentage of the population. Then the solutions are sorted according to their dominance (a solution A dominates a solution B if A is better than B in at least one objective and not worse in the others). The solutions that are dominated by fewer solutions have preference over those that are dominated by more solutions. To distinguish between solutions with the same dominance, a second metric called crowding distance is used. This metric establishes the similarity of the solutions, causing proximate solutions to have lower priority to enter the elite set. This second metric is used when the set of solutions is greater than the number of openings in the elite set. In this paper, the population size was 15 times the number of subsystems analyzed, and the elite set was defined as being composed of 15% of the population.

The crossover operator selects two elements in the population, as described previously, and combines the genes of these chromosomes to create a new member for the following generation. Figure 1 depicts an example of the crossover operation performed by the algorithm. In it, the elite individual (chromosome 1) and non-elite individual (chromosome 2) are compared position-by-position according to a random number and a parameterized probability relation. If the random number is smaller than the value established by the probability relation, the offspring will inherit the value of the gene of the elite individual. The relation illustrated in the figure indicates that the offspring has a 70% chance of inheriting the chromosome of the elite individual. The value used in the design (60%) is found according to the literature and presented in Gonçalves & Resende (2011).

The mutation utilized here is the same one proposed by Gonçalves & Resende (2011), and the percentage of new individuals generated randomly (30%) is in line with the literature.

For composition of the elements of the next generation, the proposed algorithm also utilizes the proposal of Gonçalves & Resende (2011). As a stopping criterion, we employed the number of generations without improvement. This number was configured at 5 times the number of subsystems being treated by the algorithm. It is important to mention that the improvement is based on observance of the two objectives under analysis. If the iteration improves one of the objectives, it is considered to be an improvement iteration.

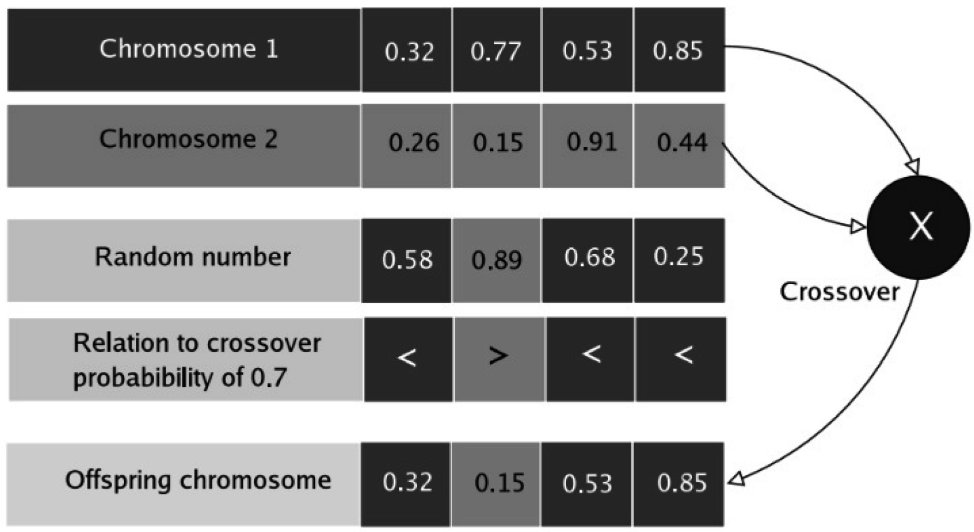

Figure 1 – Crossover operator.

Source: Adapted from Gonçalves & Resende (2011).

4 APPLICATION CASES

With the purpose of verifying the applicability of the proposed approach, below we study two cases from the literature (Valian & Valian (2013), Afonso et al. (2013), among others), with the proper considerations for the input data to satisfy the characteristics of the general problem addressed. The first test problem (Problem 1) consists of a parallel system in series, according to Figure 2a, while the second test problem (Problem 2) consists of a bridge system according to Figure 2b.

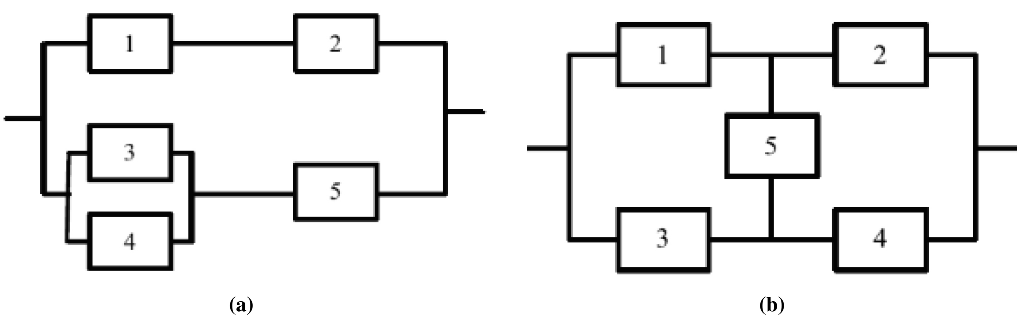

Figure 2 – Configurations of the systems for Problems 1 (a) and 2 (b).

The method was fully implemented in Octave version 6.1.0 and also adjusted with Matlab R2017a. The simulations were carried out in a laptop computer with i7-8750H processor with 2.20GHz and 16 GB of RAM. For test problems 1 and 2, we assumed that two components will be developed/improved. For each component, a maximum number of active redundancies m was permitted. Without losing generality, we considered the respective numbers of components to be the maximum number of redundancies allowed for each of them. Also without losing generality, we considered components 2 and 5 as those needing to be developed/improved. For the analyses, we considered a mission time of 1000 hours. For each component we considered an expected cost of failure CF, an acquisition cost c_a, and an initial cost c_0 for those to be developed or improved. The data of the two problems are displayed in Table 1.

Table 1 – Basic data for Problems 1 and 2.

Component	$R(t)^{(1)}$	$R0(t)^{(1)}$	c_0	c_a	CF	m	ρ
1	0.90	-	-	1	0.7	3	-
2	-	0.81	1	-	0.7	3	0.1054
3	0.85	-	-	2	0.7	3	-
4	0.83	-	-	3	0.7	3	-
5	-	0.83	2	-	1	3	0.0675

(1) Note that these reliability data can come from any probabilistic model, not being limited to the exponential distribution, which is commonly employed.

By applying the methodological procedure in Problems 1 and 2 with the data presented in Table 1, we obtained the Pareto frontiers shown in Figures 3 and 4.

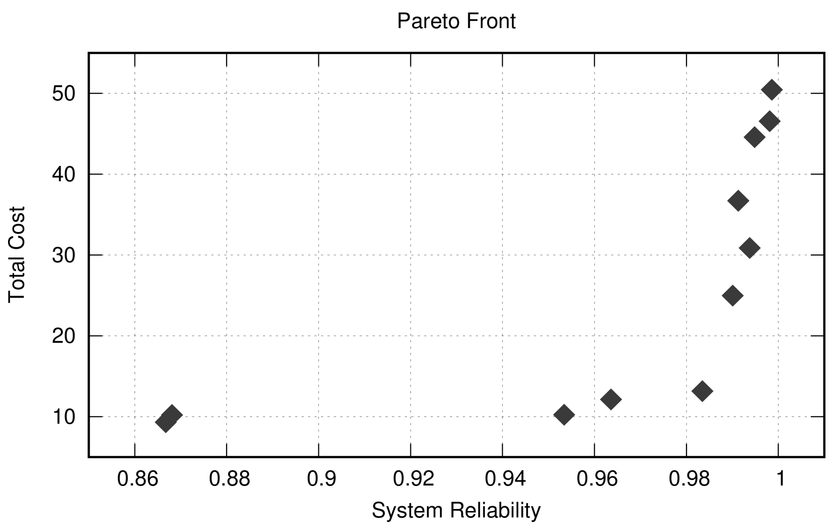

Figure 3 – Pareto frontier for Problem 1.

Based on the set of elite solutions, formed by 12 points on the frontier, we found that: (i) the least costly solution,\$9.3037, also presented the lowest reliability, 0.866755, (ii) the solution with the highest reliability, 0.998574, was also the solution with the greatest cost,\$50.4589, (iii) there was greater concentration of points on the frontier for reliability values greater than 0.98, (iv) the solution with the highest reliability was that considering all the possible redundancies for all the components, with the exception of component 3, while maintaining the reliability of

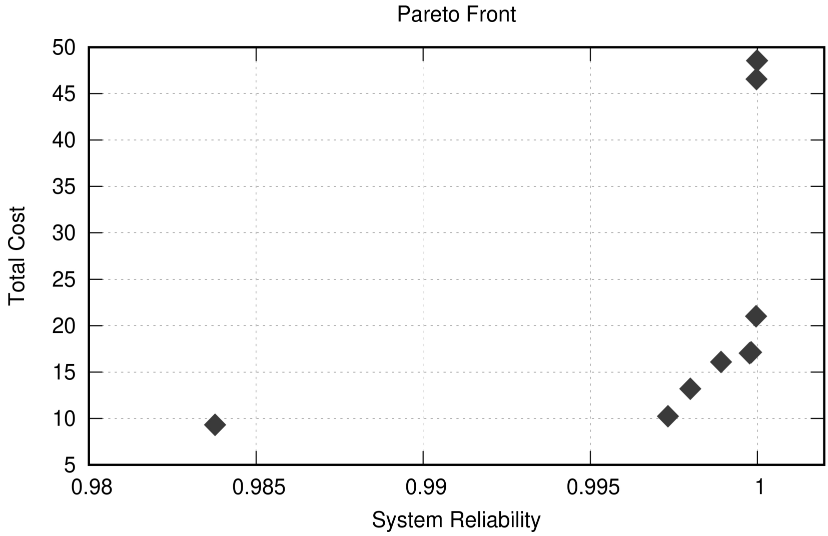

Figure 4 – Pareto frontier for Problem 2.

component 5 at its minimum value, i.e., raising the reliability of component 2 to 0.941272 with no effort to develop something better regarding component 5, and (v) the solution with least cost, and consequently lowest reliability, maintained all the components with minimum redundancy, except for components 2 and 5, which received the maximum redundancies permitted, so these components were maintained without improvements. Table 2 presents a summary of the solutions on the frontier for Problem 1.

Table 2 – Summary of the Pareto frontier for Problem 1.

Solution	Rs(t)	Total Cost	(n1, n2, n3, n4, n5)	(R2, R5)
1	0.998574	50.4589	(3, 3, 2, 3, 3)	(0.941272, 0.83)
2	0.998114	46.5611	(3, 3, 3, 1, 3)	(0.941272, 0.83)
3	0.994864	44.5744	(3, 3, 2, 1, 3)	(0.941272, 0.83)
4	0.993781	30.8609	(3, 3, 2, 3, 3)	(0.941272, 0.83)
5	0.991304	36.7169	(3, 3, 1, 3, 3)	(0.81, 0.951594)
6	0.991304	36.7169	(3, 3, 1, 3, 3)	(0.81, 0.951594)
7	0.990089	24.9764	(3, 3, 2, 1, 3)	(0.941272, 0.83)
8	0.983497	13.1452	(3, 3, 2, 1, 3)	(0.81, 0.83)
9	0.963603	12.1329	(3, 3, 1, 1, 3)	(0.825589, 0.83)
10	0.95343	10.2407	(2, 3, 1, 1, 3)	(0.81, 0.83)
11	0.868111	10.2022	(1, 3, 1, 1, 3)	(0.825589, 0.83)
12	0.866755	9.30371	(1, 3, 1, 1, 3)	(0.81, 0.83)

Considering Problem 2, based on the set of elite solutions, formed by 12 points on the frontier presented in Figure 4, we found that: (i) the least costly solution, \$9.3037, also presented the lowest reliability, 0.983777; (ii) the solution with highest reliability, 0.999996, was also the one presenting the lowest cost, \$48.5528; (iii) there was greater concentration of points on the frontier for reliability values greater than 0.996; (iv) the solution with the highest reliability was that considering all the possible redundancies for all the components, with the exception of component 4, while maintaining the reliability of component 5 at its minimum value, i.e., raising the reliability of component 2 to 0.993801 with no effort to develop something better regarding component 2; and (v) the solution with least cost, and consequently lowest reliability, maintained all the components with minimum redundancy, except for components 2 and 5, which received the maximum redundancies permitted, so these components were maintained without improvements.

Table 3 presents a summary of the solutions on the frontier for Problem 2.

Table 3 – Summary of the Pareto frontier for Problem 2.

Solution	Rs(t)	Total Cost	(n1, n2, n3, n4, n5)	(R2, R5)
1	0.999996	48.5528	(3, 3, 3, 1, 3)	(0.993801, 0.83)
2	0.999989	48.5527	(3, 2, 3, 1, 3)	(0.993801, 0.83)
3	0.999977	46.5661	(3, 3, 2, 1, 3)	(0.993801, 0.83)
4	0.999963	21.0162	(3, 3, 3, 3, 3)	(0.81, 0.83)
5	0.999963	21.0162	(3, 3, 3, 3, 3)	(0.81, 0.83)
6	0.999811	17.1189	(3, 3, 1, 3, 3)	(0.81, 0.83)
7	0.999767	17.0393	(2, 3, 3, 2, 3)	(0.81, 0.83)
8	0.998911	16.0909	(3, 2, 2, 2, 2)	(0.81, 0.83)
9	0.997996	13.2016	(2, 3, 1, 1, 3)	(0.85711, 0.83)
10	0.997325	10.2407	(2, 3, 1, 1, 3)	(0.81, 0.83)
11	0.983777	9.30371	(1, 3, 1, 1, 3)	(0.81, 0.83)
12	0.983777	9.30371	(1, 3, 1, 1, 3)	(0.81, 0.83)

5 CONCLUSIONS

The objective of this article was to present a new approach to the problem of optimal allocation of redundancy and reliability in situations where there is a mixture between components with known reliability metrics, but that need definition of redundancy, and components that need to be developed or improved. In other words, determined components will need to have their minimum reliability requirements defined along with the number of redundancies to be allocated to them.

Since previous studies have not discussed this category of problem, we developed, as a way to establish an equilibrium for the solutions, an approach with two objective functions, one for the system reliability and the other for the cost function. This last is a novelty in the literature and includes three macro-factors: (i) the first for the acquisition of the redundant components, which have defined reliability metrics but need to have redundancies allocated; (ii) the second including the cost of development or improvement, considering the numbers of redundant components to be supplied for the conceptual pilot project; (iii) and the third considering the expected cost of failures of the subsystems with their respective redundancy values. These last two factors make an important contribution to the design of new complex systems. The portion referring to the cost of development or improvement brings an estimate of costs for the development or improvement of parts of the system that needs to be developed. In turn, the portion referring to the expected cost of failure provides an estimate, for the operating time, of the financial impacts associated with possible failures of parts of the system. These information can be used, for example, to search for cheaper alternatives to the project respecting the reliability requirements.

To deal with this combinatorial problem, we considered a variation of the Biased Random Key Genetic Algorithm with Non-dominated Sorting Genetic Algorithm II, which presented satisfactory performance in terms of execution time for both problems. In other words, the Pareto frontiers were ascertained of both problems in under one minute. The solutions presented demonstrate the efficacy of the proposed approach to deal with this class of problems.

In the future, we intend to expand the problem, among others things, to cases of (i) mixtures of redundancy strategies, (ii) optimal maintenance scheduling for the mission time, and (iii) diversity of components for redundancy, as a way to avoid or reduce common cause failure.

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How to cite

GARCIA PAA, NEVES TA, JACINTO CMC, ALVAREZ GB, GARCIA VS & MOTTA GS. 2022. Proposal of an optimal redundancy and reliability allocation approach for designing complex systems. *Pesquisa Operacional*, **42** (nspe1): e263499. doi: 10.1590/0101-7438.2022.042nspe1.00263499.