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Academic Performance of 9th graders on Spatial Geometry: Impact of personal and contextual variables

Desempenho académico de alunos de 9º ano na Geometria Espacial: Impacto de variáveis pessoal e de contexto

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Abstract

This article presents and discusses results from a study investigating the quality of Spatial Geometry academic performance of 9th grade students when they are about to be exposed to more complex concepts, geometric solids, and related problems in curricular learning experiences. Participants were Portuguese public school students. We also collected information about previous mathematics grades and mother's schooling. A geometry test, a causal attribution to a school achievement test, and two reasoning tests – spatial and mechanical – were applied, and all collected data was statistically analysed and interpreted. To address the different nature of variables, we conducted a hierarchical linear regression. Results suggest that nearly 41% of variance on Spatial Geometry academic performance can be explained by the personal and contextual variables studied. The recommendation to schoolteachers involves designing learning experiences that engage students in spatial reasoning and high-order thinking skills.

Keywords: Spatial Geometry. Spatial reasoning. Mechanical reasoning. Causal attributions.

Resumo

Este artigo apresenta e discute os resultados de um estudo que investiga a qualidade do desempenho académico em Geometria Espacial de alunos do 9.º ano, quando estão prestes a ser expostos a conceitos, sólidos geométricos e problemas mais complexos, em experiências escolares de aprendizagem. Os participantes eram alunos de escolas públicas portuguesas. Foram recolhidas informações sobre a classificação escolar em matemática do ano letivo anterior e a escolaridade da mãe. Foram aplicados um teste de geometria, um teste de atribuição causal ao desempenho escolar e dois testes de raciocínio – espacial e mecânico – e todos os dados recolhidos foram analisados e interpretados estatisticamente. Para abordar a diferente natureza das variáveis, foi realizada uma regressão linear hierárquica. Os resultados sugerem que quase 41% da variação no desempenho académico da Geometria Espacial pode ser explicada pelas variáveis pessoais e contextuais estudadas. A recomendação para

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professores envolve a criação de experiências de aprendizado que envolvam os alunos em raciocínio espacial e habilidades de pensamento de nível mais elevado.

Palavras-chave: Geometria Espacial. Raciocínio espacial. Raciocínio mecânico. Atribuições causais.

1 Introduction

Spatial Geometry (SG) plays an important role in mathematics education, as the study of the Euclidean space relates directly to our real experience with and within our physical environment. In the Portuguese mathematics curriculum, the study of Spatial Geometry begins in the early years of schooling and evolves until the 9th grade: from that point on, some secondary school areas will not require spatial geometry and scientific ones will only consolidate this study by framing it in the analytical context. Therefore, it is of particular importance to understand the learning conditions and environment that are present in the conclusion of the study of Spatial Geometry.

The components of any action to learn and the predisposition to engage in it are conditioned by intrapersonal variables, such as cognitive structure, cognitive abilities, and motivational factors (AUSUBEL; NOVAK; HANESIAN, 1980). In addition, the “home functions are the most salient out-of-school context for student learning, amplifying or diminishing the school’s effect on school learning” (WANG; HAERTEL; WALBERG, 1993, p. 278). Thus, this study focuses on personal and contextual variables of learning SG. In general, a deeper understanding of the school learning and academic performance phenomena can be found if some personal or contextual variables are controlled. This allows the emergence of subtle patterns, facilitating researchers’ ultimate goal to, with a reasonable degree of certainty, support/refute theory or point to different and apparently promising research, pedagogical and didactical directions. The objective of the research presented in this article is to find relations between 9th grade students’ spatial and mechanical reasoning, their previous mathematics grades, their causal attribution to mathematics achievement, and their SG knowledge and skills prior to SG 9th grade classes. In other words, the aim is to asset in what degree these can be predicted by those personal and contextual variables. Information about students’ mothers schooling was taken into account. The following brief literature review provides this studiy’s theoretical background.

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1.1 Spatial Geometry and Measurement

As advocated by the National Council of Teachers of Mathematics (NCTM) – the world's largest mathematics education organization (NCTM, 2020) – and acknowledged on current Portuguese curricular standards, SG concepts are to be introduced in early years: children should learn throughout primary school how to locate and orientate themselves in space, and intentionally have contact with forms, their concepts, and the volume notion. For them to develop the prescribed ability to visualize and understand picture properties as well as measurement processes, teachers should ask them to identify, interpret, and describe spatial relationships, and describe, construct, and represent geometric solids, identifying their position in space, their properties and establishing geometric relations (DAMIÃO *et al.*, 2014; DGE, 2018; NCTM, 2000). Moving on to middle school and to keep developing their ability to visualize and comprehend geometrical properties, Portuguese students are formally presented to polyhedron (pyramids and prisms) and other solids (cone and cylinder); they begin to study volumes of prisms and cylinders and their characteristics such as net, lateral surfaces, relations between number of edges, vertices and faces, and to solve related problems. In order to describe spatial figures based on their properties, and to identify and draw a solid's net – and, reciprocally, to recognize a solid from its net – students should explore geometric solids' properties, engaging in different activities that teachers should conduct: using geometric and other manipulative materials, including digital technology; using measurement and drawing instruments in the construction of geometric objects; viewing, interpreting, and drawing representations of geometric figures, and building solids from two-dimensional representations and reciprocally (DAMIÃO *et al.*, 2014; DGE, 2018).

By the end of middle school, and even though the *Cavalieri* Principle is not formally introduced, 9th grade Portuguese students are asked to evolve their knowledge about volumes of solids and consolidate the SG study: in activities that should still involve the use of manipulatives and geometrical models (including from digital technology) and the visualization and interpretation of geometric figures' representations, pyramids and cones' volumes and surfaces are studied; the sphere volume and the spherical surface area are also presented. Students should engage in problem-solving situations relating to solid height, base, and lateral apothems, base radius, angle of sector, generatrix, volume and lateral and total areas. The level of difficulty may be increased by adding solid composition and decomposition (with cone and pyramid trunks; spherical wedges; circular crowns) (DAMIÃO *et al.*, 2014; DGE, 2018).

In general, however, teachers don't seem to engage students on those same spatial

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activities in classroom (CLEMENTS *et al.*, 2018), turning the didactical focus mainly to techniques and procedures (SMITH; VAN DEN HEUVEL-PANHUIZEN; TEPPO, 2011; TREVISAN; AMARAL, 2016). Regarding measurement, research suggests that students successfully learn the procedures without learning their conceptual principles, performing "relatively well on well-practiced tasks and very poorly on tasks that are conceptually simple but frame measurement in nonstandard ways" (SMITH; VAN DEN HEUVEL-PANHUIZEN; TEPPO, 2011, p. 618). In a study analysing 24 mathematics written tests with a total of 131 items, Trevisan and Amaral (2016, p. 462) found a "massive prevalence of items classified in the lower levels of the cognitive domain (remember, understand and apply) and an almost absence of items in the more elaborate levels (analyse, evaluate, and create)".

1.2 Spatial and mechanical reasoning

The ability to visualize, often required in geometry classroom activities, is object of fertile research often under the terms visualizing, visuospatial reasoning, visual reasoning, spatial reasoning, and spatial thinking (SINCLAIR *et al.*, 2016). Indeed, studies point to a relation between this ability and success in geometry (BISHOP, 2008; GOLDSMITH; HETLAND; HOYLE; WINNER, 2016; JOLY; MUNER; SILVA; PRIETO, 2011; LUBINSKI, 2010; MULLIGAN, 2015) and also Science, Technology, Engineering and Mathematics (STEM) areas (GILLIGAN; FLOURI; FARRAN, 2017; JULIÀ; ANTOLÌ, 2017; KELL; LUBINSKI, 2013). In fact, according to Lubinski (2010), individuals with STEM degrees or occupations tend to reveal salient levels of spatial ability during early adolescence, and "selecting students for advanced learning opportunities in STEM without considering spatial ability might be iatrogenic" (LUBINSKI, 2010, p. 344).

Spatial reasoning is also connected to what psychologists call *spatial visualization*, which refers to the cognitive ability to mentally manipulate complex three-dimensional figures (PRIETO; VELASCO, 2008) and that can be assessed in different tasks soliciting the identification of sections of solids, the association of solids to their nets (YILDIZ; ÖZDEMIR, 2017) or the identification of the image of a three-dimensional figure transformation. Spatial reasoning is also connected to geometric measurement (BATTISTA; WINER; FRAZEE, 2017), drawing solids and spatial geometry problem-solving (BUCKLEY; SEERY; CANTY, 2019). Searching for an overlap between geometric reasoning and visual-spatial thinking required to correctly draw solids, Goldsmith *et al*. (2016, p. 67) hypothesized that "deficient envisioning, as revealed by drawings that are substantially spatially incoherent and disorganized, may signal

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weaknesses in geometric reasoning”. This study revealed that students who performed deficient drawings, scored substantially worse than students who performed adequate/good drawings on spatial reasoning tests on cube rotation, paper folding, and water level on solids.

Mechanical reasoning – which refers to the aptitude for understanding and applying mechanical principles, in formal or non-formal learning contexts – holds a strong correlation to spatial ability (HEGARTY; SIMS, 1994) and instruments to evaluate it are usually adopted by school psychologists in counselling when searching for interests to STEM related areas.

1.3 Motivation

Dispositions to learn are connected to a broader sense of *motivation*. Attribution Theory, one of several cognitive theories of motivation, seeks to explain how an individual's perceived reasons for past success and failure contribute to his current and future motivation and success (WEINER, 1985). This author proposes three typology of causal dimensions: *causality locus*, with internal (self) or external (context) causes; *stability*, with unstable (varying over time) or stable causes; and *controllability*. Teacher's method, test difficulty, and luck are thus defined as external, unstable, and uncontrollable causes to success or failure. Intellectual ability, study method, effort, and previous knowledge are internal causes but differ in stability and controllability; intellectual ability is stable and uncontrollable, and study method is usually stable but controllable, in a sense that it can be changed by own will. In general, and according to this theory, people who attribute their success to internal, stable, and controllable factors tend to be more highly motivated, feeling competence, pride, satisfaction, confidence, and greater expectations of success (ALMEIDA; MIRANDA; GUISANDE, 2008); usually attributing failure to external and unstable factors, which prevents lowering self-esteem. On the other hand, the attribution of success to luck or other external uncontrollable factors prevents experimenting satisfaction, leading to a debilitated motivation, usually attributing failure to internal uncontrollable factors like intellectual ability. Studies point to moderate associations between these specific attributional styles and academic performance in mathematics (ALMEIDA *et al.*, 2012).

1.4 Mother's schooling

According to Berhran (1997), there's a conventional wisdom that the effects of mothers' schooling on children's education are positive, pervasive, and substantial. Recent studies include sociocultural level and point to their impact not only on academic achievement (ALVES; GOMES; MARTINS; ALMEIDA, 2017) but also to cognitive performance (ALVES *et al.*, 2016; LEMOS; ALMEIDA; COLOM, 2011). Regarding the development of geometry concepts in preschool children, Maričić and Stamatović (2018) found that children whose mothers had a college or a university degree achieved better results compared to children whose mothers took only primary or secondary education.

Overall, this study aims to investigate the impact of these students' personal and contextual variables on SG performance. For that, a hierarchical regression analysis with a measurement in SG performance as the dependent variable was conducted, taking successively sociocultural background, psychological variables (spatial and mechanical reasoning, and causal attributions to academic success) and previous academic achievement in mathematics into account.

2 Materials and Methods

Participants were 179 9th grade students belonging to nine different classes of three Portuguese northern countryside public schools. Schools were located in semi urban areas and were selected by convenience of both geographical and professional proximity to researchers.

To assess students' SG knowledge and skills, an instrument was conceived addressing 6th grade school curricular outcomes. The Geometry Test (GT) was composed of 20 items soliciting knowledge and understanding on number of vertices, edges, and faces of solids like cube, pentagonal pyramid or hexagonal prism (1a, 1b and 1c), associations between solids and their nets (2a, 2b, 2c, 2d, 2e and 2f), algebraic thinking on a given regular quadrangular prism's surface area (3), performance in drawing a solid given its net (4), volume and/or surface area's calculations of cube, prisms, and cylinders (1d, 5a, 5b and 5c), investigation about the possibility of pyramid's constructions given different nets (6a, 6b, 6c), analyzing two solids and connecting concepts of volume and capacity (7) and problem solving to determine the volume of a composed solid (8). The maximum score of GT was 20 (1 to each item), and missing or unjustified answers (where justification was required) were scored with 0. Question 4 presented the net of a regular triangular prism with squares as lateral faces. Its total score had subdivisions to take performance quality into account: 0 to a impossible drawing or drawing of any other solid but a triangular prism; 1 to a correct representation, and discounting 0.25 to each of the following faults: if the triangular prism was not regular, if it didn't present squares as lateral faces or if its invisible edges were not correctly represented. Table 1 presents GT items in

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categories sorted by knowledge and skills mobilized.

Table 1 – GT items sorted by knowledge or skills mobilized

Knowledge and Skills	GT Items
KS1 – Number of vertices, edges, and faces	1a, 1b, 1c
KS2 – Net	2a, 2b, 2c, 2d, 2e, 2f, 6a, 6b, 6c
KS3 – Measurement (volume, capacity, and surface area)	1d, 3, 5a, 5b, 5c, 7, 8
KS4 – Solid drawing skill	4

Source: prepared by the authors

To assess students' psychological characteristics, two different instruments were applied. QARE (MIRANDA; ALMEIDA, 2008) was used to assess students' motivations based on Weiner attributional theory (WEINER, 1985). For this study only two of the four QARE items were analyzed, both presenting situations about success in mathematics and six different causes to it, which students rank in order of importance (from 1 – most important to 6 – less important). The item on success in mathematics *tests* is presented as follows:

"The good results that I obtain in some mathematics tests are due to:

___ I work hard to succeed.

___ I have a good study method for mathematics.

___ I have good previous knowledge about mathematics.

___ The teacher explains the subject well.

___ I get lucky with the test questions.

___ I have good intelectual ability".

To each cause of success – effort, study method, previous knowledge, teacher, luck, and intellectual capacity – presented for importance ranking, the mean of the ranking attributed in both items was calculated and resulted in a variable used in this study, respectively *s_effort*, *s_study*, *s_prevkn*, *s_teach*, *s_luck* and *s_abil*. Spatial Reasoning test (SRT) and Mechanical Reasoning test (MRT), respectively composed of 20 and 25 multiple choice items (five options with one correct), were selected from the Reasoning Tests Battery BPR (ALMEIDA; LEMOS, 2006). MRT items involve practical problem-solving on physical and mechanical situations, and SRT items are orientation tasks taking series of cubes in rotation.

Mother's schooling was taken as an ordinal variable designated by *mothersch*, where consecutive levels differed in roundly 4 years of schooling from elementary education to a doctoral degree. The study took place after the first term of the 9th grade. Since participant schools did not hold the same distribution of mathematics topics over the school year, mathematics' classification at the end of that first term would reflect students' achievement in specific topics taught in that period, and not in mathematics in general. Therefore, to assess previous academic achievemnt in mathematics, researchers preferred to take students' 8th grade final marks. In Portugal, this evaluation lies in an ordinal scale from 1 to 5 (where 3 is considered a positive classification, the minimum to be approved), proposed by the mathematics school teacher, usually following assessment criteria defined by the school's pedagogical leadership. This variable was designated as *8th gc.*

Proper ethical standards were observed by informed consent solicitation to parents and students. Data collection took place in the first two weeks of the second school term. First, students took GT in a 60 minutes' period, submitted to basic standards of exam supervision and applied by the researcher and voluntary school teachers in their own class time. In a different day, no later than one week after, students took psychological tests in a 45 minutes' period, applied by school psychologists within the available school time. Students were not informed about the GT or reasoning tests' scores.

Statistical analysis was performed with IBM SPSS Statistics version 26.0. Evans (1996) taxonomy was adopted to qualify correlation coefficient. A value of $p < 0.05$ was considered statistically significant (two-tailed test).

3 Results

The sample was quite evenly distributed by sex, with 85 female and 94 male students (47.5% and 52.5%, respectively). Table 2 presents descriptive statistics of some variables being studied. Table 3 presents correlations between personal and contextual variables and GT categories and total scores. For all variables, distribution around mean value was approximately Gaussian, as absolute values of asymmetry and kurtosis were less than 1.3. There was no account of participants scoring more than 20 over of MRT's maximum score (25), and its mean value was below the SRT mean score. On average, participants tended to attribute their success mostly to teacher practices and their own effort, and less to luck or their own good previous knowledge of mathematics.

Table 2 – Descriptive Statistics of 8th grade classifications, MRT, SRT, QARE variables, GT categories, and total scores

	Min.	Max.	Mean	S.D.	As.	K.
s_effort	1	6	3.02	1.66	.32	-1.24
s_study	1	6	3.49	1.25	.07	-.91
s_prevkn	1	6	3.71	1.29	-.20	-.61
s_teach	1	6	2.89	1.50	.49	-.88
s_luck	1	6	4.43	1.70	-.66	-.94
s_abil	1	6	3.50	1.37	-.07	-.91
MRT	0	20	9.62	3.09	.16	.91
SRT	1	20	10.75	4.07	.08	-.48

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8^{th}gc	2	5	3.58	.98	.01	-1.02
KS1	.00	3.00	1.35	.82	.42	-.28
KS2	.50	8.00	4.94	1.55	-.27	-.10
KS3	.00	7.00	1.92	1.86	.71	-.52
KS4	.00	1.00	.65	.39	-.68	-1.13
GT_total	.50	19.25	9.10	3.69	.54	.11

Source: adapted by the authors from IBM SPSS Statistics output

Of QARE variables, some statistically significant coefficients emerged, and the attribution of success to intellectual ability was the strongest in absolute value, even though this association was weak in quality. Overall, and comparing causes attributed by students to their success in mathematics, students who obtained lower scores on GT tended to attribute their success in mathematics to other causes other than their intellectual ability; inversely, students who obtained higher scores on GT tended to refer their intellectual ability as one of the main causes of mathematics success. Another value with no statistical significance that emerged was the almost null association between the GT score and attributing success to effort. That is to say: the knowledge and skills that participants revealed on SG, when observing GT score, held no association to the importance they attribute to their own effort relative to their own success in mathematics.

Table 3 – Correlations between personal and contextual variables and GT categories and total scores

	KS1	KS2	KS3	KS4	Total
sex	-.104	$-.149^{*}$	.059	-.049	-.083
mothersch	$.261^{**}$	.098	$.280^{**}$	.138	$.276^{**}$
s_eff	.061	-.032	-.014	.049	-.006
s_study	$.152^{*}$	.010	.016	.058	.065
s_prevkn	-.136	-.058	-.136	-.038	-.118
s_teach	.118	$.204^{**}$	.065	.011	$.157^{*}$
s_luck	.025	-.034	$.338^{**}$	-.007	$.155^{*}$
s_abil	$-.230^{**}$	-.096	$-.275^{**}$	-.050	$-.253^{**}$
MRT	$.288^{**}$	$.209^{**}$	$.231^{**}$	$.264^{**}$	$.303^{**}$
SRT	$.423^{**}$	$.319^{**}$	$.419^{**}$	$.344^{**}$	$.499^{**}$
8^{th}gc	$.382^{**}$	$.221^{**}$	$.510^{**}$	$.320^{**}$	$.501^{**}$

Source: adapted by the authors from IBM SPSS Statistics output

Regarding knowledge and skills mobilized in GT, categories KS1 and KS3 held moderate associations to SRT. Comparing all categories, KS3 held the strongest association to 8^{th} grade classification. An almost moderate association was also found between this category and the attribution of success to luck, meaning that the higher the scores in this category, the less students attribute their success in mathematics to luck. On KS4, only represented in GT by question 4, quality of drawings was directly connected to the ability to visualize, as measured

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by SRT score: in general, between students who performed correct, reasonable or incoherent drawings, there were positive differences in their spatial reasoning.

Qualitative analysis of specific GT items allowed to infer the importance of spatial and mechanical reasoning to spatial geometry knowledge and skills. In general, question 6 revealed a quite reasonable association to spatial reasoning; however, regarding item 6a, mechanical reasoning seemed to play a superior importance. In fact, several students with top 8th grade classification presented well-articulated justifications but failed to see the impossibility of construction, since paper folding would result in a zero height pyramid. On item 7, students revealed poor knowledge on solids' capacity and volume, as 87% presented no or incorrect arguments about perceived differences in quantity of water on both glasses. Items 5 and 8, soliciting application of knowledge on volume and/or surface area measurements, held the strongest association to mother's schooling, 8th grade classification, and reasoning abilities measured by MRT and SRT.

To approach the association between Spatial Geometry achievement to contextual and personal variables, a hierarchical linear regression was conducted to evaluate the prediction of GT score from sex, mother's schooling, mechanical and spatial reasoning, motivation, and academic achievement. Table 4 presents coefficients of the studied variables over the three models.

Table 4 – Coefficients of hierarchical linear regression analysis

		Unstandardized Coefficients		Standardized Coefficient		
		B	Std. Er.	Beta	t	Sig.
1	(Constant)	6.371	.884		7.203	.000
	Sex	-.800	.531	-.109	-1.508	.133
	Mother sch.	1.235	.318	.280	3.890	.000
2	(Constant)	-2.585	10.794		-.239	.811
	Sex	-1.124	.478	-.153	-2.354	.020
	Mother sch.	.534	.289	.121	1.852	.066
	s_eff	.213	.528	.096	.403	.687
	s_study	.344	.557	.116	.617	.538
	s_prevkn	.120	.538	.042	.224	.823
	s_teacher	.416	.522	.170	.797	.427
	s_luck	.402	.538	.185	.748	.455
	s_abil	-.050	.541	-.018	-.092	.927
	MRT	.216	.082	.181	2.649	.009
	SRT	.347	.062	.383	5.630	.000
3	(Constant)	-4.545	10.142		-.448	.655
	Sex	-.839	.452	-.114	-1.856	.065
	Mother sch.	.222	.278	.050	.798	.426
	s_eff	.101	.496	.045	.203	.840
	s_study	.356	.523	.120	.681	.497

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s_prevkn	.117	.505	.041	.232	.817
s_teacher	.360	.490	.147	.735	.463
s_luck	.114	.508	.052	.223	.824
s_abil	.102	.509	.038	.201	.841
MRT	.176	.077	.148	2.288	.023
SRT	.296	.059	.327	5.040	.000
8th g. class.	1.326	.273	.353	4.860	.000

Source: adapted by the authors from IBM SPSS Statistics output

For the first block analysis, we analyzed sex and mother's schooling. The results of the first block hierarchical linear regression revealed a statistically significant model ($F(2,176) = 8.461$, $p < .01$). The adjusted R^2 value of 0.077 associated with this regression model suggests that sex and mother's schooling accounted for only 7,7% of the variance in Spatial Geometry knowledge and skills, which means that 92,3% of its variance could not be explained by this model.

A different outcome was found for the second block analysis. In this case, the psychological variables were added to the analysis, and the results of the second block hierarchical linear regression revealed a statistically significant model ($F(10,168) = 9.667$, $p < .001$). The adjusted R^2 value of 0.327, associated with this second regression model, suggests that the addition of psychological variables to the first block model accounts for 32,7% of the variation in GT total score, which means that 67,3% of its variance could not be explained by sex, mother's schooling, and psychological variables alone.

A different outcome was found for the third block analysis. In the third model, the 8th grade classification was added to the analysis. The results of the third block hierarchical linear regression revealed a statistically significant model ($F(11,167) = 12.119$, $p < .001$). Additionally, the adjusted R^2 value of 0.407, associated with this regression model, suggests that the addition of previous academic achievement in mathematics to the second block model, accounts for 41% of the variation in the GT total score.

4 Discussion

4.1 8th grade classification

As participants were not asked to study for the GT test but rather take it as a diagnostic evaluation, the moderate correlation of its score to the 8th grade classification is assumed expectable. However, the stronger association of 8th grade classification to GT application items and KS3 category, compared to other items, pose the hypothesis already found on literature that

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mathematics teachers are favoring some dimensions in their didactical practices and/or school grading criteria, in particular related to measurement knowledge and skills (SMITH; VAN DEN HEUVEL-PANHUIZEN; TEPPO, 2011) and also to application items (TREVISAN; AMARAL, 2016). A significant percentage of these 9th graders, who were about to conclude the space study and already had contact with the concept of volume and capacity over their scholar course, revealed a poor understanding of these when presented in a non-familiar situation, confirming Smith, van den Heuvel-Panhuizen and Teppo (2011) findings. The 8th grade classification was the academic variable entered in the third block of the hierarchical linear regression analysis, which substantially improved the model in its ability to predict the GT score.

4.2 Spatial and mechanical reasoning

SRT held a moderate positive association to KS1 category. Even though the algebraic relations between the number of vertices, edges, and faces of prisms and pyramids are a part of 6th grade geometry curricula, it is the researchers believe that most students in the beginning of the 9th grade probably don't recall those same algebraic relations, and that participants relied on their ability to mentally visualize the solids and their components to answer items on KS1 category. As soliciting associations of solids to their nets and folding activities are thought to require spatial reasoning (YILDIZ; ÖZDEMIR, 2017), a higher correlation between questions 2 and 6 scores and SRT was expected, and, in general, with items on KS2 category. Specifically regarding KS3, results confirm Battista, Winer and Frazee (2017) findings; however, another possible explanation for a positive and moderate association between SRT and KS3 may be that SRT items of BPR evaluate inductive and deductive reasoning, in cube rotation situations. Fluid reasoning – which holds inductive and deductive reasoning as major narrow abilities (MCGREW, 2009) – is heavily associated to problem solving skills, which can explain why the SRT test was moderately correlated to problem-solving items that fell into the Measurement category. Results confirm Goldsmith *et al*. (2016) findings that SRT revealed to be an important predictor of drawing skill and, in general, performace in SG as measured by GT.

Even tough mechanical reasoning held weak associations to GT categories and total score, its stronger association to folding paper items compared to SRT pose an interesting connection between Spatial Geometry and mechanical reasoning activities that should be explored in future research and pedagogical sets. Overall, these cognitive dimensions revealed statistically significant coefficients in the third model of linear regression.

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4.3 Motivation

Causal attributions to success presented weak or very weak associations with GT total score. This result can arguably be explained by the nature of the activity that involved taking GT. It is probably fair to say that mathematics academic achievement is often measured by performance in tests, to which students usually prepare themselves by taking classes and studying and, sometimes, knowing in advance the specific mathematical topics that will be addressed. In GT, not only students did not know the content of the test but also did not know, in great advance, when it was going to be applied; in addition, they were told by researchers that results would not be given to their mathematics teacher as an assessment instrument, not even for diagnostic purposes. This “relaxation” regarding the GT score’s impact on their academic mathematic achievement and consequent perception of success in mathematics can arguably be a reason to such weak correlations to attributional causes for success.

The cause of success with the strongest association to GT was intellectual capacity. Regarding GT categories, KS3 revealed the strongest associations with QARE variables and in line with literature: the higher the score in GT the less importance is attributed to luck and the more importance is attributed to intellectual ability as causes of success. This results validate the hypothesis previously posed in the 8^{th} grade classification analysis: Portuguese teachers may be privileging measurement in classroom activities and assessment, as success in mathematics is probably being understood by students as high achievement in measurement tasks.

4.4 Mother’s schooling

All GT variables were associated with mother’s schooling in a weak trend. This result arguably point to the relevance of other cultural or socioeconomic dimensions that may assume a greater relevance in students’ academic performance (ALVES; GOMES; MARTINS; ALMEIDA, 2017). It is to notice that, the associations were stronger and with statistical significance in measurement items and also in application items in this category, such as 5 or 8. Even though it is beyond the scope of this article to discuss the nature of the influence mother’s schooling has on the student’s mathematics achievement, it seems reasonable to argue that such influence is mostly present in knowledge and skills privileged by teachers didactical and assessment activities, which, for better or for worse, seem to represent what is expected from students in Spatial Geometry over 8 years of schooling.

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5 Conclusions

A main limitation of this study is that conclusions cannot be generalized with confidence, for schools were not randomly selected. Also the small number of items representing categories 1 and 4, recommends caution when reading conclusions about knowledge and skills associated.

Several conclusions can be drawn by this study: (1) academic performance on Spatial Geometry is moderately associated to spatial reasoning ability; (2) measurement skills are moderately associated to spatial reasoning ability; (3) mentally visualizing prisms and pyramids and reasoning about their number of vertices, edges and faces is moderately associated to spatial reasoning ability; (4) regarding measurement activities, students perform better in application items than in analysis ones, revealing poor understanding of capacity concept; (5) mathematics teachers tend to overvalue measurement tasks in their pedagogical activities and summative evaluation; (6) regarding major attributional causes for success and failure, self-concept of intellectual capacity holds the strongest association to Spatial Geometry academic achievement; and (7), sex, mother's schooling, attributional causes for success, spatial and mechanical reasoning and recent mathematics achievement explain, altogether, the 41% of variance in academic performance in SG.

As the last hierarchical model unveils a percentage of GT variance that can be explained by variables studied in this research, it also informs that 59% of it lies on other innumerable variables of distinct nature. Since contextual variables include teachers' scientific and didactical knowledge and their general didactical and pedagogical practices, the hypothesis regarding specific didactic and assessment activities already discussed may well be one of many variables that could not only have an impact on academic performance in SG but also improve the explanation of its variance in a linear regression model.

The ability to visualize – to understand complex objects and patterns and to mentally simulate how they will be presented if transformed (SCHNEIDER; MCGREW, 2012) – is essential to understand and interpret the two-dimensional views of three dimensional objects found in textbooks (KÖSA, 2016; VIANA, 2009). This simple fact makes probably self-evident the importance of spatial ability in teaching and learning Geometry throughout all schooling levels: as Sinclair *et al*. (2016) highlight, the Geometry Educational field produces fertile ground to, in natural contexts, see and learn about spatial reasoning in curricular motion. Furthermore, and according to Clements *et al*. (2018), several authors also agree that spatial reasoning plays a significant role in the cognitive processes related to learning not only other topics in mathematics but also other subject matter areas.

Specifically, in the Portuguese schooling system, the transition from middle school to high school presents, to 9th grade or 15-years-old students, a moment of choice between different curricular paths, each of them leading to diverse professional exits, where the *science and technology* area provides academic knowledge and skills to access University in STEM, health, and parallel areas. As the data was collected prior to Spatial Geometry 9th grade classes, STEM teachers can interpret the results presented in this study as diagnostic. Therefore, this study offers a major didactical alert to the importance of developing classroom activities that: i) consolidate knowledge of measurement foundations; ii) solicit analysis and argumentation; and iii) foster spatial and mechanical reasoning.

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