New Low-Energy Levels Calculation for ^{155}Eu

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We have revisited the low-energy calculation of odd Z ^{155}Eu in the frame of a semi-microscopic formalism as a support for the interpretation of the experimental results for the multipole mixing ratios of some electromagnetic transitions. The deformation parameters were obtained through a macroscopic-microscopic method, and the proton single particle levels, calculated with realistic Woods-Saxon potential were used as input in a quasi-particle calculation of the first few rotational band heads in the Lipkin-Nogami BCS aproximation. A better agreement is found between the experimental and calculated band heads if compared with previous evaluations and RIPL recommended values.

This work is a theoretical support for the interpretation of the experimental results for the multipole mixing ratios of observed electromagnetic transitions between low-energy levels in ^{155}Eu [1]. In order to explain the experimental results, it was necessary to calculate the energy, angular momentum and parity of the first excited states (E$\leq$ 1MeV). Due to the fact that the parameters in the potential energy are determined for several nuclei and not for a specific one, the previous calculation was not successful in the description of the first excited states. For example, in the work of Nazarewicz et al. [2], the energy of the ground state of ^{155}Eu differs from the expected value. In addition, when the parameterization proposed by Cwiok et al. [3] is used, it is found that the energy of the single particle states are in disagreement with the literature values. The IAEA RIPL Database recommended single particle levels for protons obtained with the Finite Range Droplet Model (FRDM) [4] and Hartree-Fock–BCS model [5] do not describe even the ground state angular momentum and parity. In this work, a new calculation of the ground-state and the low-energy levels in ^{155}Eu is proposed, using the macroscopic -microscopic method [6]. In this sense, the odd-proton single particle levels in a deformed potential plus residual pairing interaction were calculated in order to describe the ^{155}Eu low-energy rotational band heads (with E$\leq$ 1 MeV). The ground-state deformation parameters were obtained by minimizing the total energy [6]; the single particle energy spectra and wave functions for protons and neutrons were calculated in a deformed Woods-Saxon potential [7]. The parameters of the potential for neutrons were obtained from Ref. [8]. For protons, these parameters were adjusted in order to adequately describe the main sequence of angular momentum and parity of the low energy excited levels (band heads), as well as the proton binding energy. The residual pairing interaction was considered in the BCS prescription using the Lipkin-Nogami approximation [9, 10].

I. NUCLEAR DEFORMATION

Within the macroscopic-microscopic method in the Strutinskys formalism, the total energy of the nuclear system as a function of deformation can be expressed as [6]:

$$E_{tot}(\varepsilon,\widehat{\alpha}) = E_{macr}(\varepsilon,\widehat{\alpha}) + E_{micr}(\varepsilon,\widehat{\alpha}) \qquad (1)$$

where ε and $\widehat{\alpha}$ are the set of deformation parameters.

The bulk contribution to the total energy comes from the liquid drop model. The shell effects represent smaller variations added to the liquid drop energy E_{macr}. The microscopic portion E_{micr} can be divided into two components: the contribution associated with the shell correction energy and the pairing contribution. In order to obtain the equilibrium deformation parameters (ground-state deformation) the total energy is minimized. As the Cassini ovaloid shape parameterization was used, the adopted deformation parameters were the quadrupole moment term (ε) and the hexadecapole moment term (α_4) [6]. The calculated total energy is plotted in Figure 1, as a function of deformation parameters. It is important to note that the obtained parameters $\varepsilon = 0.23$ and $\alpha_4 = 0.030$ are in good agreement with their equivalents in other nuclear shape parameterizations, reported in previous works [11, 12].

II. SINGLE PARTICLE ENERGIES

The single particle states were calculated using the Woods-Saxon (W-S) potential. To determine the W-S potential, twelve constants should be provided: six for protons and six for neutrons.

V_0 = depth of the central potential,
a_{so} = diffuseness parameter of the spin-orbit part,
R_0 = radius parameter,
r_{0-so} = radius parameter of the spin-orbit potential,
a = diffuseness nuclear parameter,
λ = strength of the spin-orbit interaction.

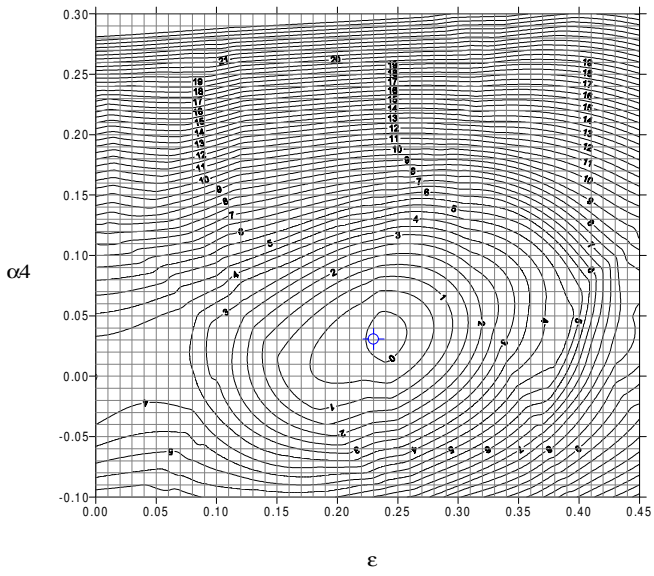

FIG. 1: Equilibrium deformation parameters obtained from macroscopic-microscopic model.

Several parameter sets have been proposed for the Woods-Saxon potential, usually determined through a global fit to various ground state nuclear properties of several β-stable nuclei in a mass number range.

The Woods-Saxon potential consists of the central part V_{cent}, the spin-orbit part V_{so} and the Coulomb potential V_{Coul} for the protons:

$$V^{ws}(r,z,\varepsilon,\widehat{\alpha}) = V_{cent}(r,z,\varepsilon,\widehat{\alpha}) + V_{so}(r,z,\varepsilon,\widehat{\alpha}) + V_{Coul}(r,z,\varepsilon,\widehat{\alpha}) \quad (2)$$

where (r,z) are cylindrical coordinates.

The central part is defined in order to describe the density distribution function

$$V_{cent}(r,z,\varepsilon,\widehat{\alpha}) = \frac{V_0}{1+e^{\frac{dist(r,z,\varepsilon,\widehat{\alpha})}{a}}} \quad (3)$$

where *dist* is equal to the distance of a given point to the nuclear surface

The depth of the central potential is parameterized by:

$$V_0 = V_0[1 \pm 0.63(N-Z)/(N+Z)] \quad (4)$$

with positive signal for protons and negative for neutrons.

The spin-orbit term is defined by

$$V_{so}(r,z,\varepsilon,\widehat{\alpha}) = \lambda\left(\frac{h}{2Mc}\right)^2 \nabla V(r,z,\varepsilon,\widehat{\alpha})(\overrightarrow{\sigma}\times\overrightarrow{p}) \quad (5)$$

where M is the nucleonic mass, the vector operator $\overrightarrow{\sigma}$ stands for the Pauli matrices and $\overrightarrow{p}$ is the linear momentum operator.

The Coulomb potential is assumed to be that corresponding to the nuclear charge $(Z-1)e$, and uniformly distributed inside the nucleus.

In order to diagonalize the Hamiltonian, the eigenfunctions of the axially-symmetric harmonic oscillator in the cylindrical coordinates were used:

$$\left|n_\rho n_z \Lambda\Sigma\right\rangle = \Psi^\Lambda_{n_\rho}(\rho)\Psi_{n_z}(z)\Psi_\Lambda(\varphi)\chi(\Sigma) \quad (6)$$

where

$$\begin{aligned}
\Psi_\Lambda(\varphi) &= \frac{1}{\sqrt{2}}e^{i\Lambda\varphi} \\
\Psi_{n_z}(z) &= N_{n_z}\left[\frac{M\omega_z}{\hbar}\right]^{1/4} e^{-\xi^2/2}H_{n_z}(\xi) \\
\Psi^\Lambda_{n_\rho}(\rho) &= N^\Lambda_{n_\rho}\left[\frac{2M\omega_\perp}{\hbar}\right]^{1/4} \eta^{\Lambda/2}e^{-\eta/2}L^\Lambda_{n_\rho}(\eta) \\
n_\perp &= 2n_\rho + \Lambda
\end{aligned}$$

Here, n_z-1 and $n_\perp-1$ are the number of nodes of the basis functions in the z-direction and the r-direction, respectively; Λ and Σ are the projections of the orbital and spin angular momenta on the symmetry axis, respectively. In the above equations,

$$\eta^{1/2} \equiv \sqrt{\frac{M\omega_\perp}{\hbar}}r, \quad (7)$$

$$\xi \equiv \sqrt{\frac{M\omega_z}{\hbar}}z, \quad (8)$$

$$N_{n_z} \equiv \left[\sqrt{\pi 2^{n_z} n_z!}\right]^{-1/2}, \quad (9)$$

$$N^\Lambda_{n_\rho} \equiv \sqrt{\frac{n_\rho!}{(n_\rho+\Lambda)}} \quad (10)$$

The energy of a given basis state is given by

$$E_{n_\rho,n_z,\Lambda} = \left(n_z+\frac{1}{2}\right)\hbar\omega_z + (n_\perp+1)\,\hbar\omega_\perp \quad (11)$$

The parameters of Woods-Saxon (W-S) potential for neutrons were obtained from ref. [7], and for protons they were adjusted in order to describe the main sequence of angular momentum and parity of the low energy single particle states, as well as the proton binding energy of ^{155}Eu. The values obtained are compared with those from Cwiok et al. [3], called Universal (see Table 1). We can notice that the main difference is in the spin-orbit coupling parameter, which in our result is almost a half of the universal one.

In Figure 2, our calculation of the single particle states as well as deformation parameters are compared with FRDM [4],

	V_0(MeV)	r_0(fm)	a(fm)	r_{0-so}(fm)	a_{so}(fm)	λ
Present Work	51.5	1.258	0.61	1.14	0.61	19.8
Universal	49.6	1.275	0.70	1.32	0.7	36.0

TABLE I: Parameters of W-S potential

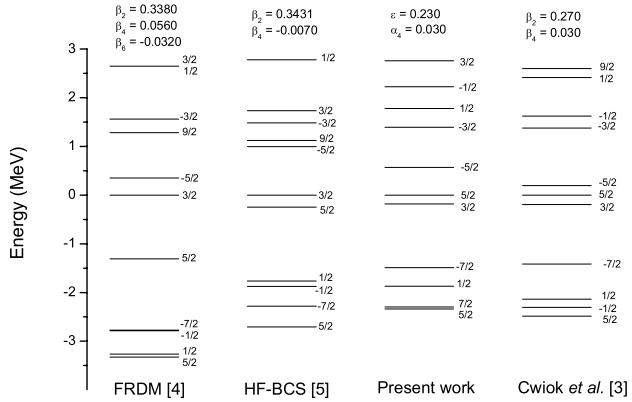

FIG. 2: Proton single particle energies of ^{155}Eu obtained from different formalisms.

HF-BCS [5] models and Cwiok et al. [3] calculations. As we can see, the recommended RIPL single particle energy and spin of Fermi level are in a total disagreement with the experimental values of ground state spin and parity. For the Universal parameters of Cwiok they are very well reproduced, but the gap between the ground state and the first excited single-proton level is underestimated.

III. QUASI-PARTICLE STATES

The pairing energy was evaluated in the usual prescription of the BCS approach. The Hamiltonian operator in the BCS model contains two parts: the first, $\hat{H}_{sp}$, corresponding to the single particle states and the second $\hat{H}_{pair}$ corresponding to the pairing interaction. If the single particle term is diagonal, the BCS operator can be written in the formalism of the second quantization as

$$\begin{aligned}\hat{H} &= \hat{H}_{sp} + \hat{H}_{pair} = \\ &\sum \varepsilon_{\Omega_i}(a_i^\dagger a_i + a_{\bar{i}}^\dagger a_{\bar{i}}) - \sum G_{ji} a_j^\dagger a_{\bar{j}}^\dagger a_{\bar{i}} a_i\end{aligned} \tag{12}$$

were ε_{Ω_i} is the single-particle energy of level i and G is the pairing strength between orbitals j and i. In the monopole pairing approximation, all the two-body matrix elements G_{ji}, are taken to be equal to a single G.

The pairing interaction was taken into account by applying the BCS theory in the Lipkin-Nogami approximation to each configuration. All residual interactions except pairing are neglected.

According to each generated configuration, in the Lipkin-Nogami approximation the pairing gap Δ, Fermi energy λ, number fluctuation constant λ_2, occupation probabilities υ_k^2 , and shifted single-particle energies ε_k are determined from the $2\times(N_2-N_1)+5$ coupled nonlinear equations for protons and neutrons:

$$N = 2\sum_{k=N_1}^{N_2}\upsilon_k^2 + 2(N_1-1), \tag{13}$$

$$\frac{2}{G} = \sum_{k=N_1}^{N_2}\left[(\varepsilon_k-\lambda)^2+\Delta^2\right]^{-1/2}, \tag{14}$$

where

$$\upsilon_k^2 = \frac{1}{2}\left(1-\frac{(\varepsilon_k-\lambda)}{\left[(\varepsilon_k-\lambda)^2+\Delta^2\right]^{1/2}}\right), \tag{15}$$

$$u_k^2 = 1-\upsilon_k^2, \quad k=N_1,N_1+1,\ldots,N_2, \tag{16}$$

$$\varepsilon_k = e_k + (4\lambda_2 - G)\upsilon_k^2, \quad k=N_1,\ldots,N_2, \tag{17}$$

$$\lambda_2 = \frac{G}{4}\times \times\left[\frac{\left(\sum_{k=N_1}^{N_2}u_k^3\upsilon_k\right)\left(\sum_{k=N_1}^{N_2}u_k\upsilon_k^3\right)-\sum_{k=N_1}^{N_2}u_k^4\upsilon_k^4}{\left(\sum_{k=N_1}^{N_2}u_k^2\upsilon_k^2\right)^2-\sum_{k=N_1}^{N_2}u_k^4\upsilon_k^4}\right] \tag{18}$$

and e_k are the known single particle energies.

The total configuration energy, according to the model, is

$$E_k = \left[(\varepsilon_k-\lambda)^2+\Delta^2\right]^{1/2} - \lambda_2, \quad ,k=N_1,N_1+1,\ldots,N_2. \tag{19}$$

The parity of this state is defined as

$$\pi = \prod_{k=1}^{N_2}\pi_{k.} \tag{20}$$

As the system of equations is solved separatelly for protons (Z) and neutrons (N), all the combinations with the following energies and quantum numbers are accounted for the nuclear system:

$$E_{K\pi} = E_{\Omega\pi}(Z) + E_{\Omega\pi}(N), \tag{21}$$

$$K = \Omega(Z) \pm \Omega(N), \tag{22}$$

$$\pi = \pi(Z)\pi(N). \tag{23}$$

where Ω is the total angular momentum projection (for protons and neutrons), and K is the projection of Ω on nuclear symmetry axis.

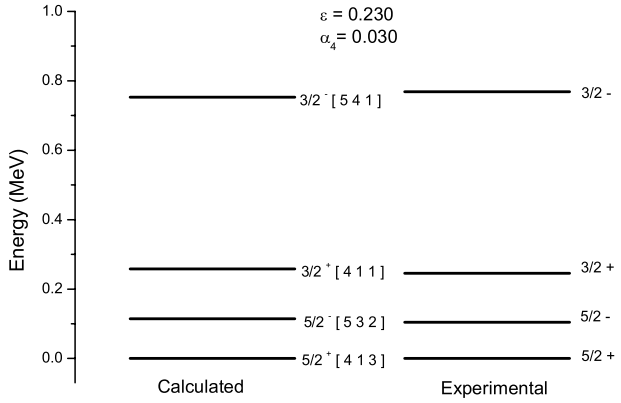

FIG. 3: Calculated and experimental band heads of ^{155}Eu .

The excitation energy is, on the other hand, calculated as the difference between the total energy of a configuration and the total energy of the ground state, where the first summation includes only blocked orbitals. Using the previously calculated proton single particle energies and spins, the rotational band heads are obtained, and are shown in Figure 3. As we can see, there exists a remarkable agreement between experimental and theoretical levels.

IV. CONCLUSIONS

We have calculated in a very successful way the excitation energy of the rotational band heads, with their spin and parity, for the nucleus ^{155}Eu, by using the macroscopic-microscopic method combined with residual pairing interaction in the BCS prescription using the Lipkin-Nogami approximation [9, 10]. We have also obtained a better set of parameters for the Woods-Saxon proton single particle potential of ^{155}Eu as well as the deformation parameters in a Cassinian ovals parametrization.

Acknowledgments

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