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Mathematical model for planning the distribution of locomotives to meet the demand for making up trains

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Modelo matemático para planejamento da distribuição de locomotivas para atendimento à demanda de formação de trens

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Abstract: The cost for locomotive distribution over the rail yards to meet the locomotive demand for train formation is very high. Thus, this paper proposes a mathematical model based on the Locomotive Scheduling Problem for locomotive distribution planning to meet the demand of the rail yards seeking to minimize the distribution costs. The proposed model presents a new formulation for the LSP with Multiple Locomotives and considers the imbalance between offer and demand of locomotives, this situation was not addressed in the literature yet. Tests on instances based on real data from the Vitória a Minas Railroad (EFVM) were solved optimally using CPLEX 12.6. The model proved to be a good tool to analyze the locomotive distribution planning. When compared with the manual planning currently held by the railroad, the results showed several gains.

Keywords: Locomotive distribution planning; Locomotive scheduling problem; Locomotive assignment problem; Railroad transport.

Resumo: *O custo operacional para distribuir locomotivas ao longo dos pátios ferroviários com o objetivo de atender às demandas de formação de trens é muito alto. Assim, este artigo propõe um modelo matemático para planejamento da distribuição de locomotivas que visa atender às requisições dos pátios, Locomotive Scheduling Problem (LSP), com o intuito de minimizar os custos de distribuição. O modelo proposto apresenta uma nova formulação para o LSP with Multiple Locomotives e considera o desbalanceamento entre oferta e demanda de locomotivas para atender às requisições de um pátio, o que ainda não havia sido tratado na literatura. Testes em instâncias com base em dados reais da Estrada de Ferro Vitória a Minas (EFVM) foram resolvidos de forma ótima utilizando o solver CPLEX 12.6. O modelo se mostrou bastante aderente ao planejamento da distribuição, e diversos parâmetros que afetam os custos da distribuição foram analisados. Os resultados mostraram ganhos em relação ao planejamento manual atualmente realizado.*

Palavras-chave: *Planejamento da distribuição de locomotivas; Locomotive scheduling proble; Locomotive assignment problem; Transporte ferroviário.*

1 Introduction

In Brazil, rail cargo transport grew between 2006 and 2014 around 30% in the transport of one ton-kilometre (TKU), reaching 307,304 million TKU in 2014. The rail network in 2013 had a length of about 30,000 kilometers (ANTT, 2015; CNT, 2013).

Due to the high investments to buy locomotives associated with their high operational costs, like maintenance and diesel oil, it is necessary to plan the use of the fleet of locomotives in order to minimize the overall costs. Thus, it is important the use of mathematical models to improve the distribution of the locomotive fleet over the rail yards to meet the demand for traction to pull the trains (Vaidyanathan et al., 2008a).

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Planning the distribution of locomotives seeks to distribute available locomotives in some rail yards to other rail yards that demands locomotives in order to pull the trains that are being making up to start a new trip. The goal is to minimize the total distribution costs. Finding a solution to the problem is complex and at the same time very important because the major railroad companies in the word expend a lot of resources distributing their locomotives. Another important point is that an optimized distribution planning can lead to a reduction in the number of locomotives to be bought, reducing the total investment of the railroad (Ahuja et al., 2006).

This distribution can be performed in two ways: 1) attaching locomotives in a train that is already circulating on the rail and therefore the locomotives to be distributed are towed by the train's locomotives and 2) traveling alone, or at most coupled to other locomotives, without any wagon pulled by them. In the first case, the locomotive is called Deadheading and the train has its own locomotives pulling the wagons and uses the idle traction capacity to pull the locomotives to be distributed. In the second case the locomotives are called Light Traveling and. Therefore, deadheading distribution is a lot cheaper than the Light Traveling distribution, where the railroad has to pay the engine driver and has to spend more diesel to make the distribution (Maposa & Swene, 2012).

Planning the distribution of locomotives in the literature is known as Locomotive Assignment Problem (LAP) (Piu & Speranza, 2014). When the LAP has a strategic or tactic vision, then it is called Locomotive Scheduling Problem (LSP) and considers the time and place that the locomotives shall be to attend all the demand, without specifically defining each locomotive, but a type of locomotive to be distributed. When the LAP has an operational view, it is called Locomotive Routing Problem (LRP) and in this case, specifically defines the route of each locomotive, considering the moment that it will be at a rail yard and the moment that it will arrive and depart the rail yard. Two published papers did a review about the LAP: Cordeau et al. (1998) and Piu & Speranza (2014).

This paper proposes a mathematical model for the LSP which is based on the train circulating plan aims to minimize the locomotives distribution cost to meet the traction demand in HP for train formation. This paper has three contributions to the study of the LSP: 1) a new mathematical formulation using a space-time vector defined in this paper as the transformation of the space-time matrix into a vector, which leads to a more simple and small model; 2) an analysis of the imbalance between offer and demand of locomotives, foreseeing the possibility of unmet demands, introducing the concept of virtual locomotives and 3) an application to a real case of a Brazilian railroad comparing the CPLEX results with the results achieved by the manual process done by the railroad.

It was proposed the use of virtual locomotives by the proposed mathematical model to address possible imbalances between offer and demand of locomotives, making more realistic the analysis and the decisions regarding the distribution of locomotives. If CPLEXs solution uses these virtual locomotives at one rail yard at a certain time, thus it indicates that the rail yard's traction demand at this time will not be met by the railroad locomotives fleet. Therefore, the use of virtual locomotives becomes an important tool for the locomotives' dispatcher because he can easily see which rail yard will not have their demand met. Thus, he can make new plans, creating new scenarios to test which is the best situation for the railroad. It is important to say that these virtual locomotives have a very high cost and so the mathematical model will only use them in the case where there are no available locomotives of the fleet to meet the demand. After the literature review, it was not found any paper that dealt with the imbalance between offer and demand of locomotives to making up trains.

The model was tested with data from Vitória a Minas Railroad (EFVM), which is one of the major railroads of Brazil. EFVM carries 140 million TKU, including iron ore and general cargo. It has a fleet of 322 locomotives serving 13 iron ore mines, 34 train making up rail yards and 26 general cargo terminals (ANTT, 2013). This paper focus on the general cargo's demand for traction and so it studied the distribution of locomotives for general cargo trains over the 13 major general cargo rail yards of EFVM. It was used CPLEX 12.6 to solve the proposed mathematical model. The solution reached by CPLEX was compared with the solution achieved by the manual planning carried out by the railroad. Gains were achieved by the mathematical model reducing the distribution costs.

This paper is divided into five sections, including this introduction. Section 2 describes the LSP problem and makes a survey of the published papers about the LSP. Section 3 presents the proposed mathematical model. In Section 4 the results and analysis are presented. In Section 5 the conclusions and futures works are presented.

2 Locomotive Scheduling Problem (LSP)

To solve the LSP, most of the published papers uses a space-time network, Figure 1. A space-time network can be viewed as a matrix with two dimensions: the horizontal dimension represents the discretized time and the vertical dimension represents the rail yards. The arcs from one rail yard to another one represent the trains with their idle capacity of traction and

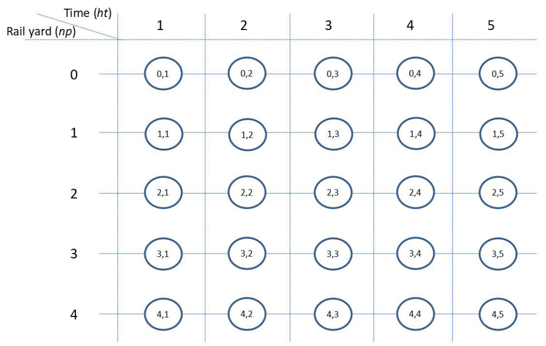

Figure 1. Space-time network.

flows associated with them represent the number of locomotives pulled by the train to be distributed. The arcs between the same rail yard at different times represent the number of locomotives parked at the rail yard through the time. The distribution of locomotives between rail yards occurs due to the imbalance between offer and demand in the rail yards (Vaidyanathan et al., 2008b). Figure 1 shows a space-time network with five rail yards and a planning horizon of five days. So, each rail yard appears five times in the space-time network of Figure 1.

The LSP can be classified in two ways: Single and Multiple Locomotives. The Single Locomotive problem considers only one locomotive pulling the train. The Multiple Locomotives problem considers more than one locomotive pulling the train, and they can be from different models. The LSP with Multiple Locomotives is a more complex problem and difficult to develop models to it (Piu & Speranza, 2014). The mathematical model proposed in this paper is classified as a LSP with Multiple Locomotives and thus this paper presents next a survey of the published papers about the LSP with Multiple Locomotives.

Holt (1973) addressed for the first time the locomotive distribution problem presenting a computer system to solve the LSP. Gohring et al. (1973) proposed a space-time network and an Out-of-kilter algorithm to find an optimal flow of locomotives in a space-time network. Florian et al. (1976) presented a mathematical model to solve the LSP applied to Canadian National Railroads (CN) aiming to find an optimal fleet of locomotives to meet all the traction demand that led to a reduction of the total investment and operation costs. They proposed a solution based on Benders decomposition.

Ziarati et al. (1997) extended the mathematical model proposed by Florian et al. (1976) including other operational restrictions including the withdrawal of locomotives for maintenance. They proposed a space-time network to solve the model and used a method consisting of a Branch and Bound procedure and Dantzig-Wolfe decomposition. Despite not having achieved the optimality of the model, they report significant financial gains up to US $ 4 million annual savings to CN. Ziarati et al. (1999) introduced in the model proposed by Ziarati et al. (1997) new cutting planes and bounds to seek the optimality of the problem, but still, they could not have found the optimal solution for the problem.

Scholz (2000) proposed a study about the locomotives distribution planning to reduce the number of locomotives used in the Swedish railroad system. He structured the planning as a Two-Dimensional Bin-Packing Problem where the vertical axis represents the locomotives and the horizontal axis represents the time, each train is represented by a rectangle where the length is the travel time and the width is equal to 1. Noble et al. (2001) analyzed the Australian State of Victoria Public Transport Corporation (PTC) where trains are cyclical, and they have to decide which locomotives should be assigned to a set of long-distance travel. They proposed an Integer Linear Programming model.

Ziarati et al. (2005) proposed a Genetic Algorithm for the LSP based on the concept of Multi-Commodity Flow Network. They addressed the problem of the Canadian National Railroads (CN) for a set of cyclic trains aiming to reduce the number of used locomotives.

Ahuja et al. (2005) proposed an Integer Linear Programming formulation for a space-time network applied to CSX railroad seeking to reduce the total cost defined as the sum of investments, Deadheading distribution costs and Light-traveling distribution costs. The model considers various practical constraints and it was solved by CPLEX 7.0 to find an initial solution and then they used a metaheuristic Very Large-Scale Neighborhood Search (VLSN) to solve the proposed model. The solution obtained was much better than the solution found by CSX.

Bacelar & Garcia (2006) studied the Vitória a Minas Railroad (EFVM) applied to the iron ore transportation and developed a mathematical model based on Ahuja's model (Ahuja et al., 2005) doing some simplifications adapting the model to the reality of EFVM, i.e., they did not consider Light Traveling trains. Their results showed to be better than the results of the plan done manually by the railroad team.

Vaidyanathan et al. (2008a) aimed to reduce the locomotive distribution costs of CSX railroad. They used the same model and the same metaheuristics as Ahuja et al. (2005), i.e., they used CPLEX to find an initial solution and VNS to solve the problem. The difference between the two papers is that Vaidyanathan et al. (2008a) addressed the issue of a set of locomotives being divided when they reached a rail yard to meet the traction demand of more than one train. They were able to reduce up to 400 locomotives for CSX.

The paper of Vaidyanathan et al. (2008b) is a continuation of Vaidyanathan et al. (2008a) proposing a method called consist flow formulation that proved to be much faster than the method proposed by Ahuja et al. (2005) and Vaidyanathan et al. (2008a). This method also incorporates some restrictions of the real world, such as minimizing the split of locomotives of a train to meet other trains demand.

Piu (2011) proposed a Mixed Integer Linear Programming mathematical model that considers various operational aspects that had not been considered before, such as: refueling of diesel oil, locomotive maintenance and uncertainties regarding the planning of the trains. Noori & Ghannadpour (2012) modeled the problem as the Vehicle Routing Problem with Multi-depots where trains are represented as customers that must be met in a time window. They treated the time windows by fuzzy method. They proposed a hybrid Genetic Algorithm to solve the problem.

Maposa & Swene (2012) presented a Mixed Integer Linear Programming mathematical model based on Ahuja et al. (2005) and solved it using the solver Lingo 10. They applied the model to the National Railroads of Zimbabwe (NRZ). The solution showed a reduction of 38 locomotives compared to the real scenario. Bouzaiene-Ayari et al. (2016) used the Approximate Dynamic Programming (ADP) framework and applied to the Norfolk Southern Railroad and Burlington Northern Santa Fe Railroad.

After this review, it can be noticed that few articles concerning the LSP were published. This paper presents a new formulation for the LSP with Multiple Locomotives that considers the imbalance between offer and demand of locomotives on a rail yard at a certain point in time.

3 Proposed mathematical model

The proposed mathematical model was developed to minimize the locomotive distribution cost meeting all the traction demand to make up train. The proposed model is classified as Locomotive Scheduling Problem (LSP) with Multiple Locomotives.

In this paper, it is proposed the space-time vector, Figure 2, generated from a space-time network to simplify the representation of the problem. First, it is necessary to define that *np* is the number of rail yards and *p* represents each rail yard and *p* ranges from *0* a *np-1*. The parameter *ht* represents the time horizon and *t* represents the discretized time ranging from 1 to *ht*. After these definitions, there are two situations that may happen: 1) $t = 1$, that represents the time equal to 1 in each rail yard *p*; and 2) $t > 1$, that represents any time greater than 1 in each rail yard p. Thus, taking Figure 1 and the two situations presented, the space-time network of Figure 1 can be transformed in to the space-time vector of Figure 2 by the formula $(t+(p\,ht))$. Considering the rail yard $p = 0$, the time equal to $t = 1$ and using the formula it defines the position 1 in the space-time vector of Figure 2, i.e., $(t+(p\,ht)) = (1+(0\times 5)) = 1$. Using the same formula and considering the rail yard $p = 2$ and the time equal to $t = 3$, comes to position 13 in the space-time vector of Figure 2, i.e., $(t+(p\,ht)) = (3+(2\times 5)) = 13$.

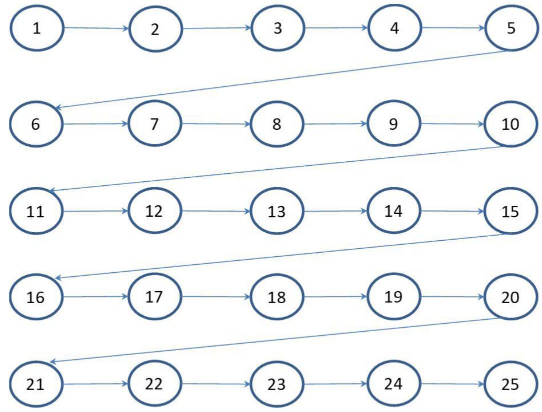

Figure 2. Space-time vector.

Based on the space-time vector previously explained, the proposed mathematical model is presented in five parts: sets, parameters, decision variables, objective function and constraints.

Sets:

K - Set of locomotives types, $K = \{KV \cup KR\}$;

KV - Set of locomotives of virtual type, $KV \subset K$;

KR – Set of locomotives of real type, $KR \subset K$;

G - Set of all trains circulating on the railroad in the analyzed time period, $G = \{GL \cup GD\}$;

GL - Set of all trains circulating on the railroad with Light Traveling locomotives, $GL \subseteq G$;

GD - Set of all trains circulating on the railroad with Deadheading locomotives, $GD \subseteq G$;

HT - Set of times of the planning horizon, defined in days, ranging from 1 to ht;

NP - Set of railroad yards, ranging from 0 to the number of railroad yards less 1, thus, $np-1$; and

N - Set of nodes of space-time vector, $i \in N$ representing the (yard, time), where the number of nodes of the set N is calculated as $(np\,ht)$.

Parameters:

cd_g - Cost of a Deadheading train, $g \in GD$;

cl_g - Cost of a Light Traveling train, $g \in GL$;

μ_{gij} - Maximum quantity of locomotives pulled by the train $g \in G$ when it circulates in the arc between node $i \in N$ and node $j \in N$;

θ_{ki} - Offer of locomotives type $k \in K$ at node $i \in N$;

α_i - Demand in HP to make up the trains at node $i \in N$;

β_k - Power in HP of the locomotive type $k \in K$;

λ - Parameter to prioritize the use of higher horsepower locomotives; and

η - Parameter of penalization of the use of virtual locomotives.

Decision variables:

x_{gkij} - Quantity of locomotive type $k \in K$ coupled to train $g \in G$ when it is circulating in the arc between node $i \in N$ and node $j \in N$;

y_{ki} - Quantity of type locomotives $k \in K$ parked and ready for use at node $i \in N$;

w_{ki} - Quantity of locomotive type $k \in K$ allocated to meet the demand at node $i \in N$.

The objective function and the constraints of the proposed mathematical model are presented next. The objective function and the constraints of the proposed mathematical model are presented next.

is proposed to avoid the use of virtual locomotives, which were proposed in this paper to generate the balance between supply and demand for locomotives. What is sought with this fourth part, which has a high penalty factor, $\eta = 1{,}000$, is to avoid the use the virtual locomotive. It is noteworthy that the use of virtual locomotives $k \in KV$ to meet the demand on the node i represents that the node i will not have its locomotive demand met.

Objective Function

Minimize:

$$\sum_{g\in GD}\sum_{k\in K}\sum_{i\in N}\sum_{j\in N} cd_g\, x_{gkij} + \sum_{g\in GL}\sum_{k\in K}\sum_{i\in N}\sum_{j\in N} cl_g\, x_{gkij} + \lambda \sum_{k\in KR}\sum_{i\in N} w_{ki} + \eta \sum_{k\in KV}\sum_{i\in N} w_{ki} \tag{1}$$

Constraints

$$y_{ki} = \theta_{kij} - \sum_{g\in G}\sum_{j\in N} x_{gkij} - w_{ki} \qquad \forall\, k\in K,\ p\in P,\ t\in HT:\ t=1,\ i=\big(t+(p\ ht)\big) \tag{2}$$

$$y_{ki} = y_{k(i-1)} - \sum_{g\in G}\sum_{j\in N} x_{gkij} + \sum_{g\in G}\sum_{j\in N} x_{gkji} - w_{ki} + \theta_{ki} \qquad \forall\, k\in K,\ p\in P,\ t\in HT: t>1,\ i=\big(t+(p\ ht)\big) \tag{3}$$

$$\alpha_i \le \sum_{k\in K} w_{ki}\,\beta_k \qquad \forall\, i\in N \tag{4}$$

$$y_{ki} \ge 0 \qquad \forall\, i\in N,\ k\in K \tag{5}$$

$$0 \le x_{gkij} \le \mu_{gij} \qquad \forall\, g\in G,\ k\in K,\ i\in N,\ j\in N \tag{6}$$

$$w_{ki} \ge 0 \qquad \forall\, k\in K,\ i\in N \tag{7}$$

The objective function, Equation (1), represents the locomotives distribution costs, first and second part, the quantity of locomotives used, part 3, and the quantity of used virtual locomotives, part 4. The objective function must be minimized. The first part calculates the cost of all Deadheading trains, $g \in GD$, it is calculated by the cost of transporting a locomotive in a Deadheading train multiplied by the quantity of the type of locomotives $k \in K$ is transported by the train $g \in GD$ traveling the arc $i \in N$ to $j \in N$. The second part calculates the cost of all Light Traveling trains, $g \in GL$, it is calculated by the cost of transporting a locomotive in a Light Traveling train multiplied by the quantity of the type locomotives $k \in K$ on the train $g \in GL$ traveling the arc $i \in N$ to $j \in N$.

The third part calculates the quantity of real locomotives $k \in KR$ used to meet the demand on the node i and, since the objective function is a minimization function, the model will tend to use larger locomotives, as there will be a reduction in the number of locomotives used to meet the demand. It was adopted $\lambda = 0{,}01$ as a weight of importance of this part in the objective function. The fourth part

Constraints (2) calculate the number of locomotives in the corresponding node, considering the balance between supply and demand, as well as locomotives that were sent to other nodes. Constraints (2) are activated only when the condition $t = 1$ is true, which is the time 1 at a certain yard. Constraints (2) represent the initial moment of planning each yard, or does not exist locomotives that can be transported to such nodes. Constraints (3) differ from Constraints (2), because represent the other times in the space-time vector, $t > 1$, at a certain rail yard. It also considers the locomotives arriving from other yards in earlier times for the calculation of flow conservation, which does not occur in Constraints (2). Constraints (3) are activated only when the condition $t > 1$ is true, which represents the time bigger than 1 of a certain yard in the space-time vector.

Constraints (4) ensure that the demand in HP at node $i \in N$ is met. That is, the number of locomotives of type $k \in K$ to meet the demand on the node i multiplied by the quantity of HP per locomotive of type $k \in K$ must be greater or equal to the requested demand, α_i. Constraints (5) ensure that the quantity of locomotives parked on the rail yard in a certain node $i \in N$ is greater or equal to zero. Constraints (6) ensure that the quantity of locomotives of type $k \in K$ on the train $g \in G$ traveling from node $i \in N$ to node $j \in N$ is greater or equal to zero and less than the maximum quantity of locomotives pulled by the train $g \in G$ traveling in the arc between node $i \in N$ to node $j \in N$. Constraints (7) ensure the quantity of the locomotives of type $k \in K$ to meet the demand at node i is greater than or equal to zero.

4 Results and analysis

This section presents the results achieved by the solver CPLEX 12.6. Ten instances were tested and data from June to August of 2015 for the general cargo trains were obtained from the computerized system of EFVM. It was established to CPLEX a time limit of 14,400 seconds (4 hours) to run each instance. It was used an Intel *i5* computer with 8 GB of memory.

Table 1 shows the 10 tested instances. Instance 1 is the instance used as a default instance. It is based on the demand coming from the train plans, considering 13 rail yards and a planning horizon of 7 days. It is considered a maximum of six locomotives per train, which is the technical limit of the railroad. All other instances were compared with it. The amount of virtual locomotives was set to 100 for all rail yard in the time equal to 1 to meet all imbalance between offer and demand.

Instances 1 to 7 use data from the first week of June of 2015. Instance 1 considers the offer of virtual locomotives and sets as 6 the maximum number of locomotives per train. Instance 2 is similar to Instance 1, but sets as 4 the maximum number of locomotives allowed per train. Instance 3 is similar with Instance 1, but sets as 8 the maximum number of locomotives per train.

Instance 4 is similar to Instance 1, but considers an increase of 10% in the interval between trains, i.e., an increasing in the transit time between the origin and destination that generates a reduction in the number of trains in the same period of time. Instance 5 is similar to Instance 1, but considers an increase of 10% in the interval between trains and set as 8 the maximum number of locomotives per train. Instance 6 is similar to Instance 1, but considers a reduction of 10% in the interval between trains, i.e., reducing the transit time between the origin and destination that generates an increase in the number of trains in the same time period. Instance 7 is similar to Instance 1, but considers a 10% reduction in the interval between trains and set as 8 the maximum number of locomotives per train.

Instances 8 to 10, Group B, represent real instances and so they are compared with the results obtained by the railroad that currently uses a manual locomotive distribution planning. The three instances use data from the first week of July of 2015, the fourth week of July of 2015 and the fourth week of August of 2015, respectively. It was set as 6 the maximum number of locomotives per train, the same limit used actually by the railroad.

The cost to make up a Light Traveling train was obtained with the operation team of the railroad. The same was done for the Deadheading trains. For confidentiality reasons, these values cannot be disclosed. However, the collected data shows that the cost of a Light Traveling train are much higher than the cost of a Deadheading train. This happens because the costs of the diesel used, and the locomotive engineer are very high. It is important to mention that a Light Traveling train does not bring any revenue for the railroad. The power of the locomotives is defined in amount of HP and it was considered for all instances the same locomotive types used by EFVM to pull the general cargo trains: B-36 with 3,600 HP, DDM-45 with 3,600 HP and DASH-8/9 with 4,000 HP.

Table 2 presents the results obtained by CPLEX for each instance and CPLEX was able to solve

Table 1. Instances.

Group	Instance	Variation of the Instances	Number of trains / week
A	1	Standard	156
	2	Maximum 4 locomotives per train	156
	3	Maximum 8 locomotives per train	156
	4	+10% interval between trains	148
	5	+10% interval between trains and at maximum 8 locomotives per train	148
	6	-10% interval between trains	174
	7	-10% interval between trains and at maximum 8 locomotives per train	174
B	8	1st week of July/2015	162
	9	4th week of July/2015	157
	10	4th week of August/2015	132

optimally all instances at a relative small amount of time, 23,65 seconds. The first and second columns, respectively, represent the group analysis and the instance. Column 3, 4 and 5 show CPLEX's performance indicators: OF (Objective Function), GAP and Execution time. Column 6 shows the total cost of the sum of Deadheading trains and Light Traveling trains. Columns 7, 8 and 9, respectively, show the number of distributed locomotives that travels between the rail yards as Deadheading, Light Traveling and Virtual, respectively.

Table 2 and Figure 3 show the number of locomotives in Deadheading trains, Light Traveling trains and the total cost found by CPLEX for the instances of Group A. The total cost is represented by the two first parts of the mathematical model's objective function presented in Section 3. Instance 1, which is the default instance, was compared with Instances 2 to 7 where there were changes in various parameters as previously seen.

The analysis is done comparing the distribution cost and the number of locomotives used to meet the demand. For the distribution cost, it was first analyzed the impact of the reduction in maximum number of locomotives allowed per train. When this number decreases from 6 to 4 locomotives, no impact occurs in the total distribution cost, the dashed line in Figure 3. It can also be seen that the quantity of locomotives distributed as Deadheading and as Light Traveling remained the same. This comparison was made between Instance 1 and 2. When the number of locomotives per train goes from 6 to 8, there is also no impact on the total distribution cost, dashed line in Figure 3. It can also be seen that the quantity of locomotives distributed as Deadheading and as Light Traveling remained the same. This comparison was done between Instance 1 and 3.

Table 2. CPLEX results.

Group	Instance	OF	GAP (%)	Execution Time (s)	Total Cost Light + Dead (US$)	Number of distributed locomotives		
						Dead	Light	Virtual
A	1	74,792.60	0.0	14.01	1,790.00	19	16	73
	2	74,792.60	0.0	18.17	1,790.00	19	16	73
	3	74,792.60	0.0	13.81	1,790.00	19	16	73
	4	75,492.40	0.0	16.33	1,490.00	9	14	74
	5	75,492.40	0.0	15.73	1,490.00	9	14	74
	6	82,812.85	0.0	23.65	1,810.00	21	16	81
	7	82,812.85	0.0	15.60	1,810.00	21	16	81
B	8	79,827.32	0.0	17.99	1,723.94	22	15	78
	9	80,034.81	0.0	17.13	1,997.69	29	17	72
	10	67,834.53	0.0	18.41	1,256.78	35	11	67

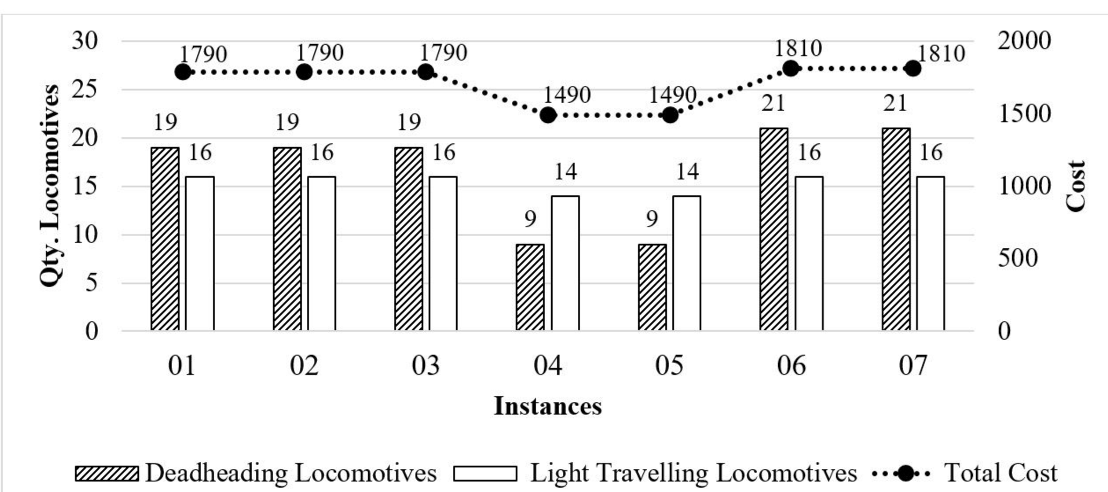

Figure 3. Number of locomotives distributed as Deadheading and Light Traveling and the Total Cost (Group A).

Another analysis was done about the reduction of circulating trains in the railroad caused by an increase of 10% in the transit time. As expected, with the reduction of circulating trains, there was a decrease in the demand for locomotives and therefore the distribution cost was reduced, dashed line in Figure 3. This comparison was done between Instances 1 and 4.

When there was a reduction of circulating trains in the railroad caused by an increase of 10% in the transit time, but there was also an increase of the maximum number of locomotives allowed per train from 6 to 8, there was a decrease in the demand for locomotives and, therefore, the distribution cost was reduced, dashed line in Figure 3, and there was also a reduction in the quantity of locomotives distributed as Deadheading and as Light Traveling. This comparison was done between Instances 1 and 5. When the comparison is done between Instance 4 and 5, the increase of the maximum number of locomotives allowed per train did not cause any variation of the distribution cost.

It was analyzed the situation where there was an increase of circulating trains in the railroad caused by a decrease of 10% in the transit time. In this situation, there was an increase of circulating trains leading to an increase in the demand for locomotives and, therefore, an increase of the distribution cost, dashed line in Figure 3. This comparison was done between Instances 1 and 6.

Another situation analyzed was the increase of circulating trains caused by a decrease of 10% in transit time, but considering an increase from 6 to 8 in the maximum number of locomotives allowed per train. In this situation, there has been an increased in the demand for locomotives and, therefore, the increase in the distribution cost, dashed line in Figure 3. However, comparing Instance 6 and 7, the increase in the maximum number of locomotives allowed per train did not cause changes in distribution cost.

To evaluate the real case of EFVM, it was compared CPLEX results for Group B, Instances 8 to 10, with the EFVM results achieved by the manual planning, which will be called as Real Case.

In Table 3, Column 1 and 2 represent, respectively, the group and the instance number. Columns 3, 4 and 5 show, respectively, the number of distributed locomotives in Deadheading trains, Light Traveling trains and unmet demand achieved by railroad, Real Case. Columns 6, 7 and 8 show, respectively, the number of distributed locomotives in Deadheading trains, Light Traveling trains and unmet demand achieved by CPLEX. The unmet demand is represented by the virtual locomotives as explained before. Columns 9, 10 and 11 represent the difference between CPLEX's results and Real Case's results, respectively, for the number of locomotives distributed in Deadheading trains, Light Traveling trains and the unmet demand.

Comparing the results from CPLEX with the Real Case, Table 3, it can be seen that in the Real Case there were no locomotives being distributed by Light Traveling trains. This happened due to a railroad policy that forbidden the making up of this type of train. For this reason, in the Real Case, in Instances 8, 9 and 10 there were, respectively, an unmet demand of 101, 98 and 87 locomotives. By contrast, since CPLEX allows Light Traveling trains, therefore, there was the use of this type of train. This led to a reduction, when comparing CPLEX with the Real Case, of 23, 26 and 20 in the unmet demand of locomotives in Instances 8, 9 and 10, respectively.

These results show that the allowance, eventually, of the use of Light Traveling trains by the railroad could bring interesting gains in meeting the demand for locomotives to make up trains, reducing the unmet demand and consequently leading to a better service for the clients. The gain of CPLEX against the Real Case, Figure 4, was not only from distributed locomotives in Light Traveling trains but there was also an increase of distributed locomotives in Deadheading trains, 4, 2 and 4 locomotives, respectively, in Instances 8, 9 and 10. This shows that CPLEX was able to use more and better Deadheading trains in the same period to meet the demand of the Real Case.

To make a more effective comparison between CPLEX and the Real Case, CPLEX was run without allowing making up Light Traveling trains. These results can be seen in Table 4 that has the same columns structure as Table 3.

Even when it was set to zero the number of Light Traveling trains in CPLEX and in the Real Case, Table 4, CPLEX was able to distribute the locomotives using more Deadheading trains, 4, 2 and 4, respectively,

Table 3. Comparison between CPLEX × Real Case results.

Group	Instance	Number of distributed locomotives						Difference between CPLEX × Real Case Number of distributed locomotive		
		Real Case			CPLEX					
		Dead	Light	Unmet	Dead	Light	Unmet	Dead	Light	Unmet
B	8	18	0	101	22	15	78	4	15	-23
	9	27	0	98	29	17	72	2	17	-26
	10	31	0	87	35	11	67	4	11	-20

in Instances 8, 9 and 10 than the Real Case. Thus, the CPLEX was able to reduce the unmet demand in all three instances, and even without using Light Traveling trains it distributed better the locomotives avoiding the unmet demand for locomotives to make up trains. It was reduced the unmet demand in 7, 6 and 4 locomotives for Instances 8, 9 and 10, respectively. These reductions can be seen in Figure 5.

Thus, CPLEX, running the mathematical model, can make a better locomotive distribution to meet the locomotive demand with a lower distribution cost. More than that, the mathematical model becomes a standard for locomotive distribution to the railroad that today does the locomotive distribution planning manually, based on the experience of few employees without a standard to be followed.

This paper also proposed the introduction of virtual locomotives as a method to do the analysis of the imbalance between offer and demand for locomotives when the offer is smaller than the demand in a rail

Table 4. Comparison between CPLEX × Real Case results, excluding Light Traveling trains.

Group	Instance	Number of distributed locomotives						Difference between CPLEX × Real Case Number of distributed locomotive		
		Real Case			CPLEX					
		Dead	Light	Unmet	Dead	Light	Unmet	Dead	Light	Unmet
B	8	18	0	98	22	0	91	4	0	-7
	9	27	0	96	29	0	90	2	0	-6
	10	31	0	85	35	0	81	4	0	-4

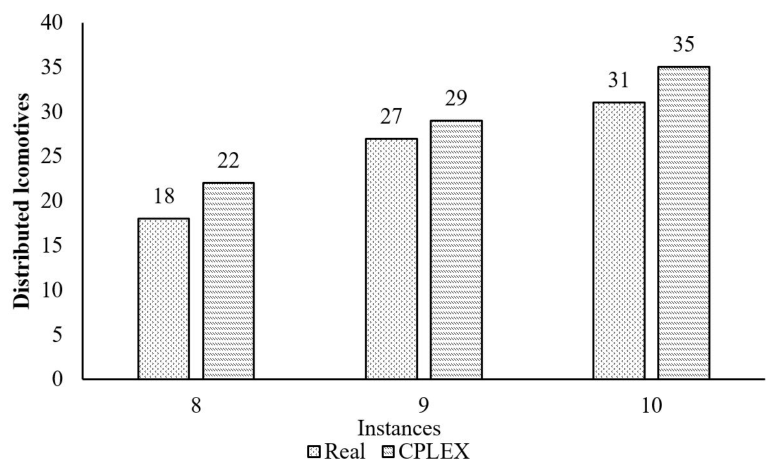

Figure 4. Number of distributed locomotives in Deadheading trains.

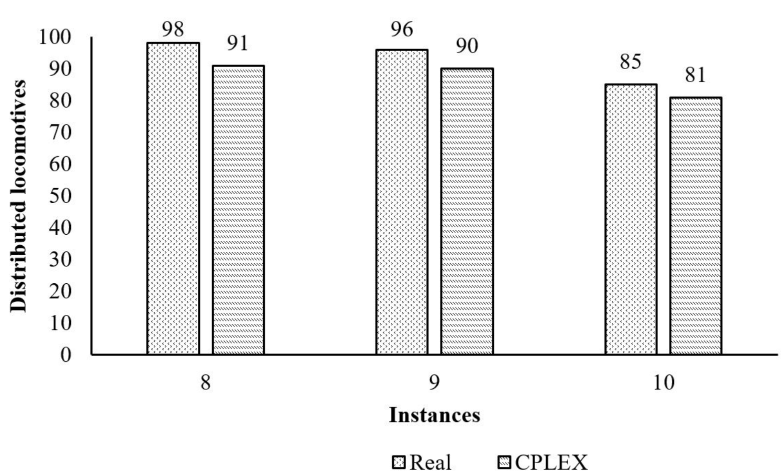

Figure 5. Unmet demand for locomotives.

yard at a certain time. Thus, this method enables the railroad distributor to anticipate any unmet demand and look for alternative scenarios to prevent these unmet demands, seeking for scenarios where all demand is met.

Thus, it can be said that the proposed mathematical model can bring benefits for the short-term operational planning and can be even a tool to help the decision-making process of locomotive distribution planning considering different locomotive types, imbalance demands, unmet demands seeking to provide locomotives to meet all the contracted demand for rail transport. It is also important to highlight that the proposed mathematical model might be used by any railroad because for all railroads need there is a need to plan properly the locomotives distribution to meet the demand for making up trains.

5 Conclusions

This paper presented a new mathematical formulation using a space-time vector for the LSP that makes the model smaller and simply1. The model aims to minimize the total locomotive distribution cost to meet the demand for making up trains. Moreover, it is provided in the objective function of the proposed mathematical model a mechanism to deal with the imbalance between offer and demand, foreseeing the possibility of unmet demand, solved by the virtual locomotives that were introduced in this paper.

It was also presented instances that were compared with the manual planning currently carried out in EFVM. The results achieved by CPLEX compared with the manual planning showed gains in terms of reducing the unmet demand and a lower distribution cost. CPLEX results showed that sometimes it is necessary the use of Light Traveling trains to meet the locomotive demand, although, the cost may rise a little bit but maybe the reduction in the unmet demand justifies.

The proposition of the virtual locomotive enables the distributor to perform a locomotive distribution planning checking possible imbalances between offer and demand in a given rail yard at a certain time. Thus, he can anticipate and propose new scenarios in order to avoid a possible unmet demand for making up trains, thereby achieving a better locomotive distribution.

The proposed mathematical model can be used as a management tool for the locomotive distribution planning in rail yards to service the locomotive demand for train formation in any railroad, both in Brazil and in the world. It is suggested for future work the development of a heuristic or metaheuristic for the proposed model seeking to solve bigger instances.

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REFERENCES

Agência Nacional de Transportes Terrestres – ANTT. (2013). *Acompanhamento das concessões ferroviárias: relatório anual*. Brasília: ANTT.

Agência Nacional de Transportes Terrestres – ANTT. (2015). *Evolução do transporte ferroviário de cargas*. Brasília: ANTT.

Ahuja, R. K., Liu, J., Orlin, J. B., Sharma, D., & Shughart, L. A. (2005). Solving real-life locomotive-scheduling problems. *Transportation Science*, 39(4), 503-517. http://dx.doi.org/10.1287/trsc.1050.0115.

Ahuja, R. K., Shughart, L. A., & Liu, J. (2006). *An optimization-based approach for locomotive planning*. Gainesville: Innovative Scheduling.

Bacelar, A., & Garcia, A. S. (2006). An optimization approach to the locomotive scheduling problem in brazilian railways. In: *Proceedings of the Conference on Railway Engineering* (pp. 493-500). Melbourne: Railway Technical Society of Australasia.

Bouzaiene-Ayari, B., Cheng, C., Das, S., Fiorillo, R., & Powell, W. B. (2016). From single commodity to multiattribute models for locomotive optimization: a comparison of optimal integer programming and approximate dynamic programming. *Transportation Science*, 50(2), 366-389. http://dx.doi.org/10.1287/trsc.2014.0536.

Confederação Nacional do Transporte – CNT. (2013). *Transporte e economia: o sistema ferroviário brasileiro*. Brasília: CNT.

Cordeau, J. F., Toth, P., & Vigo, D. (1998). A survey of optimization models for train routing and scheduling. *Transportation Science*, 32(4), 380-404. http://dx.doi.org/10.1287/trsc.32.4.380.

Florian, M., Bushell, G., Ferland, J., Guerin, G., & Nastansky, L. (1976). The engine scheduling problem in a railway network. *Infor*, 14(2), 121-138. http://dx.doi.org/10.1080/03155986.1976.11731632.

Gohring, K. W., Mcbrayer, R. N., & Mcgaughey, R. S. (1973). Planning locomotive and caboose distribution. *Rail International*, 3, 151-158.

Holt, J. N. (1973). Locomotive scheduling by computer bashpeak. *Rail International*, 4, 1053-1058.

Maposa, D., & Swene, S. D. (2012). Locomotive scheduling in freight transport at Mpopoma Train Station in Bulawayo for the Southern Region, Zimbabwe. *Internacional*

Journal of Economics and Management Sciences, 1(12), 104-116.

Noble, D. H., Al-Amin, M., & Mills, R. G. J. (2001). Production of locomotive rosters for a multi-class multi-locomotive problem. *The Journal of the Operational Research Society*, 52(11), 1191-1200. http://dx.doi.org/10.1057/palgrave.jors.2601210.

Noori, S., & Ghannadpour, S. F. (2012). Locomotive assignment problem with train precedence using genetic algorithm. *Journal of Industrial Engineering International*, 8(1), 1-13. http://dx.doi.org/10.1186/2251-712X-8-9.

Piu, F. (2011). A mixed integer programming approach to the locomotive assignment problem. In: *Proceedings of The 11th Swiss Transport Research Conference* (pp. 1-42). Lausanne: EPFL.

Piu, F., & Speranza, M. G. (2014). The locomotive assignment problem: a survey on optimization models. *International Transactions in Operational Research*, 21(3), 327-352. http://dx.doi.org/10.1111/itor.12062.

Scholz, V. (2000). *Knowledge-based locomotive planning for the Swedish Railway* (Technical Report). Sweden: Swedish Institute of Computer Science. Retrieved in 2015, March 31, from http://citeseerx.ist.psu.edu/viewdoc/summary?doi=10.1.1.34.7764

Vaidyanathan, B., Ahuja, R. K., & Orlin, J. B. (2008a). The locomotive routing problem. *Transportation Science*, 42(4), 492-507. http://dx.doi.org/10.1287/trsc.1080.0244.

Vaidyanathan, B., Ahuja, R. K., Liu, J., & Shughart, L. A. (2008b). Real-life locomotive planning: new formulations and computational results. *Transportation Research Part B: Methodological*, 42(2), 147-168. http://dx.doi.org/10.1016/j.trb.2007.06.003.

Ziarati, K., Chizari, H., & Nezhad, A. M. (2005). Locomotive optimization using artificial intelligence approach. *Indian Journal of Science and Technology*, 29, 93-105.

Ziarati, K., Soumis, F., Desrosiers, J., & Solomon, M. M. (1999). A branch-first, cut-second approach for locomotive assignment. *Management Science*, 45(8), 1156-1168. http://dx.doi.org/10.1287/mnsc.45.8.1156.

Ziarati, K., Soumis, F., Desrosiers, J., Gélinas, S., & Saintonge, A. (1997). Locomotive assignment with heterogeneous consists at CN North America. *European Journal of Operational Research*, 97(2), 281-292. http://dx.doi.org/10.1016/S0377-2217(96)00198-1.