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Modeling Diameter Distributions of Mixed-Oak Stands In Northwestern Turkey

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FOREST MANAGEMENT

ABSTRACT

Background: Diameter distribution models are one of the most important components of growth and yield models. Diameter distribution models, based on the Weibull function, were developed for even-aged mixed-oak stands (Turkey oak, Sessile oak, and Hungarian oak) in northwestern Turkey. Two modeling methods were considered. Weibull parameters were recovered from either equation predicting Dq and Dvar (method of moments) or equations predicting Dq and D_{90} (hybrid method). For each modeling method, three estimation methods were considered: (a) Least Squares method, (b) CDF Regression method in which regression coefficients were estimated separately for each species, and (c) CDF Regression method in which regression coefficients were simultaneously estimated for all species.

Results: Results indicated that the hybrid method coupled with the CDF Regression estimation method yield best results in this study. Similar results were obtained when the regression coefficients were estimated either separately for each species or simultaneously for all species.

Conclusion: The proposed models enable one to predict diameter distribution of a given mixed-oak species stand in northwestern Turkey, using limited stand information. These models are useful tools for the inventory and management of mixed-oak stands.

Keywords: Stand structure, Weibull function, Parameter prediction, CDF Regression, Mixed-stands

HIGHLIGHTS

Diameter distribution models were developed for even-aged mixed-oak stands.
Two modelling and three estimation methods were used for estimation of Weibull parameters.
The hybrid method coupled with the CDF Regression estimation method yielded best results.
The models are useful tools for the inventory and management of mixed-oak stands.

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INTRODUCTION

Mixed natural forests form a significant part of forestland in Turkey covering about 9 million ha, almost 41% of total forestlands (GDF 2018). Several studies concerning growth and yield of mixed stands have been carried out in Turkey, but none of the research conducted to date has emphasized on mixed stands of oaks. Oak species have many ecological and economical values both for wildlife and humans. Oaks provide living space for other organisms such as mycorrhizal fungus, other beneficial microorganisms, and insects. Oak species are important for protecting soils against erosion, for their valuable wood for furniture, manufacture and fuelwood, and oaks are used in other purposes such as cork and tannin production (Yaltırık, 1984). Oak species occupy about 29.2% of the nation's forest area (GDF, 2018). They can form either pure oak stands or mix with coniferous species. A greater part of these forests is intensively managed as pure even-aged stands. Mixed natural forests of oak species occupy 2.4 million ha, almost 25% of total mixed natural forests of Turkey (GDF, 2018).

Turkey has recently adopted the principles of multipurpose and ecologically based forest management. Management decisions should be based on information about both current and future resource conditions, requiring growth and yield models (Zhang et al. 2003). However, available information on forest growth and yield in Turkey lacks sufficient detail for the development sound management plans in the existing complex forest systems. Detailed information is required to design effective management plans for the development of mixed stands.

Forest growth and yield models can be divided into four broad categories: whole-stand models, size-class models, diameter-distribution models, and individual-tree models (Burkhart and Tomé, 2012). Diameter distribution models are useful tools for predicting stand growth and yield, updating forest inventory, and planning forest management activities (Liu et al. 2014). Forest managers are interested in estimating the number of trees in each diameter class in a stand, because diameter partly determines the industrial uses for which the wood is suitable and thus the price. Detailed information on variability of tree diameters in forest stands is a key indicator in sustainable forest management and is needed to assess the carbon sequestered by the tree components in forest ecosystems (Fonseca et al. 2009). Diameter distributions also provide information on stand structure, age structure, stand stability, and also to enable planning of silvicultural treatments (Palahí et al. 2007; Borders et al. 2008; Gorgoso et al. 2012). Furthermore, tree diameter is an important factor in harvesting because it determines the type of machines used and how they perform during felling and transport of the wood (Gorgoso et al. 2012).

Various probability density functions (PDF) such as normal, log-normal, gamma, beta, Johnson's SB, and Weibull functions have been used to describe diameter distribution in forest stands (Liu et al. 2009). The Weibull function has been the most widely used PDF for describing diameter distributions because of its flexibility and relative simplicity (Poudel and Cao, 2013). The Weibull function has been used to describe the diameter distributions of common birch (Gorgoso et al. 2007), sitka spruce and other conifer species (Rennolls et al. 1985), scots pine and Norway spruce (Maltamo et al. 1995), Maritime pine (González, 1997), loblolly pine (Matney and Sullivan 1982; Clutter et al., 1984; Bullock and Burkhart, 2005), European beech (Nord-Larsen and Cao, 2006), longleaf pine (Jiang and Brooks 2009), Austrian black pine (Stankova and Zlatanov, 2010), cork oak (Calzado-Carretero and Torres-Alvarez, 2013), Bormullerian fir (Sakıcı and Gülsunar, 2012), and juniper (Diamantopoulou et al. 2015).

The parameters of the Weibull PDF can be predicted directly from stand characteristics such as age, site quality, and stand density (parameter prediction method), or recovered from predicted diameter moments and/or percentiles (parameter recovery method) (Poudel and Cao, 2013). Cao (2004) estimated the regression coefficients in the parameter prediction method by minimizing the sum of squared differences between the observed and predicted cumulative probability. He termed this new approach the CDF (Cumulative distribution function) regression (CDFR) method, which produced better goodness-of-fit statistics than other methods. The CDFR technique was also found by Newton and Amponsah (2005) and Cao and McCarty (2006) to yield the best goodness-of fit statistics among the methods tested. This technique was applied with satisfactory results to even-aged beech (Nord-Larsen and Cao, 2006), loblolly pine plantations (Poudel and Cao, 2013), and uneven-aged pine-oak mixed forests (Sun et al. 2019). Jiang and Brooks (2009), however, found that for young longleaf pine plantations, the hybrid method by Bailey et al. (1989) provided better results than the CDFR method.

A myriad of diameter-distribution models has been developed over the years. Diameter distribution models are appropriate for plantations or even-aged stands of a single species and may not be suitable for mixed-species stands having distributions of highly irregular shapes. Various flexible approaches have been tried to solve this problem including use of segmented distributions (Cao and Burkhart, 1984), distribution-free models (Borders et al. 1987), and nonparametric statistical methods (Maltamo and Kangas, 1998). As indicated by Maltamo (1997), diameter distributions in mixed stands can be modeled in two ways. First, it is possible to estimate the distribution for the entire growing stock of a certain stand if the growing stock is from a unimodal distribution. However, in multimodal cases, using unimodal statistical functions to model the entire growing stock may be inadequate (Cao and Burkhart, 1984) because they tend to oversimplify the complex stand structures (Maltamo et al. 2000). The second possibility is to estimate the distributions separately for different tree species, and then to add them together for the entire growing stock. More accurate results were obtained with the second alternative for mixed stands of scots pine and Norway spruce (Maltamo, 1997). Little (1983) reported that the three-parameter Weibull function met specified standards of goodness of fit as a model for diameter distribution of western hemlock and Douglas-fir.

Tham (1988) described the structure of mixed stands by fitting the Johnson's S_B distribution, first to each of three species and then to the entire stand. Cao and Burkhart (1984) used a segmented distribution approach for irregular thinned stands and suggested that it could be applied to mixed stands. Liu et al (2002) suggested using a finite mixture model for characterizing the diameter distribution of mixed-species stands. This approach simultaneously estimates the proportion and component diameter distribution of different tree species in mixed-species stands. A disadvantage of this approach is that it may not predict each species component as accurately as fitting the component data separately.

Forest modelers have attempted to use modern statistical methods and techniques to describe the diameter distributions of multi-species forest stands. Maltamo and Kangas (1998) and Maltamo et al. (2000) utilized nonparametric statistical methods to describe multimodal distributions. However, these studies assumed that all species come from the same or similar distributions and ignored the relationships and differences among species.

The objectives of this study were to: (1) develop diameter distribution models for even-aged natural mixed oak stands in Turkey using the Weibull distribution, by estimating the distributions separately for different tree species and then adding them together for the entire growing stock; (2) evaluate two approaches to predict the parameters of the Weibull function for diameter distributions of mixed-oak stands; and (3) evaluate three methods of model fitting for each of the above prediction approaches.

MATERIAL AND METHODS

Data

The data used in this study was from 112 sample plots established in natural, even-aged, and mixed-species stands of Turkey oak (*Quercus cerris* L.), Sessile oak (*Quercus petraea* (Matt.) Liebl), and Hungarian oak (*Quercus frainetto* Ten.) in the Bilecik Region of northwestern Turkey (Figure 1). Turkey oak is native to southern Europe and Asia Minor (including Turkey). It is usually one of the dominating deciduous tree species in mixed forest stands in Turkey. Sessile oak is native to most of Europe. This species is one of the most economically and ecologically important deciduous forest tree species in Europe. Hungarian oak is native to the Balkan Peninsula, southern Italy, and north-west Turkey.

The species composition of sample plots is shown in Table 1. These pure and mixed stands are labeled as species group 1 to 7. Turkey oak occurred in 68.75%, sessile oak in 84.82%, and Hungarian oak in 52.68% of the plots. Species group 3, which is supposed to include plots that had only *Quercus frainetto*, contained no plot. Figure 2 shows the diameter distribution of each species group for all plots.

The sample plots were selected so as to capture the whole range of variation in site and stand density. Sample plots were circular in shape. Plot size ranged from 400 to 1200 m^2 in order to achieve a minimum of 30 trees per plot. Two perpendicular diameters outside-bark (1.3 m above ground level) were measured for each tree to the nearest 0.1 cm and then averaged to obtain diameter at breast height (*dbh*, cm). Total heights (*H*, m) of trees in the each plot were measured to the nearest 0.5 m with a Blume-Leiss hypsometer. The following stand variables were calculated from each plot: quadratic mean diameter (*Dq*), number of trees per hectare (*N*), stand basal area (*B*), dominant height (*H*), and minimum diameter (*Dmin*). Summary statistics are shown by oak species in Table 2. A total of 3,006 trees were measured in the 112 sample plots. Diameter of the individual trees ranged from 6.5 to 54.8 cm with mean 19.0 cm and standard deviation 7.3 cm.

Weibull distribution

The cumulative density function (CDF) and probability density function (PDF) of the three-parameter Weibull distributions are as follows:

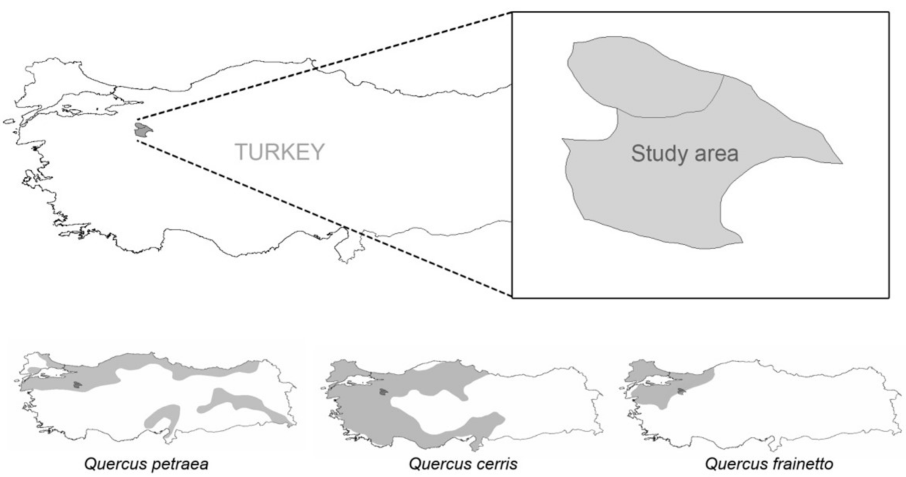

Figure 1. Distribution of three oaks stands in northwestern Turkey.

Table 1. Species composition of sample plots.

Species Group	Quercus cerris	*Quercus petraea*	*Quercus frainetto*	Number of plots	Percent
1	✓			2	1.79
2		✓		1	0.89
3			✓	0	0.00
4	✓	✓		50	44.64
5	✓		✓	15	13.39
6		✓	✓	34	30.36
7	✓	✓	✓	10	8.93

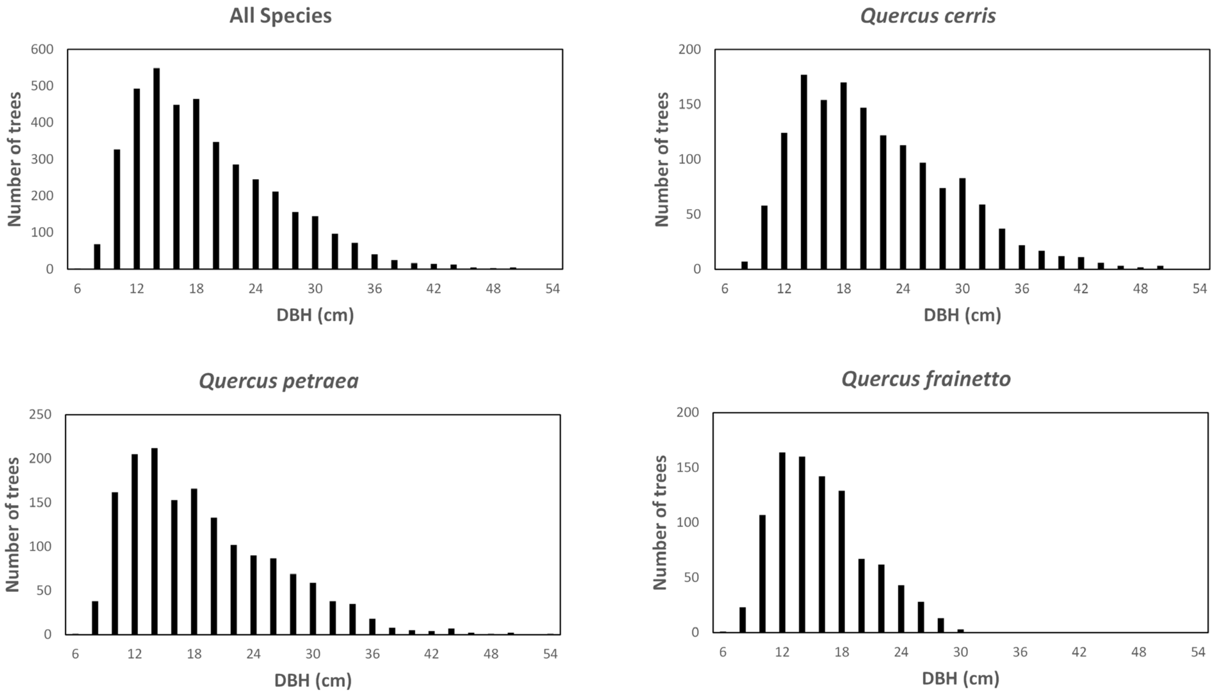

Figure 2. Diameter distributions for all plots, by species.

Table 2. Summary statistics of stand-level variables for mixed-stands of oaks.

Variable	*Quercus cerris*		*Quercus petraea*		*Quercus frainetto*		All	
Number of trees/ha	341.02	(214.39)	289.00	(225.15)	299.83	(200.82)	637.53	(233.51)
Basal area (m^2/ha)	13.21	(11.54)	9.06	(8.97)	6.46	(5.92)	20.17	(14.49)
Quadratic mean *dbh* (cm)	21.70	(6.74)	19.66	(6.84)	15.98	(3.57)	19.48	(6.56)
Minimum diameter (cm)	15.37	(5.58)	14.44	(5.77)	11.07	(2.94)	12.88	(5.20)
Dominant height (m)							18.59	(7.72)
Plot Size (ha)							0.06	(0.02)

Cumulative distribution function (CDF):

$$F_s(x) = 1 - exp\left\{-\left(\frac{x - a_s}{b_s}\right)^{c_s}\right\} \quad (1)$$

Probability density function (PDF):

$$f_s(x) = \left(\frac{c_s}{b_s}\right)\left(\frac{x - a_s}{b_s}\right)^{c_s - 1} exp\left\{-\left(\frac{x - a_s}{b_s}\right)^{c_s}\right\} \quad (2)$$

where *x* is tree diameter, a_s is the location parameter, b_s is the scale parameter and c_s is the shape parameter for species *s*.

Modeling methods

The minimum diameter (*Dmin*) was predicted separately for each species:

$$Dmin_s = f\left(H, N, B, Dq, \frac{N_s}{N}, \frac{B_s}{B}, \frac{Dq_s}{Dq}\right) + \varepsilon \quad (3)$$

where subscript *s* denotes species *s*.

The backward elimination technique was applied to ensure that all independent variables in the model were significant at the 5% level. As suggested by Poudel and Cao (2013), we preferred the backward elimination approach because variables tend to perform well in groups that might be missed by the forward and stepwise approaches.

The Weibull location parameter, a_s, for species *s* was computed as:

$$a_s = \max(5, \widehat{D}min_s/2) \quad (4)$$

where the ^ symbol denotes predicted value.

The location parameter was constrained to be greater or equal to 5.0 cm, which is the lower bound of the smallest 2-cm diameter class (or 6-cm class) in the data set.

Modeling method 1: Using *Dq* and *Dvar*

For each species, the diameter variance (*Dvar*) was predicted separately as follows:

$$Dvar_s = f\left(H, N, B, Dq, \frac{N_s}{N}, \frac{B_s}{B}, \frac{Dq_s}{Dq}\right) + \varepsilon \quad (5)$$

As with the $Dmin_s$ model, the backward elimination technique was also applied here.

The scale and shape parameters b_s and c_s, were solutions of equations involving Dq_s and $\widehat{D}var_s$ as follows (Poudel and Cao 2013):

$$b_s = -\frac{a_s G_{1s}}{G_{2s}} + \left[\left(\frac{a_s}{G_{2s}}\right)^2 (G_{1s}^2 - G_{2s}) + \frac{Dq_s^2}{G_{2s}}\right]^{0.5} \quad (6)$$

$$0 = b_s^2(G_{2s} - G_{1s}^2) - \widehat{D}var_s \quad (7)$$

where $G_{is} = \Gamma(1 + i/c_s), i = 1, 2; \Gamma(\bullet)$ is the complete gamma function.

Modeling method 2: Using *Dq* and D_{90}

The *90*th percentile (D_{90}) was predicted separately for each species:

$$D_{90s} = f\left(H, N, B, Dq, \frac{N_s}{N}, \frac{B_s}{B}, \frac{Dq_s}{Dq}\right) + \varepsilon \quad (8)$$

The scale and shape parameters, b_s and c_s, were solutions of equations involving Dq_s and $\widehat{D}_{90s}$ as follows:

$$b_s = \frac{\widehat{D}_{90s} - a_s}{[-ln(1 - 0.90)]^{1/c_s}} \quad (9)$$

$$c_s = b_s^2 G_{2s} + 2a_s b_s G_{1s} + a_s^2 - Dq_s^2 \quad (10)$$

Estimation methods

In the first phase, parameters to predict *Dmin* were estimated by use of nonlinear regression. In the second phase, the following three estimation methods were used to obtain parameters to predict *Dq* and *Dvar* for each of the two modeling methods.

Estimation method a: Least Squares

For each species, the coefficients of equations (5) or (8) were obtained by minimizing the sum of the squared differences between observed and predicted *Dvar* (modeling method 1) or D_{90} (modeling method 2). This least squares method is often called the Parameter Recovery method (Burkhart and Tomé, 2012, Weiskittel et al., 2011).

Estimation method b: CDF Regression - Separate estimation

This CDF Regression approach was developed by Cao (2004).The coefficients for predicting *Dvar* (equation 5) or D_{90} (equation 8) were obtained separately for each species by minimizing sum of the squared differences between observed and predicted cumulative probabilities:

$$\text{minimize} \sum_{i=1}^{m} \frac{\sum_{j=1}^{n_{si}} (F_{sij} - \widehat{F}_{sij})^2}{n_{si}} \quad (11)$$

where $F_{sij}=((j-0.5))/n_{si}$ is observed cumulative probability of tree *j* of species *s* in plot *i*, *j* is rank (from smallest to largest) of that tree in terms of *dbh* for species *s* in plot *i*, $\widehat{F}_{sij} = 1 - exp\left\{-\left(\frac{x_{sij} - a_s}{b_s}\right)^{c_s}\right\}$ =value of the Weibull CDF evaluated at x_{sij},

x_{sij} is *dbh* of tree *j* of species *s* in plot *i*,

n_{si} is number of trees of species *s* in plot *i*, and

m is number of plots.

The Weibull location parameter a_s was predicted from equations (3) and (4) as shown earlier. For modeling method 1, the scale and shape parameters b_s and c_s, were solutions of equations (6) and (7). Similarly for modeling method 2, b_s and c_s were solutions of equations (9) and (10).

Table 3. Regression equations for predicting the minimum diameter (*Dmin*), diameter variance (*Dvar*) and the 90_{th} diameter percentile (D_{90}), by species. All coefficients are statistically different from zero at α=0.05.

Dependent Variable	Species[1/]	Regression Equation	R^2
Dmin	1	$Dmin_1 = \frac{Dq_1}{1 + \exp[a_{01} + a_{11}\ln(N_1)]}$	0.8935
	2	$Dmin_2 = \frac{Dq_2}{1 + \exp[a_{02} + a_{12}\ln(N_2)]}$	0.9385
	3	$Dmin_3 = \frac{Dq_3}{1 + \exp[a_{03} + a_{13}\ln(N_3)]}$	0.7757
Dvar	1	$Dvar_1 = \exp[b_{01} + b_{11}\ln(N) + b_{21}\ln(B) + b_{31}\ln(N_1)]$	0.3857
	2	$Dvar_2 = \exp[b_{02} + b_{12}\ln(N_2) + b_{22}\ln(B_2)]$	0.5449
	3	$Dvar_3 = \exp[b_{03} + b_{13}\ln(N) + b_{23}\ln(B) + b_{33}\ln(N_3) + b_{43}\ln(B_3)]$	0.4986
D_{90}		$D_{90,1} = \exp[c_{01} + c_{11}\ln(N_1) + c_{21}\ln(B_1)]$	0.9487
		$D_{90,2} = \exp[c_{02} + c_{12}\ln(N_2) + c_{22}\ln(B_2)]$	0.9537
		$D_{90,3} = \exp[c_{03} + c_{13}\ln(N_3) + c_{23}\ln(B_3)]$	0.8669

[1/] Species: 1 = *Quercus cerris*, 2 = *Quercus petraea*, and 3 = *Quercus frainetto*.

Table 4. Evaluation statistics by method and species. Below each statistic is its relative ranking in parentheses. For each row, a bold, italic number denotes the best method for that row.

Statistic	Species	Method					
		1a	1b	1c	2a	2b	2c
AD	*Q. cerris*	1.4761	0.9158	0.9167	0.8879	0.8651	**0.8389**
		(6.00)	(1.60)	(1.61)	(1.39)	(1.21)	(1.00)
	Q. petraea	0.9705	0.9256	0.9258	1.0896	**0.8916**	0.9028
		(2.99)	(1.86)	(1.86)	(6.00)	(1.00)	(1.28)
	Q. frainetto	0.8070	0.7826	**0.7697**	0.9794	0.8000	0.8148
		(1.89)	(1.31)	(1.00)	(6.00)	(1.72)	(2.07)
–lnL	*Q. cerris*	58.163	54.862	54.875	54.498	54.400	**54.331**
		(6.00)	(1.69)	(1.71)	(1.22)	(1.09)	(1.00)
	Q. petraea	45.040	44.837	44.839	45.338	**44.759**	44.771
		(3.42)	(1.67)	(1.69)	(6.00)	(1.00)	(1.11)
	Q. frainetto	**42.180**	42.247	42.215	42.633	42.204	42.232
		(1.00)	(1.74)	(1.39)	(6.00)	(1.27)	(1.57)
KS	*Q. cerris*	0.1890	**0.1712**	0.1713	0.1749	0.1723	0.1719
		(6.00)	(1.00)	(1.02)	(2.05)	(1.30)	(1.19)
	Q. petraea	**0.2039**	0.2055	0.2055	0.2123	0.2048	0.2052
		(1.00)	(1.96)	(1.93)	(6.00)	(1.51)	(1.78)
	Q. frainetto	0.2024	0.1996	**0.1982**	0.2162	0.2030	0.2040
		(2.18)	(1.41)	(1.00)	(6.00)	(2.33)	(2.62)
EI	*Q. cerris*	478.89	459.83	459.89	459.89	**457.77**	458.12
		(6.00)	(1.49)	(1.50)	(1.50)	(1.00)	(1.08)
	Q. petraea	391.83	391.04	391.07	394.65	**390.03**	390.25
		(2.95)	(2.10)	(2.13)	(6.00)	(1.00)	(1.23)
	Q. frainetto	408.82	409.33	409.19	412.14	**408.37**	408.55
		(1.61)	(2.28)	(2.09)	(6.00)	(1.00)	(1.25)

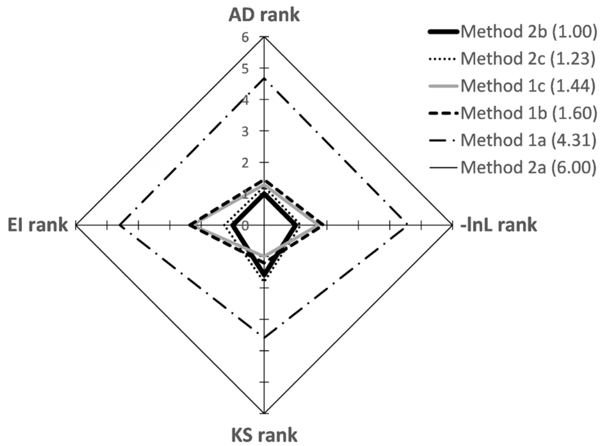

Figure 4. Overall comparison of the six methods. The overall ranking is shown in parentheses for each method. The method (2b) resulting in the smallest area inside the box represents the best method.

Estimation method c: CDF Regression - Simultaneous estimation

This is similar to estimation method *b*, except that the coefficients of equations (5) or (8) were obtained simultaneously for all three species by minimizing the total squared differences between observed and predicted cumulative probabilities:

$$\text{minimize} \sum_{i=1}^{m} \sum_{s=1}^{3} \frac{\sum_{j=1}^{n_{si}} (F_{sij} - \hat{F}_{sij})^2}{n_{si}} \quad (12)$$

Evaluation of Methods

A two-fold validation technique was used to evaluate the methods. In the first phase, coefficients obtained from one group were used to predict for the other group. In the second phase, predictions from both groups were pooled to compute the evaluation statistics. The method that produces the lowest values for each of the following evaluation statistics is the best method for that criterion.

The Anderson-Darling (AD) statistic

$$AD_{si} = -n_{si} - \sum_{j=1}^{n_{si}} (2j-1)\left[\ln(u_{sj}) + \ln\left(1 - u_{s,n_{i}-j+1}\right)\right]/n_{si} \quad (13)$$

where $u_{sj} = \hat{F}_{sij} = 1 - exp\left\{-\left(\frac{x_{sij} - a_s}{b_s}\right)^{c_s}\right\}$, n_{si} is number of trees of species *s* in the *i*th plot, and the x_{sij}'s are diameters, sorted in ascending order for species *s* in plot *i* $i\ (x_{si1} \le x_{si2} \ldots \le x_{si,n_{si}})$.

The Kolmogorov-Smirnov (KS) statistic

$$KS_{si} = \max\{\max_{1\le j\le n_{si}}[(j/n_{si}) - u_{sj}], \max_{1\le j\le n_i}[(u_{sj}-(j-1))/n_{si}]\} \quad (14)$$

Negative log-likelihood (–lnL) statistic

$$-\ln L_s = \sum_{j=1}^{n_{si}} \left[\ln(b_s) - \ln(c_s) + (1-c_s)\ln\left(\frac{x_{sij}-a_s}{b_s}\right) + \left(\frac{x_{sij}-a_s}{b_s}\right)^{c_s}\right] \quad (15)$$

where $-\ln L_s$ is the negative value of the log-likelihood function of the Weibull distribution.

Error Index (EI)

$$EI_{si} = \sum_{k=1}^{m_{si}} |n_{sik} - \hat{n}_{sik}| \quad (16)$$

where n_{ik} and $\hat{n}_{ik}$ are, respectively, observed and predicted number of trees per ha of species *s* in the *k*th diameter class in plot *i*, and m_{si} is the total number of diameter classes of species *s* in the *i*th plot.

Ranking of methods

For each statistic and each oak species group, a relative rank (between 1 and 6) was computed for each of the six combinations (two modeling methods × three estimation methods). The relative rank method (Poudel and Cao, 2013; Özçelik et al. 2019; Sun et al. 2019) was used in this study. This ranking method assigns relative ranks of 1 and *k*, respectively, for the best and worst methods, where *k*=6 is number of methods being evaluated. The remaining methods have ranks as real number between 1 and *k*. This scheme considers both magnitude and order of the evaluation statistics, and therefore should offer more information than the traditional ordinal ranks. The relative rank of method *i* is defined as:

$$R_i = \frac{(m-1)(S_i - S_{min})}{S_{max} - S_{min}} \quad (17)$$

where R_i= relative rank of method *i*, S_i= goodness-of-fit statistics produced by method *i*, S_{min}= minimum value of the goodness-of-fit statistics, and S_{max}= maximum value of the goodness-of-fit statistics.

RESULTS AND DISCUSSION

Regression equations for each species

The general model used in this study for equations (3), (5), and (8) includes some of common stand-level variables such as *H*, *N*, *B*, D_q for each species in mixed-species stands of oaks. The final models for *Dmin*, *Dvar*, and D_{90} varied depended on the species (Table 3). Equations for *Dvar* included variables for total stand density (*N* and *B*), whereas those for *Dmin* and D_{90} did not. Because stand variables often perform well in groups, the backward elimination approach applied in this study has the advantage of keeping these sets of variables intact, in contrast to the forward and stepwise approaches.

Values of R^2 ranged from 0.78 to 0.94 for *Dmin* and from 0.87 to 0.95 for D_{90}. In contrast, R^2 for *Dvar* ranged from 0.39 to 0.54, slightly lower that the R^2 values between 0.47 to 0.58 obtained by Sun et al. (2019).

Evaluation

The evaluation statistics were computed separately for each oak species group. Table 4 shows the evaluation statistics and their relative ranks by oak species, prediction approach, and fitting method. According to these results, the Least Squares method was inferior to the two CDF Regression methods; this was consistent with findings from Cao (2004) and Poudel and Cao (2013).

Summing the relative ranks across the four evaluation criteria and three species groups yields the overall rankings, displayed graphically in the radar chart (Figure 3). Each method is represented by a quadrilateral; its area is smallest for the best method and largest for the worst method. The two Parameter Recovery estimation methods (1a and 2a) were the worst, scoring 4.31 and 6.00, respectively. The remaining methods (1b, 1c, 2b, and 2c) produced similar results, ranking between 1.00 and 1.60. The best overall method was method 2b, which involves the prediction of D_{90} (modeling method 2) with coefficients estimated from the CDF Regression method, separately for each species (estimation method b).

Modeling methods: *Dvar* vs. D_{90}

Modeling method 1 is based on the method of moments that involves *Dq* and *Dvar*, whereas modeling method 2 is a hybrid method that utilizes both a moment (*Dq*) and a percentile (D_{90}). Overall, the hybrid method was the better approach for both estimation methods b and c (Figure 3). The exception was for estimation method a: method 1a ranked higher than method 2a (4.31 vs. 6.00). The low R^2 values (from 0.39 to 0.54) explained the difficulty in predicting *Dvar* from other stand variables (Table 3), and therefore brought about low goodness-of-fit values from the resulting Weibull distributions. The reverse was true for D_{90}, with high R^2 values (from 0.87 to 0.95).

Estimation methods

The sum of the ranks across the four evaluation criteria and three species groups revealed that the Least Squares method (a) was a distant third among the three estimation methods with a sum of 95.20. The CDF Regression method in which coefficients were estimated separately for each species (method b) scored slight better (rank sum = 35.53) than method c with simultaneous estimation for all species (rank sum = 36.13). Results showing that the Parameter Recovery approach was inferior to the CDF Regression approach were also reported by Cao (2004) and Poudel and Cao (2013).

Figure 3 shows that the difference in performance between methods 1b and 1c, and between methods 2b and 2c were negligible. This means that estimating the coefficients separately for each species (method b), which is a simpler procedure, is preferable to simultaneously estimating for all species (method c). These results are similar to those obtained in the studies by Maltamo (1997) and Sun et al. (2019).

Forest managers are able to predict future volume yields more accurate predict using proposed models for mixed-oaks stands in northwest Turkey and also have financial insight into the stand's future value.

CONCLUSIONS

Diameter distribution models, based on the Weibull function, were developed for even-aged mixed-oak stands (Turkey oak, Sessile oak, and Hungarian oak) in northwest Turkey. Two modeling methods were considered. Weibull parameters were recovered from either equations predicting *Dq* and *Dvar* (method of moments) or equations predicting *Dq* and D_{90} (hybrid method). For each modeling method, three estimation methods were considered: (a) Least Squares method, (b) CDF Regression method in which regression coefficients were estimated separately for each species, and (c) CDF Regression method in which regression coefficients were simultaneously estimated for all species.

Results indicated that the hybrid method coupled with the CDF Regression estimation method yield best results in this study. Also, similar results were obtained when the regression coefficients were estimated either separately for each species or simultaneously for all species.

The diameter distribution models, as outlined in this study, allow successful prediction of diameter distribution for a given mixed-oak species stand in northwestern Turkey, using stand-level information. This research has significantly increased our knowledge of diameter distributions of mixed oak stands in Bilecik Region. In fact, this is the first study of this type on mixed oak stands in Turkey. These models should play an important role in planning and inventorying mixed-oak stands. Nevertheless, results might be different for other mixed species stands.

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