






Multifractal analysis of weeds in a no-tillage system in the Pre-Amazon region¹

Análise multifractal das plantas-daninhas em sistema de semeadura direta na região Pré-Amazônia

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HIGHLIGHTS:

Weeds present varying spatial variability scales.

Euphorbia hirta L. exhibited high density and abundance.

Commelina benghalensis L. exhibited the greatest heterogeneity among the evaluated weeds.

ABSTRACT: Weeds have several mechanisms of dispersal and reproduction, resulting high spatial variability. The objective of this study was to assess the scale and spatial heterogeneity of weeds using multifractal analysis in a no-tillage system in the Pre-Amazon region. Sampling was conducted in a commercial soybean (*Glycine max*) production plot in the Mata Roma, Maranhão, Brazil, comprising 1,071 points marked on a 10 × 10 m grid. Data were analyzed using descriptive statistics and multifractal analysis through the box-counting method. Weeds showed varying degrees of multifractality, resulting in different scales and spatial heterogeneity in the study area. *Euphorbia hirta* and *Turnera subulata* exhibited asymmetry of branches to the left in the singularity spectrum, indicating dominance of high measurement values.

Key words: spontaneous plants, degree of multifractality, generalized dimension, singularity spectrum

RESUMO: As plantas-daninhas possuem diversos mecanismos de dispersão e reprodução descrevendo elevada variabilidade espacial. O objetivo deste estudo foi avaliar a heterogeneidade de escala e espacial de plantas-daninhas utilizando análise multifractal em sistema de semeadura direta na região Pré-Amazônia. A amostragem foi realizada em uma parcela de produção comercial cultivada com soja (*Glycine max*). Na área de estudo foram demarcados 1.071 pontos, em malha regular de 10 × 10 m, no município de Mata Roma (Maranhão, Brasil). Os dados foram analisados por meio da estatística descritiva e análise multifractal empregando o método box-counting. As plantas-daninhas apresentaram diferentes graus de multifractalidade, descrevendo maior ou menor heterogeneidade de escala e espacial na área de estudo. *Euphorbia hirta* e *Turnera subulata* apresentaram assimetria dos ramos do espectro de singularidade para a esquerda, indicando domínio de valores elevados de medidas.

Palavras-chave: plantas espontâneas, grau de multifractalidade, dimensão generalizada, espectro de singularidade



INTRODUCTION

Weeds in agricultural production areas present high spatial variability (Chiba et al., 2010; Siqueira et al. 2016; Silva et al., 2022), and can compromise crop yields when not properly managed (Gazziero et al., 2015; Siqueira et al., 2016; Caetano et al., 2018; Castro et al., 2021; Silva et al., 2021; Silva et al., 2022; Osunleti et al., 2022).

Spatial variability of weeds has been described through different methods. Chiba et al. (2010) and Siqueira et al. (2016) analyzed the spatial variability of weeds using geostatistical tools, whereas Silva et al. (2022) evaluated scale properties of weeds using multifractal analysis. Multifractal analysis characterizes the structure of a system or object, allowing for the evaluation of the distribution of measurement values at different scales, thus describing the spatial variability (Hentschel & Procaccia, 1983; Halsey et al., 1986; Evertsz & Mandelbrot, 1992; Posadas et al., 2009). Therefore, it enables the assessment of the heterogeneity of a system (Vidal-Vázquez et al., 2013; Bertol et al., 2017; Santos et al., 2019; Leiva et al., 2019; Silva & Siqueira, 2020; Leiva et al., 2021; Siqueira & Silva, 2022; Siqueira et al., 2022; Silva et al., 2022). According to Dafonte et al. (2015), it is possible to determine whether the structure of a system is monofractal or multifractal.

Therefore, the hypothesis in this work is that the spatial distribution of weeds constitutes multifractal systems. The objective of this study was to evaluate the scale and spatial heterogeneity of weeds using multifractal analysis in a no-tillage system in the Pre-Amazon region.

MATERIALS AND METHODS

The study was conducted in a 10-hectare area (Figure 1) in Mata Roma, state of Maranhão, Brazil ($3^{\circ}42'26.56''S$, $43^{\circ}11'19.56''W$, and average altitude of 130 m). The soil of the area was classified as a Latossolo Vermelho-Amarelo típico by the Brazilian Soil Classification System (Santos et al., 2018), which corresponds to a Typic Hapludox (United States, 2022). The region presents an Aw climate, according to the Koppen classification, with two well-defined seasons: a dry season from June to November and a rainy season from December to May, with an annual rainfall depth of 1,835 mm, temperatures ranging from 23 to 36 °C, and a mean relative air humidity is 80%. The study area has been managed with crop rotation since 2007, with soybean [*Glycine max* (L.) Merrill] and maize (*Zea mays* L.) crops under no tillage system, with subsoiling up to a depth of 0.32 m when necessary, every five years.

Sampling was conducted when the area was cultivated with soybeans, sown on December 23, 2016. Weeds were sampled on January 12, 2017, at 1,071 points distributed in a 10 × 10 m grid (Figure 2), considering weeds within circular plots with a useful area of approximately 1 m² (1.126 m diameter). Weeds were counted and identified following the procedures described by Gazziero et al. (2015). A total of 21,217 individuals were identified: *Euphorbia hirta* L. (10,021), *Spigelia anthelmia* L. (4,399), *Spermacoce verticillata* L. (2,024), *Cenchrus echinatus* L. (1,968), *Turnera subulata* Sm. (1,586), *Commelina benghalensis* L. (925), *Sida rhombifolia* L. (147),



Figure 1. Map of Brazil with the location of the study area in Mata Roma, Maranhão, Brazil

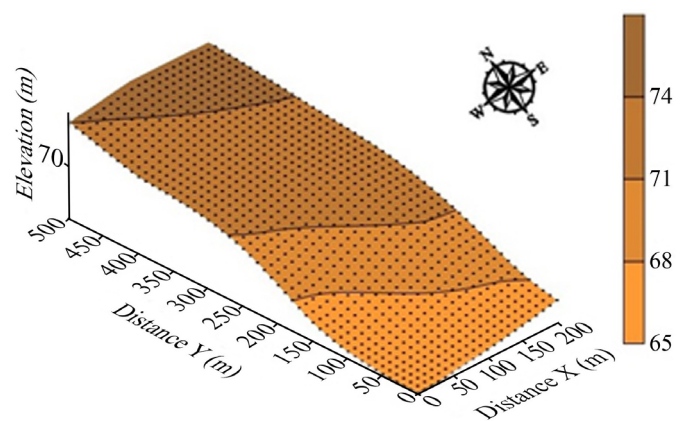


Figure 2. Altimetric map and sampling arrangement (10 × 10 m) for weed sampling in an area under no-tillage system in Mata Roma, Maranhão, Brazil

Desmodium tortuosum (Sw.) DC. (83), and *Scoparia dulcis* L. (64). *Sida rhombifolia* L., *Desmodium tortuosum* (Sw.) DC., and *Scoparia dulcis* L. were grouped into a category called Other Weeds (OW) due to their lower expression in the study area.

The ecology among species was evaluated according to Küchler et al. (1976), considering: Density = total number of individuals per species / total number of sampling points in the area (plants m⁻²); Relative Density = species density × 100 / sum of densities of all specimens (%); Frequency = number of sampling points containing the species / total number of sampling points obtained in the area; Relative Frequency = species frequency × 100 / sum of frequencies of all specimens (%); Abundance = total number of individuals per species / total number of sampling points containing the species; Relative abundance = species abundance × 100 / sum of abundances of all specimens (%); and Importance Value Index (IVI) = relative density + relative frequency + relative abundance (%). Richness was determined by the total number of taxonomic groups identified at each sampling point.

Data of the weed species *E. hirta*, *S. anthelmia*, *S. verticillata*, *C. echinatus*, *T. subulata*, *C. benghalensis* and OW were subjected to descriptive statistical analysis to obtain the main statistical moments: mean (\bar{x}), variance, standard deviation, coefficient of variation (CV%), asymmetry, kurtosis, and D (maximum deviation from the normal distribution using the Kolmogorov-Sminorv test, with $p \geq 0.01$). The coefficient of variation was classified as low (CV < 12%), moderate (12% < CV < 60%), or high (CV > 60%), according to Warrick & Nielsen (1980).

The multifractal analysis was conducted using the Non-linear Analysis Scaling System (NASS) software (Oliveira et al., 2022), and the box-counting method, which allows for the subdivision of the geometric support (δ) into grids of size (L) and successive moments (N = 2, 3, 4, 5, 6, 7 ...), thus the segments are evaluated successively tending to infinity, $n \rightarrow \infty$ (Evertsz & Mandelbrot, 1992).

In heterogeneous systems, the contents of the grids can be quantified following the scale properties (Eq. 1 - Table 1), using a probability distribution (P), which enables the estimation of the scale properties (ϵ) in the i_{th} region or spatial location (Posadas et al., 2009); thus, the Hölder exponent (α_i) can vary in the interval ($\alpha_{-\infty}$, $\alpha_{+\infty}$). The partition function ($\mu(q, \epsilon)$) (Eq. 2) of order q is confirmed based on the scale

properties, where N(ϵ) is the number of segments with size ϵ and the statistical moments q are defined by $-\infty < q < \infty$. Furthermore, the partition function is dimensioned as $\epsilon^{\tau(q)}$ (Eq. 3), where the exponent $\tau(q)$ is the moment correlation exponent of order q, also known as the mass function (Halsey et al., 1986). In this sense, multifractal sets are characterized through the generalized dimension (D - Eq. 4) for moments of order q in a Dq distribution (Hentschel & Procaccia, 1983); when q is replaced by 0, 1, and 2, it is possible to determine the dimensions of capacity (Eq. 5), information (Eq. 6) and correlation (Eq. 7), respectively.

The dimension spectra or singularity spectra (q) are defined by Equations 8 and 9 (Chhabra & Jensen, 1989), which specify that the scale properties of the partition function reflect the contribution of individual segments. The degree of multifractality (Δ - Equation 10) and asymmetry (AI - Equation 11) of the data were determined considering the values of D_q and α where: D is the generalized dimension at times $q = -5$ and $q = 5$; AI is the asymmetry of the system; α_0 is the value of $f(\alpha)$ in interval 0; α_5 is the value of $f(\alpha)$ in the interval $q = 5$; and α_{-5} is the value of $f(\alpha)$ in the interval $q = -5$ (Halsey et al., 1986).

RESULT AND DISCUSSION

The diversity parameters presented in Table 2 are indicators used to characterize the ecology among species. Density, frequency, and abundance express the participation of different species, the spatial distribution of each specimen, and concentration of species in the study area, respectively (Küchler et al., 1976; Siqueira et al., 2016; Caetano et al., 2018; Castro et al., 2021). The importance value index (IVI) is intended to characterize which species have a greatest influence within the weed community (Caetano et al., 2018), thus, the higher the IVI (Table 2), the higher the positive species rate at the sampling points.

The weed species with the highest density (D) and relative density (RD - Table 2) were *E. hirta* (D = 9.36 plants m⁻² and RD = 47.23%), *S. anthelmia* (D = 4.11 plants m⁻² and RD = 20.73%), *S. verticillata* (D = 1.89 plants m⁻² and RD = 9.54%), *C. echinatus* (D = 1.84 plants m⁻² and RD = 9.28%), and *T. subulata* (D = 1.48 plants m⁻² and RD = 7.48%). The highest D and RD of the species *E. hirta* reflect its ecological characteristics. According to Gazziero et al. (2015), *E. hirta* is an annual, latex-bearing, semi-prostrate, pigmented plant with rapid seed reproduction; furthermore, plants of the genus *Euphorbia* are among the most worrying in soybean crops,

Table 1. Equations used in the multifractal analysis process

	$P_i(\epsilon) \sim \epsilon^{\alpha_i}$ (1)
Partition function	$\mu(q, \epsilon) = \sum_{i=1}^{N(\epsilon)} P_i^q(\epsilon)$ (2)
	$\mu(q, \epsilon) \sim \epsilon^{\tau(q)}$ (3)
Generalized dimension	$D_q = \lim_{\epsilon \rightarrow 0} \left(\frac{1}{q-1} \frac{\log \sum_{i=1}^{N(\epsilon)} P_i^q(\epsilon)}{\log(\epsilon)} \right)$ (4)
	$D_0 = \lim_{\epsilon \rightarrow 0} \frac{\log(N(\epsilon))}{\log(\epsilon)}$ (5)
	$D_1 = \lim_{\epsilon \rightarrow 0} \frac{\sum_{i=1}^{N(\epsilon)} \mu_i(\epsilon) \log(\mu_i(\epsilon))}{\log(\epsilon)}$ (6)
	$D_2 = \lim_{\epsilon \rightarrow 0} \frac{\log(C(\epsilon))}{\log(\epsilon)}$ (7)
Spectrum of singularities	$f(q) = \lim_{\epsilon \rightarrow 0} \frac{1}{\log(\epsilon)} \sum_{i=1}^{N(\epsilon)} \mu_i(q, \epsilon) \log[\mu_i(q, \epsilon)]$ (8)
	$\alpha(q) = \lim_{\epsilon \rightarrow 0} \frac{1}{\log(\epsilon)} \sum_{i=1}^{N(\epsilon)} \mu_i(q, \epsilon) \log[P_i(\epsilon)]$ (9)
Degree of multifractality (Δ) and asymmetry (AI)	$\Delta = D_{-5} - D_5$ (10)
	$AI = \frac{\alpha_0 - \alpha_5}{\alpha_{-5} - \alpha_0}$ (11)

P - Probability distribution; i - Spatial location; ϵ - Scale; α - Hölder exponent; $\mu(q, \epsilon)$ - Partition function; N(ϵ) - number of segments with size ϵ ; q - Statistical moment; $\tau(q)$ - Correlation exponent at the moment of order q; D - Generalized dimension; C(ϵ) - Correlation function; f - Singularity function; α - Uniqueness spectrum; Δ - Degree of multifractality; AI - Asymmetry

Table 2. Ecological variables for weed species in Mata Roma, Maranhão, Brazil

Variable	D	RD	F	RF	A	RA	IVI
<i>Cenchrus echinatus</i> L.	1.84	9.28	0.97	17.08	1.89	7.65	34.00
<i>Commelina benghalensis</i> L.	0.86	4.36	0.81	14.23	1.06	4.31	22.90
<i>Desmodium tortuosum</i> (Sw.) DC.	0.08	0.39	0.05	0.80	1.69	6.87	8.06
<i>Euphorbia hirta</i> L.	9.36	47.23	1	17.54	9.36	37.92	102.69
<i>Scoparia dulcis</i> L.	0.06	0.30	0.06	1.05	1	4.05	5.40
<i>Sida rhombifolia</i> L.	0.14	0.69	0.10	1.69	1.43	5.78	8.16
<i>Spermacoce verticillata</i> L.	1.89	9.54	1	17.54	1.89	7.66	34.74
<i>Spigelia anthelmia</i> L.	4.11	20.73	0.91	15.95	4.52	18.31	54.99
<i>Turnera subulata</i> Sm.	1.48	7.48	0.81	14.13	1.84	7.45	29.05
Total	19.81	100	5.70	100	24.67	100	300

D - Density (plants m⁻²); RD - Relative density (%); F - Frequency; RF - Relative frequency (%); A - Abundance; RA - Relative abundance (%); IVI - Importance value index (%)

as they have reproduction and dispersal strategies that make their management difficult. Similarly, Caetano et al. (2018) and Castro et al. (2021) evaluated weeds in areas of the Brazilian Cerrado biome with soybean crops under no-tillage system and reported predominance of *E. hirta*.

Osunleti et al. (2022) evaluated weed control methods and reported *S. anthelmia* as the species with the highest density and relative density in the treatments evaluated, mainly due to its high degree of tolerance to oxyfluorfen at a rate of 0.36 kg ha⁻¹. The other weed species found in the present work had less expressiveness, which is consistent with the results of Chiba et al. (2010), Gazziero et al. (2015), Siqueira et al. (2016), Caetano et al. (2018), Castro et al. (2021), Silva et al. (2021), and Silva et al. (2022).

The species with the highest frequency were *E. hirta* (F = 1) and *S. verticillata* (F = 1); frequency close to one denotes uniformity in the distribution of weeds in the study area; *E. hirta* and *S. verticillata* were found in all sampling points. The species with the highest abundance was *E. hirta* (A = 9.36 - Table 2), with a high concentration of weed plants, resulting in a higher importance value index (IVI = 102.69). According to Freitas et al. (2021) *Euphorbiaceae* is a family of species with short cycles, tiny inflorescences, and a high potential for seed production. This explains the high incidence of *E. hirta* in the study area, as described by the ecological parameters (Table 2).

The three species with the lowest IVI [*S. rhombifolia* (8.16%), *D. tortuosum* (8.06%), and *S. dulcis* (5.40%)] were grouped into the category Other Weeds (OW). According to Carvalho & Carvalho (2009) and Gazziero et al. (2015), species in the OW category have later germination relative to the soybean cycle, explaining the occurrence of lower IVI for these species.

The statistical parameters of weeds in the study area are shown in Table 3. The species *E. hirta* had the highest mean number of individuals (\bar{x} = 9.36 plants per m²), followed by *S. anthelmia* (4.52 plants per m²) and OW (\bar{x} = 2.85 plants per m²). These are similar results to those found in other studies. Samuel & Rastogi (2022) evaluated the ecology of *E. hirta* and found 4.11 plants per m². Silva et al. (2022) evaluated multiscale properties of weed plants and found a mean of 11.35 plants per m² for the OW category, involving five species: *Bidens pilosa* L., *Amaranthus deflexus* L., *Ipomoea grandifolia* (Dammer) O'Donnell, *Acanthospermum australe* (Loerfl.) Kuntze, *Digitaria insularis* (L.) Fedde, *Euphorbia heterophylla* L., and *Parthenium hysterophorus* L.

The coefficients of variation (CV%; Table 3) of the evaluated weeds were classified as moderate (12 % < CV < 60 %), according to Warrick & Nielsen (1980); except for the OW

category (CV = 81%) which presented a high CV (> 60%). The OW category encompassed three weed species [*D. tortuosum*, *S. dulcis*, and *S. rhombifolia*], with high heterogeneity in the study area and cluster distribution (Chiba et al., 2010; Siqueira et al., 2016; Castro et al., 2021; Silva et al., 2022), as well as distinct ecological processes (Gazziero et al., 2015).

The weeds had a lognormal frequency distribution (Table 3), according to the Kolmogorov-Smirnov normality test (D-KS, p < 0.01), which is consistent with the median to high CV and the asymmetry and kurtosis values.

Table 4 presents the results of the multifractal analysis for moments of order q in the interval from q = -5 to q = +5. In multifractal systems, the capacity dimension (D₀), information dimension (D₁), and correlation dimension (D₂) follow the pattern: D₀ > D₁ > D₂ (Posadas et al., 2009; Vidal-Vázquez et al., 2013; Dafonte et al., 2015; Bertol et al., 2017; Leiva et al., 2019; Silva & Siqueira, 2020; Leiva et al., 2021; Siqueira et al., 2022; Silva et al., 2022). Therefore, the variables represent multifractal systems, as they present the pattern D₀ > D₁ > D₂, except for OW, which represents a monofractal system. According to Dafonte et al. (2015), a monofractal system present the following dimension pattern: D₀ ≈ D₁ ≈ D₂. OW represents a monofractal system due to the characteristics of the species in this class, as there are three species that occur in the study area with independent spatial patterns, resulting in high variability (CV = 81.00%) and comprising a chaotic system.

The capacity dimension (D₀ = 1.995) remained constant for all evaluated weed species, with values close to 2, indicating that almost all boxes/scales are filled with measurement values, as described by Posadas et al. (2009). The information dimension (D₁) measures the degree of heterogeneity in the system (Siqueira et al., 2022), and values close to 2 represent a relatively uniform distribution of measurement values across scales, whereas values close to 1 represent sets that have concentrated irregularities (Leiva et al., 2019; Silva et al., 2022). The highest and lowest D₁ values (Table 4) were found for Richness (D₁ = 1.992) and for *C. benghalensis* (D₁ = 1.913); despite the numerical difference in D₁, in both cases, the values are close to 2, indicating uniformly distributed systems. Mathematically, the values of correlation dimension (D₂ - Table 4) are associated with the correlation function (Hentschel & Procaccia, 1983) and describe how measurements are distributed in boxes/scales. The results showed that D₂ values ranged from 1.818 (*C. benghalensis*) to 1.988 (Richness), indicating low irregularity in the data series.

The degree of multifractality (Δ - Table 4) describes systems with higher or lower heterogeneity (Vidal-Vázquez et al., 2013; Siqueira & Silva, 2022). The highest degree of multifractality

Table 3. Descriptive statistics for the number of weed plants

Weeds	\bar{x}	Variance	SD	CV (%)	Skew	Kurtosis	D-KS
<i>Cenchrus echinatus</i> L.	1.89	0.21	0.46	24.00	-0.44	1.32	0.05 Ln
<i>Commelina benghalensis</i> L.	1.06	0.06	0.25	23.00	3.55	10.66	0.06 Ln
<i>Euphorbia hirta</i> L.	9.36	20.03	4.48	48.00	1.64	6.43	0.05 Ln
<i>Spermacoce verticillata</i> L.	1.89	0.20	0.45	24.00	-0.45	1.40	0.05 Ln
<i>Spigelia anthelmia</i> L.	4.52	4.28	2.07	46.00	0.49	-0.50	0.05 Ln
<i>Turnera subulata</i> Sm.	1.84	1.08	1.04	57.00	1.66	5.13	0.06 Ln
OW	2.85	5.38	2.32	81.00	3.76	15.65	0.16 Ln

OW - Other weeds [*Sida rhombifolia* L., *Desmodium tortuosum* (Sw.) DC, and *Scoparia dulcis* L.]; \bar{x} - mean (plants per m²); SD - Standard deviation; CV - Coefficient of variation (%); D-KS - Maximum deviation from the normal distribution using the Kolmogorov-Smirnov test at p ≥ 0.01

Table 4. Multifractal parameters of the attributes of the study area images

Weeds	q	q _±	Δ	D ₀	D ₁	D ₂	α ₋₅	α ₅	α ₀	AI	
<i>Cenchrus echinatus</i> L.	-5	5	0.255	1.995	1.986	1.980	2.532	1.953	2.007	0.102	
<i>Commelina benghalensis</i> L.	-5	5	0.388	1.995	1.913	1.818	2.227	1.657	2.055	2.324	
<i>Euphorbia hirta</i> L.	-5	5	0.124	1.995	1.984	1.971	2.118	1.845	2.005	1.401	
<i>Spermacoce verticillata</i> L.	-5	5	0.191	1.995	1.986	1.981	2.319	1.955	2.006	0.164	
<i>Spigelia anthelmia</i> L.	-5	5	0.094	1.995	1.988	1.980	2.151	1.935	2.002	0.448	
<i>Turnera subulata</i> Sm.	-5	5	0.219	1.995	1.967	1.942	2.167	1.805	2.024	1.529	
Richness	-5	5	0.071	1.995	1.992	1.988	2.163	1.969	1.998	0.172	
Abundance	-5	5	0.086	1.995	1.988	1.980	2.122	1.907	2.001	0.780	
OW				monofractal							

Richness - Species richness per point; Abundance - Abundance of weeds per point; OW - Other weeds; Δ - degree of multifractality; D₀ - Capacity dimension; D₁ - Information dimension; D₂ - Correlation dimension; α₋₅, α₅, α₀ are the singularity spectra for the moments q = -5, q = 5 and q = 0 respectively; AI - Asymmetry

was found for *C. benghalensis* (Δ = 0.388 - Table 4), indicating that this species occurs in the study area at a low density (D = 0.86 - Table 2). In the present study, the multifractality values (Δ) reflected heterogeneous systems with greater or lesser complexity for the biological systems under study (Silva et al., 2022).

The species with the highest occurrence in the study area showed a lower degree of multifractality (Δ) due to the homogenous distribution of these species in the experimental plot. Silva et al. (2022) evaluated the multifractality of weeds and reported that the degree of multifractality represents the complexity of the ecological dynamics of weeds, reinforcing the findings of the present study.

The diversity indices Richness and Abundance showed the lowest degree of multifractality (Δ = 0.071 and Δ = 0.086, respectively), as expected, since these indices represent measures with certain uniformity across boxes.

C. benghalensis had the highest Hölder exponent (α₀ = 2.055) and asymmetry (AI = 2.324) values. Thus, this species system had the highest multifractality/heterogeneity. Overall, the Hölder exponent (α₀) values found for the other weed species varied slightly from one species to another, indicating that the colonization process by weeds in the study area is structured, however, with different spatial variability scales among species. Contrastingly, asymmetry (AI) values showed a high variation in the study area, with the highest value found for *C. benghalensis* (AI = 2.324) and the lowest for *C. echinatus* (AI = 0.102). The presence of positive asymmetry (AI) indicates greater variability at scales corresponding to low measurement values (Siqueira et al., 2022; Silva et al., 2022), denoting a more frequent occurrence of low measurement values throughout the study area.

The generalized dimension (Figure 3A) at moment q (D_q) is a decreasing function with a sigma curve shape. The graph for *C. benghalensis* demonstrates that at times q = 0 to q = 5 there is a difference from the other weeds, showing higher heterogeneity, which is consistent with the result found for the degree of multifractality (Table 4 - Δ = 0.388). Richness had the lowest variation for both positive (q = 0 to q = 5) and negative (q = 0 to q = -5) moments, denoting greater system homogeneity and lower multifractality, as shown by the degree of multifractality (Table 4 - Δ = 0.071).

The mass exponent or Rényi graph (Figure 3B) shows multifractal behavior for all evaluated variables. According to Santos et al. (2019), linear graphs do not represent multifractal patterns, whereas nonlinear functions correspond

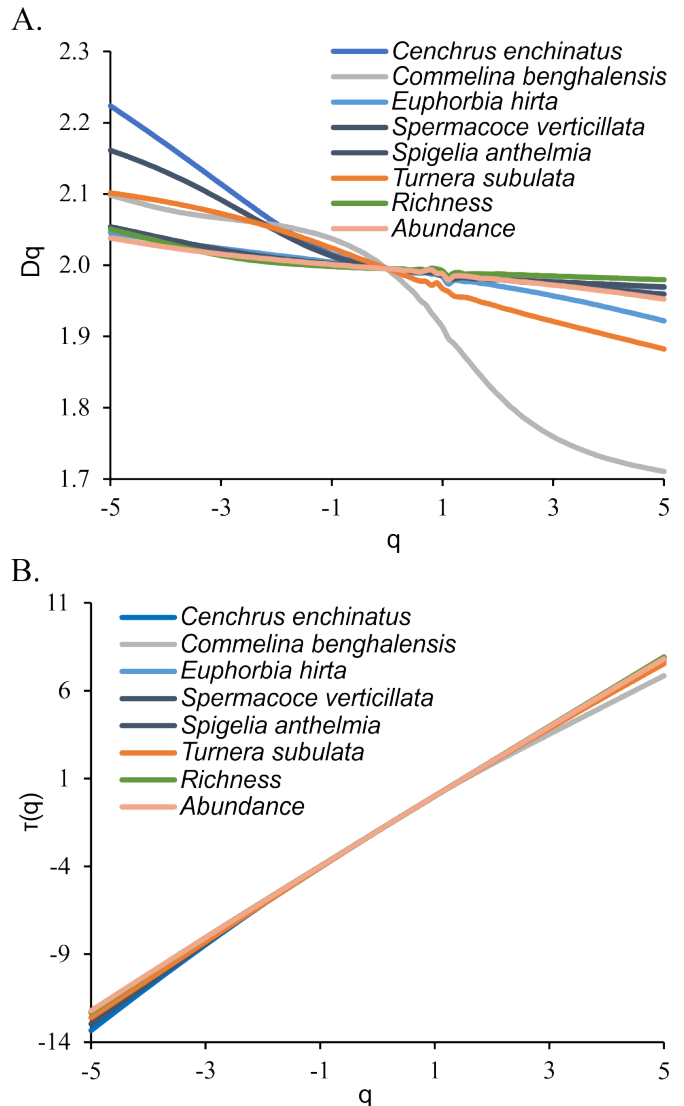


Figure 3. Generalized dimension graph (D_q vs q - A) and mass exponent graph (τ(q) vs q - B). q = statistical moment; D_q = generalized dimension for moments of order q; τ(q) = correlation exponent at the moment of order q

to multifractal systems, i.e., mass exponent graphs do not exhibit linear functions, but they present a certain curvature.

Weed plants (*C. echinatus*, *S. verticillata*, and *S. anthelmia*) and ecological variables (Richness and Abundance) (Figure 4) have a multifractal spectrum with asymmetry of the branches to the right, indicating dominance of low measurement values. The uniqueness spectrum for *C. benghalensis*, *E. hirta*, and *T. subulata* exhibits asymmetry to the left, indicating dominance of high measurement values. Information on the domain of of

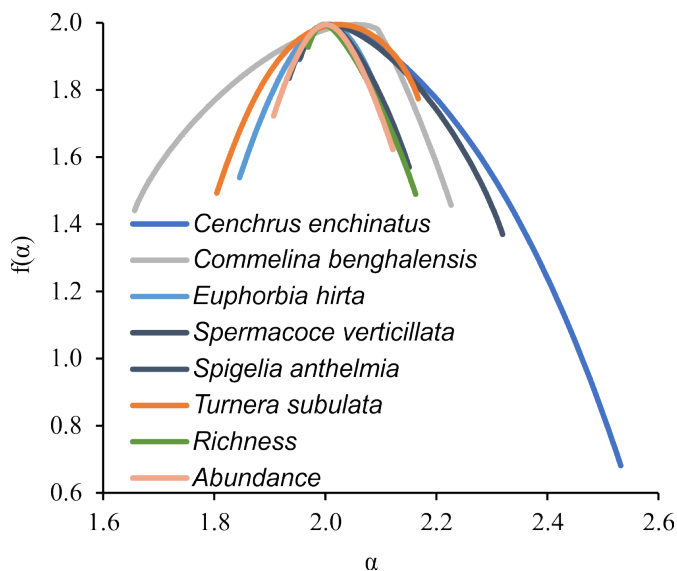


Figure 4. Singularity spectrum for weeds identified in soybean crops under no-tillage system (α - uniqueness spectrum; $f(\alpha)$ - singularity function for moments of order α ;) -

weed values allows for effective rate of production inputs, avoiding waste, thus preserving the environment.

The results found in this study are consistent with findings from other studies, such as those by Vidal-Vázquez et al. (2013), Silva & Siqueira (2020), and Silva et al. (2022), who evaluated the uniqueness spectrum and described the predominance of high and low values related to the left and right branches of the spectrum, respectively. The singularity spectrum (Figure 4) displays descending and concave parabolas (Bertol et al., 2017) confirming the multifractal behavior of the data (Dafonte et al., 2015).

The results showed that the promising use of multifractal analysis for studying weed plants, as it was possible to identify multifractal patterns related to ecological processes of the different species under study, including seed dispersal ability, dormancy period, and reproduction with a high disseminule production capacity (Gazziero et al., 2015; Freitas et al., 2021). The generalized dimension graph provides information on the spatial variability of measurement values, describing greater and lesser heterogeneity in the system (Posadas et al., 2009; Leiva et al., 2019). The singularity spectrum graph proved to be effective in evaluating the domain of measurement values (low or high), highlighting spatial distribution patterns that would not be characterized by other spatial analysis methods. The identification of spatial distribution patterns of weeds on a multifractal scale enables the development of increasingly precise management strategies.

CONCLUSIONS

1. Weeds exhibited varying degrees of multifractality (*C. benghalensis* - $\Delta = 0.388$, *C. enchinatus* - $\Delta = 0.255$, *T. subulata* - $\Delta = 0.219$, *S. verticillata* - $\Delta = 0.191$, *E. hirta* - $\Delta = 0.124$, and *S. anthelmia* - $\Delta = 0.094$), resulting in greater or lesser scales and spatial heterogeneity in the study area.

2. *Euphorbia hirta* and *Turnera subulata* presented asymmetry of branches to the left in the singularity spectrum, indicating dominance of high measurement values.

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