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Theoretical modeling of low-attenuation lamb wave modes generation in three-layer adhesive joints using angle beam transducer

Abstract

In this paper, at first the attenuation of Lamb waves in three-layer adhesive joints, including two elastic plates bonded together by a viscoelastic adhesive layer, is investigated using Global matrix method and then suitable incidence angle is theoretically calculated to generate low-attenuation Lamb waves using angle beam transducer. Theoretical boundary value problem in three-layer adhesive joints with perfect bond and traction-free boundary conditions on their outer surfaces is solved to find a combination of frequencies and modes with lowest attenuation. Characteristic equation is derived by applying continuity and boundary conditions in three-layer joints using Global matrix method. Phase velocity dispersion curves and attenuation intensity plot in high and low frequencies are obtained with numerical solution of this equation by a computer code for a three-layer joint, including an aluminum repair patch bonded to the aircraft aluminum skin by a layer of viscoelastic epoxy adhesive. To validate the numerical solution results of characteristic equation, wave structure curves are plotted for a special mode in two different frequencies in the adhesive joint. Also, transducer incidence angle is calculated in terms of frequency for different modes using theoretical method to generate Lamb wave modes with low attenuation level by angle beam transducer. These modes are recognizable by transducers in inspections with Lamb waves because of low attenuation level.

Keywords

Three-layer adhesive joints; viscoelastic; lamb wave generation; attenuation; transducer incidence angle.

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1 INTRODUCTION

The applications of adhesive joints in plates and pipes to increase life, improve stiffness, protect against corrosion, and electrical insulation have been caused the widespread use of viscoelastic polymer material like epoxy with plates and pipes as adhesive and coating. An example of these joints' application is a three-layer adhesive joint including an aluminum patch bonded to a surface, like aircraft aluminum skin, by a viscoelastic epoxy adhesive layer. Repair patches are used to extend the life of the aircraft. Ultrasonic guided waves are used to inspect these adhesive joints. Lamb waves have applications in non-destructive inspection of elastic-viscoelastic multi-layer joints and plates. Some modes of these waves have frequencies with minimum attenuation and are recognizable in inspection by transducer, and they can also detect the defects in the structures. Low-attenuation Lamb waves can be produced in multi-layer structures using angle beam transducers for inspection purposes.

Different studies have been carried out to obtain propagated modes and frequencies in multi-layer structures (or dispersion curves). Thomson (1950) and Haskell (1953) first investigated the equations of elastic waves propagation in planar multi-layers with arbitrary number of layers using transfer matrix method or Thomson-Haskell method. They introduced a transfer matrix that shows the relationship between displacement and stress in bottom of a layer in comparison to their values in top of the same layer. One of the difficulties when using transfer matrix method is the instability of the solution whenever the product of frequency in thickness increases. Dunkin (1965) introduced delta operator technique to solve this difficulty. Knopoff (1964) was the first to use Global matrix method to investigate the propagation of elastic waves in multi-layers. In this method a global matrix is used which is derived from putting together the equations of continuity and boundary conditions in all the layers. In investigated studies, the effect of wave energy attenuation in materials is not taken into consideration.

Watson (1972) obtained the complex roots of the characteristic equation in earth layers and showed that imaginary part of the wave number is the same as the attenuation in multilayered. Hosten and Castaings (1993) applied the transfer matrix method in multilayered anisotropic and damping media. The use of this method in high frequencies is accompanied with numerical instability. Castaings and Hosten (1994) applied delta operator technique to improve the stability of transfer matrix method in multilayered anisotropic damping plates. Lowe (1995) presented a summary of the matrix methods for modeling the propagation of ultrasonic waves in multilayered media. Both global matrix method and transfer matrix method are used in this study. These techniques can be used to obtain attenuation and phase velocity dispersion curves in viscoelastic materials. Pan et al. (1999) investigated the propagation of ultrasonic guided waves in gas pipelines with a thick coating to choose the suitable mode for inspection. Both the effect of coating thickness and the effect of coating damping on dispersion curves and mode shapes were investigated and the modes being the least affected by coating thickness and coating damping were identified. To model the viscoelastic behavior, the coating is assumed a linear standard solid. Seifried et al. (2002) investigated the propagation of guided waves in multilayered adhesive structures by taking into consideration the low stiffness and viscoelastic behavior of adhesive layer. To better understand the guided waves behavior and to obtain dispersion curves, they used analytical, experimental and transient FEM simulation methods.

Simonetti (2004) investigated the propagation of Lamb wave in elastic plates coated with viscoelastic materials, and considered the viscoelastic coatings effect on dispersion properties of Lamb wave propagation in elastic plates. To do this, he used Superposition Partial Bulk Waves (SPBW) method to model the wave. Simonetti and Cawely (2004) investigated the propagation of shear horizontal (SH) waves in an elastic plate coated with viscoelastic material. Material damping causes an excessive reduction of applied signal in ultrasonic test. In this research, SH wave dispersion curves for metal plates coated with viscoelastic layers are obtained using SPBW method. Barshinger and Rose (2004) investigated the propagation of guided waves in elastic hollow cylinders with viscoelastic coating using experimental and analytical methods. Wave equation is solved using theoretical boundary value problem and the best modes are specified. In this research, Global matrix method is used to obtain the roots of characteristic equation. It should be noted that in this paper, the viscoelastic characteristics of the coating are obtained using the transient wave propagation method. Shorter (2004) investigated the wave propagation in linear viscoelastic laminates using spectral finite element method or semi-analytical finite element method (SAFE). In this reference, damping loss factor is estimated for waves in low frequencies, and also stiffness matrix is assumed to be real. Birgersson et al. (2005) investigated damping loss factor using SAFE method and taking into consideration the complex stiffness matrix. Bartoli et al. (2006) investigated the wave propagation in viscoelastic waveguides with an arbitrary cross-section. To model ultrasonic wave propagation in different waveguides, SAFE method is used. The results of group velocity and phase velocity dispersion curves (for undamped media), attenuation and energy velocity (for damped media), and cross-section mode shapes are obtained which are used in non-destructive inspection. The results accuracy is validated compared to the SPBW method. Marzani et al. (2008) used SAFE method to analyze wave propagation in viscoelastic axisymmetric waveguides. The results accuracy of the dispersion curves is validated compared to the SPBW method. Puthillath and Rose (2010) inspected the titanium repair patches bonded to the aircraft aluminum skin using ultrasonic guided waves. They plotted the wave structures using a theoretical method and selected the mode shape with maximum in-plane displacement for inspection, although they didn't take into consideration the effect of material damping.

In the present study, the propagation of Lamb waves in elastic-viscoelastic three-layer joints, including two elastic plates bonded together with a layer of viscoelastic adhesive, is investigated using Global matrix method and considering viscoelastic layer damping effect. Then, the suitable incidence angle is theoretically calculated to generate Lamb wave mode with low attenuation using angle beam transducer. Also, wave structure is plotted for a specific mode in two different frequencies to verify that continuity and boundary conditions are satisfied and also to explain the attenuation behavior of waves in joints. Adhesive damping causes the excessive reduction of sending signal amplitude in ultrasonic test; so, modes and frequencies with minimum attenuation should be specified. Because these waves travel the maximum possible distance in joints and can detect the different defects namely interfacial defects.

2 THEORETICAL MODELING OF LAMB WAVES PROPAGATION IN THREE-LAYER ADHESIVE JOINTS

Lamb waves are propagated in thin plate-like mediums in which planar dimensions are far greater than the thickness of plate and wavelength of the same order with plate thickness (Su and Ye, 2009). Free upper and lower surfaces in plate guide movement of these waves. Lamb waves have infinite modes and their propagation properties depend on wave entry angle, frequency, and structure geometry. Figure 1 shows Lamb wave propagation in an adhesive joint which is comprised of three layers. The first and the third layers, which are elastic and isotropic, are bonded together by the second layer which is an isotropic viscoelastic layer. The layers are perfectly bonded together and the free surfaces at the top and the bottom of the three layers are traction-free. Layers thickness is shown by h_1, h_2, and h_3. A local Cartesian coordinate system is used to investigate the propagation of Lamb waves in the three layers. Because of the propagation of the Lamb waves in three-layer joint, the problem is investigated as plain strain, and also the wave propagation is considered harmonic. In each layer, Lamb wave are comprised of shear and longitudinal waves superposition (Rose, 2004). L+ and L- show the propagation of longitudinal waves downwards and upwards the plate, and S+ and S- show the propagation of shear waves downwards and upwards the plate, respectively.

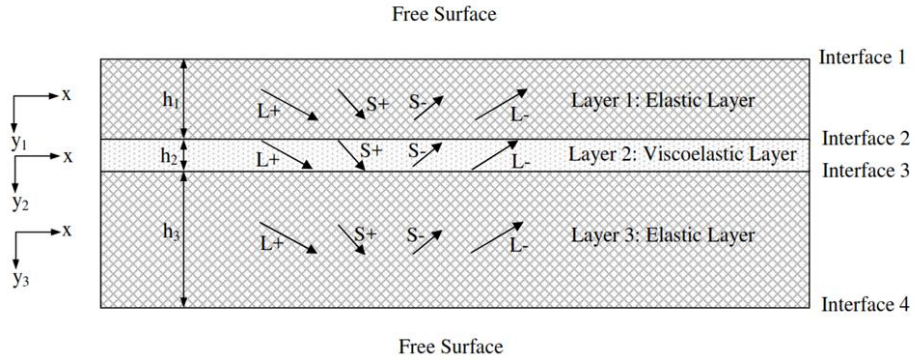

Figure 1: The propagation of Lamb wave in an elastic-viscoelastic three-layer adhesive joint.

Assuming that the wave propagation in three-layer adhesive joints in terms of time is harmonic, stress-strain equations of viscoelastic layer are similar to those in elastic layer, except that material properties of viscoelastic layer are complex numbers and a function of frequency (Christensen, 2010). This dependency between elastic and viscoelastic material in harmonic state is called Alfrey's Correspondence Principle (Flugge, 1975 and Ferry, 1980). Also, Navier's equation of motion in viscoelastic layer is similar to the elastic layer and is expressed by Eq. (1):

$$\mu\nabla^2\mathbf{u} + (\lambda + \mu)\nabla(\nabla.\,\mathbf{u}) = \rho\frac{\partial^2\mathbf{u}}{\partial t^2} \tag{1}$$

In Eq. (1), λ, μ, and ρ are Lame constants and density, respectively. Lame constants in viscoelastic material are complex numbers and a function of frequency which are measured using experimental methods such as ultrasonic tests (Barshinger and Rose, 2004). In Eq. (1), the displacement field, $\mathbf{u}$, can be decomposed as a combination of the gradient of a scalar potential field, Φ, and the curl of a vector potential field, $\mathbf{H}$ (Helmholtz decomposition) (Rose , 2004 and Graff, 1991):

$$\mathbf{u} = \nabla\Phi + \nabla\times\mathbf{H}, \quad \nabla.\mathbf{H} = \mathbf{0} \tag{2}$$

Substituting Eq. (2) in Eq. (1), scalar and vector equations are obtained respectively:

$$\nabla^2\Phi = \frac{1}{C_1^2}\frac{\partial^2\Phi}{\partial t^2}, \quad C_1 = \sqrt{\frac{\lambda + 2\mu}{\rho}} \tag{3}$$

$$\nabla^2\mathbf{H} = \frac{1}{C_2^2}\frac{\partial^2\mathbf{H}}{\partial t^2}, \quad C_2 = \sqrt{\frac{\mu}{\rho}} \tag{4}$$

Eq. (3) shows the propagation of longitudinal wave, and Eq. (4) shows the propagation of transverse wave in structures, and C_1 and C_2 quantities are longitudinal and shear wave velocities in medium, respectively. Since Lame constants in viscoelastic material are complex numbers and a function of frequency, wave velocities are also complex numbers and a function of frequency.

Using Cartesian coordinate system, the potential vector, $\mathbf{H}$, can be defined as the Eq. (5):

$$\mathbf{H} = H_x\mathbf{e_x} + H_y\mathbf{e_y} + H_z\mathbf{e_z} \tag{5}$$

Since this problem is assumed as a plain strain, then the equation $u_z = \partial/\partial z = 0$ should be satisfied. This happens when H_x and H_y components equal zero and only H_z remains. The scalar potential function, Φ, should also be a function of x and y.

The solutions of Eqs. (3) and (4) for a harmonic wave propagates along the positive x direction, are assumed as Eqs. (6) and (7):

$$\Phi = f(y)e^{i(kx-\omega t)} \tag{6}$$

$$H_z = h_z(y)e^{i(kx-\omega t)} \tag{7}$$

In Eqs. (6) and (7), k and ω are wave number and angular frequency, respectively.

Substituting Eqs. (6) and (7) in Eqs. (3) and (4) and taking into consideration that the two components of vector potential function are equal to zero, and after solving the differential equations, the solutions are obtained as:

$$\Phi = \{A_{(L+)}e^{i\alpha y} + A_{(L-)}e^{-i\alpha y}\}e^{i(kx-\omega t)}, \qquad \alpha^2 = \left(\frac{\omega^2}{C_1^2} - k^2\right) \tag{8}$$

$$H_z = \{A_{(S+)}e^{i\beta y} + A_{(S-)}e^{-i\beta y}\}\, e^{i(kx-\omega t)}, \qquad \beta^2 = \left(\frac{\omega^2}{C_2^2} - k^2\right) \tag{9}$$

The solutions of Eqs. (8) and (9) are known as the partial waves solution. The four terms obtained from Eqs. (8) and (9) show the longitudinal waves propagation, L, and transverse waves propagation, S, upwards and downwards the layer. Constant values show the amplitude of propagated waves; for instance, $A_{(L+)}$ shows the longitudinal wave amplitude propagates towards the bottom of the layer.

Substituting vector and scalar potential functions from Eqs. (8) and (9) in Eq. (2), the displacement field in adhesive joint is obtained in terms of unknown constants of the shear and longitudinal wave amplitudes:

$$u_x = i\{k(A_{(L+)}e^{i\alpha y} + A_{(L-)}e^{-i\alpha y}) + \beta(A_{(S+)}e^{i\beta y} - A_{(S-)}e^{-i\beta y})\}e^{i(kx-\omega t)} \quad 10)$$

$$u_y = i\{\alpha(A_{(L+)}e^{i\alpha y} - A_{(L-)}e^{-i\alpha y}) - k(A_{(S+)}e^{i\beta y} + A_{(S-)}e^{-i\beta y})\}e^{i(kx-\omega t)} \quad (11)$$

$$u_z = 0 \quad (12)$$

Eqs. (10) and (11) can also be expressed as Eqs. (13) and (14):

$$u_x = U_x e^{i(kx-\omega t)} \quad (13)$$

$$u_y = U_y e^{i(kx-\omega t)} \quad (14)$$

In Eqs. (13) and (14), U_x and U_y are unattenuated displacement amplitudes.

Using Hooke and strain-displacement relations, stresses in the adhesive joint can be obtained in terms of the unknown constants of shear and longitudinal wave amplitudes:

$$\sigma_{xx} = \mu\{(2\alpha^2 - k^2 - \beta^2)(A_{(L+)}e^{i\alpha y} + A_{(L-)}e^{-i\alpha y}) - 2k\beta(A_{(S+)}e^{i\beta y} - A_{(S-)}e^{-i\beta y})\}e^{i(kx-\omega t)} \quad (15)$$

$$\sigma_{yy} = \mu\{(k^2 - \beta^2)(A_{(L+)}e^{i\alpha y} + A_{(L-)}e^{-i\alpha y}) + 2k\beta(A_{(S+)}e^{i\beta y} - A_{(S-)}e^{-i\beta y})\}e^{i(kx-\omega t)} \quad (16)$$

$$\sigma_{zz} = -\lambda\{(\alpha^2 + k^2)(A_{(L+)}e^{i\alpha y} + A_{(L-)}e^{-i\alpha y})\}e^{i(kx-\omega t)} \quad (17)$$

$$\sigma_{xy} = -\mu\{(2k\alpha)(A_{(L+)}e^{i\alpha y} - A_{(L-)}e^{-i\alpha y}) + (\beta^2 - k^2)(A_{(S+)}e^{i\beta y} + A_{(S-)}e^{-i\beta y})\}e^{i(kx-\omega t)} \quad (18)$$

$$\sigma_{xz} = 0, \quad \sigma_{yz} = 0 \quad (19)$$

In order to obtain Lamb waves dispersion curves for elastic-viscoelastic three-layer adhesive joint, continuity and boundary conditions should be applied.

3 FORMULATION OF CONTINUITY AND BOUNDARY CONDITIOND USING GLOBAL MATRIX METHOD

Global matrix method is a suitable method for formulation of problems concerning multi-layers. Continuity and boundary conditions are needed for this formulation. Using this method, continuity and boundary conditions can be shown as matrices and vectors. This method can simultaneously consider effects of material damping and wave leakage to the environment. In this method a global matrix is used to describe all the continuity and boundary conditions, and when it comes to numerical stability, it is better than other matrix methods (Lowe, 1995).

Figure 2 shows the boundary conditions of a three-layer adhesive joint including stress and displacement continuity in layers interfaces and traction-free conditions in up and bottom surfaces of the elastic-viscoelastic three-layer adhesive joint.

The bond between layers is perfect, and there's no shear and normal stress on free-surfaces at the top and bottom of the three-layer. This condition is shown in vector Eq. (20). Continuity of interfaces conditions include continuity of displacement components, and shear and normal stresses components. As an example, continuity between m and m+1 layers are shown by vector Eq. (21).

$$\begin{Bmatrix} \sigma_{yy} \\ \sigma_{xy} \end{Bmatrix}_{\substack{\text{Free}\\ \text{Surface}}} = \mathbf{0} \tag{20}$$

$$\begin{Bmatrix} u_x \\ u_y \\ \sigma_{yy} \\ \sigma_{xy} \end{Bmatrix}_{\substack{\text{Layer=m}\\ \text{Interface=m+1}}} = \begin{Bmatrix} u_x \\ u_y \\ \sigma_{yy} \\ \sigma_{xy} \end{Bmatrix}_{\substack{\text{Layer=m+1}\\ \text{Interface=m+1}}} \tag{21}$$

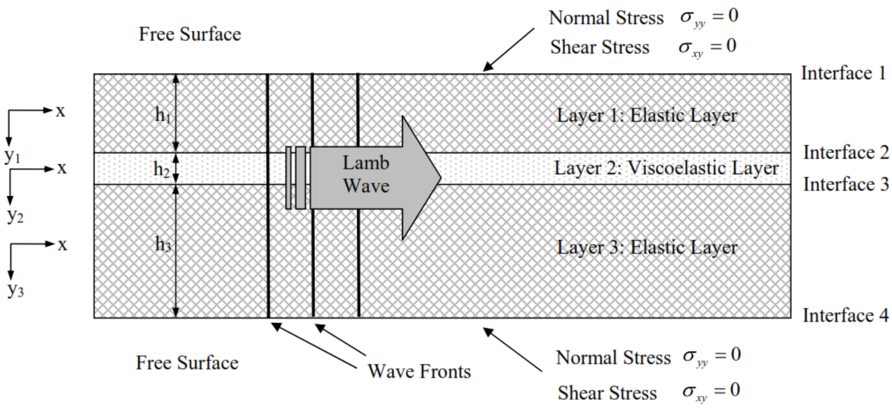

Figure 2: Continuity and boundary conditions in an elastic-viscoelastic three-layer adhesive joint.

Before applying continuity and boundary conditions, a vector relation for displacement and stress in each layer is necessary which is obtained using Eqs. (10), (11), (16), and (18) and is shown by vector Eq. (22):

$$\begin{Bmatrix} u_x \\ u_y \\ \sigma_{yy} \\ \sigma_{xy} \end{Bmatrix} = \mathbf{D} \begin{Bmatrix} A_{(L+)} \\ A_{(L-)} \\ A_{(S+)} \\ A_{(S-)} \end{Bmatrix} e^{i(kx-\omega t)} \tag{22}$$

In which $\mathbf{D}$ is the layer matrix and is expressed as Eq. (23):

$$\mathbf{D} = \begin{bmatrix} ike^{i\alpha y} & ike^{-i\alpha y} & i\beta e^{i\beta y} & -i\beta e^{-i\beta y} \\ i\alpha e^{i\alpha y} & -i\alpha e^{-i\alpha y} & -ike^{i\beta y} & -ike^{-i\beta y} \\ \mu(k^2-\beta^2)e^{i\alpha y} & \mu(k^2-\beta^2)e^{-i\alpha y} & 2\mu k\beta e^{i\beta y} & -2\mu k\beta e^{-i\beta y} \\ -2\mu k\alpha e^{i\alpha y} & 2\mu k\alpha e^{-i\alpha y} & \mu(k^2-\beta^2)e^{i\beta y} & \mu(k^2-\beta^2)e^{-i\beta y} \end{bmatrix} \tag{23}$$

Before applying continuity and boundary conditions using Eq. (22), layer matrix in interfaces of each layer is calculated. This is achieved from Eq. (23), by substituting $y = -h/2$ for layer top interface, and $y = h/2$ for layer bottom interface. These two new layer matrices are shown by $\mathbf{D}_t$ and $\mathbf{D}_b$, respectively, in which the subscripts t and b show the top and bottom interfaces of layer, respectively. Local coordinate system is used to derive these matrices, which are shown in Figure 2, and therefore can be derived for all layers by substituting material properties and thickness.

Now, we express three-layer joint continuity and boundary conditions in the form of a global matrix which is shown in Eq. (24). $\mathbf{A}_m$ and $\mathbf{0}$ vectors in this matrix are shown by Eq. (25):

$$\begin{bmatrix} (\mathbf{D}_{1t})_{34} & \mathbf{0} & \mathbf{0} \\ \mathbf{D}_{1b} & -\mathbf{D}_{2t} & \mathbf{0} \\ \mathbf{0} & \mathbf{D}_{2b} & -\mathbf{D}_{3t} \\ \mathbf{0} & \mathbf{0} & (\mathbf{D}_{3b})_{34} \end{bmatrix} \begin{Bmatrix} \mathbf{A}_1 \\ \mathbf{A}_2 \\ \mathbf{A}_3 \end{Bmatrix} = \begin{Bmatrix} \mathbf{0} \\ \mathbf{0} \\ \mathbf{0} \end{Bmatrix} \tag{24}$$

$$\mathbf{A}_m = \begin{Bmatrix} A_{(L+)m} \\ A_{(L-)m} \\ A_{(S+)m} \\ A_{(S-)m} \end{Bmatrix}, \qquad \mathbf{0} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix} \tag{25}$$

In Eq. (24) the subscript 34 shows the rows 3 and 4 of the layer matrix.

Global matrix method is a $4n \times 4n$ system of equations, in which n is number of layers, and the global matrix for an elastic-viscoelastic three-layer is 12×12. In order to the nontrivial solution to exist, the determinant of global matrix should become zero. This is shown by Eq. (26), which is called characteristic or dispersion equation of Lamb waves. With the aid of the roots of this equation, attenuation and phase velocity dispersion curves are plotted in terms of frequency.

$$\begin{vmatrix} (\mathbf{D}_{1t})_{34} & \mathbf{0} & \mathbf{0} \\ \mathbf{D}_{1b} & -\mathbf{D}_{2t} & \mathbf{0} \\ \mathbf{0} & \mathbf{D}_{2b} & -\mathbf{D}_{3t} \\ \mathbf{0} & \mathbf{0} & (\mathbf{D}_{3b})_{34} \end{vmatrix} = 0 \tag{26}$$

4 NUMERICAL SOLUTION METHOD OF CHARACTERISTIC EQUATION

Characteristic equation roots in the three-layer adhesive joint are obtained using numerical solution method. In characteristic equation, frequency, ω, is the independent variable, and wave number, k, is the dependent variable. The wave number in a desired frequency is obtained by solving this equation. To find characteristic equation roots, computer code is written in Matlab software. These roots are shown by curves called dispersion curves.

Finding complex roots of a characteristic equation concerning a three-layer adhesive joint of which at least one layer is viscoelastic, is a difficult task. In linear viscoelasticity, if harmonic wave propagation is desired, transverse and longitudinal velocities, and Lame constants of viscoelastic layer, are complex and a function of frequency. The transverse and longitudinal velocities are calculated from Eqs. (27) and (28) (Christensen, 2010):

$$C_1(i\omega) = \frac{1}{\dfrac{1}{c_1(\omega)} + i\dfrac{\alpha_1(\omega)}{\omega}} \tag{27}$$

$$C_2(i\omega) = \frac{1}{\dfrac{1}{c_2(\omega)} + i\dfrac{\alpha_2(\omega)}{\omega}} \tag{28}$$

In Eqs. (27) and (28), c_1 and c_2 are bulk velocities of longitudinal and transverse waves, and α_1 and α_2 are bulk attenuations of longitudinal and transverse waves of viscoelastic layer. Bulk attenuation and velocity values for viscoelastic material can be calculated in terms of frequency, using experimental test such as ultrasonic test (Barshinger and Rose, 2004).

Before introducing a method for finding the attenuation and phase velocity numerical results, wave number should be defined in terms of imaginary and real parts. Eq. (29) shows the wave number as complex (Blanc, 1993):

$$k = k_R + ik_I = \frac{\omega}{c_{ph}} + ik_I \tag{29}$$

Eq. (29) enables us to solve the viscoelastic characteristic equation in terms of attenuation, k_I, and phase velocity, c_{ph}, instead of wave number, k. In this case, the attenuation and phase velocity dispersion curves are obtained directly.

One solution method for finding the viscoelastic characteristic equation roots is taking into consideration the minimum of characteristic equation absolute value. In this case, the problem becomes three dimensions in which the characteristic equation absolute value is a function in terms of the attenuation and phase velocity. In this method we seek to find minimum value of this function. The main issue in this method is finding all the roots.

Figure 3 shows a minimization process of characteristic equation absolute value in order to find characteristic equation complex roots.

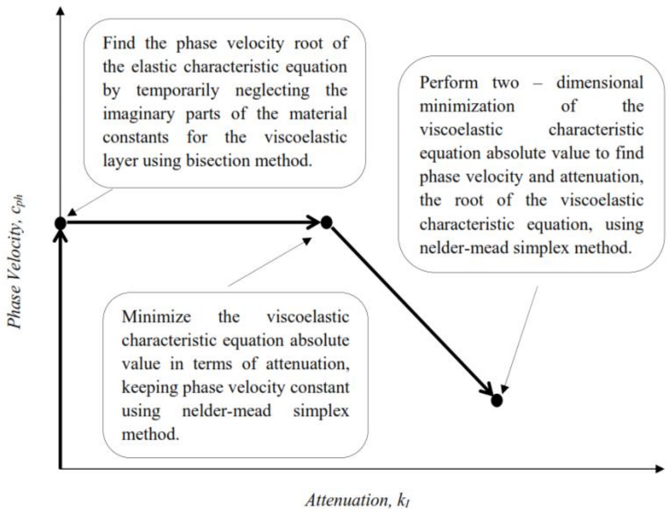

Figure 3: The process of minimization in order to find characteristic equation complex roots.

Using the process shown in Figure 3, a computer code can be written to find characteristic equation roots. This process can be applied for all desired frequencies, and attenuation and phase velocity can be obtained in terms of frequency. Attenuation constant can be converted to attenuation in decibel per length unit, using Eq. (30). This conversion magnifies the attenuation values.

$$\alpha_0(\mathrm{dB\,m^{-1}}) = 20\log_{10}^{(e^{-1000k_I})} \tag{30}$$

5 THEORETICAL MODELING OF LAMB WAVE MODE GENERATION

Different methods exist to generate and receive ultrasonic guided waves. Viktorov (1967) was the first to evaluate the dispersive properties of Lamb wave he also investigated generation methods of Lamb waves. He investigated four methods to generate Lamb waves in plates. These methods can also be used in other structures.

One method to generate Lamb wave is using a longitudinal wave transducer on a plexiglass wedge, which is also called angle beam transducer. Figure 4 shows the Lamb wave generation method in an elastic-viscoelastic three-layer adhesive joint using an angle beam transducer. In this method, according to Snell's law transducer incidence angle depend on wedge velocity and Lamb wave phase velocity. Eq. (31) shows Snell's law, in which θ_i is the plexiglass wedge angle, c_{plexi} is the longitudinal wave velocity of the wedge, and c_{ph} is the Lamb wave phase velocity in adhesive joint.

$$\theta_i = \sin^{-1}(\frac{c_{plexi}}{c_{ph}}) \tag{31}$$

In order to theoretical modeling of the Lamb wave generation, at first attenuation and phase velocity in the joint are obtained, then a combination of modes and frequencies which have low attenuation are selected, and finally the suitable incidence angle for generation is calculated using Snell's law and phase velocity of these modes. These angles are used in inspections.

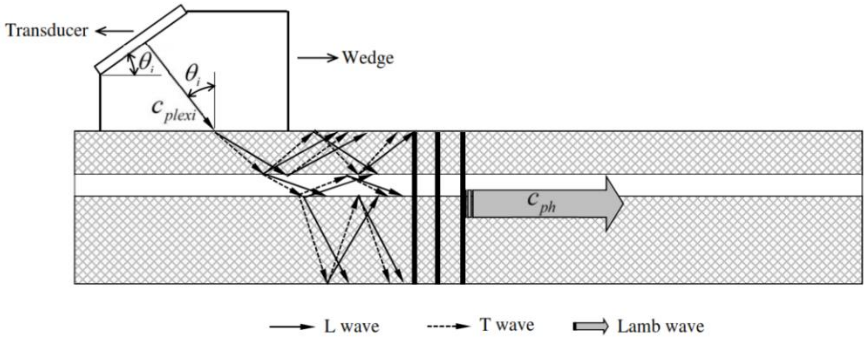

Figure 4: Lamb wave generation with an angle beam transducer.

6 DISCUSSION OF RESULTS FOR A SPECIFIC APPLICATION

Solving the characteristic equation by a computer code for a three-layer adhesive joint, including an aluminum repair patch bonded to the aircraft aluminum skin with a viscoelastic epoxy adhesive layer, the attenuation intensity plot and phase velocity dispersion curves in high and low frequencies for this specific application are generated. Also, acceptable attenuation level is calculated for ultrasonic inspection using a single transducer of the adhesive joint with 200 mm length and suitable modes are selected. Geometric and acoustic properties of elastic-viscoelastic three-layer adhesive joint can be seen in Table 1. Aluminum and Mereco Epoxy 303 acoustic properties are picked up from (Barshinger and Rose, 2004).

Wave structure for a mode in two different frequencies is plotted to validate numerical solution results. Finally, transducer incidence angle is plotted in terms of frequency for different modes, and suitable wedge angles are selected to generate low-attenuation Lamb wave modes in the adhesive joint with 200 mm length.

Layer	Material	c_1 ($\mathrm{km\,s^{-1}}$)	α_1/ω ($\mathrm{s\,km^{-1}}$)	c_2 ($\mathrm{km\,s^{-1}}$)	α_2/ω ($\mathrm{s\,km^{-1}}$)	ρ ($\mathrm{g\,cm^{-3}}$)	h (mm)
1	Aluminum	6.35	-	3.13	-	2.7	1.6
2	Mereco 303 Epoxy	2.39	0.0070	0.99	0.0201	1.08	0.66
3	Aluminum	6.35	-	3.13	-	2.7	3.175

Table 1: Geometric and acoustic properties of an elastic-viscoelastic three-layer adhesive joint.

6.1 Phase velocity dispersion curves and attenuation intensity plot

Figure 5 shows the phase velocity dispersion curves in terms of frequency for different modes in the elastic-viscoelastic three-layer adhesive joint, the properties of which are shown in Table 1. The lamb wave modes are identified with M and numbers in Figure 5. Investigating this curve it can be seen that in the frequency intervals of 150 kHz to 250 kHz only three modes of M1, M2, and M3 propagate, and other modes don't propagate in these low frequencies. 250 kHz frequency is cutoff frequency of M4 mode; because, M4 mode doesn't propagate in frequencies lower than this frequency. Also, in the frequency intervals of 275 kHz to 575 kHz only four modes of M1, M2, M3, and M4 propagate, and 575 kHz is the cutoff frequency of M5 mode.

The attenuation is shown superimposed on the Lamb wave dispersion curves with the intensity plot in Figure 6. In this paper, acceptable range of attenuation is calculated for ultrasonic inspection of the adhesive joint with 200 mm length using a single transducer. The suitable range of attenuation depends on wave propagation distance in a wave round-trip to transducer and on the signal to noise ratio (SNR). In inspection with guided waves, SNR is a measure for detecting small defects and is the ratio of reflected signal from defects to return signal from grains (as noise) to transducer. Minimum identifiable SNR in guided wave test is 6 dB (Barshinger and Rose, 2004). In guided wave test, defect signal is usually 20 dB higher than the noise signal; therefore, a 14 dB signal can be lost because of the guided wave mode attenuation, and if attenuation is more than which, defects are not detectable. The distance that a wave travels in a round-trip to transducer is twice the length of the plate and equal to 400 mm, and maximum attenuation that the wave can have in a round-trip equals -14 dB / 0.4 m or -35 dB $\mathrm{m^{-1}}$; therefore, the suitable range of attenuation is from 0 to -35 dB $\mathrm{m^{-1}}$.

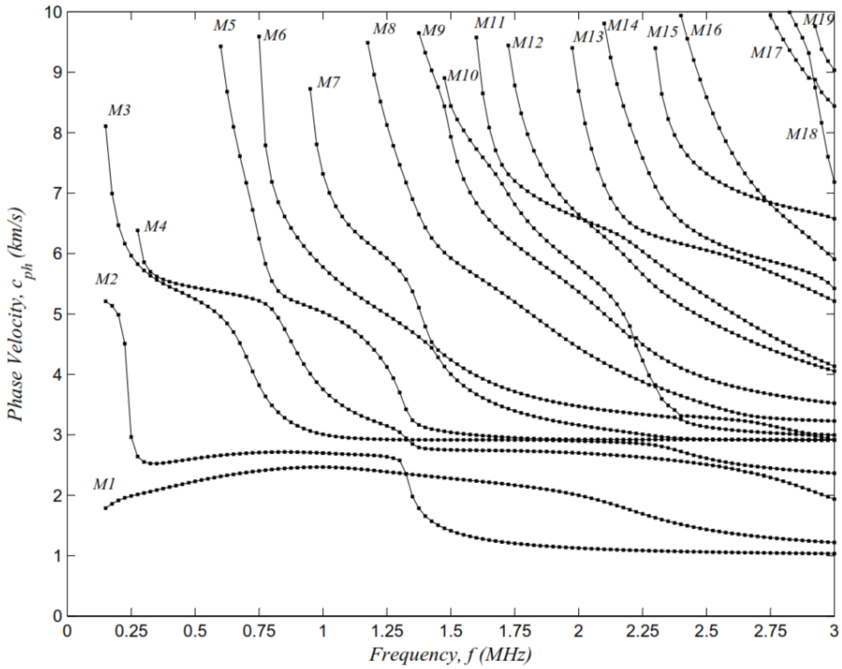

Figure 5: Phase velocity dispersion curves in terms of frequency in the three-layer adhesive joint: aluminum-epoxy-aluminum.

The modes with acceptable attenuation level (0 to -35 dB m^{-1}) are selected for inspection in high and low frequencies. Generating these modes, adhesive joint inspection can be carried out to find the defects. From curves in Figure 6, it can be seen that M1 mode in frequency range of 150 kHz to 500 kHz has a suitable attenuation level for inspection, and in frequencies higher than 500 kHz, a sudden and excessive increase can be seen in attenuation. This mode in high frequencies is not suitable for inspection. Attenuation in M2 mode in low frequencies, in the range of 150 kHz to 250 kHz increases extremely, and has a sudden and excessive increase in frequencies higher than 800 kHz. M2 Mode in frequency range of 325 kHz to 800 kHz and M3 mode in frequency ranges of 150 kHz to 675 kHz and 1.2 MHz to 1.775 MHz have suitable attenuation levels for inspection of the adhesive joint with 200 mm length. M3 mode in 500 kHz has an attenuation equal to -5.1 dB m^{-1} which is the lowest attenuation level in frequency range of 0 to 3 MHz. M5 mode has a suitable attenuation level in high frequencies and is suitable for inspection in 1.9 MHz to 3 MHz frequency range, and it also has negligible attenuation about -0.27 dB m^{-1} in frequencies near 3 MHz.

6.2 Validation of numerical solution results

One method to validate the numerical solution results of characteristic equation, which are the same as attenuation and phase velocity, is the investigation of the interfacial continuity equations and boundary conditions in the adhesive joint. The wave structure of propagated modes in the three-layer adhesive joint is plotted to validate whether interfacial continuity equations and boundary conditions are satisfied. Wave structure curves are the same as stress and displacement amplitudes across three-layer adhesive joint thickness.

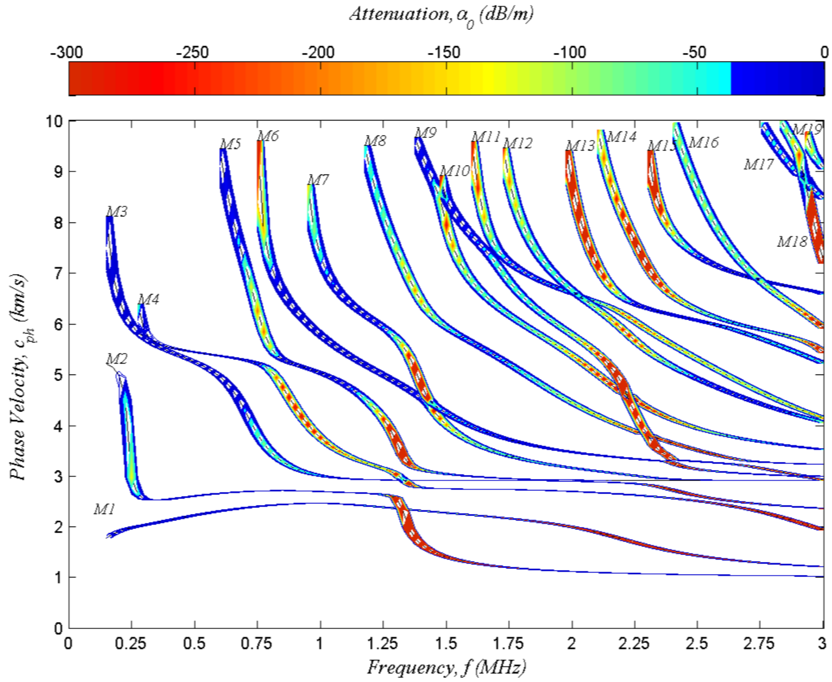

Figure 6: Variation of the attenuation superimposed over the Lamb wave dispersion curves for the three-layer adhesive joint: aluminum-epoxy-aluminum.

The curve in Figure 7 is the M2 mode wave structure in 500 kHz frequency with an attenuation of -13.5 dB m^{-1}. As the Figure 7 shows, shear and normal stresses don't exist in free surfaces at the top and bottom of the three layers; also, interfacial continuity conditions including continuity of shear and normal stresses and displacement components, are satisfied. M2 mode wave structure in 1.25 MHz frequency with the attenuation of -222.8 dB m^{-1} is also plotted in the curves of Figure 8, in which continuity and boundary conditions are also satisfied. Because attenuation level is high in wave structure curve of Figure 8, most of the displacement exists in the viscoelastic layer.

6.3 Transducer incidence angles to generate low-attenuation Lamb wave modes

In this section, at first the transducer incidence angle curves in terms of frequency for generating lamb wave modes in the three-layer adhesive joint is plotted using Eq. (31) and phase velocity values, these curves are shown in Figure 9. Then the suitable incidence angles are specified to generate low-attenuation modes by transducer. Investigating Figure 9 it can be seen that the incidence angles for generating some modes such as M1 mode don't exist, the generation of which is impossible by the transducer for all the frequency ranges. M2 mode has a low attenuation level and a suitable incidence angle for inspection in 150 kHz to 200 kHz and 650 kHz to 800 kHz frequency ranges. To generate this mode in 200 kHz frequency with low attenuation level of -7.34 dB m^{-1}, the transducer incidence angle should be 32.4 degree. Simultaneously investigating the attenuation values and transducer incidence angle it can be seen that M3 mode in 150 kHz to 675 kHz frequency range, M4 mode in 300 kHz to 725 kHz frequency range, and M5 mode in 875 kHz to 1.075 MHz and 1.9 MHz to 3 MHz frequency ranges have low attenuation level and suitable incidence angle for the generation of Lamb wave. Transducer incidence angle should be 30.6 degree to generate M3 mode in 500 kHz frequency and low attenuation level of -5.1 dB m^{-1}, and 66 degree to generate M5 mode in 3 MHz frequency and negligible attenuation level of -0.27 dB m^{-1}.

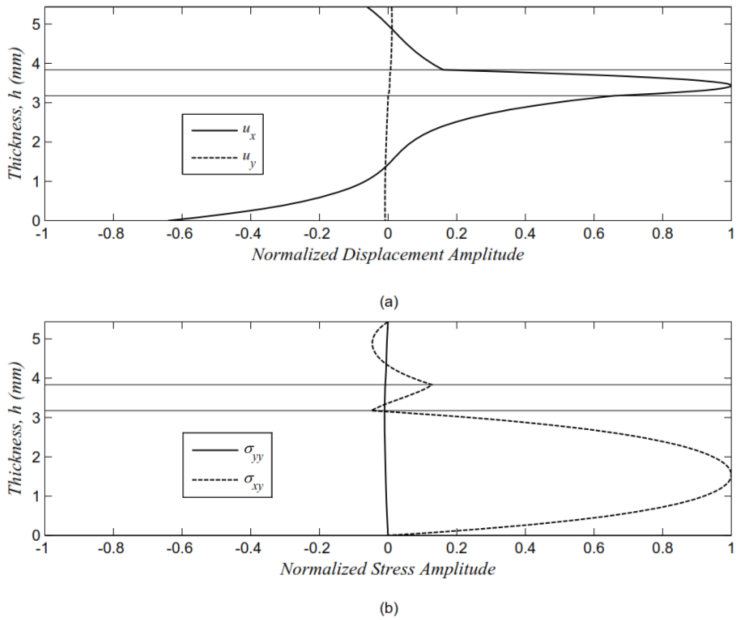

Figure 7: M2 mode wave structure in 500 kHz frequency with the attenuation of -13.5 dB m-1 in a three-layer adhesive joint (a) normalized displacement wave structure (b) normalized stress wave structure.

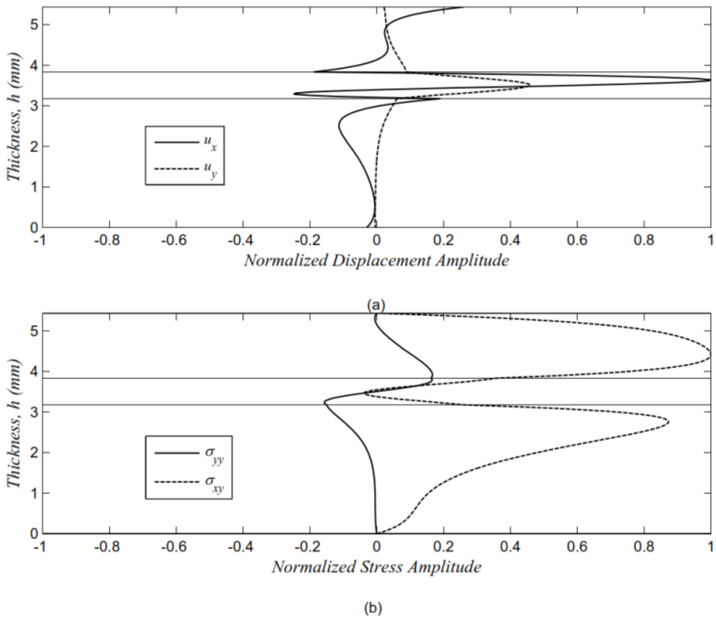

Figure 8: M2 mode wave structure in 1.25 MHz frequency with the attenuation of -222.8 dB m-1in a three-layer adhesive joint (a) normalized displacement wave structure (b) normalized stress wave structure.

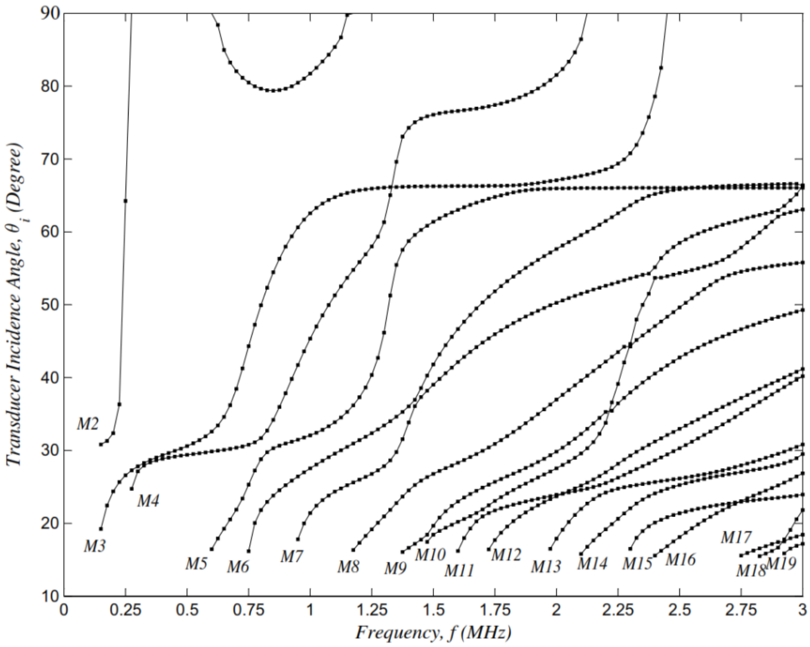

Figure 9: Transducer incidence angle curves in terms of frequency for generating Lamb wave modes in the three-layer adhesive joint: aluminum-epoxy-aluminum.

7 CONCLUSIONS

The obtained results of Lamb wave theoretical modeling in three-layer adhesive joint with a viscoelastic adhesive can be used in inspections using ultrasonic guided waves in three-layer structures. The results of the present paper can be summarized as:

1. Lamb waves have many modes, some of which don't propagate in frequencies lower than the cutoff frequency.

2. Some modes, such as M1, M2, and M3, have an acceptable attenuation level for inspection with guided waves in low frequencies and some others, such as M5, in high frequencies.

3. Investigating the wave structure curves it can be seen that the interfacial continuity and boundary conditions is satisfied in adhesive joint. This result validates the numerical solution results of characteristic equation.

4. Transducer incidence angle obtained from theoretical modeling of Lamb wave mode generation can be used to inspect adhesive joints and generate Lamb wave with low attenuation level in joints.

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