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Flow behavior of unsteady incompressible Newtonian fluid flow between two parallel plates via homotopy analysis method

Abstract

An analysis has been performed to study the problem of the flow of incompressible Newtonian fluid between two parallel plates where the upper plate is impermeable and can move up or down and the lower one is fixed and has a porous surface. The governing equations for this problem are reduced to an ordinary form and is solved using Homotopy Analysis Method (HAM) and numerically by fourth order Runge–Kutta technique. Also, Velocity fields have been computed and shown graphically for various values of physical parameters. As an important outcome, HAM is able to solve a large class of nonlinear problems effectively, more easily and accurately; and thus it has been widely applicable in engineering and physics.

Keywords

Homotopy Analysis Method; parallel porous plates; unsteady flow.

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1 INTRODUCTION

The problem of unsteady time-dependent flow between parallel plates has many crucial applications in science and technology. Among them are hydrodynamic lubrication, aerodynamic heating, polymer technology, petroleum industry and biomechanics. Many researchers have investigated such flows with different geometries and different flow conditions (Sharma and Singh, 2008; Ishizawa, 1966; Hamza, 1999; Rashidi et al., 2008; Hasanzadeh et al., 2013).

In recent decades many attempts have been made to develop analytical methods for solving such nonlinear equations. One of them is the perturbation method (Nayfeh, 2000), which is strongly dependent on a so called small parameter to be defined according to the physics of the problem. Since these equations cannot be solved via the conventional analytical techniques, recent attempts have been focused on constructing an analytical solution for these equations using the advanced developed methods such as Adomian's Decomposition Method (ADM) (Sheikholeslami, 2012), Homotopy Perturbation Method (HPM) (He, 2006; Rahimi et al., 2012), Variational x'Iteration Method (VIM) (He, 2007), Differential Transformation Method (Ganji and Azimi, 2013), Homotopy Aanalysis Method (HAM) (Liao, 2003; Liao, 2012; Abbasi et al., 2014; Ganji et al., 2014) and Least Square Method (LSM) (Hatami et al., 2013; 2014).

Liao introduced the basic idea of Homotopy in topology to propose a general analytical method for nonlinear problems, namely the Homotopy Analysis Method (Hatami and Ganji, 2013; 2014), that does not need any small parameter. This method has been successfully applied to solve many types of nonlinear problems (Hatami et al., 2014; Hasanzadeh et al., 2013; Aziz, 2006).

The aim of this study is to investigate, the effect of physical parameters on Flow behavior of unsteady incompressible Newtonian fluid flow between two parallel plates. In addition, the convergence of the series solution is also explicitly discussed. Obtaining the analytical solution of the models and comparing with numerical result reveal the capability, effectiveness and convenience of HAM.

2 PROBLEM STATEMENT AND MATHEMATICAL FORMULATION

We consider an incompressible two dimensional flow of Newtonian fluid between two parallel infinite rectangular plates in Cartesian coordinates. These two plates are placed a distance $a(t)$ at apart from each other and t denotes time.

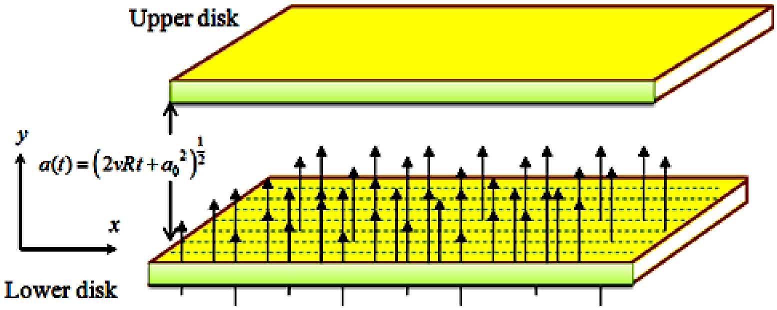

Figure 1: Schematic diagram and the coordinate system for the considered flow.

We also consider that the upper plate which is at $y = a(t)$ at $=$ is moving toward the lower plate with velocity $\dot{y} = \dot{a}(t)$ and the lower porous plate which is at $y = 0$ is fixed. The equations of motion for this flow are (Hasanzadeh et al., 2013):

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0\,, \tag{1}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial P}{\partial x} + \nu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right), \tag{2}$$

$$\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=-\frac{1}{\rho}\frac{\partial P}{\partial x}+\nu\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right), \tag{3}$$

where ρ denotes density, P denotes pressure and ν denotes kinematic viscosity. For boundary conditions we have:

$$y=a(t)\ \rightarrow\ u(x,y,t)=0\ ,\ v(x,y,t)=\dot{a}(t) \tag{4}$$

$$y=0\ \rightarrow\ u(x,y,t)=0\ ,\ v(x,y,t)=K\,\dot{a}(t) \tag{5}$$

As we can see for the axial velocity we have no slip boundary condition. Also as the distance between two plates varies with $\dot{a}(t)$ and the lower plate is stationary we can write for the upper plate that $v=\dot{a}(t)$. For the lower plate we insert a constant parameter like K which is a candidate for the strength of the suction or injection. Here $K>0$ corresponds to suction and $K<0$ corresponds to injection. An assumption made by Singh et al. (1990) proposes that,

$$u=\frac{C-x}{a(t)}\dot{a}(t)\ f'(\eta)\ ,\ v=\dot{a}(t)f(\eta)\ ,\ \eta=\frac{y}{a(t)} \tag{6}$$

Substituting these changes in Eq. (1) to Eq. (3), Equation of continuity will be satisfied automatically and the Navier-Stokes equations of motion reduce to:

$$\frac{1}{C-x}\frac{\partial P}{\partial x}=\frac{\rho\dot{a}^2}{a^2}\left(\frac{1}{R}f'''-f\,f''+\eta f''+f+f'^2-Sf'\right) \tag{7}$$

$$\frac{\partial P}{\partial \eta}=\rho\dot{a}^2\left(\frac{1}{R}f'''-f\,f'+\eta f''-Sf\right) \tag{8}$$

Where

$$R=\frac{a\dot{a}}{\upsilon}\ ,\ S=\frac{a\ddot{a}}{\dot{a}^2} \tag{9}$$

Now differentiating Eq. (7) with respect to η gives,

$$f''''=R\left(f\,f'''+f'\,f''+\eta f'''-2f''+S\,f''\right) \tag{10}$$

Similarity solution exists only when R and S are constants. Now integrating the first equation of (9) we can find the distance between two plates as,

$$a\left(t\right)=\sqrt{\left(2\upsilon Rt+a_0^2\right)} \tag{11}$$

From this equation, we can say that when $R > 0$ the upper plate moves away from the lower plate and when $R < 0$, it moves toward it. Combining Eq. (9) with Eq. (11) we see that $S = -1$. Equation (10) will then be written as,

$$f'''' = R\left(f f''' - f' f'' - \eta f''' - 3f''\right) \tag{12}$$

With the boundary conditions,

$$\begin{aligned} y = 0: \quad & f' = 0; \quad f = K. \\ y = 1: \quad & f' = 0; \quad f = 1. \end{aligned} \tag{13}$$

Eq. (12) with boundary conditions (13) will be solved by analytical method.

3 IMPLEMENTATION OF THE HOMOTOPY ANALYSIS METHOD

For HAM solutions, we choose the initial guess and auxiliary linear operator in the following form:

$$f_0\left(\eta\right) = \left(-2 + 2K\right)\eta^3 + \left(3 - 3K\right)\eta^2 + K \tag{14}$$

$$L(f) = f'''', \tag{15}$$

$$L\left(\frac{1}{6}c_1\,\eta^3 + \frac{1}{2}c_2\,\eta^2 + c_3\,\eta + c_4\right) = 0\,, \tag{16}$$

Where $c_i (i = 1,2,3,4)$ are constants. Let $P \in \left[0,1\right]$ denotes the embedding parameter and $\hbar$ indicates non–zero auxiliary parameters. We then construct the following equations:

Zeroth –order deformation equations

$$\left(1 - P\right)L\left[F(\eta;p) - f_0\left(\eta\right)\right] = p\hbar H\left(\eta\right)N\left[F\left(\eta;p\right)\right] \tag{17}$$

$$F(0;p) = K\,,\ F'(0;p) = 0\ ,\ F(1;p) = 1\ ,\ F'(1;p) = 0 \tag{18}$$

$$N\left[F\left(\eta;p\right)\right] = \frac{d^4F\left(\eta;p\right)}{d\eta^4} - R\left[F\left(\eta;p\right)\frac{d^3F\left(\eta;p\right)}{d\eta^3} - \frac{dF\left(\eta;p\right)}{d\eta}\frac{d^2F\left(\eta;p\right)}{d\eta^2} - \eta\frac{d^3F\left(\eta;p\right)}{d\eta^3} - 3\frac{d^2F\left(\eta;p\right)}{d\eta^2}\right] \tag{19}$$

For $p = 0$ and $p = 1$ we have

$$F\left(\eta;0\right) = f_0\left(\eta\right),\ F\left(\eta;1\right) = f\left(\eta\right) \tag{20}$$

When p increases from 0 to 1 then $F(\eta;p)$ varies from $f_0(\eta)$ to $f(\eta)$. By Taylor's theorem and using equation (20), $F(\eta;p)$ can be expanded in a power series of p as follows:

$$F(\eta;p) = f_0(\eta) + \sum_{m-1}^{\infty} f_m(\eta)p^m \ , \quad f_m(\eta) = \frac{1}{m!}\frac{\partial^m\left(F(\eta;p)\right)}{\partial p^m}\bigg|_{p=0} \tag{21}$$

In which $\hbar$ is chosen in such a way that this series is convergent at $p=1$, therefore we have through equation (26) that

$$f(\eta) = f_0(\eta) + \sum_{m-1}^{\infty} f_m(\eta) \ , \tag{22}$$

mth –order deformation equations

$$L\left[f_m(\eta) - \chi_m \ f_{m-1}(\eta)\right] = \hbar \ H(\eta) R_m(\eta) \tag{23}$$

$$F_m(0;p) = 0 \ , \ F_m'(0;p) = 0 \ , \ F_m(1;p) = 0 \ , \ F_m'(1;p) = 0 \tag{24}$$

$$R_m(\eta) = f_{m-1}'''' + 3f_{m-1}'' + \eta f_{m-1}''' - \sum_{k=0}^{m-1}\left[R\left(f_{m-1-k} \ f_k''' - f_{m-1-k}' \ f_k''\right)\right] \tag{25}$$

$$\chi_m = \begin{cases} 0, & m \le 1 \\ 1, & m > 1 \end{cases}, \tag{26}$$

Now we determine the convergency of the result, the differential equation, and the auxiliary function according to the solution expression. So let us assume:

$$H(\eta) = 1 \tag{27}$$

We have found the answer by maple analytic solution device. The first deformation is presented below

$$\begin{aligned} f_1(\eta) = {} & R\left(-\frac{4}{35}\hbar K + \frac{2}{35}\hbar + \frac{2}{35}\hbar K^2\right)\eta^7 + R\left(-\frac{1}{5}\hbar K^2 + \frac{2}{5}\hbar K - \frac{1}{5}\hbar\right)\eta^6 + \\ & + R\left(-\frac{1}{5}\hbar K + \frac{3}{10}\hbar K^2 - \frac{1}{10}\hbar\right)\eta^5 + R\left(\frac{3}{4}\hbar - \frac{1}{4}\hbar K - \frac{1}{2}\hbar K^2\right)\eta^4 + \\ & + R\left(\frac{43}{70}\hbar K^2 + \frac{1}{14}\hbar K - \frac{24}{35}\hbar\right)\eta^3 + R\left(-\frac{19}{70}\hbar K^2 + \frac{13}{140}\hbar K + \frac{5}{28}\hbar\right)\eta^2 \end{aligned} \tag{28}$$

The solutions $f(\eta)$ were too long to be mentioned here, therefore, they are shown graphically.

4 CONVERGENCE OF THE HAM SOLUTION

As pointed out by Liao (2003, 2012), the convergence region and rate of solution series can be adjusted and controlled by means of the auxiliary parameter $\hbar$. To influence of $\hbar$ on the convergence of solution, we plot the so-called $\hbar$-curve of $f'''(0)$, as shown in Figures 2 and 3. The solutions converge for $\hbar$ values which are corresponding to the horizontal line segment in ħ curve.

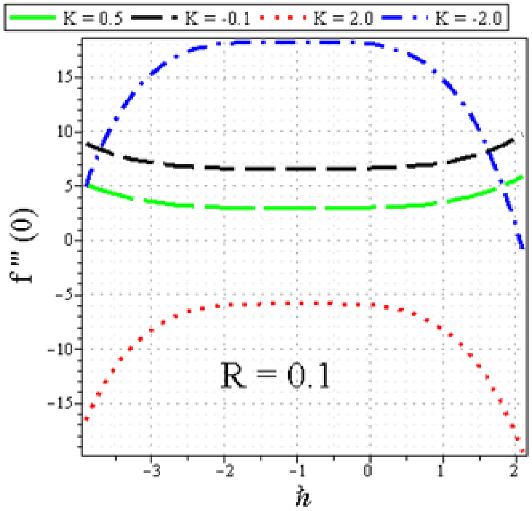

Figure 2: The $\hbar$ - validity for $R = 0.1$ and different value of K .

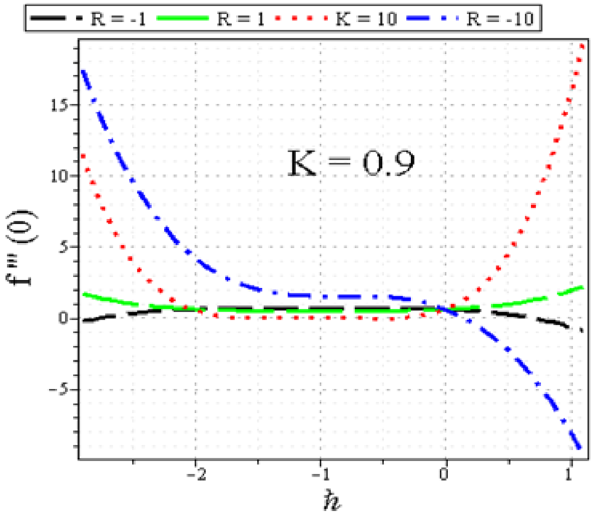

Figure 3: The $\hbar$ - validity for $K = 0.9$ and different value of R .

In order to investigate the range of admissible values of the auxiliary parameter $\hbar$, for various quantities of R and K, the curves of $\hbar$ were derived 6th-order approximations. Figures 2 and 3 shows a typical $\hbar$ curve for $f'''(0)$ which presentation admissible values for auxiliary parameter $\hbar$. In our case study, it is easy to discover that $\hbar = -1$ is suitable value which is used for values of $-2.0 < K < 2.0$ ($M \geq 0$) and $-10 < R < 10$ ($0 \leq T \leq 6$).

5 RESULTS

In the present study HAM method is applied to obtain an explicit analytic solution of the flow of incompressible Newtonian fluid between two parallel plates (Figure 1). First, a comparison between the applied methods, obtained by the numerical method and HAM for different values of active parameters is shown in Figures 4 till 6. The numerical solution is performed using the algebra package Maple 16.0, to solve the present case. The package uses a fourth order Runge–Kutta procedure for solving nonlinear boundary value (B-V) problem by Aziz (2006). Validity of HAM is shown in Table 1. In these tables, the %Error is defined as:

$$\%\,\mathrm{Error} = \left| f\left(y\right)_{\mathrm{NUM}} - f\left(y\right)_{\mathrm{HAM}} \right| \tag{29}$$

The results are proved to be precise and accurate in solving a wide range of mathematical and engineering problems especially Fluid mechanic cases. This accuracy gives high confidence to us about validity of this problem and reveals an excellent agreement of engineering accuracy. This investigation is completed by depicting the effects of some important parameters to evaluate how these parameters influence on this fluid.

From a physical point of view, Figures 4 to 6 are prepared in order to see the effects of the K and R flow parameters on the velocity distribution. Figure 4 is the graphical representation on the velocity distribution in both cases of approaching ($R<0$) and receding ($R>0$) plates for different values of the K number.

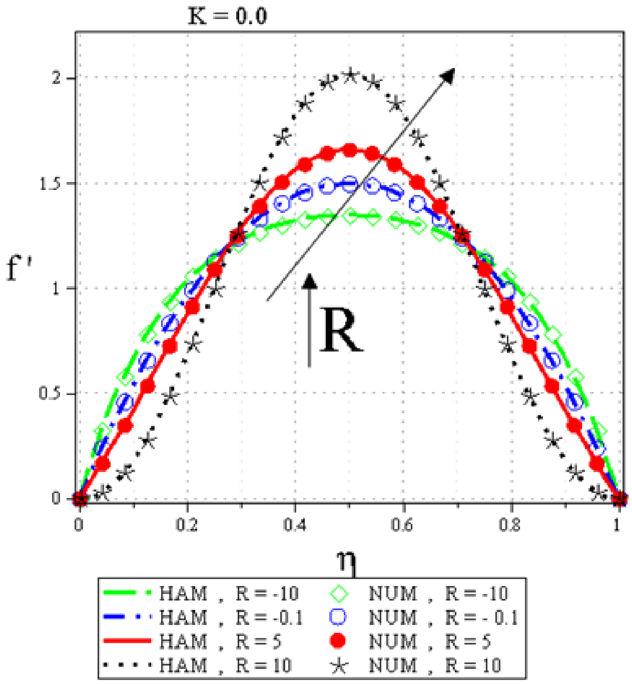

Figure 4: Dimensionless velocities predicted by HAM and numerical method (NUM) in different R number when $K = 0.0$, $\hbar = -1.0$.

According to the obtained solution, It can be seen that when $K = 0$, in the case of approaching plates $R < 0$, increasing the magnitude of R will damp the maximum value of f'. By contrast, when the plates recede each other everything is reversed e.g. we have an increase in the middle region.

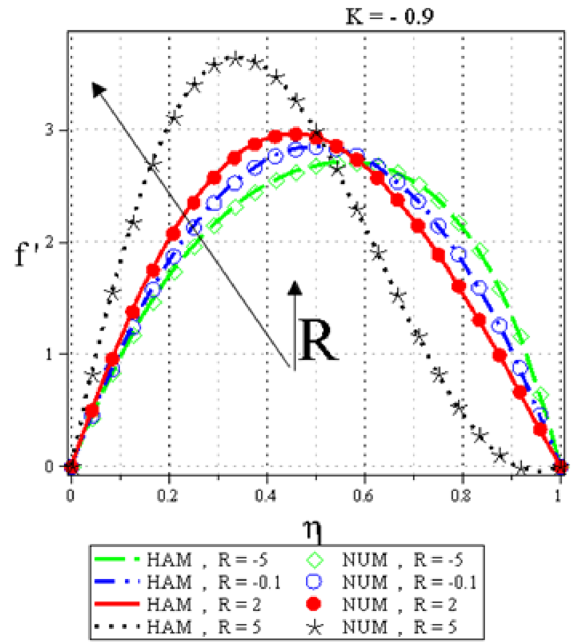

Figure 5: Dimensionless velocities predicted by HAM and numerical method (NUM) in different R number when $K = -0.9$, $\hbar = -1.0$.

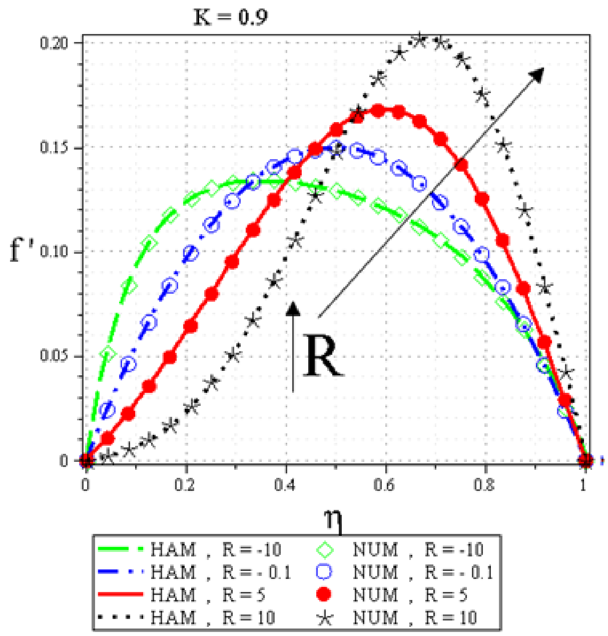

Figure 6: Dimensionless velocities predicted by HAM and numerical method (NUM) in different R number when $K = 0.9$, $\hbar = -1.0$.

Moreover, Figures 5 and 6 have been prepared for the variations of flow number R on the distribution of velocity for $K < 0$ and $K > 0$ respectively. As we seen that, in the case of

$K < 0$, near the fixed plate the increase in R will increase the flow but near the moving plate this reverses. In addition, Figure 6 shows that we have the same trend like the injection case but in the opposite direction. The increase of flow near the wall region is completely obvious but the flow near the upper plate will not change sensitively with the increase of R.

η	$f(\eta)$			$f'(\eta)$		
	HAM	NUM	Error	HAM	NUM	Error
0.00	-0.300000000	-0.300000000	0.00E+00	0.000000000	0.000000000	0.00E+00
0.05	-0.291354676	-0.291354676	5.00E-10	0.345993417	0.345993396	2.13E-08
0.10	-0.265475699	-0.265475701	1.90E-09	0.687476197	0.687476163	3.44E-08
0.15	-0.222869707	-0.222869711	3.80E-09	1.013210723	1.013210683	4.00E-08
0.20	-0.164599515	-0.164599521	5.50E-09	1.312281007	1.312280969	3.80E-08
0.25	-0.092254286	-0.092254293	7.35E-09	1.574636006	1.574635976	3.00E-08
0.30	-0.007893760	-0.007893768	8.82E-09	1.791579558	1.791579538	2.00E-08
0.35	0.086030154	0.086030144	9.96E-09	1.956181991	1.956181985	6.00E-09
0.40	0.186770002	0.186769991	1.07E-08	2.063594851	2.063594855	4.00E-09
0.45	0.291392456	0.291392445	1.10E-08	2.111256426	2.111256446	2.00E-08
0.50	0.396895398	0.396895387	1.07E-08	2.098982036	2.098982067	3.10E-08
0.55	0.500326605	0.500326595	9.70E-09	2.028938543	2.028938584	4.10E-08
0.60	0.598898367	0.598898359	8.00E-09	1.905508152	1.905508196	4.40E-08
0.65	0.690092719	0.690092714	4.70E-09	1.735051163	1.735051202	3.90E-08
0.70	0.771752572	0.771752569	3.60E-09	1.525581730	1.525581765	3.50E-08
0.75	0.842154803	0.842154802	1.40E-09	1.286374302	1.286374324	2.20E-08
0.80	0.900062378	0.900062379	1.00E-09	1.027521158	1.027521170	1.20E-08
0.85	0.944753585	0.944753585	2.00E-10	0.759463805	0.759463805	1.00E-10
0.90	0.976027672	0.976027670	1.60E-09	0.492521856	0.492521847	8.80E-09
0.95	0.994187374	0.994187376	2.10E-09	0.236443433	0.236443428	5.00E-09
1.00	1.000000000	1.000000000	1.00E-09	4.00000E-09	0.000000000	4.00E-09

Table 1: The results of HAM and Numerical methods for $f(\eta)$ and $f'(\eta)$ for K = -0.3 and Re = 4.

6 CONCLUSIONS

In this investigation, the analytical approach called Homotopy Analysis Method (HAM) has been successfully applied to find the most accurate analytical solution for the velocity distributions of unsteady incompressible Newtonian fluid flow between two parallel plates. Furthermore, the obtained solutions by proposed methods have been compared with the direct numerical solutions generated by the symbolic algebra package Maple 16. In addition, the effects of different physical parameters, such as K, and R on the velocity profiles of the problem have been investigated. The following main points can be concluded from the present study:

- The comparison shows that the HAM solutions is highly accurate and provide the rapid achievement to compute the flow velocities. Also according to the previous publications this methods is a powerful technique for finding analytical solutions in science and engineering problems.

- The results show that the reversal flow will take place near the wall region above a critical value of R in both case of injection and suction.

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