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**Artigos Gerais**

# Plastic optical fiber as a tool for experimenting with simple pendulum

*(Fibra óptica plástico como um instrumento para a experimentação com o pêndulo simples)*

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In this paper an experiment with plastic optical fiber to control the motion of a simple pendulum was described. The monitoring system of pendulum position is based on bend loss in plastic optical fiber which depends of bend radius. As a recording device a PC sound card was used. With this measurement technique we are able to visualize theoretical expectation and to extract some of the parameter values from the recorded data as the position, velocity and acceleration of pendulum. The angular displacements, velocity and acceleration are plotted using Mathematica, an available symbolic computer program that allows us to plot easily the obtained dependence as well as their theoretical behavior. Good agreement between measurements and theoretical prediction was observed.
**Keywords:** simple pendulum, optical fiber, PC sound card.

Neste trabalho uma experiência com fibra óptica de plástico para controlar o movimento de um pêndulo simples foi descrito. O sistema de monitoramento da posição do pêndulo é baseado em curva perda de fibra óptica de plástico, que depende do raio de curvatura. Como um dispositivo de gravação de uma placa de som do PC foi usado. Com esta técnica de medição de que somos capazes de visualizar expectativa teórica e extrair alguns dos valores dos parâmetros a partir dos dados registados como posição, velocidade e aceleração do pêndulo. O deslocamento angular, velocidade e aceleração são plotados usando Mathematica, um programa de computador disponível simbólico que nos permite plotar facilmente a dependência obtidos, bem como seu comportamento teórico. Boa concordância entre as medidas e predição teórica foi observada.
**Palavras-chave:** pêndulo simples, fibra ótica, placa de som do PC.

## 1. Introduction

The pendulum is of great importance in science and education. It was used by Galileo as an accurate and simple timekeeper and by Newton to prove the equivalence between gravitational and inertia mass. Now it is set up in the laboratory as a pedagogical instrument used by teachers and students to measure the local acceleration of gravity and to study linear and nonlinear oscillations. The simple pendulum is one of the most popular examples analyzed in textbooks [1], physics education journals [2-8] and undergraduate courses, being also one of the nonlinear systems most studied not only in advanced, but also in introductory university courses of classical mechanics. The simple pendulum is usually the first example of a nonlinear problem and it is the first time that students analyze and understand the differences between linear and nonlinear physical problems. Several approximation schemes have been developed to investigate the situation for large amplitude oscillations of a simple pendulum [2-8]. An exact solution for the nonlinear pendulum is given in [7]. Our motivation in this work here was to understanding the motion of simple pendulum as a simple harmonic oscillator that is of great importance in physics because many more complicated systems can be treated to a good approximation as harmonic oscillators. Also, pendulum motion is accurately described by theory, but direct measurement of its position, velocity and acceleration in any point of its trajectory is rather very difficult.

Guided by this demands we proposed an experiment with a simple pendulum that was equipped with plastic optical fiber. Although many of the applications of optical fibers are based on their capacity to transmit optical signals with low losses, it can also be desirable for the optical fiber to be strongly affected by a certain physical parameter of the environment. In this way, it can be used as a sensor of such a parameter. There are many strong arguments for the use of POFs as sensors. In addition to their easiness to handle and low price, they present the advantages common to all multi- mode optical fibers. Specifically, we can men-

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tion that the flexibility and small size of optical fibers enable a great sensitivity to be achieved without having to occupy a big volume. Moreover, it has been proved that a POF can be employed to detect a great variety of parameters, including temperature, humidity, pressure, presence of organic and inorganic compounds, wind speed, and refractive index. On the other hand, POF-based optical sensors eliminate the risk of electric sparks in explosive environments, and they can be read from remote positions. The mechanisms allowing us to detect a physical parameter by using POFs are very diverse, although most of them are based on light intensity modulation. Some of the most important kinds of POF-based sensors are summarized in [9].

The experiment with plastic optical fiber that is proposed is based on the light power radiated by an optical fiber due to its bending. With the specific positioning of POF on the pendulum structure, one can exploit bending losses for accurate determination of pendulum position and by that extract all the other variables of motion.

## 2. Theoretical discussion

### 2.1. Theory of a simple pendulum

A simple pendulum is an idealized body consisting of a point mass suspended by a light inextensible cord. When pulled to one side of its equilibrium position and released, the pendulum swings in a vertical plane under the influence of gravity. The motion is periodic and oscillatory.

Figure 1 shows a pendulum of length $l$, particle mass $m$, making an angle $\theta$ with the vertical. The forces acting on $m$ are the gravitational force $mg$and the tension in the cord $T$.

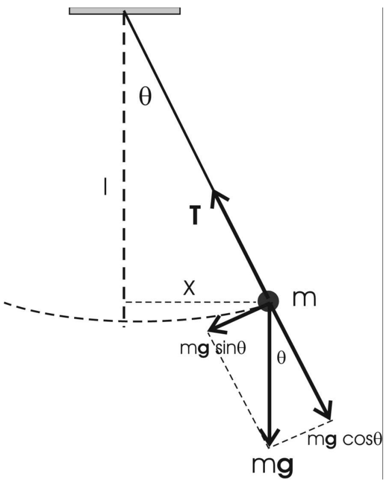


Figure 1 - The particle of mass $m$is displaced to the angle $\theta$ and the released at rest. The system oscillates under the action of gravity. The forces acting on the mass $m$are shown

Let us choose axes tangent to the circle of motion and along the radius. Resolve $mg$ into a radial component of magnitude $mg\cos\theta$ and a tangential component of magnitude $mg\sin\theta$. The radial components of the forces supply the necessary centripetal acceleration to keep the particle moving on a circular arc. The tangential component is the restoring force acting on $m$ tending to return it to the equilibrium position. Hence, the restoring force is

$$F = -mg\sin\theta. \tag{1}$$

Notice that the restoring force is not proportional to the angular displacement $\theta$, but to the $\sin\theta$ instead. The resulting motion is, therefore, not simple harmonic. The nonlinear oscillations of a simple pendulum are described by the following differential equation

$$\frac{d^2\theta}{dt^2} + \frac{g}{l}\sin\theta = 0, \tag{2}$$

where $g$ is the acceleration of gravity, $l$is the length of the pendulum, and $\theta$ is the elongation of the angular displacement. The oscillations of the pendulum are subject to the initial conditions

$$\theta(0) = \theta_0, \ \left(\frac{d\theta}{dt}\right)_{t=0} = 0, \tag{3}$$

where $\theta_0$ is the amplitude of oscillation. The system oscillates between symmetric limits $[-\theta_0, \theta_0]$. The periodic solution $\theta(t)$ of Eq. (2) depends on the amplitude $\theta_0$. For $\theta = 0$, we have $T = T_0$ and the period, $T_0$, of a linear oscillation of a simple pendulum is given by $T_0 = 2\pi/\omega$, where $\omega = \sqrt{g/l}$. The motion of the nonlinear pendulum depends on amplitude $\theta$ of the angle of displacement, and the relationship between the amplitude and the corresponding period $T$ describes the dynamic of this nonlinear motion. The solution of Eq. (2) is expressed in terms of elliptic integrals, and could be solved either numerically or by using approximations. In most cases, there are no analytical solutions to this differential equation.

However, if the angle $\theta$ is small, $\sin\theta$ is very nearly equal to $\theta$ in radians. For small displacement, therefore, the restoring force is proportional to the displacement and is oppositely directed. This is exactly the criterion for simple harmonic motion. Hence, assuming $\sin\theta \approx \theta$, we obtain the following differential equation

$$\frac{d^2\theta}{dt^2} + \frac{g}{l}\theta = 0. \tag{4}$$

The periodic solution of Eq. (4) is given by

$$\theta(t) = \theta_0 \sin(\omega t + \alpha), \tag{5}$$

where $\omega = \sqrt{g/l}$ is angular frequency, and $\alpha$is called the phase constant. In this case, the period of oscillation depends on the length of the pendulum and the acceleration due to gravity, and is independent of the amplitude $\theta_0$. The period of simple pendulum when its amplitude is small is

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{m}{mg/l}} = 2\pi\sqrt{\frac{l}{g}}. \tag{6}$$

The period $T$ is independent on the mass of the suspended particle. The speed of the mass is given by $v = ds/dt$ or, since $\theta = ls$, by $v = ld\theta/dt$. If we substitute$\theta$ from Eq. (5) we obtain

$$v = \theta_0 l\omega \cos(\omega t + \alpha). \tag{7}$$

Comparing Eq. (7) with Eq. (5), we see that the speed $v$oscillates with the same frequency as the angle $\theta$, but is out of phase with $\theta$ by $\pi/2$. Clearly this makes sense; the speed is zero when the angle is greatest. Now we can easily calculate the linear acceleration of mass, by deriving with time the former equation (Eq. (7))

$$a = -\theta_0 l\omega^2 \sin(\omega t + \alpha). \tag{8}$$

As we can see, the linear acceleration also follows the same periodical behavior, but it opposes velocity for the fixed angle of $\pi/2$. If we make plot of all three variables, scale it to 1 and place them on one figure, we can easily notice their mutual phase shift, as shown on the Fig. 2.

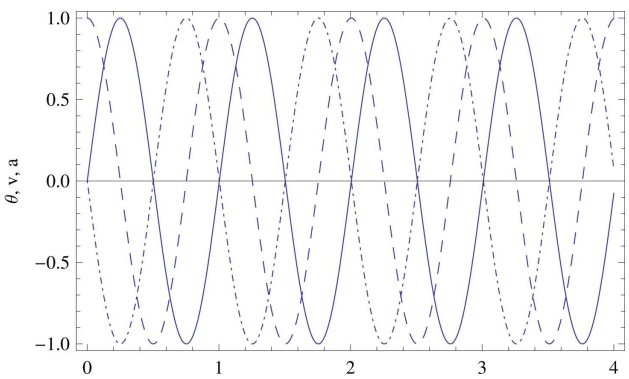


Figure 2 - Basic variables of pendulum motion. Full line present the angle $\mu$, dashed line mass velocity $v$ and dash-dotted line present the linear acceleration $a$.

### 2.2. Bend loss in large core multi-mode plastic optical fiber

The problem of bending losses from fibers has received considerable theoretical study by a number of authors. A general method has been developed by [10, 11] who computed the electric and magnetic fields at a large distance from the curved optical waveguide, and thus found the radiated Poynting vector. From the radiated power per unit length, an effective attenuation coefficient can be determined. Bend loss depends of bend radius, fiber parameters such as numerical aperture and core radius, and the launching conditions of the input beam [12]. The model that proposed by Gloge in [13] shows that the power loss attenuation coefficient of a guided mode propagating at an angle $\mu$ to the axis for a step index fiber is given by

$$\alpha = 2n_1 k\left(\theta_c^2 - \theta^2\right) \exp\left[-\frac{2}{3} n_1 k R \left(\theta_c^2 - \theta^2 - 2a/R\right)^{3/2}\right], \tag{9}$$

where $n_1$ is the refractive index of the fiber core, $k$ is the propagation constant ($k = 2\pi/\lambda$, where $\lambda$ is the free space wavelength of the light), $\theta_c$ is the critical angle (*i.e.* the maximum value of $\theta$ for guiding modes in the unperturbed fiber), $a$ is the radius of the fiber core, and $R$ is the radius of the bend measured to the axis of the fiber.

Bend loss has also been usefully exploited as a transduction mechanism in fiber optic sensors [14]. Most of the oscillations in the bend loss could form the basis of a number of applications, which include the using the rapid rise in the loss to sense changes in the radius of an object to which the fiber is attenuated. In this work, the fact of the bend radius dependence on the bend loss suggests the possibility of designing a simple sensor for monitor the motion of simple pendulum.

## 3. Experimental setup

In this section we describe the experimental equipment used in the laboratory and the method followed to conduct the experiment. Figure 3 shows the experimental setup. As we can see support board (1) bears the electronic circuit (5) and stands as a hanging point for pendulum. Plastic optical fiber (4) presents necessary sensor to determine inclination angle during the motion by moving together with the pendulum cord. It is fixed by two parallel pieces of pertinax (3) to the hanging steal wire. In this way we have achieve stability of fiber and avoid its torsion and non-symmetrical bending. Positioning in this way assure bending of fiber just on two places, symmetrical to the cord, which produce clear and harmonic signal on the amplifier output. The choice of placing the optical fiber is subject of discussion and could be done in many different ways. Also, on the support board we can see the multi-turn resistor potentiometer (6) and voltmeter (9). On the right-hand side and below there are voltage supply (8) and PC computer (7) for data acquisition.

The basic idea of this measurement is connected with bending losses which are described in the chapter B. Light directed in the input side of the optical fiber is slightly attenuated on the output side, when the pendulum move away from its equilibrium position, thus bending the POF for some angle. This presents direct connection between elongation angle and amount of light on the output sensor. In this case as an emitter we use LED (Light Emitted Diode) MFOE71 diode which has maximum of radiation in the infrared spectrum ($\lambda_m$ = 850 nm). As a counterpart a coupled photo detector MFOD72 is employed, with maximum sensitivity in same spectral band (around 820 nm) for measuring the light arriving on the output side of optical fiber. This specially selected photo couple enables us to measure slight deviations in light intensity. However, choice of photo couple is rather arbitrary and one can use different types with different sensitivity and wavelengths.

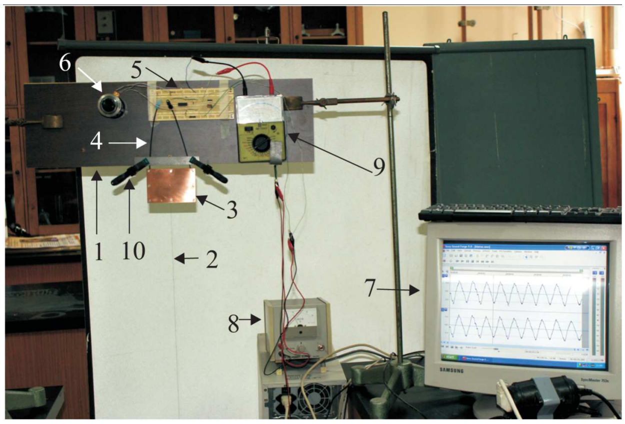


Figure 3 - Experimental arrangement for the simple pendulum. 1 - support board, 2 - hanging cord, 3 - pertinax boards, 4 - POF, 5 - electronic circuit together with the photo couple, 6 - multi turn resistor potentiometer, 7 - PC computer, 8 - power supply, 9 - analog voltmeter, 10 - clamps for fixing the POF on the support board.

Simple flowchart diagram that presented on the Fig. 4, give us clear picture of proposed idea. First box (1) presents the pendulum with the optical fiber sensor supplied from the input side (box 2) with constant source of light (emitter LED). Output side of fiber is terminated with LED detector (box 3), connected with the constant voltage source through the resistor. A small voltage across LED $U_{led}$ is compared with the fixed voltage $U_{ref}$ by the differential amplifier (box 4), their difference amplified and sent to the computer sound card (box 5) in order to be recorded.

We conduct an experiment in the following order. Measurements are taken with the properly selected resistor and power supply to establish the constant light source with the infrared LED diode. We use supply voltage of 5 V and resistor of 100 Ω to achieve the current in the LED about 30 mAmps. Such generated light is directed into an input side of the optical fiber. The output side of the fiber is connected with receiving diode to the 5 V supply via the 2 kΩ resistor assuring the voltage across diode about 0.5 V with fully alighted LED. By swinging the pendulum for some angle the periodic oscillation are established. Angular displacement designated by the angle $\mu$ represents as a change of voltage on the receiving LED. This rather very small signal is amplified and recorded by computer sound card. We choose differential amplifier made from the integrated circuit containing the operational amplifiers. Choice of this type of amplifier is rather necessary for the reason to be able to separate the direct and superposed alternative component of voltages applied on the receiving LED. We could also use serial capacitor and normal single input amplifier. However, capacitor could additionally deviate this slow changing signal and by that produce systematic error in our measurements.

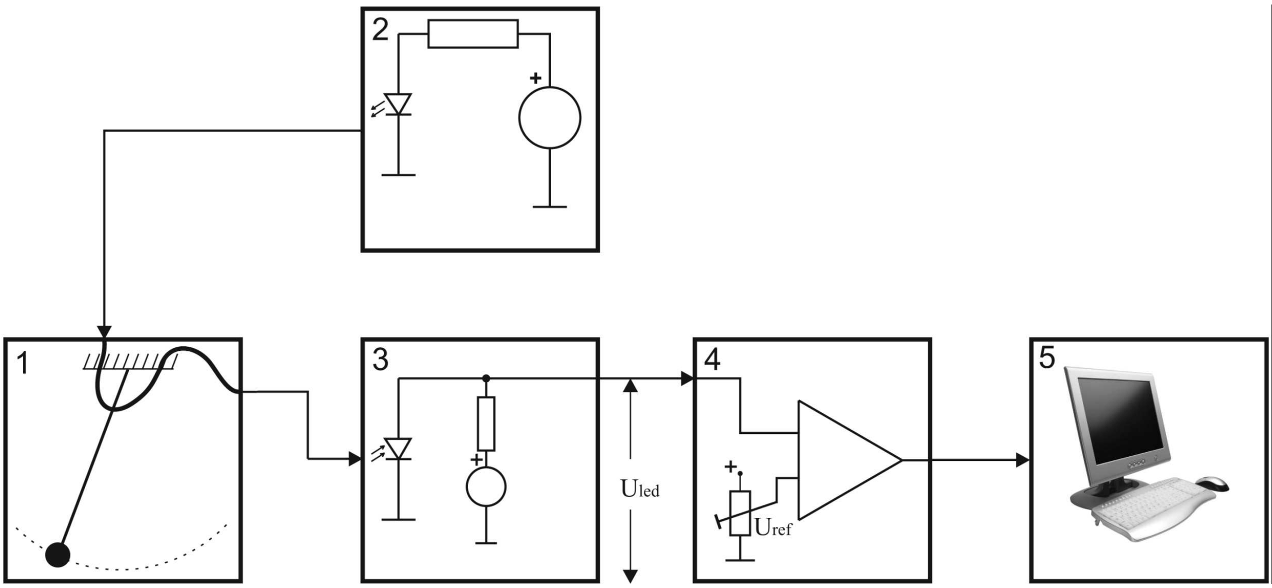


Figure 4 - Experiment flow chart.

In order to use the differential amplifier one need to setup the referential voltage, $U_{ref}$ on one input. For this purpose a voltage divider is used in the form of multi turn variable resistor, which allows high precision in voltage selection. We state here that ability to vary $U_{ref}$ is rather necessary, for the reason that the process of tuning and fixating the fiber, demand its bending in number of places, influencing the amount of output light and by that voltage on input side of amplifier. In such case voltage difference can be easily greater then necessary, thus producing the saturation in the amplified output signal. Solution is to setup input voltage difference in proper range to avoid output saturation by using the multi turn variable resistor.

Signal from the amplifier output is passed to the recording device, like digital oscilloscope or computer sound card. In this experiment our choice is sound card, mainly because computers are almost always present in the physical laboratory. Also, the process of recording the signal is not so complicated and demands basic skills within the usual operating systems. This approach gives us opportunity to compare experimental data with the theoretical prediction. Software used for this purpose is freely available to download from the Internet. We have chosen Sony Sound forge, but others with necessary functionality are also good enough, for example Creative Lab Studio or simply Sound Recorder from the Windows operating system. These programs allow us to record the signal, but in order to analyze it, compare with theoretical prediction and derive some useful information we need software with programming abilities, like Wolfram Research - Mathematica 7, or MathWorks - Mathlab. Our choice is Mathematica, mainly because former experience connected with this program package.

## 4. Results and discussion

After the initial setup one need to adjust amplifier reference voltage and write down starting value for observed deflection angle. First of all is done by using the installed voltmeter, which measure output signal. The difference on the amplifier input may overwhelm expected values thus putting the output signal in saturation. Therefore we adjust reference voltage to keep the output in measurement range. Initial value of deflection angle $\theta$ could be measured by using simple trigonometry. With the pendulum swinging we start recording and as a result we get "wav" file containing evolution of output voltage during the time of measurement. This file can be converted into an arrays for $x$ and $y$ data by using standard Mathematica functionality and presented in the form of $y(x)$ dependence shown on the Fig. 5. This signal presents voltage variation on the amplifier output and has an initial value of amplitude equal to the measured value of $\theta_m$. It is easy to recognize almost ideal periodical curve with the small deviations from sinus function.

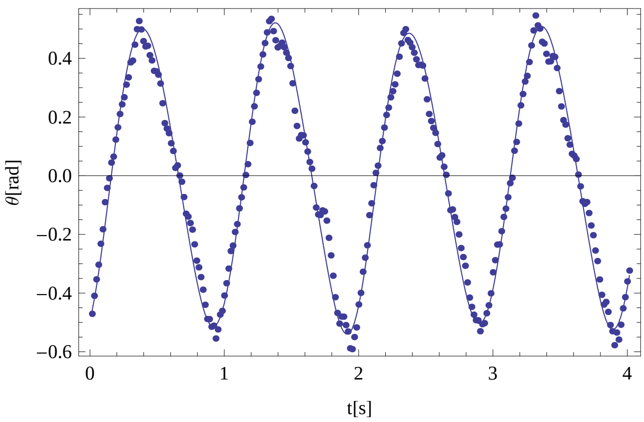


Figure 5 - Angular displacement as a function of $t$, acquired by the PC-sound card (dots). Full line is the fit obtained by *rms* method.

Output file consists more data points then we actually need to create the evolution curve. To simplify calculations we do not need all elements, but rather select every 500-th of them, thus producing reduced arrays. Also, data are slightly perturbed by the superposed noise from the amplifier, as well as with the random errors in our measurement process. To avoid this insufficiency we smooth extracted data by using *rms* method, which find best fit curve by minimizing the *rms* value.

Once we measured deflection angle evolution it can be compared with the results of Eq. (5), and used to calculate evolution of linear velocity and acceleration. This can be done similar as in the theoretical discussion in $x$ 2.1, but we need to account discrete derivation with the finite time intervals. As a result we have

$$v = l \cdot \left( \frac{\theta_{i+1} - \theta_i}{t_{i+1} - t_i} \right), \quad a = \frac{v_{i+1} - v_i}{t_{i+1} - t_i}. \tag{10}$$

We designate with $i$an $i$-th member of data arrays and $l$is the length of pendulum hanging wire. By using this method we can generate evolutions presented in the figures below. Figure 6a, b shows evolutions of linear velocity and acceleration. From the graphics we can see small deviations from the theoretical prediction. Those are mostly noticeable at the peaks of amplitude from one side for the $v$ and at the minimum values for the acceleration $a$. Observed variations are caused by lack of precision in our realization of pendulum device and measuring process. As we spoke earlier in the chapter III, we use two pertinax plane boards for fixating the optical fiber to the hanging cord of pendulum. Because the optical fiber is thicker then the hanging wire, it makes a gap between two pertinax boards, allowing a small oscillations of cord in the gap, with amplitude depending on the swinging side. This can produce elongation slightly higher in one side then in the other. Additionally, flimsy attached mass at the end of pendulum wire cause it's wobbling in the points of maximum elongations. As a overall result we see some deviations from the ideal theoretical prediction. These insufficiencies in proposed experiment are very interesting, showing a good sensitivity of our instrumentation, therefore we intentionally left them incorrect.

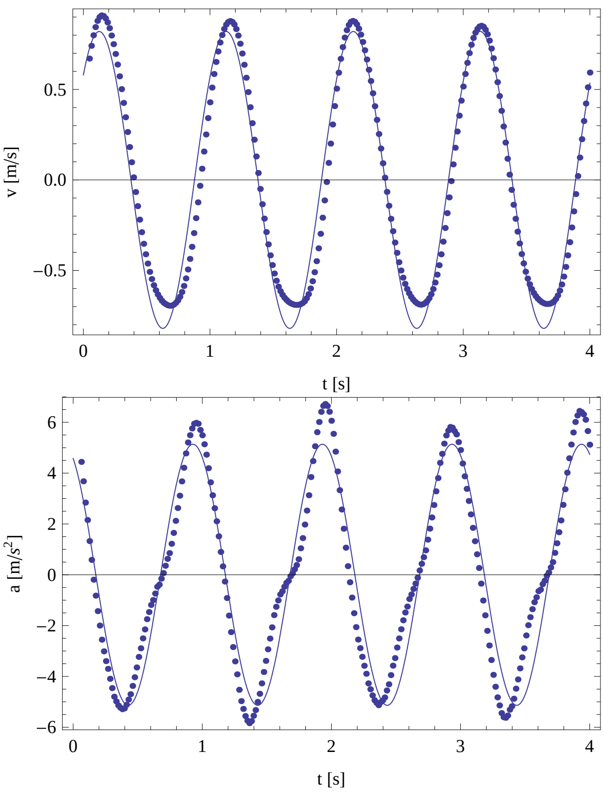


Figure 6 - Evolution of velocity (top) and acceleration (bottom) of the pendulum, both graphs present their experimental (dots) and theoretical (full line) data.

On the Fig. 7, we presented all three dependencies scaled to 1. These variables are not comparable in it's values, but rather to be used for examining their behavior depending on the pendulum position. We can see that at the instant of amplitude from the null position, acceleration has its maximum and angular velocity is rather small or zero, while in the points close to the null position acceleration tends to be zero and velocity is at its highest. Such behavior is expected and in a good agreement with a given theory of pendulum motion II and Fig. 2.

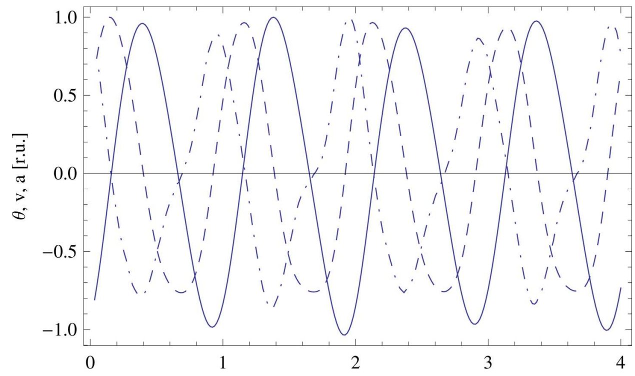


Figure 7 - Comparison of measured angular displacement $\mu$ (solid line), velocity $v$(dashed line) and acceleration $a$(dash-dotted line) of simple pendulum. All three quantities are scaled to 1.

## 5. Conclusion

In almost all courses of phenomenological physics studied at faculty level or high school, idealized pendulum is always present. It provides a straightforward introduction to linear/nonlinear oscillations and presents an irreplaceable tool for investigating properties of gravitational fields. This experiment has huge importance in science and it must be educationally and methodologically well treated in the schools and universities. We have presented original method to monitor the motion of simple pendulum which uses plastic optical fiber as a sensor. All steps from positioning and accommodating the sensor, through transport and handle the acquired signal and data recording and analyzing are described. Angular displacement, velocity and acceleration of pendulum were plotted as a function of time. Comparison of measurements and theoretical prediction was presented. Good agreement is observed between experimental results and theoretical prediction.

Derived results show that this system work good enough to be used in the physical laboratory for practical demonstration of the mathematical pendulum, as well as the application of optical fiber as a sensor. Sensitivity of measurement revealed some interesting features which can not be monitored by human eye or reflex, like small deviation from the theoretical prediction caused by insufficiency in pendulum realization.

With this experimental setup we can easily record amplitude attenuation due to the friction force, which can be applied for determining of the attenuating coefficient. Also, this equipment can be used in many different situation, so we plan for future work to apply it for monitoring elliptical motion of pendulum. Same principle with slight modification, mainly in sensor shape and position can be applied to monitor elastic oscillation of body on the hanging spring.

The main educational goal of presented experiment is comparison of theoretical prediction and measurements. We believe that present study may be a suitable and fruitful exercise for teaching and better understanding the behavior of the simple pendulum in undergraduate courses on classical mechanics.

## 6. Acknowledgment

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