

A downward buoyant force experiment

(Um experimento em que a força de empuxo aponta para baixo)

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In hydrostatics, the Archimedes principle predicts an upward force whenever a body is submerged in a liquid. In contrast to common sense, this physical law is not free of exceptions, as for example when the body touches the container. This is more evident when a rectangular block less dense than the liquid rests on the bottom, with no liquid underneath it, a case in which a downward force is expected, according to a recent work by the first author. In the present work, we describe a simple, low-cost experiment which allows the detection of such force. This counterintuitive result shows the inadequacy of Archimedes' principle for treating "contact" cases.

Keywords: hydrostatics, Archimedes' principle, Buoyant force.

Em hidrostática, o princípio de Arquimedes prevê uma força vertical para cima sempre que um corpo encontra-se submerso em um líquido. Contrariamente ao senso comum, esta lei física não está livre de exceções, como, por exemplo, quando o corpo está em contato com o recipiente. Isto fica mais evidente quando um paralelepípedo menos denso do que o líquido encontra-se em repouso no fundo do recipiente, sem nenhum líquido abaixo dele, um caso em que uma força para baixo é esperada, de acordo com um trabalho recente do primeiro autor. Neste trabalho nós apresentamos um experimento simples e de baixo custo que permite a detecção de tal força. Este resultado contra-intuitivo mostra que o princípio de Arquimedes é inadequado para casos em que há contato do corpo com o recipiente.

Palavras-chave: hidrostática, princípio de Arquimedes, força de empuxo.

1. Introduction

In everyday life, it is easy to observe the buoyancy phenomenon for a solid body lighter than a liquid. The common sense says that when a 'light' body is immersed in a liquid it will be pushed upwards until floating. At school, this kind of knowledge gains the status of a scientific law when we learn the celebrated *Archimedes' Principle* (AP) [1,2]:

When a body is fully or partially submerged in a fluid, a buoyant force from the surrounding fluid acts on the body. This force is directed *upward* and has a magnitude equal to the weight of the fluid displaced by the body.

Our confidence in this law is indeed reinforced in introductory physics courses [1,2] or courses of fluid me-

chanics for engineers [3], in which AP is presented as a law free of exceptions. However, when a body touches any of the container's walls the net hydrostatic force is not necessarily vertical, as indicated for the smaller block at the left of Fig. 1. This is indeed more evident in the so-called 'bottom case', also indicated in Fig. 1, in which the net hydrostatic force points *downward*. This counterintuitive situation was firstly reported in an experimental work by Jones and Gordon, in which a piece of cork glued to a thin aluminum disc was pressed on a large aluminum block, both immersed in water [4].² On noting that the metallic surfaces stuck together even after the water was removed (*i.e.*, in air), Ray and Johnson soon complemented this experiment by placing the objects into an evacuated cell and verifying that they were effortless separated, showing that the force holding the surfaces together is not *adhesive* (*i.e.*, intermolecular attraction between the surfaces), but due to atmospheric pressure [5].

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²In the words of those authors, "this experiment generates a great deal of interest, particularly if the students are drawn into a discussion to predict the result".

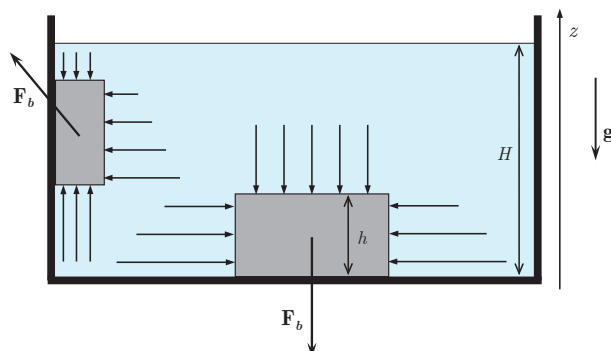


Figure 1 - The hydrostatic forces acting on rectangular blocks in contact to the walls of a container. The larger block, with height h , represents the ‘bottom’ case. The arrows indicate the pressure forces exerted by the liquid on the surface of each block. The net force exerted by the liquid in each block — *i.e.*, the ‘buoyant’ force, as defined in the text —, is represented by \mathbf{F}_b . As usual, the z -axis points upwards.

In a more recent work, Valiyov and Yegorenkov discussed the existence of a BF in the ‘bottom case’ and suggest an experiment in which a table-tennis ball is attached to a thin glass plate which is placed on a large glass block [6]. When the set is fully submerged in water the ball would remain below the waterline, which led those authors to conclude (correctly, in our opinion) that AP as stated above is deficient. Their article is followed by a comment by Kibble, who argues that these experiments are out of the scope of AP because the word ‘immersed’ would mean *completely surrounded by the liquid* [7]. However, his opinion is deficient because it excludes the case of a solid body floating in a denser liquid with its emerged portion exposed to air (or vacuum), a case in which AP of course works! Kibble indeed points out that, due to an adhesive force, the thin glass plate and the large glass block could be treated as a single object for which the weight is greater than the BF, which is *incorrect* since adhesive forces were ruled out by Ray and Johnson’s experiment [5].

In an experimental work by Bierman and Kincanon (2003), the validity of AP as stated in both the original and modern texts is reconsidered [8]. By cutting a hole in the bottom of a rubber-lined bucket and putting an aluminum block over the hole, they overcame the problem of water seepage. They then measured the downward force exerted by the liquid on the block for different depths using a PASCO sensor, showing that it *increases with depth*, instead of being a constant (as predicted by AP).

In another recent work [9], Graf investigated the validity of AP as originally stated by Archimedes (287-212 b.C.) in his *On Floating Bodies, Book I* [10]. For a body less dense than the liquid, treated in Propositions 4 to 6, Graf noted that the term ‘buoyant force’ does

³This differs from the definition adopted by Mungan in a recent work [12]. There, in order to keep the direction of the BF upward in all static cases, he proposes that the BF could be defined as *the negative of the weight of the fluid displaced by the body*. For a body fully surrounded by a liquid, both definitions are equivalent.

not appear, but in Proposition 6, namely [10]

If a solid lighter than a fluid be forcibly immersed in it, the solid will be driven upwards by a force equal to the difference between its weight and the weight of the fluid displaced,

an upward force is mentioned which, in modern words, would be the *apparent weight*. He also points out that the proofs by Archimedes were only for static cases in which *there is fluid under the body*, so our ‘bottom case’ was overlooked. It is only for *a body denser than the liquid*, treated in Proposition 7, that Archimedes speaks of a body in contact to the bottom, but he again mentions only the apparent weight. Graf then devises a method to measure the apparent weight directly with a waterproof scale, establishing that this weight is the same whether or not there is liquid under the block. For *a block less dense than the liquid*, however, he argues that “the apparent weight would be *negative* and provision would have to be made to keep the object in contact to the scale.”

By suspecting that the above conclusion by Graf could be wrong, we have developed a simple experiment to test it. Since the experiment by Jones and Gordon requires the complement by Ray and Johnson [5], which demands a vacuum pump (not available in most schools and undergraduate labs), here in this work we present a simpler, cheaper experimental setup for checking the direction of the buoyant force acting upon a block of wood immersed in water and touching the bottom of an aquarium.

2. Basic theory for the ‘bottom case’

Let us start by defining the buoyant force as *the net force that a fluid exerts on the part of the surface of a body that is effectively in contact to the fluid*.³ For a solid body with a volume V fully submerged in a liquid with a density ρ , as we are interested here, AP predicts an *upward* force with a magnitude

$$F_b = \rho V g, \quad (1)$$

i.e., the weight of the liquid displaced by the body, g being the local acceleration of gravity [1, 2, 11]. Note that, according to AP, the BF should not vary with depth.

For a rectangular block resting on the bottom of a container filled with a liquid, however, given that the block is fully submerged and assuming that no liquid seeps under the block, the horizontal forces cancel out and the net force exerted by the liquid is expected to point *downward*, as indicated in Fig. 1. In Ref. [13],

the first author predicts a magnitude $p_{top} A$ for that force, p_{top} being the pressure at the level of the top of the block, whose area is A . From Stevinus law [1], this yields

$$\mathbf{F}_b = -[p_0 + \rho g (H - h)] A \hat{\mathbf{k}}, \quad (2)$$

where $\hat{\mathbf{k}}$ is the unit vector pointing upwards, p_0 is the atmospheric pressure, H is the height of the liquid column above the bottom of the block, and h is the height of the block, as indicated in Fig. 1. Since the liquid pressure in any point of the bottom is $p_b = p_0 + \rho g H$, one can write the force in Eq. (2) as Ref. [13]

$$\mathbf{F}_b = -(p_b A - \rho V g) \hat{\mathbf{k}}, \quad (3)$$

where V is the volume of the block. In our experiment, a technique similar to that introduced in Ref. [8] is employed in order to reduce the liquid seepage under the block, as indicated in Fig. 2. While no liquid seeps in the rectangular hole, the net hydrostatic force exerted on the block will be⁴

$$\mathbf{F}_b = -(p_{top} A - p_0 A_{air} - p_b A_{seep}) \hat{\mathbf{k}}, \quad (4)$$

where $A_{air} (< A)$ is the area of the portion of the base that is in contact to air and $A_{seep} (\ll A)$ is the area of the bottom of the block in contact to liquid under the block.⁵ If no liquid seeps under the block, then Eq. (4) yields a *downward* force given by

$$F_b = p_{top} A - p_0 A_{air} = p_0 A + \rho g (H - h) A - p_0 A_{air} = \rho g H A - \rho g V + p_0 (A - A_{air}). \quad (5)$$

More generally, if only a fraction $0 < f < 1$ of the region between the rectangles indicated in Fig. 2 (whose area is $A - A_{air}$) is in contact to the liquid, then the net force exerted by the fluids on the block is⁶

$$\begin{aligned} \mathbf{F}_b &= -[p_{top} A - p_0 A_{air} - p_b f (A - A_{air})] \hat{\mathbf{k}} \\ &= -[\rho g (H - h) A + p_0 A - p_0 A_{air} - f (p_0 + \rho g H) \cdot (A - A_{air})] \hat{\mathbf{k}} \\ &= -[(1 - f) \rho g H A + (1 - f) p_0 (A - A_{air}) + f \rho g H A_{air} - \rho g V] \hat{\mathbf{k}}. \end{aligned} \quad (6)$$

From this result, one easily deduces that a *downward* BF arises whenever

$$H > H_{min} = \frac{V - (1 - f) (A - A_{air}) p_0 / (\rho g)}{A - f (A - A_{air})}, \quad (7)$$

where H_{min} is the depth for which the force in Eq. (6) is null. Let us check the limits $f \rightarrow 0$ and $f \rightarrow 1$. For $f \rightarrow 0$, F_b is that given in Eq. (5), hence $H_{min} = h - (1 - A_{air}/A) p_0 / (\rho g)$. Since $A_{air} < A$, this height is of course less than h , which means that, being the block fully submerged, the force points *downward*. For $f \rightarrow 1$, on the other hand, F_b reduces to $\rho g (A_{air} H - V)$ and then

$$H_{min} = \frac{V}{A_{air}} = \frac{A}{A_{air}} h. \quad (8)$$

In this case, $H_{min} > h$ and depths $H \leq H_{min}$ must be avoided in seeking for a *downward* BF.

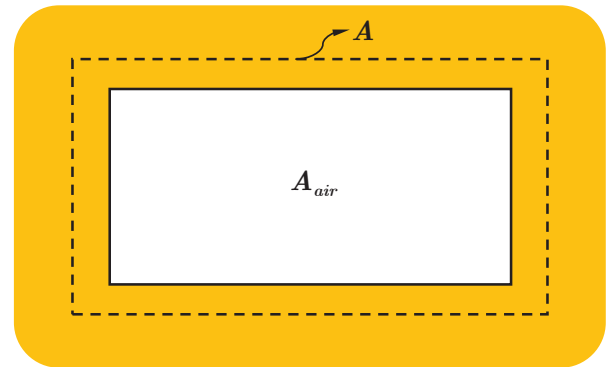


Figure 2 - Our technique for reducing the contact between the base of our block and the liquid under the block. In a rubber pad, we cut a rectangular hollow with an area A_{air} somewhat smaller than the area A of the base of the block, indicated by the dashed rectangle.

3. Our experiment

We have developed a simple, low cost experiment to investigate the net force exerted by water on a block of wood immersed in it and resting on the bottom of an

⁴Since $p_0 \approx 10^5$ N/m², this *downward* force can be strong enough to make it difficult to detach the block from the bottom. This makes it dangerous, *e.g.*, to step on the drain of a deep tank or swimming pool when it is open for emptying.

⁵Note that $A_{seep} = A$ only at the end of each run, when the liquid wets the base of the block and moves it upwards.

⁶At this point, we are assuming the liquid seeps under the block but does not touch the part of its bottom which is just above the hollow.

aquarium. Our experiment allows for checking whether that force is directed *upward*, as predicted by AP, or *downward*, as predicted in Ref. [13].

Our block is a rectangular one made of wood, with a mass of 229.6 grams and dimensions 14.0 cm \times 7.0 cm \times 3.3 cm (hence a density of 0.7 grams/cm³). It was put on a rubber pad with a rectangular hollow whose length and width are 0.5 cm narrower than the corresponding quantities in the base of the block (namely, 13.5 cm \times 6.5 cm), as illustrated in Fig. 2. The pad has a thickness of 5 mm. This technique is similar to that introduced by Bierman and Kincanon in Ref. [8]. Though water slowly seeps under the block due to small roughnesses of both the wood and rubber surfaces, the part of the base of the block just above the hole, whose area is $A_{air} = 87.75 \text{ cm}^2$ (thus smaller than $A = 98 \text{ cm}^2$), has initially no water touching the block (only air). As a consequence, the hydrostatic pressure of water acts only on the small area $A - A_{air} = 10.25 \text{ cm}^2$, which is not sufficient for pushing the block up. We have used a plastic bottle connected to a hose to slowly fill the glass box with tap water (density $\rho = 1.00 \text{ g/cm}^3$), as shown in Fig. 3. In order to avoid the block disturbance and reduce the water seepage, we slightly pressed down the top of the block with one of our fingers, but only until the water-line to attain the depth $H = H_{min} \approx 4 \text{ cm}$, according to Eq. (8). For $H > H_{min}$ the water itself presses the block down and we could safely remove the finger. In all runs, the block remained at the bottom for more than a minute, until water to fulfill the hollow under the block, after which it floated. This crucial experiment for AP shows the presence of a ‘buoyant’ force pressing the block down during all time prior to its detachment from the bottom of the aquarium.

On presenting this counterintuitive experiment to our first-year students, we have noted that most of them suspected of some ‘trick’, *i.e.* that we could be using some other source of downward force. This revealed their trust in AP, but we ruled out these suspects by positioning the glass box in a manner to leave the part below the block out of the table, as shown in Fig. 3, which allowed them to see the water seeping under the block. In fact, some students asked us to examine the block of wood, seeking for a magnet or some other trick. They also suspected that the block could be ‘heavier’ than water, which was ruled out at the end of the experiment, when the hollow was finally occupied by water and the block was pushed upwards until floating.

4. Conclusions

We have drawn the readers attention to the fact that the upward force predicted by AP is not always equal to the hydrostatic force exerted by a liquid on a body immersed in it, as observed in our experiment with a block of wood in contact to the bottom of an aquarium,

in which the force \mathbf{F}_b stated in Eq. (2) points *downward* for all $H > H_{min}$, contrarily to AP. Our result for the BF agrees to Bierman and Kincanon experiment, in which the force exerted by the liquid in a block denser than it is shown to point *downward* [8]. It would not be surprising that Archimedes, one of the greatest geniuses of the ancient world, had enunciated his original propositions with remarkable precision and insight, however there are some instances which he did not realize. One of these instances is shown here to be an *exception* to AP. We suggest the inclusion of this experiment as an interesting complement to the apparent weight experiment routinely reproduced by students in introductory physics labs.



Figure 3 - One of us (GTB) using a pet bottle and a hose to (slowly) fulfill the glass box with water. The hose enters the box from a distance of the block, in view to avoiding any disturbance in the block equilibrium and reducing the liquid seepage. Note, at the left, the vertical rule used for measuring H .

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