



Explaining school mathematics performance from symbolic and nonsymbolic magnitude processing: similarities and differences between typical and low-achieving children

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Abstract

Magnitude processing is one of the most central cognitive mechanisms that underlie persistent mathematics difficulties. No consensus has yet been reached about whether these difficulties can be predominantly attributed to deficits in symbolic or nonsymbolic magnitude processing. To investigate this issue, we assessed symbolic and nonsymbolic magnitude representations in children with low or typical achievement in school mathematics. Response latencies and the distance effect were comparable between groups in both symbolic and nonsymbolic tasks. The results indicated that both typical and low achievers were able to access magnitude representation via symbolic and nonsymbolic processing. However, low achievers presented higher error rates than typical achievers, especially in the nonsymbolic task. Furthermore, measures of nonsymbolic magnitude explained individual differences in school mathematics better than measures of symbolic magnitude when considering all of the children together. When examining the groups separately, symbolic magnitude representation explained differences in school mathematics in low achievers but not in typical achievers. These results suggest that symbolic magnitude is more relevant to solving arithmetic problems when mathematics achievement is particularly low. In contrast, individual differences in nonsymbolic processing appear to be related to mathematics achievement in a more general manner. **Keywords:** symbolic number processing; nonsymbolic number processing; cognition; child development; dyscalculia.

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Introduction

Mathematics difficulties (MD) are currently defined as persistent and severe difficulties in acquiring specific abilities related to mathematics that cannot be attributed

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to emotional or educational inadequacies, lack of general intelligence, or sensorimotor impairment (American Psychological Association, 1994; Butterworth, 2005). The terminology and diagnostic criteria for MD vary widely. Mathematics learning difficulties or disabilities as research criteria refer to performance on standardized achievement tests below cut-off scores at the 25th to 35th percentiles (Geary & Hoard, 2005; Jordan, Hanich, & Kaplan, 2003). In contrast, developmental dyscalculia is usually defined by more stringent criterion (i.e., the 10th percentile; Murphy, Mazzocco, Hanich, & Early, 2007).

One influential model of number processing attributes the main deficits encountered in MD to core magnitude representation difficulties (Piazza et al., 2010). Feigenson, Dehaene, & Spelke (2004) proposing that infants and adult humans share an approximate number system dedicated to representing number magnitude in an abstract form. According to the core deficit hypothesis, developmental dyscalculia is caused by a deficit that is specific to the approximate number system and is characterized by low

performance on tasks that assess number magnitude, such as nonsymbolic numerosity tasks (Dehaene, 1992, 2009). Recently, Piazza and colleagues (2010) showed that the acuity of the approximate number system is impaired in children with MD. Similarly, some authors have suggested that the deficits observed in MD constitute an impairment in the “number module,” a system dedicated to processing sets of objects and operating on them (Butterworth, Varma, & Laurillard, 2011). The approximate number system interacts with symbolic notational systems. Many authors argue that learning the symbolic number system may be at least as important for explaining deficits in MD as a deficit in the more basal competencies related to the approximate number system (e.g., Rousselle & Noël, 2007). Furthermore, Mussolin, Mejias, & Noel (2010) proposed a Two-Factor Theory of developmental dyscalculia that relates arithmetic achievement to both symbolic and nonsymbolic number representations. According to this theory, children are born with a nonsymbolic number sense and learn in school to map exact numerical symbols onto the internal number representations. Accordingly, dyscalculic children may initially have a weak number sense, and this deficit may prevent them from benefiting from the increasing precision yielded by symbolic numbers (Mussolin et al., 2010).

If the core magnitude deficit hypothesis is correct, then deficits in both symbolic and nonsymbolic tasks should be observed in MD (henceforth designating both of the experimental groups in the present study and MD in general) compared with typically achieving children (henceforth the control group in the present study) because magnitude representations are accessed in both tasks. However, deficits may be more pronounced in the nonsymbolic task because it more directly measures the approximate number system. Moreover, the nonsymbolic task should predict school mathematics achievement better than the symbolic task. However, if the disconnection hypothesis is true, then not only should one observe much more pronounced deficits in symbolic magnitude processing in the MD group, but this task should also be a better predictor of school mathematics achievement than nonsymbolic magnitude processing, regardless of the group. Finally, possible differences in basic magnitude comparisons between the MD group and the sample with MD associated with language difficulties (MD+L group) were also examined. If deficits in phonological and verbal abilities affect learning to connect symbols and magnitudes, a more severe deficit in symbolic magnitude processing should be observed in the MD+L group than in the MD group.

Based on the hypothesis that the approximate number system has a continuous distribution in the population (Halberda, Mazocco, & Feigenson, 2008), no categorical difference in nonsymbolic representation should exist between children with dyscalculia and typically developing children. To explore the nature of nonsymbolic representation, two criteria were used to

classify children with MD in the present study: a liberal criterion (25th percentile) and a more conservative criterion (10th percentile). Using these two criteria, one can investigate how mathematics learning difficulties are associated with the severity of deficits in magnitude processing. Moreover, the numerical ranges used in symbolic and nonsymbolic tasks were chosen to prevent interference from factors such as subitizing (Piazza et al., 2010) in the nonsymbolic task and the unit–decade compatibility effect in the symbolic task. To avoid the occurrence of subitizing processes in the nonsymbolic task, the numerical interval used in the nonsymbolic task in the present study was above the subitizing range (1–4). Moreover, with regard to the symbolic task, we chose a simple version of the task, including single digits only, to avoid interference from more complex aspects of symbolic magnitude processing such as familiarity with round decade numbers (e.g., 10, 20, 30, and so on; Brysbaert, 1995) and the unit–decade compatibility effect (Nuerk, Weger, & Willmes, 2001).

Methods

Participants

Children participated only after written informed consent was obtained from their parents and orally from the children. Participants were recruited from schools in two southeastern Brazilian cities, Belo Horizonte and Mariana. Approximately 82% of the children attended public schools. Recruitment was conducted in two phases. In the screening phase, 16 schools, eight of which were public schools, were randomly selected from seven of a total of 10 school districts in the two cities. Classrooms were then selected in these schools, and the families were invited to participate in the project. The study was approved by the local research ethics committee of the Federal University of Minas Gerais.

A total of 1643 children were screened for MD. Subjects included in the screening sample had a mean age of 9.75 years (SD = 1.95 years; range = 6 years; mean = 117.7 months; SD = 23.3 months). The mean formal schooling was 3.39 years (SD = 1.66 years; range = 6 years), and 50.9% of the subjects were female. Testing was conducted in groups in the classrooms. Two instruments were used in the screening phase: the arithmetic and single word spelling subtests of the Brazilian School Achievement Test (Teste do Desempenho Escolar [TDE]; Stein, 1994). After screening, more detailed individual testing was conducted. The parents of all of the children were invited to a meeting where the second phase of the research (i.e., an individual assessment of approximately three sessions, 1 h each) was explained. Four hundred twenty-four parents did not attend the meeting, 110 parents did not allow their children to participate in the second phase of the study, 123 children presented only isolated spelling difficulties, 581 participants were not paired controls, and two children

presented genetic disorders. Mothers were interviewed with regard to the developmental, learning, and health histories of their children (Entrevista Semi-Estruturada para Diagnóstico em Psiquiatria da infância, K-SADS; Brasil, 2003). Testing was conducted in quiet rooms dedicated by the schools for the study. Children were assessed using Raven's Coloured Progressive Matrices (Angelini, Alves, Custódio, Duarte, & Duarte, 1999), the reading subtest of the TDE, a simple reaction time task, and two computer tasks to assess magnitude comparisons. Children with performance below the 25th percentile on Raven's Coloured Progressive Matrices (Angelini et al., 1999) were excluded from the sample.

One hundred sixty-eight pupils participated in the second phase of the study that consisted of individual neuropsychological testing. The mean age was 10.10 years (SD = 1.93 years) and 127.07 months (SD = 23.42 months). The 53 children with performance below the 25th percentile in arithmetic but above the 25th percentile in spelling were assigned to the MD group. The 26 children with performance below the 25th percentile in both arithmetic and spelling were assigned to the MD+L group. The 89 children with performance above the 25th percentile in all of the tests were assigned to the control group. All groups were matched with regard to intelligence, age, and school grade (Table 1). Mean formal schooling was 3.81 years (SD = 1.72 years), and 49.7% of the participants were female. Table 2 presents the group performance in reading, spelling, and arithmetic and the mean age and intelligence scores.

Psychological instruments

Brazilian School Achievement Test (Teste do Desempenho Escolar; TDE; Stein, 1994). The TDE is

the standardized test of school achievement most widely used in Brazil. The TDE comprises three subtests: arithmetic, single-word spelling, and single-word reading. Norms are provided for school-aged children between the first and sixth grades. The arithmetic subtest is composed of three simple orally presented word problems and 45 written arithmetic calculations of increasing complexity. The spelling subtest consists of 34-word dictation of increasing syllabic complexity. The single-word reading subtest of the TDE consists of 75 stimuli that must be read aloud by the participant. Reliability coefficients (Cronbach's α) are 0.87 or higher. The children were instructed to work as hard as they could without a time limit.

Simple Reaction Time Task. The computerized reaction time task is a simple task in which a picture of a wolf (9.31 cm height, 11.59 cm length) is displayed in the center of a black screen for a maximum of 3000 ms. The participant is instructed to press the space bar on the keyboard as fast as he or she can at the moment he or she sees the wolf. Trials are terminated with the first key press. The task had 30 trials with an intertrial interval of 2000, 3500, 5000, 6500, or 8000 ms. This task was used to control possible differences in basic processing speed unrelated to numerical tasks.

Symbolic Magnitude Comparison Task. In the symbolic number comparison task, Arabic numerals from 1 to 9 were presented on the computer screen (2.12 cm height, 2.12 cm length). The visual angle of the stimuli was 2.43° in both the vertical and horizontal dimensions. The children were instructed to decide whether the magnitudes were larger or smaller than 5. Arabic numerals were presented in white on a black background. If the presented number was lower than 5, the child had to press a predefined key on the left side of the keyboard with the left hand. If the stimulus was

Table 1. Individual assessment sample descriptions

Measure	Control ($n = 89$)	MD ($n = 53$)	MD+L ($n = 26$)	χ^2	df	p
Sex (Female)	58.40%	43.40%	38.50%	4.81	2	.090
School (Public)	77.30%	86.50%	88.50%	2.78	2	.249

Control, typical achievers; MD, mathematics difficulties; MD+L, mathematics and language difficulties.

Table 2. Sample descriptions (age and intelligence) and performance in reading, spelling, and arithmetic (Z scores)

Measure	Control ($n = 89$)		MD ($n = 53$)		MD+L ($n = 26$)		$F(2, 165)$	p
	Mean	SD	Mean	SD	Mean	SD		
Age (years)	10.19	1.66	9.68	1.95	10.65	2.57	2.45	.09
Raven	.19	.54	-.01	.78	-.03	.79	2	.139
Spelling (TDE)	.35	.44	.15	.46	-1.45	1.62	11.13	< .0001
Reading (TDE)	.39	.3	0	.35	-1.38	1.84	10.1	.001
Arithmetic (TDE)	.54	.73	-.42	.42	-1.46	.5	20.28	< .0001

Control, typical achievers; MD, mathematics difficulties; MD+L, mathematics and language difficulties.

higher than 5, then the key to be pressed was located at the right side of the keyboard and had to be pressed with the right hand. The number 5 was not presented on the computer screen. Numerical distances between stimuli and the reference number (5) varied from 1 to 4, with each numerical distance presented the same number of times. Between trials, a fixation point (i.e., a cross designed with the same size and color as the stimuli) appeared on the screen. The task comprised eight learning trials and 80 testing trials. The maximum stimulus presentation time was 4000 ms with an intertrial interval of 700 ms.

Nonsymbolic Magnitude Comparison Task. In the nonsymbolic magnitude comparison task, the participant was instructed to compare two simultaneously presented sets of dots, indicating which one contained the larger number. Black dots were presented on a white circle on a black background. In each trial, one of the two white circles contained 32 dots (i.e., reference numerosity), and the other one contained 20, 23, 26, 29, 35, 38, 41, or 44 dots. Each dot set magnitude was presented eight times, each time in a different configuration. The task comprised eight learning trials and 64 testing trials. Perceptual variables were randomly varied such that individual dot size was held constant in half of the trials, whereas the size of the area occupied by the dots was held constant in the other half of the trials (Dehaene, Izard, & Piazza, 2005). The maximum stimulus presentation time was 4000 ms with an intertrial interval of 700 ms. Between each trial, a fixation point (i.e., a 3-cm diameter crossprinted in white) appeared on the screen. If the child judged that the right circle presented more dots, then a predefined key on the right side of the keyboard had to be pressed with the right hand. If the child judged that the left circle contained more dots, then a predefined key on the left side of the keyboard had to be pressed with the left hand.

Computer tasks were programmed with the Neurobehavioral Systems presentation software. Participants were seated approximately 50 cm from the computer screen.

Data analysis

Response time data were trimmed, eliminating in two iterative steps all of the responses that were more extreme than three standard deviations from the individual means and reaction times that were faster than 200 ms. Error data were arcsine-transformed to correct for skewness. The distance effect was calculated for performance accuracy (error rates) and speed (reaction time). Reaction times and error rates with large distances (6, 9, and 12 for the nonsymbolic task and 2, 3, and 4 for the symbolic task) were subtracted from those with small distances (3 for the nonsymbolic task and 1 for the symbolic task). To verify the presence of a distance effect, a series of mixed-design analyses of variances (ANOVAs) was conducted for each task and each group for reaction times and error rates, with group as the between-subjects factor. When the assumption of sphericity was not met and epsilon estimates were less

than 0.75, the Greenhouse-Geisser correction was used. When main or interaction effects were significant, more specific Bonferroni Least Significant Difference *post hoc* comparisons were performed.

Results

Simple reaction time measure

Hand, foot, and eye dominance were assessed using the standard protocol of Lefevre & Diament (1982). All analyses were conducted by merging the data from left- and right-handed boys and girls. No significant differences were found among the three groups with regard to reaction times in the simple reaction time task ($F(2,165) = 1.61$, $MSE = 56256.67$, $p = .204$, $\eta^2 = .024$). Mean reaction times were 418.9 ms ($SD = 131.43$ ms) in the control group, 457.75 ms ($SD = 173.19$ ms) in the MD group, and 500.53 ms ($SD = 337.33$ ms) in the MD+L group. These results indicate no significant baseline differences in simple reaction times.

Group comparisons in the symbolic task

A one-way ANOVA was conducted to compare the three groups (MD, MD+L, and control) in the symbolic task. No significant differences were found among groups in overall reaction time ($F[2,115] = 2.62$, $MSE = 205135$, $p = .076$, $\eta^2 = .037$). Mean reaction times were 907 ms ($SD = 323$ ms) in the MD group, 916 ms ($SD = 223$ ms) in the MD+L group, and 851 ms ($SD = 262$ ms) in the control group. Bonferroni *post hoc* comparisons indicated no significant differences among the three experimental groups. Reaction times for each distance and each group in the symbolic task are presented in Figure 1A.

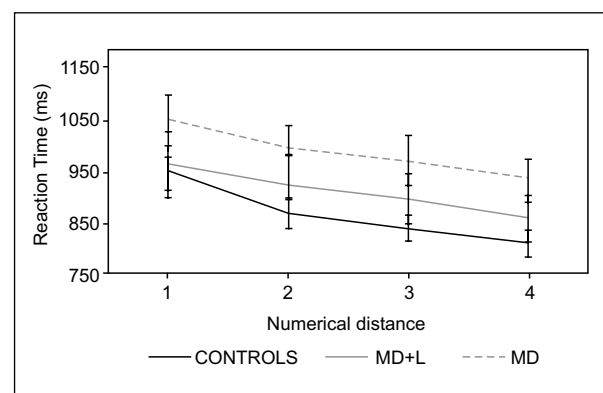


Figure 1A. Reaction time as a function of numerical distance in the MD, MD+L, and control groups in the symbolic task.

One-way ANOVA revealed significant differences in total error rates among the three groups in the symbolic task ($F[2,139] = 3.33$, $MSE = .073$, $p = .039$, $\eta^2 = .036$). Error rates were 11% in the control group, 16.38% in the MD+L group, and 18.37% in the MD group. However, pairwise comparisons revealed no significant differences among groups ($p > .05$). Error rates for each distance are presented in Figure 1B.

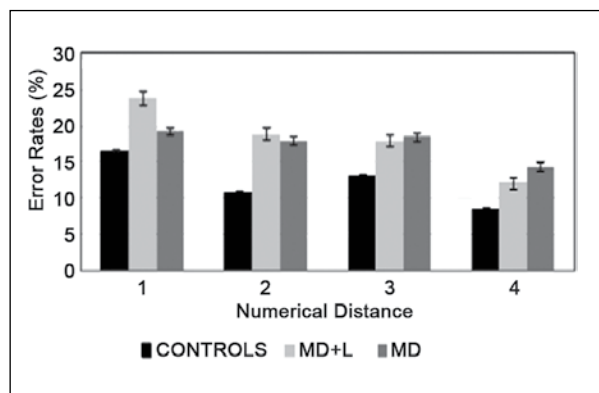


Figure 1B. Error rates (%) and standard errors as a function of numerical distance in the MD, MD+L, and control groups in the symbolic task.

Group comparisons in the nonsymbolic task

The analysis of the perceptual control factors did not reveal any differences in overall correct responses for size (mean = 78%) and luminance (mean = 79%, $p = .262$, Cohen's $d = .02$). Consequently, we assumed that children processed the numerosity characteristics of the stimuli, so the perceptual factors were not considered in the subsequent analyses.

A one-way ANOVA was conducted to assess the presence of statistically significant differences in response latency and accuracy across the three groups. The MD group presented numerically higher total reaction times (mean = 1242 ms, SD= 423 ms) than the MD+L group (mean = 1201 ms, SD = 347 ms) and control group (mean = 1191 ms, SD = 294 ms), but this difference did not reach statistical significance ($F [2, 160] = .318$, $MSE = 38479.47$, $p = .728$, $\eta^2 = .004$). Figure 2A shows the reaction times for each distance and each group in the nonsymbolic task.

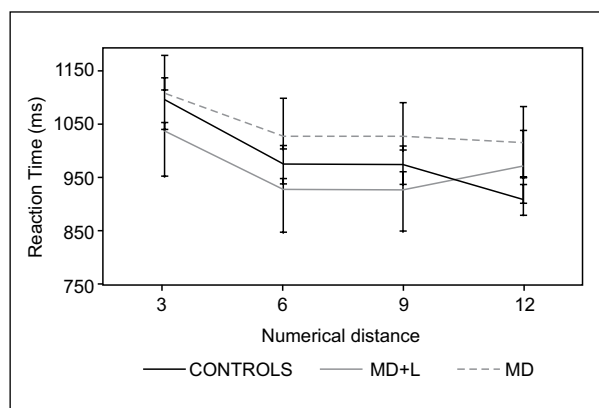


Figure 2A. Reaction time (mean and standard error) as a function of numerical distance in the MD, MD+L, and control groups in the nonsymbolic task.

With regard to total errors in the nonsymbolic task, one-way ANOVA revealed significant differences among the three groups ($F [2, 160] = 9.04$, $MSE = 0.133$, $p < .0001$, $\eta^2 = .113$). The MD group presented higher error

rates (26%) than the MD+L (25%) and control (18%) groups. Corrected Bonferroni comparisons between the MD and control groups revealed a significant effect of group ($t [141] = -4.15$, $p < .05$, Cohen's $d = .70$). Bonferroni *post hoc* comparisons revealed no significant difference between the MD+L and control groups or between the MD and MD+L groups. Figure 2B shows the error rates for each distance and each group in the nonsymbolic task.

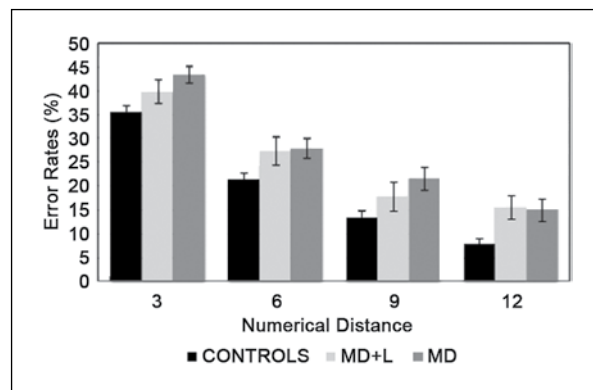


Figure 2B. Error rates (%) and standard errors as a function of numerical distance in the MD, MD+L, and control groups in the nonsymbolic task.

Distance effect

The distance effect was analyzed by repeated-measures ANOVAs calculated separately for each task for reaction times and error rates, with group as the between-subjects factor.

Distance effect for the symbolic task. A significant distance effect on reaction times was found in the symbolic task ($F [2, 141] = 70.93$, $MSE = 380986$, $p < .001$, $\eta^2 = .350$), but no significant interaction with group was observed ($F [2, 141] = 1.07$, $MSE = 5739$, $p = .346$, $\eta^2 = .016$). This indicates that the three groups presented comparable distance effects in the symbolic task. Reaction times for each distance and each group in the symbolic task are shown in Figure 1A.

A significant distance effect was found when considering error rates ($F [1, 141] = 29.26$, $MSE = .428$, $p < .001$, $\eta^2 = .180$), but no significant interaction with group was observed ($F [2, 142] = 1.28$, $MSE = .020$, $p = .281$, $\eta^2 = .019$). This indicates that the distance effect for errors in the symbolic task was comparable across groups. Bonferroni *post hoc* comparisons did not reveal any significant differences among groups with regard to either response latencies or accuracy. Error rates in the symbolic task are shown in Figure 1B for each distance and each group.

Distance effect in the nonsymbolic task. Similar to the results found for the symbolic task, a significant main effect of distance on reaction time was found in the nonsymbolic task ($F [1, 141] = 45.96$, $MSE = 658316$, $p < .001$, $\eta^2 = .245$). This indicates a robust distance effect on response latencies (Figure 2A). No significant interaction between distance and group was

found ($F [2, 141] = 1.87, MSE = 26765, p = .158, \eta^2 = .026$), indicating a similar distance effect in all three groups.

With regard to error rates, a significant distance effect was found ($F [2, 141] = 309.47, MSE = 4.26, p < .001, \eta^2 = .685$), but no significant interaction with group was observed ($F [2, 141] = .733, MSE = .010, p = .482, \eta^2 = .010$). These results indicate that the distance effect was comparable in all three groups. Error rates for each distance and each group in the nonsymbolic task are shown in Figure 2B.

Conservative selection criterion

To evaluate the generalizability of the results to more severe forms of MD, analyses were repeated in subsamples selected according to a much stricter criterion for MD (i.e., 10th percentile). Considering the stricter criterion, the MD group comprised seven children, and the MD+L group comprised 12 children. The groups were matched with regard to intelligence and age. Generally, the results replicated those obtained for the entire sample. The analysis that considered the strict group classification criterion revealed no significant differences among groups ($p > .05$) with regard to simple reaction times. The mean reaction times were 436.15 ms (SD = 81.15) in the control group, 448.29 ms (SD = 86.42) in the MD group, and 365.42 ms (SD = 116.82) in the MD+L group.

No differences in reaction time were found in either the nonsymbolic task ($F [2, 48] = 1.499, MSE = 112736, p = .234, \eta^2 = .061$) or symbolic task ($F [2, 48] = 2.30, MSE = 122231, p = .112, \eta^2 = .095$). However, specific comparisons among groups revealed significant differences between the MD group (mean = 1078 ms, SD = 157 ms) and MD+L group (mean = 840 ms, SD = 142 ms) in reaction times in the symbolic task ($t_{14} = 3.11, p = .008, \text{Cohen's } d = 1.58$).

With regard to error rates, significant differences among the three groups were observed in the nonsymbolic task ($F [2, 48] = 4.35, MSE = .047, p = .019, \eta^2 = .159$). Corrected Bonferroni t -tests revealed significant differences between the MD group (mean = 31%) and control group (mean = 20%, $t(36) = -2.96, p = .005, \text{Cohen's } d = 1.29$). No significant differences were found when comparing the MD+L group (mean = 27%) with the other two groups ($p > .05$). Importantly, no significant differences in error rates were found in the symbolic task ($F [2, 48] = 0.527, MSE = 0.025, p = .594, \eta^2 = .023$).

A significant distance effect was found on reaction time in the nonsymbolic task ($F [1, 46] = 22.24, MSE = 246849, p < .001, \eta^2 = .326$), but no significant interaction between distance and group was observed ($F [2, 46] = .188, MSE = 2090, p = .829, \eta^2 = .008$), indicating a similar distance effect in all three groups. Similarly, a significant distance effect was found on reaction time in the symbolic task ($F [1, 46] = 18.65, MSE = 100069, p = .0001, \eta^2 = .308$), but no significant interaction between distance and group was observed

($F [2, 46] = 3.03, MSE = 16294, p = .059, \eta^2 = .126$), indicating a similar distance effect in all three groups.

Significant distance effects were found on error rates in the nonsymbolic task ($F [1, 46] = 92.54, MSE = 1.11, p < .001, \eta^2 = .668$) and symbolic task ($F [1, 46] = 12.06, MSE = .111, p = .001, \eta^2 = .214$). However, no significant distance \times group interaction was observed in the nonsymbolic task ($F [2, 46] = 2.93, MSE = .035, p = .063, \eta^2 = .113$) or symbolic task ($F [2, 46] = .001, MSE = .000128, p = .999, \eta^2 = .000$).

Predictive effects of symbolic and nonsymbolic numerical representations on arithmetic achievement

To investigate the role of nonsymbolic and symbolic tasks in arithmetic achievement, a multiple linear regression was calculated with performance in the arithmetic achievement task (TDE) as the dependent variable. Age, gender, intelligence, school grade, distance effect, mean total reaction time, and the arcsine of the total error rate were the independent variables in both the symbolic and nonsymbolic tasks. The intervening variables (i.e., age, gender, intelligence, and school grade) were inserted in the model using the enter method, and the experimental variables were inserted in the model using the stepwise method.

In the model that considered three groups together, the arcsine of the total error rate in the nonsymbolic task was the only experimental variable that remained in the model ($B = -9.15, SE = 3.91, t [127] = -0.151, p = .02$). The other experimental variables were excluded from the model (Table 3).

Linear multiple regressions were conducted for each group separately. In the control group, all experimental variables were excluded from the model. The only experimental variable that showed a marginally significant result was the distance effect for error rates in the nonsymbolic task ($B = .135, t [127] = 1.79, p = .07$; Table 3). In the MD group, the only experimental variable that remained in the model was the distance effect for error rates in the symbolic task (Table 3). In the MD+L group, the experimental variable that remained in the model was the arcsine of total errors in the symbolic task (Table 3).

Discussion

In the present study the relative impact of symbolic and nonsymbolic magnitude processing on the development of mathematics learning disabilities was investigated. Children with isolated or associated math learning difficulties and normal mathematics achievers matched with regard to general intelligence and processing speed were selected from a large population and evaluated for symbolic and nonsymbolic magnitude processing. No significant difference was found between groups in response latencies in either the symbolic or nonsymbolic task. In contrast, there were more errors in the MD group than in the control group, for both the symbolic task and

Table 3. Linear multiple regression (enter and stepwise methods) for the Control, MD, and MD+L groups separately, with arithmetic achievement as the dependent variable

Group	Measure	B	SE	Beta	T	p
Control	Age	.399	.926	.087	.431	.71
	Gender	.724	1.068	.053	.678	.5
	School grade	3.153	.895	.71	3.52	.001
	Intelligence	.871	1.026	.066	.849	.399
	Distance effect nonsymbolic	5.83	3.257	.135	1.791	.078
MD	Age	.302	.356	.083	.848	.402
	Gender	.032	.474	.003	.068	.946
	School grade	3.25	.352	.868	9.22	0
	Intelligence	.203	.399	.028	.598	.554
	Distance effect symbolic	5.25	2.43	.138	2.16	.03
MD+L	Age	1.31	1.1	.493	1.189	.269
	Gender	2.67	1.307	.243	2.046	.075
	School grade	4.51	1.28	1.47	3.526	.008
	Intelligence	.325	.793	.053	.41	.692
	Error symbolic	-11.7	4.88	-.382	2.39	.038

Control, typical achievers; MD, mathematics difficulties; MD+L, mathematics and language difficulties.

especially the nonsymbolic task. Although the same trend was observed in the comparison between the MD+L and control groups, the statistical comparison did not reach significance. Moreover, no differences between the MD and MD+L groups were found in basic magnitude processing. A closer inspection of the distance effect in the symbolic and nonsymbolic tasks revealed that children in all of tested groups presented a distance effect on both reaction times and error rates in both symbolic and nonsymbolic tasks. Finally, a series of regression models revealed that both symbolic and nonsymbolic magnitudes appeared to specifically contribute to differences between children with and without MD. These results are discussed in more detail below.

With regard to the symbolic task, response latencies did not differ across groups. In response accuracy, significant differences were revealed by the one-way ANOVA when comparing the three groups, but these differences could not be replicated in the *post hoc* comparisons. Generally, one could argue that a failure to find differences in response latencies and accuracy between groups is attributable to a lack of sensitivity of our symbolic magnitude task. However, several studies have reported significant differences in reaction times in the symbolic task (De Smedt & Gilmore, 2011; Iuculano et al., 2008; Landerl, Bevan, & Butterworth, 2004; Landerl & Kölle, 2009; Rousselle & Noël, 2007). However, other studies that used the same set of stimuli and a comparable set of instructions as the present study failed to find significant differences in reaction times (Landerl, Fussenegger, Moll, & Willburger, 2009; Mussolin et al., 2010). With regard to differences between groups in response accuracy in the symbolic task, results have also been quite mixed. Two studies that used one-digit numbers (Iuculano et al., 2008; Rousselle & Noel, 2007) and one study that used two-digit numbers found higher error rates in the MD group (Landerl & Kollé, 2009). Two other studies that used one-digit numbers failed to find such differences (De Smedt & Gilmore, 2011; Mussolin et al., 2010).

The symbolic task used in the present study is comparable to the one used in most of the studies mentioned above. Failure to find significant differences in the present study may be attributable to task difficulty alone. Moreover, the significant distance effect found on response latencies and accuracy in all of the groups indicates that the MD and MD+L groups were able to access magnitudes in the symbolic task at speed and accuracy levels comparable to the control group. Again, with regard to response latencies, one could argue that the lack of differences among groups is attributable to very low task difficulty. Our results again appear to refute this hypothesis because error rates in the symbolic task were substantial in all of the groups, ranging from 11% in the control group to 16% and 18% in the MD+L and MD groups, respectively. This indicates that children with and without MD in our study systematically activated magnitude representation in the symbolic task to a comparable extent.

A significant distance effect was found on both reaction times and error rates in all groups, regardless of their mathematics achievement. This indicates that all children correctly understood the task instructions and were able to access number magnitudes independently of its format. Moreover, nonsignificant interactions between distance and group indicated that the distance effect was equally pronounced in the three groups. With regard to response latencies and accuracy, conflicting distance effect results have been published for MD and control groups. Rousselle and Noël (2007) found that children with MD had a smaller distance effect than control subjects in the symbolic task. Mussolin et al. (2010) found that children with MD had a larger distance effect than control subjects. Discrepancies among studies increase further when evaluating the study by Landerl et al. (2009). These authors showed that although MD children were specifically slower than typically developing children they did not show a deviant or atypically large distance effect. Therefore, definitive conclusions from the literature about a

difference between MD children and control subjects with regard to the distance effect are still elusive. To some extent, absence of significant differences in the symbolic task between the MD and control groups and inconsistencies in the literature do not fit the predictions derived from the disconnection hypothesis. According to this view, robust differences between MD children and control subjects should be observed foremost in the symbolic task. However, these findings are compatible with the core deficit hypothesis that predicts stronger deficits in the nonsymbolic task.

In the nonsymbolic task, differences in response latencies among groups also did not reach significance. These results are consistent with most studies that compared reaction times between children with mathematics learning difficulties and controls in the nonsymbolic task (De Smedt & Gilmore, 2011; Iuculano et al., 2008; Landerl & Kölle, 2009; Piazza et al., 2010; Rousselle & Noël, 2007; but see Landerl et al., 2009). Importantly, some studies that failed to report significant differences in response latencies in the nonsymbolic task used a set of numerical stimuli in the range of 1 to 9 (Iuculano et al., 2008; Mussolin et al., 2010; De Smedt & Gilmore, 2011). At least partially, failure to find significant differences among groups in these studies may be attributable to the occurrence of more perceptual phenomena such as subitizing (Mandler & Shebo, 1982; Piazza et al., 2010) that may have masked the occurrence of a distance effect in the numerical range of 1 to 4. However, in the present study the numerical range used in the nonsymbolic task was far beyond the subitizing range. For this reason, negative results obtained in the present study may not be attributable to confounds by the more perceptual aspects of magnitude processing. Interestingly, an inspection of the studies that used stimuli within the same magnitude range as the present study reveals contradictory evidence. Landerl et al. (2009) found differences in reaction times between children with and without MD, but Rousselle and Noël (2007), Landerl and Kölle (2009), and Piazza et al. (2010) failed to report such differences. These previous studies and the present study used comparable stimulus sets, task instructions, and selection criteria for sample assignment. Therefore, explaining why only Landerl et al. (2009) were able to find significantly slower response latencies in MD children is difficult. Future studies should use more sensitive methods (e.g., the Weber fraction) to investigate possible deficits in magnitude processing in MD (Piazza et al., 2010; Mazzocco, Feigenson, & Halberda, 2011).

In contrast to the nonsignificant results obtained in the symbolic task, significantly higher error rates were observed in the nonsymbolic task in the MD group than in the control group. These findings indicate a deficit in nonsymbolic representation in the MD group compared with the control group (Landerl et al., 2009; Mazzocco et al., 2011; Mussolin et al., 2010; Piazza et al., 2010). Although differences between the MD group and control group in the nonsymbolic task can be

conciliated with the disconnection hypothesis, they also better fit the predictions of the core deficit hypothesis because deficits observed in the symbolic task failed to reach significance.

To investigate the stability of our results with regard to symbolic and nonsymbolic magnitude processing across more or less conservative criteria for the classification of mathematics learning difficulties, we selected a subsample of children who presented a more severe form of MD. Generally, these comparisons confirmed the pattern of results found with the more liberal classification criterion (i.e., 25th percentile). The MD group presented higher error rates in the nonsymbolic task compared with the control group. A further inspection of the effect size found in this comparison revealed more robust differences between the control group and the subsample with severe MD (Cohen's $d = 1.29$) than between the control group and the entire MD sample (Cohen's $d = 0.70$). Interestingly, Mazzocco et al. (2011) found the same effect size when performing the same comparison between controls and MD children with regard to the acuity of the approximate number system using the Weber fraction. These results suggest that differences in accuracy in nonsymbolic magnitude processing reflect a deficit in core magnitude representation (Piazza et al., 2010). A possible exception to the pattern of effects found when using the more conservative classification was the comparison between the MD+L and control groups. However, further analyses of these data using an inverted efficiency score (Iuculano et al., 2008) did not confirm the existence of any differences. Apparently, the MD+L group tended to respond more quickly than the MD group in the symbolic task, but also committed more errors. These differences between groups were far from significant in the nonsymbolic task. This may indicate a particular deficit in the MD+L group when manipulating symbolic stimuli. Together our results for response latencies, accuracy, and the distance effect in symbolic and nonsymbolic tasks are compatible with a deficit in the approximate number system in MD children, independent of the classification criterion used to classify them. These analyses corroborate the existence of a statistically significant deficit in nonsymbolic magnitude processing, whereas differences between the MD and control groups in symbolic magnitude comparison were less pronounced. To what extent is performance in symbolic and nonsymbolic tasks related to more general mathematics achievement?

Regression models showed that nonsymbolic magnitude processing explained individual differences in school mathematics better than measures of symbolic processing when considering all of the children together. Similar but less robust results were obtained when considering the regression model for the control group only. No predictor related to number processing reached significance in the control group. However, error rates in the nonsymbolic task were the only variable specifically related to magnitude processing

that approached significance. Importantly, discrepant results were obtained by Holloway and Ansari (2009). After controlling for general abilities, these authors found that the distance effect that arose from reaction times in the symbolic task was the best variable to explain mathematical achievement in a group of typically achieving 6- to 8-year-old children. However, the nonsymbolic task in that study included magnitudes in the range of 1 to 9, which can be discriminated easier and consequently activate the approximate number system less than higher numerical intervals. This may have reduced the demands on the approximate number system and reduced the sensitivity of the nonsymbolic task to individual differences in mathematics achievement in the study by Holloway and Ansari (2009). Together our results suggest that nonsymbolic magnitude processing can explain mathematics achievement independently of the actual level of performance presented by children. Moreover, the results suggest that nonsymbolic number magnitude is a more specific predictor of mathematics achievement than symbolic magnitude.

Analysis of the regression models for each group separately revealed another pattern of results. Symbolic processing was the only variable specifically related to number processing that explained individual differences in low mathematics achievers. Are these results consistent with the disconnection hypothesis? Our results indicate that symbolic magnitude is more important for explaining mathematics learning in children with particularly low mathematics achievement. However, the MD group should then present more pronounced deficits in the symbolic task, and these deficits should explain mathematics achievement better than performance in the nonsymbolic task. The MD group was not significantly impaired in symbolic processing. Although ANOVA suggested the presence of significant differences between groups and the absolute difference in errors between the control and MD groups was approximately 7%, pairwise comparisons did not reveal a significant difference. Furthermore, performance in the symbolic task explained mathematics performance only for the MD group and not for the control group. Considering these results, our data cannot be interpreted as consistent with an access deficit. However, the present results may be consistent with Halberda et al. (2008), in which deficits in nonsymbolic representation may interfere with symbolic mathematics learning. We found that the MD and MD+L groups committed more errors in the symbolic task, but this difference was not sufficiently large to reach statistical significance. This suggests that symbolic representation is not significantly impaired in MD.

Considering that symbolic representation is slightly preserved in MD, this representation is used by MD children as a compensatory strategy to deal with the pronounced impairment observed in nonsymbolic representation. Considering that nonsymbolic representation is an important predictor of mathematics achievement demonstrated in the global regression

model, symbolic representation may be used as a kind of “compensatory number line” utilized to solve mathematical problems. For children with preserved nonsymbolic processing (i.e., the control group), this representation predicts arithmetic performance. This result favors the hypothesis that lower acuity of the approximate number system is a core deficit of MD (Dehaene & Cohen, 1997; Halberda et al., 2008). However, the disconnection hypothesis also predicts more pronounced deficits in symbolic tasks than in nonsymbolic tasks, which was not observed in our study.

In conclusion, children with MD appeared to be able to access magnitude representations via both symbolic and nonsymbolic tasks, but they were less accurate than typically achieving children in nonsymbolic processing than in symbolic processing. Generally, nonsymbolic processing also appears to predict mathematics achievement more accurately than symbolic processing. However, when considering low-achieving children only, symbolic processing predicted mathematics achievement more accurately than nonsymbolic processing. For these reasons, basic magnitude representations of the approximate number system appear to be a more general predictor of mathematics achievement, whereas symbolic magnitude representation appears to compensate for deficits in the approximate number system.

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Competing Interests

The authors declare that they do not have any financial or nonfinancial competing interests.

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