Study on Minimum Shear Reinforcement of Reinforced Concrete Flat Slabs

Estudo sobre Armadura Mínima de Cisalhamento de Lajes-Cogumelo de Concreto Armado

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Abstract

In a preliminary study that aims to define the minimum shear reinforcement of flat slabs that, leading to a punching shear failure surface that crosses that reinforcement, can avoid a sudden failure, nine slabs with 1800 mm x 1800 mm x 130 mm dimensions were tested. Their main variables were the number of shear reinforcement layers of the stud type, the cross-section area of that reinforcement in each circumferential layer and the diameter of the studs. In the attempt to define the minimum punching reinforcement of the slabs, a parameter κ, equal to the total force in the transverse reinforcement inside a truncated cone bounded by the shear crack divided by the punching strength of a similar slab without shear reinforcement, was used. The values of κ of the slabs varied between 0.27 and 1.03 and the punching failure surface crossed the shear reinforcement when κ was smaller than around 0.70. The value of κ equal to 0.6 was defined as close to the minimum for slabs with effective depth of 90 mm and flexure reinforcement ratio of 1.38%. In comparison with the slab with no shear reinforcement, slabs having κ of that order had a punching strength and rotation increase of about 10% and 30%, respectively.

Keywords: flat slabs, minimum shear reinforcement, reinforced concrete

Resumo

Dando início a estudo que visa definir a armadura transversal mínima de lajes-cogumelo de concreto armado que, levando a uma superfície de ruptura por punção atravessando essa armadura, pode evitar uma ruptura brusca, foram testadas nove lajes de dimensões 1800 mm x 1800 mm x 130 mm. Suas principais variáveis foram o número de camadas de armadura de cisalhamento do tipo *studs*, a área dessa armadura em cada camada circunferencial e o diâmetro dos *studs*. Na tentativa da definição da armadura mínima de punção das lajes, utilizou-se o parâmetro κ, igual à força total da armadura transversal situada dentro de tronco de cone de ruptura delimitado pela fissura de cisalhamento dividida pela resistência à punção de laje semelhante sem armadura de cisalhamento. Os valores de κ das lajes variaram de 0,27 a 1,03 e ocorreu superfície de ruptura cruzando as camadas de *studs* quando κ foi menor que cerca de 0,70. Definiu-se o valor de κ igual a 0,6 como próximo daquele mínimo para lajes com altura efetiva igual a 90 mm e taxa de armadura de flexão de 1,38%. Para as lajes com valor de κ dessa ordem, em relação à laje sem armadura transversal, os acréscimos de resistência à punção e de rotação foram, respectivamente, cerca de 10% e 30%.

Palavras-chave: lajes-cogumelo, armadura transversal mínima, concreto armado

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1. Introduction

The use of flat slabs in concrete structures has become commonplace in recent years, due to the reduction in execution time and costs as well as the flexibility of use of constructed spaces it provides.

In the region of slab-column connections of these slabs, there are stresses that may cause failure by punching with load below that of flexural failure. Shear reinforcement is generally used in this region aiming at increasing the strength and the ductility of the slabs and one of the types of this reinforcement is the one consisting of studs. The objective of the experimental study described here was to provide an initial contribution towards the establishing of the minimum transverse reinforcement for reinforced concrete flat slabs, of the stud kind, that, leading to a punching failure surface that crosses this reinforcement, prevents from a sudden failure. The main variables considered in the study were the number of shear reinforcement layers, the cross-section area of this reinforcement in each layer and the diameter of the studs.

In the attempt to define that minimum reinforcement, it was used the transverse reinforcement index $\kappa = (A_{sw}f_{yw})_{ef.}/P_{uLref}$, where $(A_{sw}f_{yw})_{ef}$ is the total force in the transverse reinforcement inside an assumed failure truncated cone and $P_{uLref.}$ is the failure load of the reference slab, without shear reinforcement. Figure 1 shows the shear crack which bounds the failure truncated cone [1]. Although P_{uLref} may be calculated using formula of any code of practice adopted, in the presented analysis it was chosen to consider for that load the experimental value obtained.

2. Experimental programme

2.1. Characteristics of the slabs and test method

The main variables of the 9 slabs tested were:
- Number of shear reinforcement layers (2 to 4)
- Cross-section area of shear reinforcement in each layer, that defined the number of reinforcement lines (4 to 8)
- Diameter of the studs (4.2 mm and 5.0 mm)

Figure 1 – Representation of the truncated cone bounded by the shear crack

$$\text{Radius of the truncated cone} = \frac{\text{side of the column}}{2} + (d-x)\cot\theta$$

$$\theta \sim 25°$$

The slabs were split into two groups. Group 1 had the number of lines (8) and the transverse reinforcement area per layer (157 mm^2) constant, and the number of transverse reinforcement layers was varied (2, 3, or 4). These slabs, together with those that had internal punching failure, tested by other authors, led to the definition of the numbers of layers and lines of the transverse reinforcement and also of the diameter of this reinforcement of the slabs in Group 2. All slabs in this group had 3 transverse reinforcement layers, varying the number of lines (4, 5, or 6) and the reinforcement area per layer (55.4 mm^2, 69.3 mm^2 or 83.1 mm^2), that was always smaller than that adopted in Group 1 slabs as a punching failure surface crossing this reinforcement was intended. Figure 2 and Table 1 show the characteristics of the tested slabs.

The load was applied upward to the bottom from a jack located at the centre of the slab and a metallic plate with a 150 mm diameter circular cross-section simulated the column. The reaction to loading was made by a set of metallic beams anchored in the strong floor and supported on eight steel plates, positioned between the metallic beams and the slab under study, 825 mm away from the centre of the slab (Figure 3).

The measurement of the displacements on the upper face of the slab was made with dial gauges and displacement transducers (linear potentiometers). Measuring points were distributed on a straight line over the slabs in x and y directions (Figure 4). For the measurement of the strains in the flexural and shear reinforcement resistance strain gauges were used.

The measuring of the applied load was made by a load cell connected to an automatic data acquisition system. Load values were increased in steps of around 10 kN, 20 kN or 25 kN, until the failure load, defined as being the highest load recorded during the test.

2.2. Materials

The CA-50 steel bars used in the flexural reinforcement had 6.3 mm and 12.5 mm diameters. For shear reinforcement, CA-60 steel bars with 4.2 mm and 5.0 mm diameters were used. In order to obtain their mechanical properties, four samples of each type of bar were tested. The average values of yield stress and tensile strength obtained were, respectively, 708 MPa and 725 MPa (4.2 mm), 624 MPa and 757 MPa (5.0 mm), 594 MPa and 714 MPa (6.3 mm), 555 MPa and 673 MPa (12.5 mm).

The concrete mix design aimed a 28 days compressive strength of 35 MPa. During each concrete casting cylindrical specimens were moulded for the compression, tensile and modulus of elasticity tests. The mean values obtained in the specimens related to the first and second concreting were, respectively: 38.0 and 38.5 MPa for compression strength, 3.28 and 3.56 MPa for splitting tensile strength, and 21.6 and 25.3 GPa for modulus of elasticity.

3. Results and discussion

3.1. Failure load and mode and shear reinforcement index

The failure load and mode of the tested slabs, together with the shear reinforcement index, are shown in Table 2. All slabs had a punching failure. From the slabs with transverse reinforcement, those of Group 1 had a punching failure surface outside the shear reinforcement region (Figure 5) and those of Group 2 had punching failure surface crossing this region (Figure 6). Tables 1 and
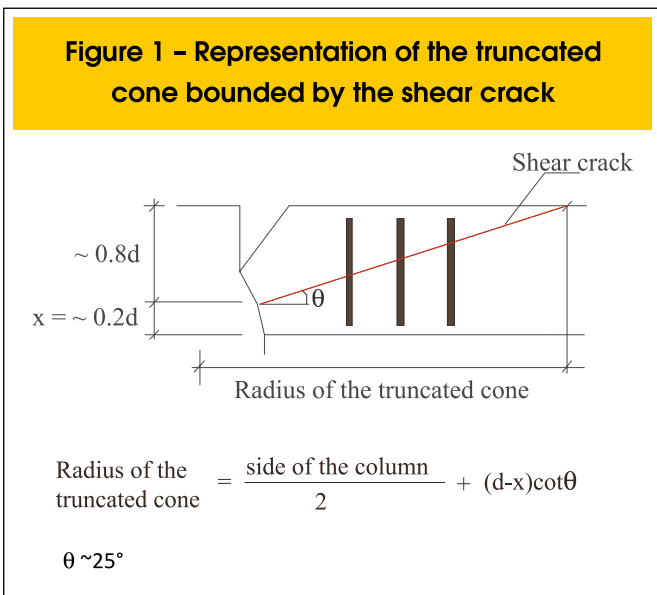
2 show that not all reinforcement layers are inside the assumed truncated cone.

In Group 1, the failure load increments in relation to reference slab L1-A were between 12% and 25% and in Group 2, these increments ranged from 5% to 10%. Slab L1-A was considered as a reference in the two groups as its strength was closer to the results from similar
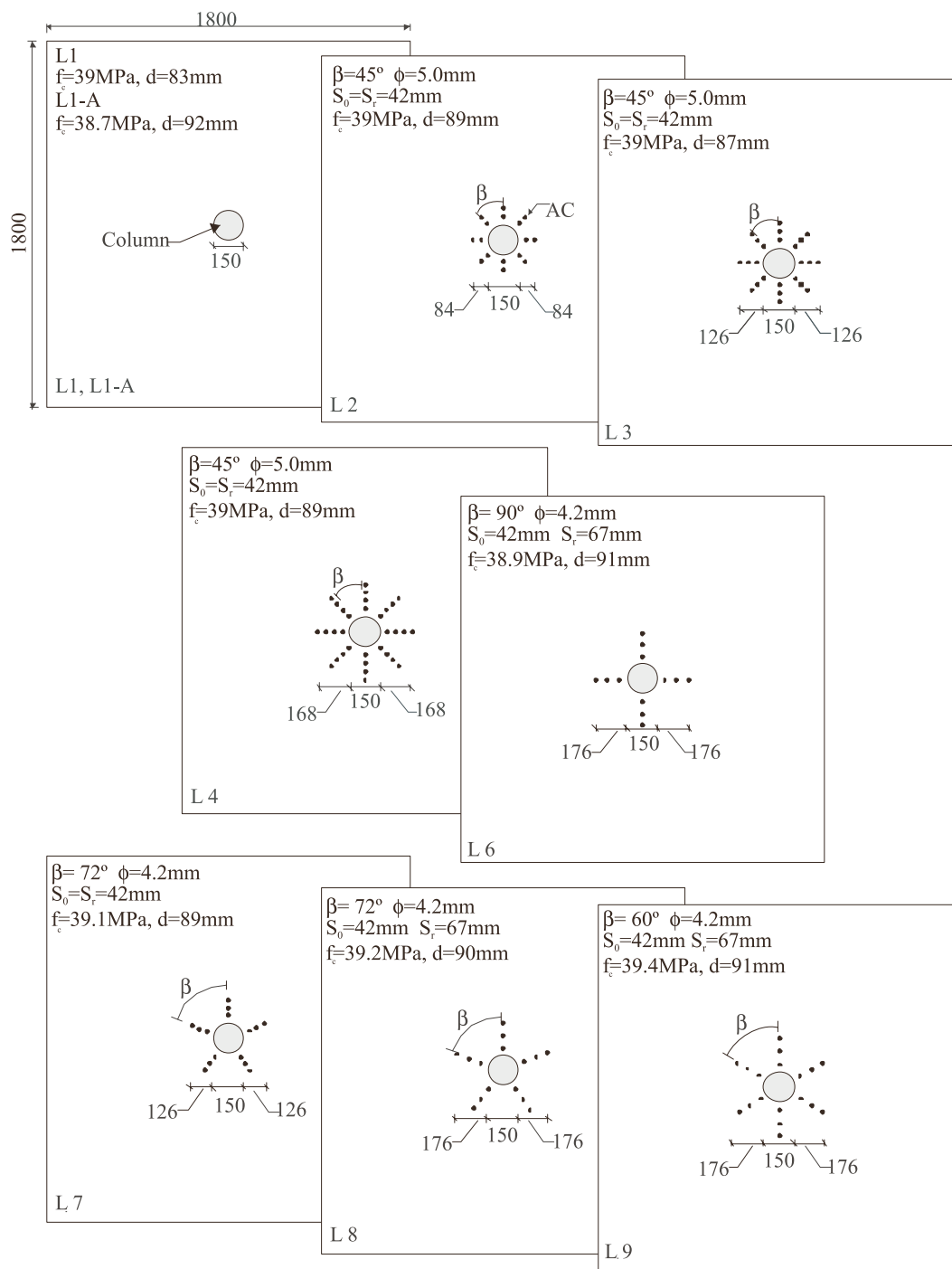

Figure 2 – Characteristics of the tested slabs

	Slabs	f_c (MPa)	d (±2mm)	Shear reinforcement						
				lines number	layers number	$A_{sw}f_{yw}$ (kN)	S_r (mm)	ϕ (mm)	f_{yw} (MPa)	angle between lines
Group 1	L1	39.0	83	-	-	-	-	-	-	-
	L2	39.0	89	8	2	98	42	5.0	624	45°
	L3	39.0	87	8	3	98	42	5.0	624	45°
	L4	39.0	89	8	4	98	42	5.0	624	45°
Group 2	L1-A	38.7	92	-	-	-	-	-	-	-
	L6	38.9	91	4	3	39	67	4.2	708	90°
	L7	39.1	89	5	3	49	42	4.2	708	72°
	L8	39.2	90	5	3	49	67	4.2	708	72°
	L9	39.4	91	6	3	59	67	4.2	708	60°

Table 1 – Characteristics of the tested slabs

S_0 – distance between the innermoststud and the column face = 42 mm
S_r – studs radial spacing
ρ – flexural reinforcement ratio = 1.38%
f_c – concrete compressive strength at the slab testing day
d – slab effective depth
h – slab depth = 130 mm
ϕ – studs diameter
f_{yw} – shear reinforcement yielding stress
$A_{sw}f_{yw}$ – force in one shear reinforcement layer

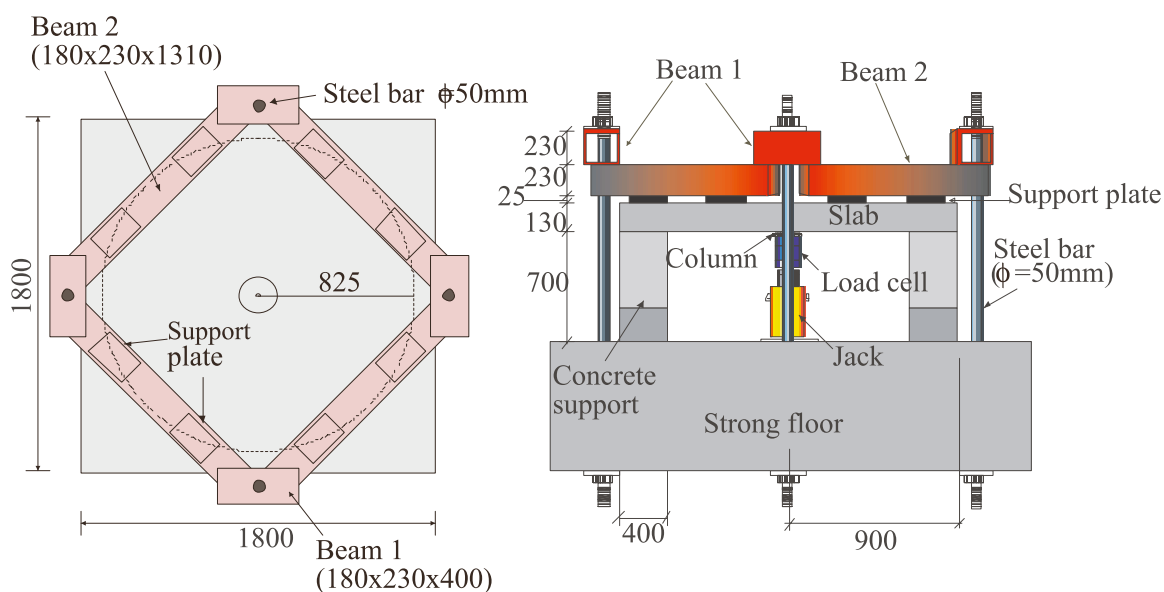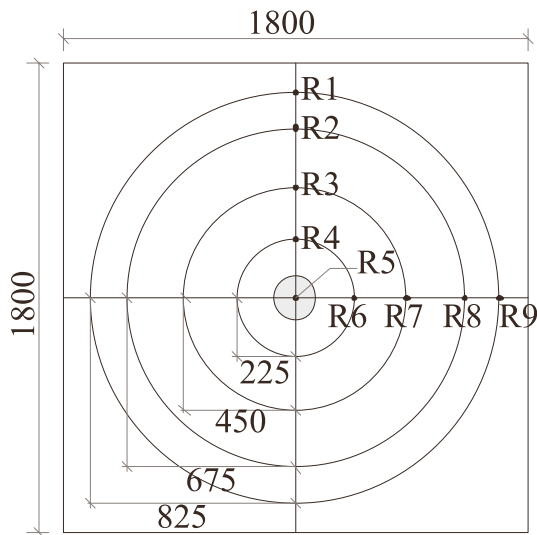**Figure 3 – Test arrangement (dimensions in mm)**

Figure 4 – Position of displacement gauges on the slabs (dimensions in mm)

Figure 7 shows the values of P_u/P_{Lref} as a function of the κ index for the slabs of this and other studies that had the same kind of reinforcement ([2], [3], [4] and [5]), where $P_{Lref.}$ is the failure load of the reference slab in each study. Slabs L2, L10 and L11 tested by Gomes [5] had punching failure surface crossing the shear reinforcement region. Slab L3 had two failure surfaces, one crossing the shear reinforcement region e other immediately outside that region. Slab L9 showed signs of flexural failure with yielding of almost all bending reinforcement, great rotations and some concrete crushing in the compressed region. Figure 7 suggests that κ values until around 0.7 lead to punching failure surface crossing the transverse reinforcement region, while higher κ values cause punching failure surface outside this region.

3.2. Vertical displacements and rotations

For loading levels over 100 kN, the vertical displacements varied almost linearly in relation to the distance to the centre of the slab and the highest values were found in the central region, where the load was applied. As a result of that, the rotation of slab ψ, in radians, was obtained dividing the vertical displacement at the centre of the slab by the distance from this point to the circumference that passes through the centres of the support plates, with 825 mm radius.

Figure 8 shows the graphs of vertical displacement for two slabs (one without and one with transverse reinforcement) as a function of the distance to the centre, for different load values, while Figure 9 presents the central vertical displacements of the slabs as a function of the load. In Figure 8 one can see that the displacements are practically equal in directions x and y, fact that occurred

slabs in previous works ([2] and [3]), and its effective depth was more similar to the one of the other slabs tested in this work.

Table 2 – Failure load and mode and shear reinforcement index of the tested slabs

| | Slabs | f_c (MPa) | d (±2mm) | Shear reinforcement | | | P_u (kN) | κ | Failure mode |
				n° of lines	n°of lines inside the assumed truncated cone	$(A_{sw}f_{yw})_{ef}$ (kN)			
Group 1	L1	39.0	83	-	-	-	203	-	P
	L2	39.0	89	8	2	196	321	0.69	E
	L3	39.0	87	8	3	294	325	1.03	E
	L4	39.0	89	8	3	294	357	1.03	E
Group 2	L1-A	38.7	92	-	-	-	286	-	P
	L6	38.9	91	4	2	78	300	0.27	I
	L7	39.1	89	5	3	147	303	0.51	I
	L8	39.2	90	5	2	98	309	0.34	I
	L9	39.4	91	6	2	118	315	0.41	I

$(A_{sw}f_{yw})_{ef}$ – force in the shear reinforcement inside theassumed failure truncated cone
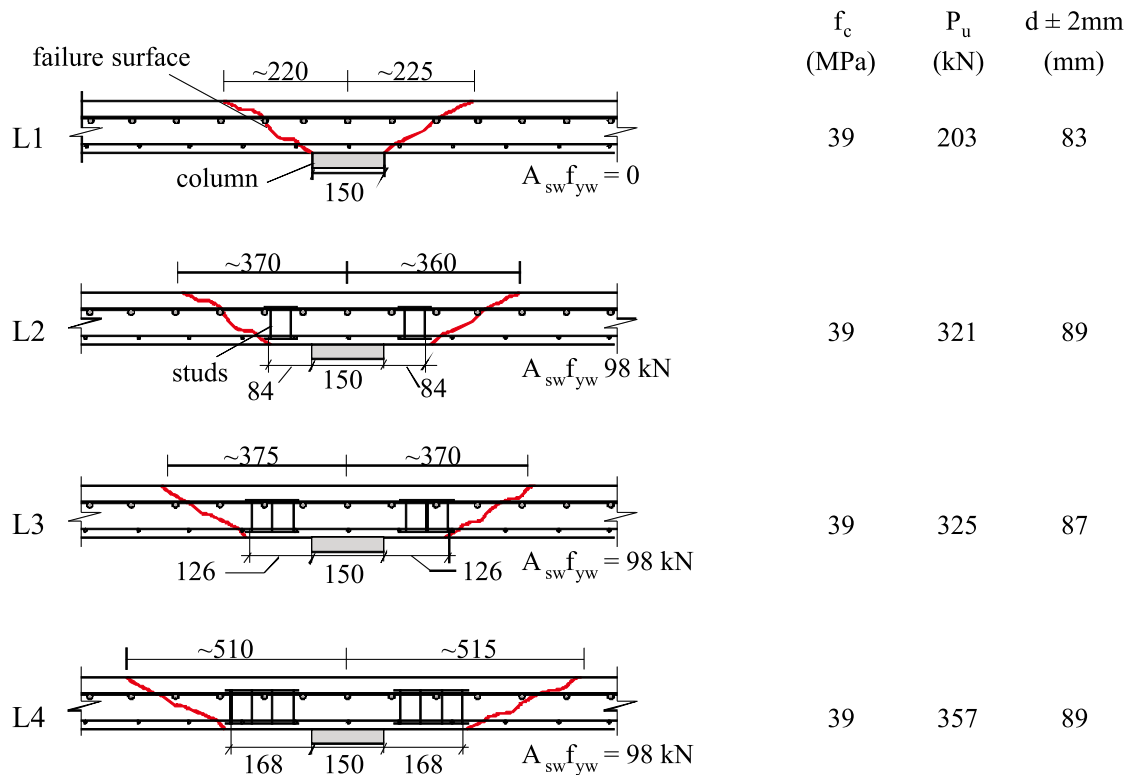P_u – Failure load
κ – Shear reinforcement index, equal to $(A_{sw}f_{yw})_{ef}/P_{uL1-A}$
P – Punching failure
E – Punching failure, with failure surface outside the layers of studs
I – Punching failure, with failure surface crossing the layers of studs

Figure 5 – Failure surface of Group 1 slabs (dimensions in mm)

	f_c (MPa)	P_u (kN)	$d \pm 2mm$ (mm)
L1	39	203	83
L2	39	321	89
L3	39	325	87
L4	39	357	89

in all slabs, and in Figure 9 it is noticed the behaviour differential between slabs of groups 1 and 2.

Figure 10 presents the values for ψ_u/ψ_{Lref} as a function of κ for the slabs tested in this and in other works that had the same type of reinforcement, where ψ_{Lref} is the rotation of the reference slab of each work carried out. In the case of the slabs in Group 2 of this study, ψ_u and ψ_{Lref} refer to the failure load, while in the other cases they refer to a load near to the ultimate one (from around 85% to 95% of the failure load).

The highest values of these relations for the slabs tested by Gomes [5] suggest that they depend on the effective depth of the slab that, in these slabs, was around 160 mm.

3.3. Strains

a) Flexural reinforcement

The highest strain values in flexural reinforcement were found in the central region of the slabs. Flexural reinforcement of Group 1 slabs reached yielding with a load near 200 kN and, in the slabs with transverse reinforcement, the region where there was yielding of the flexural reinforcement included that outside the shear reinforcement (Figure 11, slab L2). In Group 2 slabs, the yielding of the longitudinal reinforcement did not occur in all the slabs and, in those where there was yielding, this took place in the loading region and for a load around 250 kN (Figure 12, slab L8).

b) Shear reinforcement

The shear reinforcement strains of the slabs that had punching failure surface crossing the layers of studs (L6, L7, L8 and L9) were higher than those of the slabs with punching failure surface outside these layers (L2, L3 and L4), reaching the yielding strain. In the shear reinforcement strains graph of slab L9 (Figure 13) it can be seen that they were greater for the studs located nearer the loading region, which was also found in the remaining slabs.

3.4. Cracks

Radial cracks were the first ones to be visually noted, for a load between 45 kN and 55 kN. The first radial crack appeared with 19% of the failure load for the L1-A, with around 14% of the failure load for the slabs with shear reinforcement in Group 1, and with around 16% of the failure load for the slabs of Group 2 with studs. At the time the first radial cracks appeared, the obtained strains at the points of flexural reinforcement monitored were below 0.35 mm/m in the slabs with punching failure surface outside the region of shear reinforcement and below 0.20 mm/m in those with punching failure surface inside this region.

For a load between 75 kN and 100 kN, the circumferential cracks appeared in all slabs and, from this load level on, the elements of the shear reinforcement started to show bigger strains. The circumferential cracks appeared under 26% of the failure load for slab L1-A, under around 22% of the failure load for the slabs with shear reinforcement of Group 1, and under 33% to 40% of the failure loads for the slabs of Group 2 with punching reinforcement.

3.5. Evaluation of the shear reinforcement index for slabs with punching failure crossing the layers of studs.

Several variables may affect the failure load and the rotation of reinforced concrete flat slabs such as the flexural reinforcement ratio, effective depth, compressive and tensile strength of the concrete, perimeter of the loaded area, existence of holes, etc.

Table 3 shows the relations between the rotations (ψ_u/ψ_{L1-A}) and loads (P_u/P_{L1-A}) of each slab tested as a function of the shear reinforcement index κ. It is possible to see that there is a tendency of the rotation and the load decreasing when the κ index decreases. The highest increases in rotation and in failure load occurred in the slabs L4 and L3, with the highest shear reinforcement index (κ = 1.03). The smallest increases in rotation and failure load occurred in slab L6, with the smallest shear reinforcement index (κ = 0.27).

The slabs with punching failure surface external to the shear reinforcement layers (Group 1) showed, in relation to slab L1-A, rotation increments of at least 46% and failure load increments of at least 12%. In slabs with punching failure surface crossing the region of shear reinforcement (Group 2) these increments were of at least 8% and 5%, respectively. Slab L6, with the smallest number of lines (4) of shear reinforcement, angle between lines of 90° and radial spacing between layers of 67 mm, presented the smallest increases for rotation and load.

Slab L9, with index κ=0.41 presented a rotation increase of 29%, nearly the same of L7 (30%) with κ=0.51, and one load increase equal to 10%, higher than that of L7 (6%). The radial distance between the layers of the studs in slab L9 was equal to that of slabs L6 and L8 (~0.75d) but, due to its number of lines of six, higher than that of the remaining slabs of Group 2, the circumferential spacing between the outermost elements of slab L9 was of 251 mm (~2.8d), higher than that of slab L7 (~2.62d), but smaller than

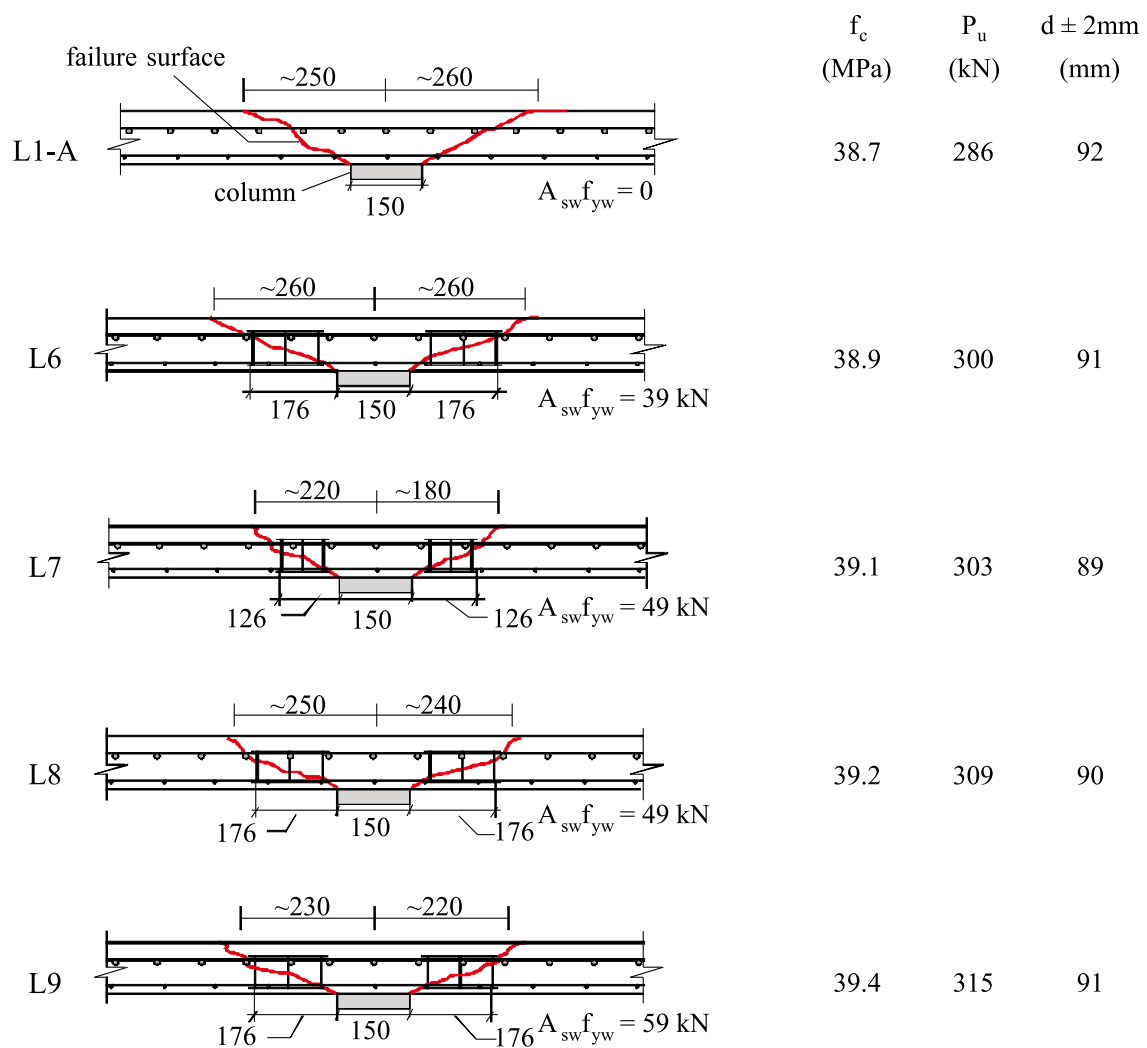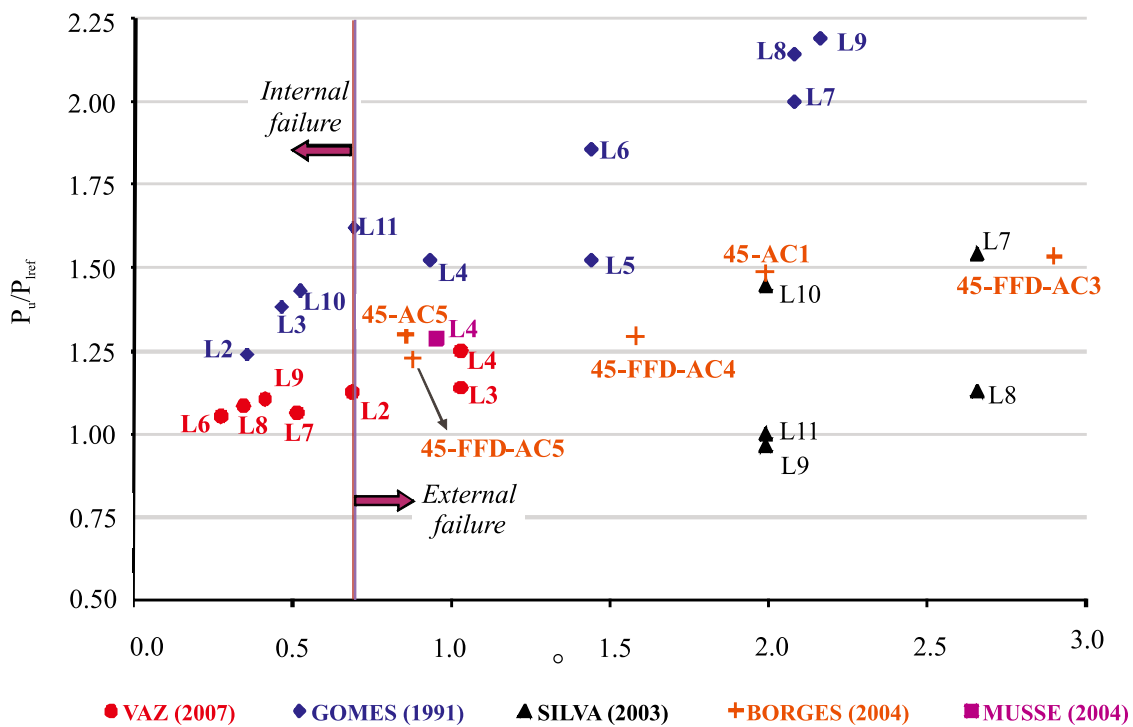## Figure 6 – Failure surface of Group 2 slabs (dimensions in mm)

	f_c (MPa)	P_u (kN)	$d \pm 2mm$ (mm)
L1-A	38.7	286	92
L6	38.9	300	91
L7	39.1	303	89
L8	39.2	309	90
L9	39.4	315	91

Figure 7 – Values of $P_u/P_{Lref.}$ as a function of κ for the slabs of this and other researches

that of slabs L6 (~3.95d) and L8 (~3.28d).

Comparing the behaviour of slabs L7 and L9 one finds that the circumferential spacing does not seem to have a significant influence on the increase of rotation (near 30% for the two slabs). However, the number of lines of shear reinforcement in slab L9 (six) higher than that of slab L7 (five) may have caused its highest gain in failure load (L9 = 10%, L7 = 6%) in spite of its larger radial and circumferential spacing values.

These results and those from other authors (Figures 7 and 10) indicate the possibility of defining a minimum shear reinforcement considering a value of κ around 0.6. For slabs with effective depth and other characteristics similar to those of the slabs tested in this study and κ of this order, there should be a gain in failure load of around 10% and a gain in rotation of around 30%. The slabs tested by Gomes [5] suggest that these increments are greater in slabs with a higher effective depth.

4. Final considerations

The results of this and other works that are more detailed by Vaz [6] indicate that the minimum shear reinforcement to avoid the sudden failure showed by flat slabs without this reinforcement can be defined from the parameter κ adopted here. The results of the analyzed tests point out that the value of κ corresponding to the minimum reinforcement should be around 0.5 to 0.7. However, additional studies are needed given that the increase of the failure load and rotation related to a certain value of κ probably depends on the effective depth, radial and circumferential spacing and layers of the shear reinforcement of the slab.

5. Acknowledgements

6. References

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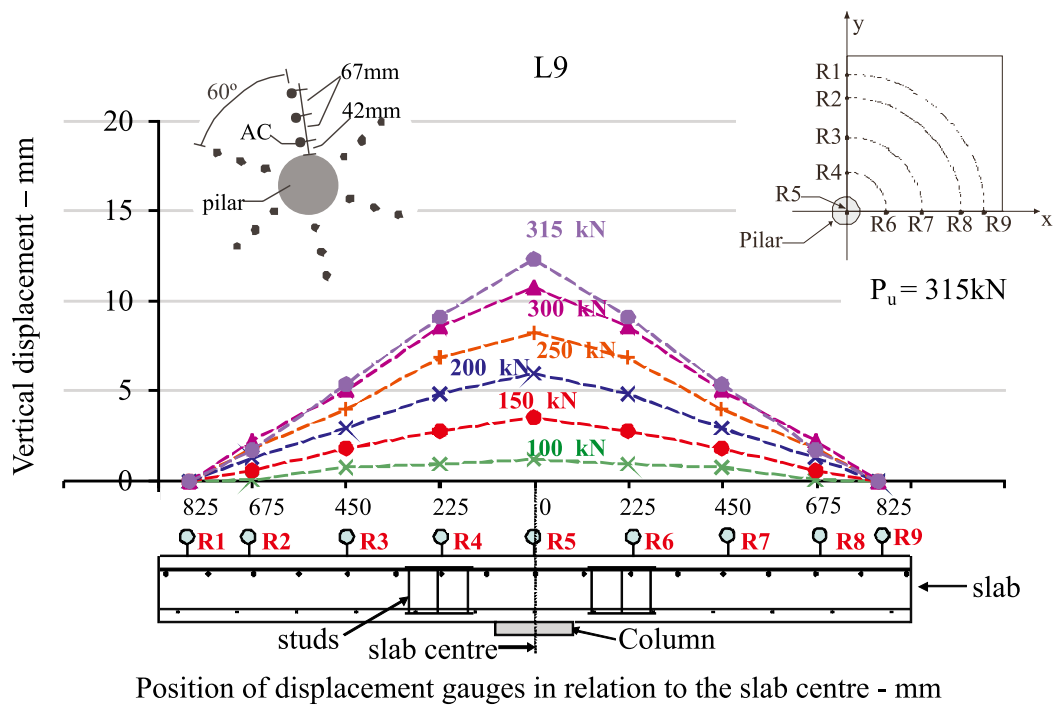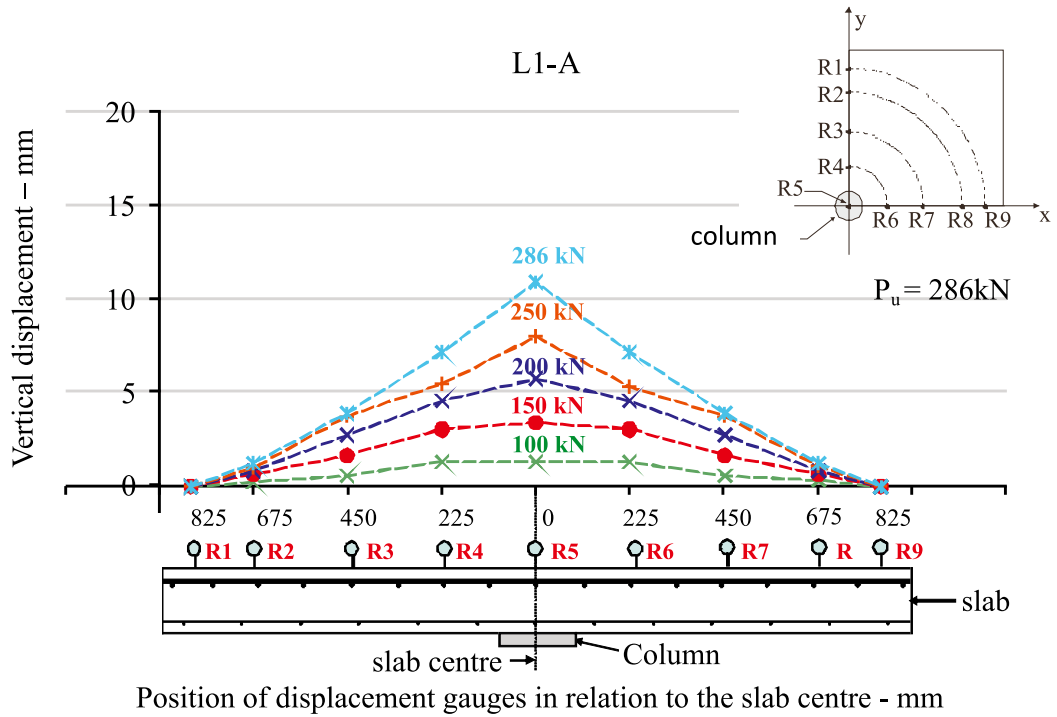**Figure 8 – Vertical displacement of slabs L1-A and L9 as a function of the distance to the centre**

L1-A

P_u = 286kN

Position of displacement gauges in relation to the slab centre - mm

L9

P_u = 315kN

Position of displacement gauges in relation to the slab centre - mm

Figure 9 – Vertical centre displacements of slabs as a function of the loads
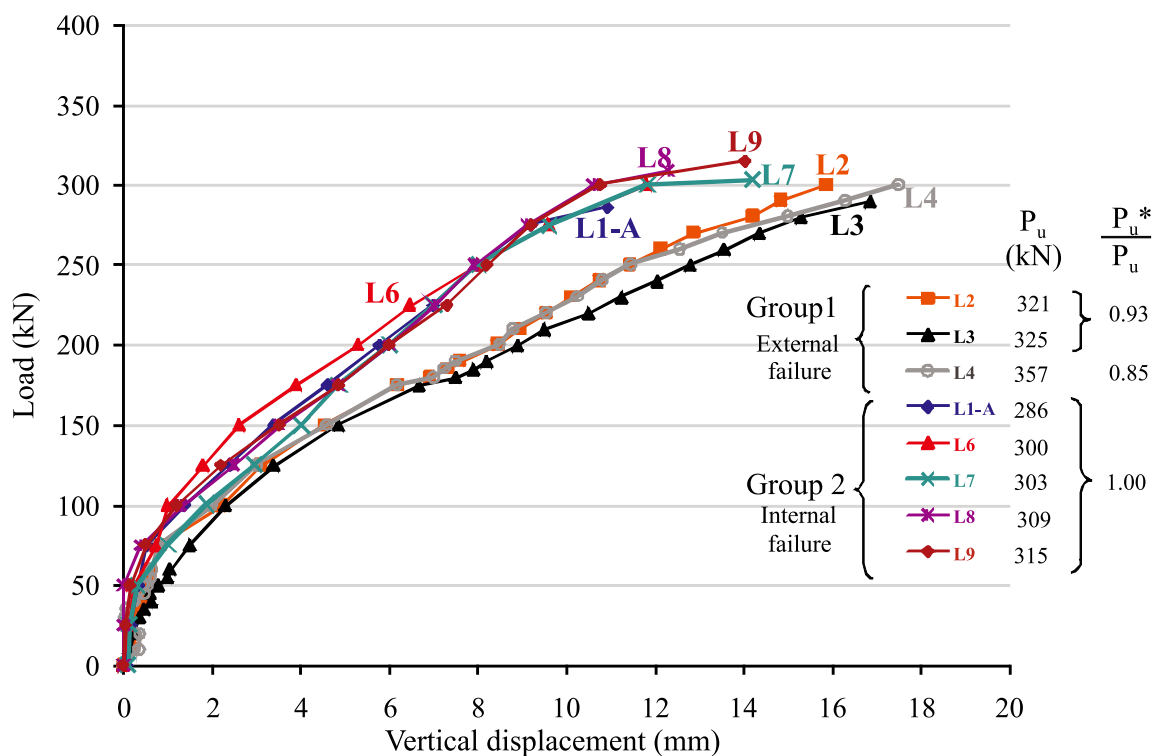
Figure 10 – Values of Ψ_u/Ψ_{Lref} as a function of κ for the slabs of this and other researches
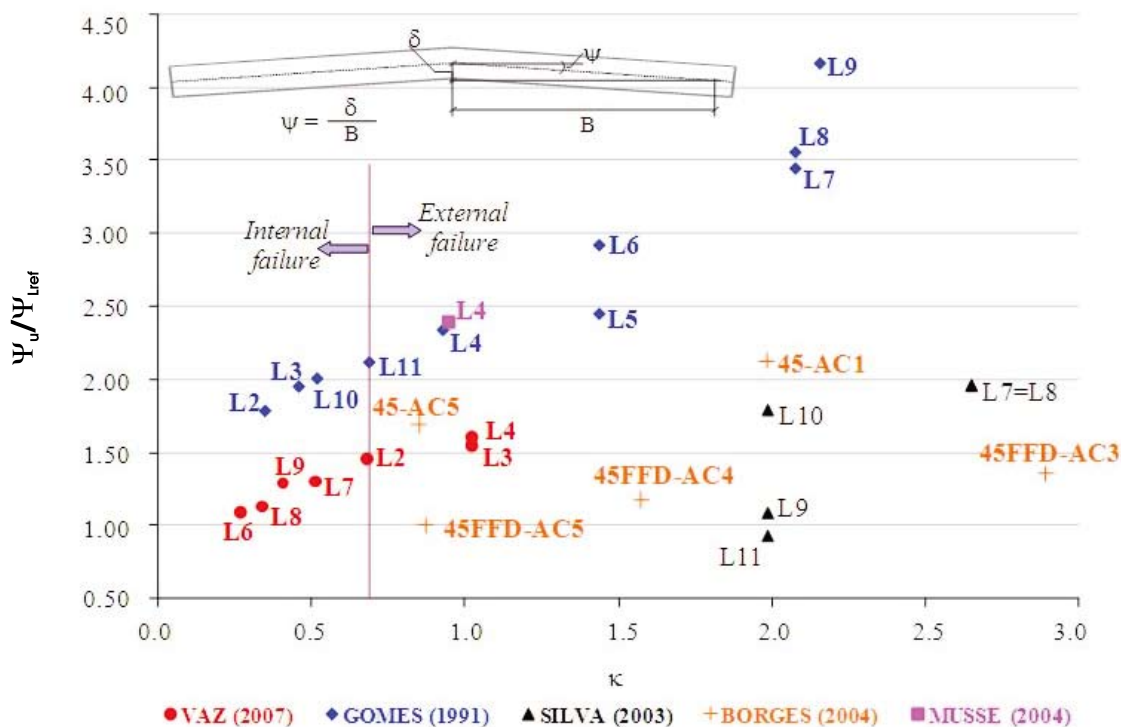
Figure 11 – ($\varepsilon/\varepsilon_y$) as a function of distance to the centre of slab L2
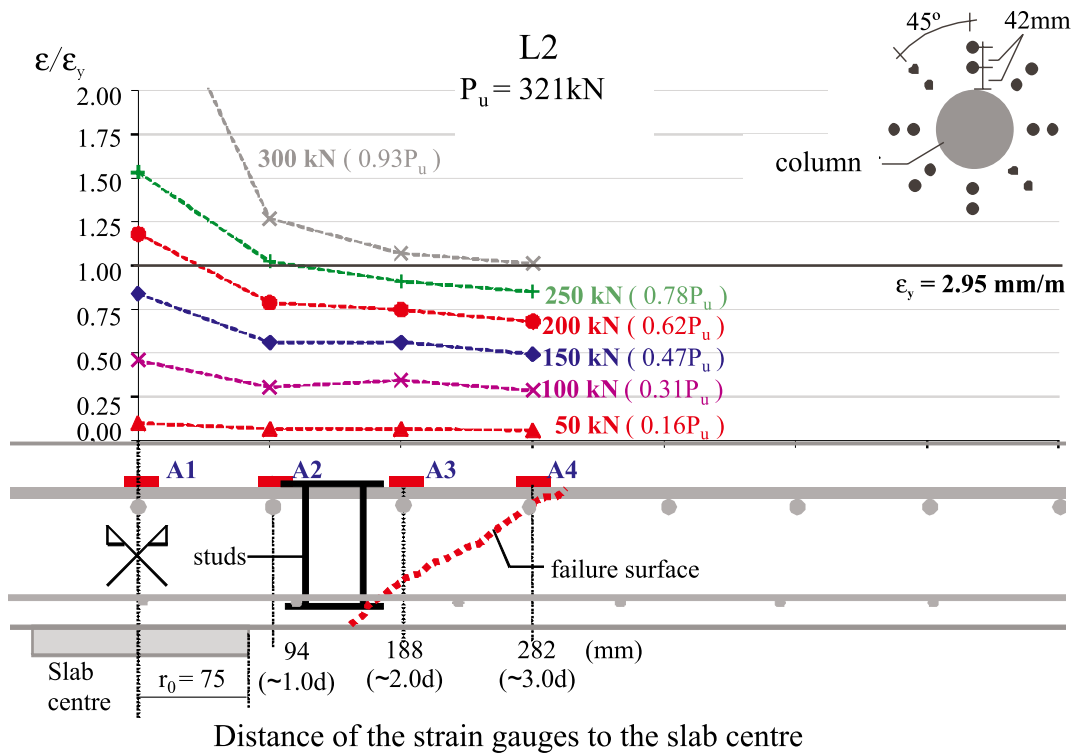

Figure 12 – ($\varepsilon/\varepsilon_y$) as a function of distance to the centre of slab L8
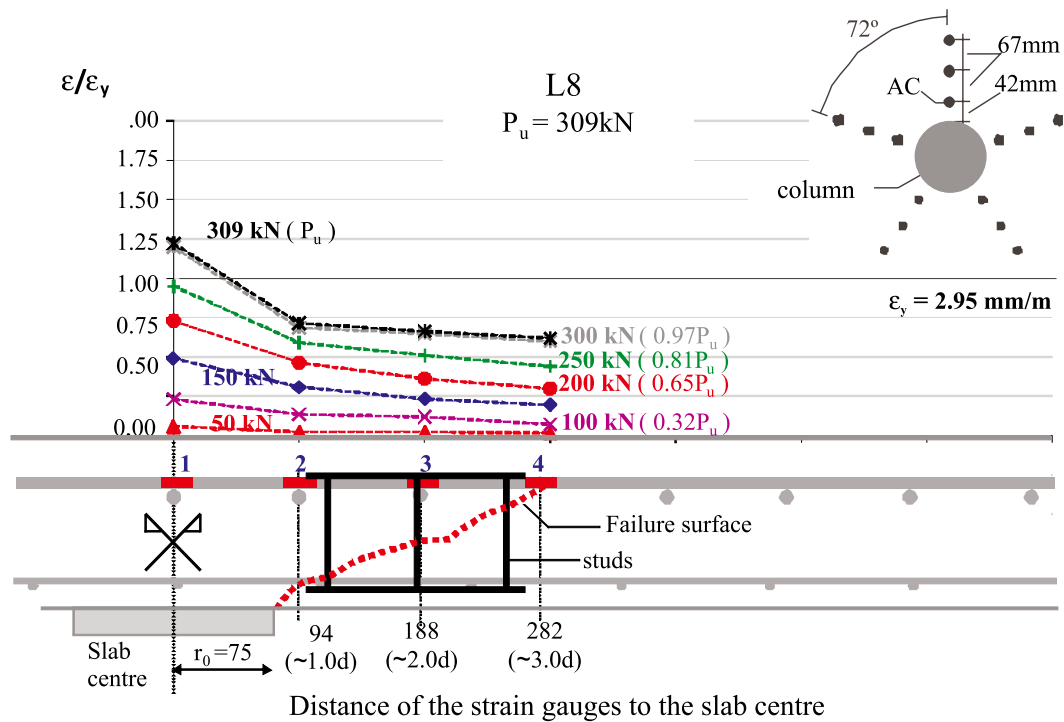
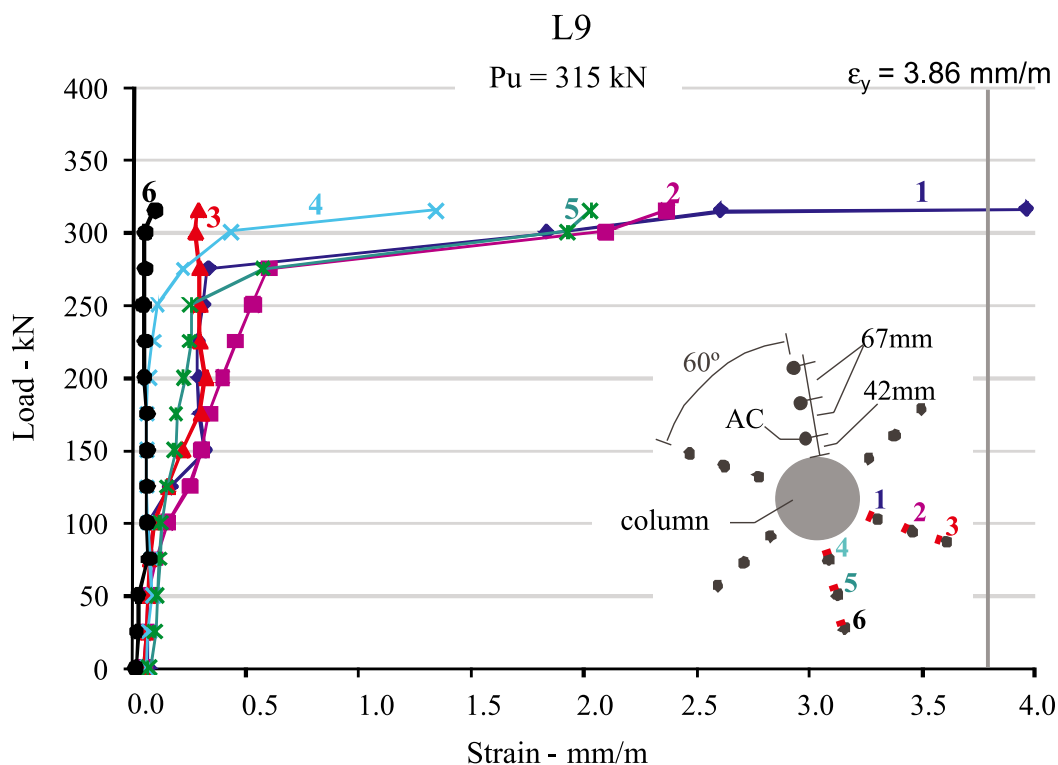## Figure 13 – Load – shear reinforcement strain relationship of slab L9

Table 3 – Ratios between rotation and failure load of the slabs with shear reinforcement and of the reference slab L1-A, $\Psi u/\Psi L1$-A and P_u /P_{L1-A}, and shear reinforcement index κ

	Slabs	ψ_u (x 10^{-4}rad)	P_u (kN)	$\dfrac{\psi_u}{\psi_{uL1-A}}$	$\dfrac{P_u}{P_{uL1-A}}$	κ	Failure surface
Group 1	L4	212	357	1.61	1.25	1.03	E
	L3	204	325	1.55	1.14	1.03	E
	L2	192	321	1.46	1.12	0.69	E
Group 2	L7	172	303	1.30	1.06	0.51	I
	L9	170	315	1.29	1.10	0.41	I
	L8	149	309	1.13	1.08	0.34	I
	L6	143	300	1.08	1.05	0.27	I

P_{uL1-A} = 286 kN; ψ_{uL1-A} = 132 x 10^{-4}rad