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Analysis of second order effects: case study

Análise de efeitos de 2ª ordem: estudo de caso

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Abstract

This paper presents a nonlinear static analysis of a reinforced concrete plane frame. It has as main objective is to realize a global stability verification of a plane frame, by using geometric stiffness matrix. In order to obtain first and second order combined effects, equilibrium and kinematic relations were studied in the deformed geometric configuration. These results were obtained by using geometric stiffness matrix and multiplying horizontal forces by Gamma-Z coefficient. Both procedures disclosed very similar results in the study, indicating that Gamma-Z can be used to study equilibrium and kinematic relations in deformed geometrical configuration of the structure.

Keywords: nonlinear analysis, instability, second order analysis, Gamma-Z.

Resumo

Neste artigo apresenta-se a análise estática não linear de um pórtico plano de concreto armado. Tem-se como objetivo geral realizar a análise de verificação de estabilidade global de um pórtico plano, com utilização da matriz de rigidez geométrica. Para a obtenção dos efeitos combinados de primeira e segunda ordem, o equilíbrio e as relações cinemáticas foram estudadas na configuração geométrica deformada. Estes resultados foram obtidos por meio de utilização da matriz de rigidez geométrica e por meio da multiplicação dos esforços horizontais (característicos) pelo coeficiente Gama-Z. Ambos os procedimentos apresentaram resultados muito próximos, no estudo, o que indica que o Gama-Z pode ser utilizado para o estudo do equilíbrio e das relações cinemáticas na configuração geométrica deformada da estrutura.

Palavras-chave: análise não linear, instabilidade, análise de segunda ordem, Gama-Z.

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1. Introduction

In geometric linear analysis, or first order analysis, efforts are determined through the structure's equilibrium. This equilibrium and kinematic relationships are studied in the structure's initial geometric configuration, i.e., undeformed configuration.

When the structure is subjected to horizontal forces (e.g. wind action), these forces cause horizontal displacement that, due to structure's flexibility, can cause additional effects added to those determined in first order analysis (1st order).

The additional effects are called second-order effects (2nd order), which must be determined considering materials' nonlinear behavior and deformed configuration in equilibrium analysis [1], [2]. These considerations are denominated physical and geometric nonlinear analysis [3]. Total efforts are, then, equal to the sum of 1st and 2nd order efforts'.

Thus, many structures need equilibrium and kinematic relationships to be used in the structure's deformed configuration [4]. Thus, global stability verification becomes a requirement in project design of reinforced concrete buildings, which aims to ensure structure's safety in relation to an ultimate limit state of instability and, to thereby verification, there are some simplified procedures called global stability parameters [5]. There are also more sophisticated procedures, as disclosed in references [6–8], the process $P-\Delta$ and methods using structure's geometric stiffness matrix [9].

1.1 Justification

Nonlinear or 2nd order analysis require knowledge, understanding and consideration of physical and geometric nonlinearities, besides numerical methods' use to structure discretization and equations' resolution that govern the problem. Thus, this study is justified by the presentation of a simplified approach (approximate) to equilibrium and kinematic relations' assessment in the deformed configuration of equilibrium and to perform qualitative and quantitative analysis of the phenomenon.

2. Objectives

2.1 Main objective

Perform global stability control analysis of a particular plane frame case, using geometric stiffness matrix.

2.2 Specific objectives

- Check the need of 2nd order effects' consideration;
- Calculate 2nd order efforts;
- Compare 2nd order results obtained from the geometric stiffness matrix, with those calculated by the approximate procedure.

3. Simplified procedures to 2nd order effect verification

The Brazilian Code NBR 6118 [2] introduces two simplified procedures to verify the need for 2nd order effects' consideration, Alpha parameter (α) and Gamma-Z coefficient (γ_z). These processes are briefly discussed below.

3.1 Alfa instability parameter

Its use is only intended to make an assessment of the building's stability, being Alfa instability parameter calculated by equation **(1)**.

$$\alpha = H_{tot}\sqrt{\frac{\sum N_k}{\sum E_c I_c}} \tag{1}$$

in which, H_{tot} is the structure's total height, $\sum N_k$ is the sum of service vertical loads and $\sum E_c I_c$ is the sum of the bracing elements stiffness.

According to the NBR 6118 [2], 2nd order effects must be considered if $\alpha > \alpha_1$, being $\alpha_1 = 0,5$ in structures composed only by frames, in accordance with the standard code.

3.2 Gamma-Z coefficient (γ_z)

The γ_z coefficient is a simplified assessing process of global stability and 2nd order effects [5], [10], [11] and is also known as 1st order effects' multiplier. NBR 6118 [2] recommends that if $\gamma_z \leq 1,10$ the structure is classified as fixed nodes and, therefore, 2nd order effects might be disregarded. To $\gamma_z > 1,10$ it should consider the effects and, in this situation, the structure is classified as mobile nodes [2]. The coefficient is calculated by equation **(2)**.

$$\gamma_z = \frac{1}{1-\dfrac{\Delta M_{Tot,d}}{M_{1,Tot,d}}} \tag{2}$$

in which:

$\Delta M_{Tot,d} \rightarrow$ The sum of vertical design forces products' acting by their respective 1st order displacements;

$M_{1,Tot,d} \rightarrow$ Moment that tends to overturn the structure.

According to reference [12], it is possible to correlate α parameter and γ_z coefficient by a cubic equation. However, γ_z coefficient turns α parameter less important, because with γ_z use is possible to evaluate the building stability and estimate 2nd order effects. Nonetheless, it is important to relativize this information, since other consulted references do not mention it. Reference [13] reports that there are special cases in which γ_z may not be applied or may result in errors above acceptable limits.

4. 2nd Order effects analysis

Second order effects take into account structure deformation (geometric nonlinearity) and nonlinear behavior of reinforced concrete sections (physical or material nonlinearity). The choice of the most suitable procedure to be used depends on various factors, such as structure's displacements and rotations' magnitude, normal active forces' level, structure's sensitivity to 2nd order effects, among others. Geometric stiffness matrix's use is one of the possible alternatives that can replace, with advantages, the $P-\Delta$ process. Other procedures also were developed, such as Two Cycles Iterative Method, Fictitious Side Load Method, Iterative Gravity Load Method and Negative Stiffness Method, which can be verified in reference [14].

4.1 Geometric stiffness matrix

Geometric stiffness matrix $\left[K_G\right]$ is one of three matrixes that comprises the secant matrix $\left[K_S\right]$ which relates applied forces to the displacements [5], [15]. The two other plots are classic linear elastic stiffness matrix $\left[K_E\right]$ and the matrix that expresses axial forces resulting from nodal displacements perpendicular to bars' axis $\left[K_I\right]$ [16].

Geometric stiffness matrix, for a plane frame element (beam element), is given by equation **(3)**, in which P is axial force on the element and l is bar length [17]. Geometric stiffness matrix takes into account the interaction between axial force and bending moment on the bar for structures formed by prismatic bars subjected to moderate rotations. Moreover, as it turns out, geometric matrix depends not only of the element geometry, but also of the active internal efforts P. For a nonlinear geometrical analysis, the full $\left[K_S\right]$ may be adopted, equation **(4)** or only $\left[K_G\right]$ and $\left[K_E\right]$ - equation **(5)** [16].

$$\left[K_G\right]=\frac{P}{l}\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{6}{5} & \frac{l}{10} & 0 & -\frac{6}{5} & \frac{l}{10} \\ 0 & \frac{l}{10} & 2\frac{l^2}{15} & 0 & -\frac{l}{10} & -\frac{l^2}{30} \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{6}{5} & -\frac{l}{10} & 0 & \frac{6}{5} & -\frac{l}{10} \\ 0 & \frac{l}{10} & -\frac{l^2}{30} & 0 & -\frac{l}{10} & 2\frac{l^2}{15} \end{bmatrix} \tag{3}$$

$$\left[K_S\right]=\left[K_E\right]+\left[K_G\right]+\left[K_I\right] \tag{4}$$

$$\left[K_S\right]=\left[K_E\right]+\left[K_G\right] \tag{5}$$

4.2 Approximate procedure (simplified)

This procedure consists in multiplying horizontal actions by the γ_z coefficient, if it is greater than 1,10 (mobile nodes structure). Thus, are calculated, in an approximate way, the results of 1st and 2nd order effects in the structure. However, to make a smoother transition between the cases, NBR 6118 [2] recommends to use $0{,}95\times\gamma_z$. In this article, it is justified the use of full γ_z to be able to compare results among different performed analyzes.

The procedure is performed to each one of the combinations of the actions, as shown in equations **(11)** and **(12)**, in which the γ_z value used must correspond to the combination in analysis. It is worth to remember that this procedure is treated as a simplified approach (approximate) in order to evaluate equilibrium and kinematic relations' in the deformed configuration of the structure.

5. Method

In this article, there were carried out numerical studies of qualitative character, as it intends to investigate the relations among studied variables accurately. It is used a plane frame with 14 nodes and 18 bars, Figure 1. The study consists of numerical analysis, which were performed by programming (script) in MatLab[1] and Mix System[2].

For the actions wind forces were considered, as well as the forces resulted from the structural elements' weight and using loads' (accidental loads).

In the analysis with α parameter, only actions due to wind were used, with the characteristic values, in order to determine the maximum structure displacement. With the sum of these loads, it was possible to obtain an equivalent distributed load which cause the same displacement at the top in a fictitious column. Thus, E_cI_c value was obtained, which is an equivalent value.

Numerical analysis of 2nd order effects (nonlinear geometric analysis) were made with Mix System, using secant matrix given by equation **(5)**. The 2nd order analysis' results were taken as a reference to comparison with the approximate procedure.

5.1 Materials' physical characteristics

For the frame, it was used concrete with compressive strength characteristic $f_{ck}=30MPa$. Secant stiffness of structural elements is treated differently for beams and columns, so that, in a simplified form, the nonlinearity of the materials can be considered, a result of nonlinear relations between stress and deformation and of reinforced concrete behavior. This procedure, which is consistent with NBR 6118 [2], consists in reducing the stiffness values of each structural element type. Thus, for beams with different compression and tension reinforcement and pillars, it is used value given by equations **(7)** and **(8)**, respectively. In which I_c is the moment of inertia of the gross concrete section and $\alpha_E=1$ (granite and gneiss).

$$E_{ci}=\alpha_E 5600\sqrt{f_{ck}} \tag{6}$$

$$(EI)_{SEC}=0{,}4\,E_{ci}I_c \tag{7}$$

$$(EI)_{SEC}=0{,}8\,E_{ci}I_c \tag{8}$$

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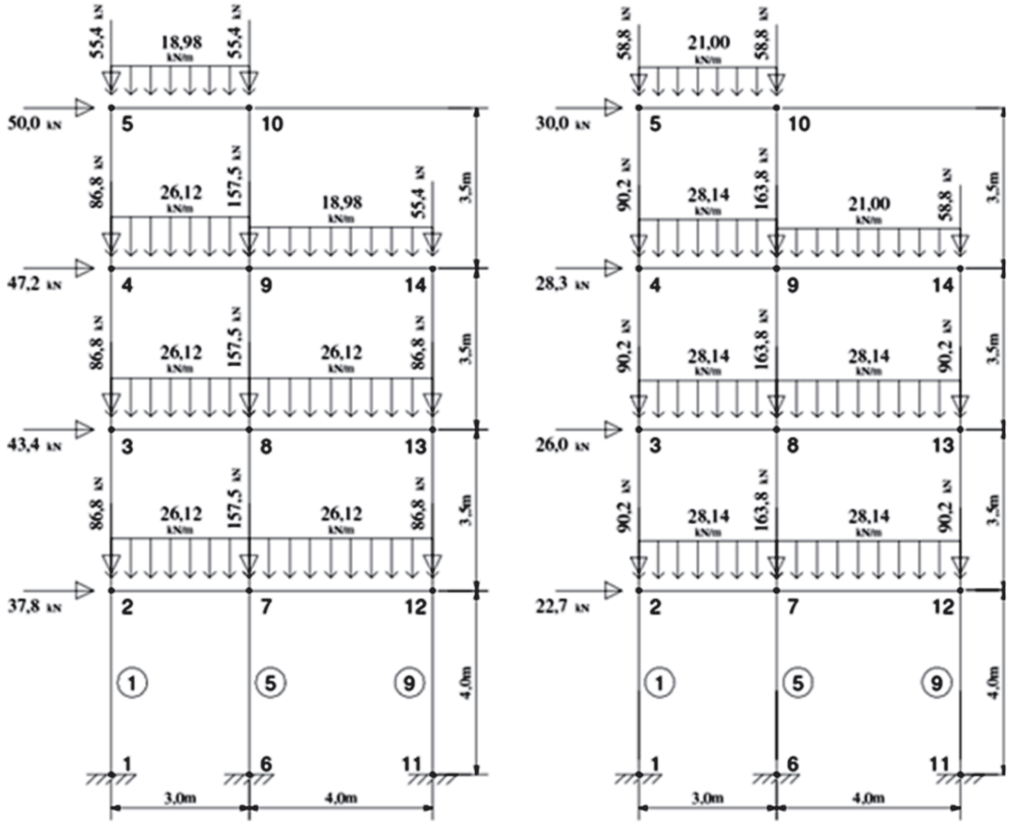
[1] *http://www.mathworks.com - Student version.*

[2] *Licensed Software to Federal University of Santa Catarina. Mix System is a system developed by Engineer Ricardo Sergio Pinheiro Medeiros and marketed by TQS Informática Ltda.*

Figure 1 – Combinations 1 and 2



A Combination 1

B Combination 2

5.2 Plane frame geometric characteristics

Pillars' sections are rectangular with 30 × 25 cm dimensions, where the 25 cm dimension is the one on the bending plan of the plane frame. To simulate rigid diaphragm effect, the beams (cross-section of 15 × 40 cm) are simulated with cross-sectional area of $6 \times 10^5 cm^2$, fictitious increase, trick that enables to obtain equal horizontal displacements along pavement points.

5.3 Actions

In this paper, were used permanent and accidental loads. In the analysis, were used two loads' combinations for ultimate limit state. The first load case considers the wind as main accidental action, equation **(9)** where $\psi_0 = 0,7$ (commercial buildings). The second case considers wind action as a secondary accidental action, equation **(10)**, with $\psi_0 = 0,6$. In these equations, *"g"* index refers to permanent loads, *"q"* to vertical accidental loads, *"V"* to wind action (horizontal loads) and *"k"* to characteristic values of each action. Combinations used in the approximate procedure are presented in equations **(11)** and **(12)**.

$$F_d = 1,4F_{gk} + 1,4V_k + 1,4\psi_0 F_{qk} \tag{9}$$

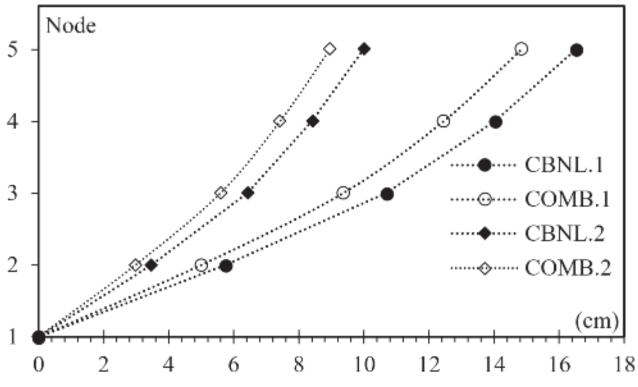
$$F_d = 1,4F_{gk} + 1,4F_{qk} + 1,4\psi_0 V_k \tag{10}$$

$$F_d = 1,4F_{gk} + \gamma_z 1,4V_k + 1,4\psi_0 F_{qk} \tag{11}$$

$$F_d = 1,4F_{gk} + 1,4F_{qk} + \gamma_z 1,4\psi_0 V_k \tag{12}$$

Figure 1 shows used values in each combination (final values). For the approximate procedure, it was used MatLab script, where only wind actions on Figure 1 (a) and Figure 1 (b) are multiplied by γ_z coefficient.

Figure 2 – Horizontal displacements (cm)



6. Results and discussions

6.1 Alpha and Gamma-Z (a, γ_z)

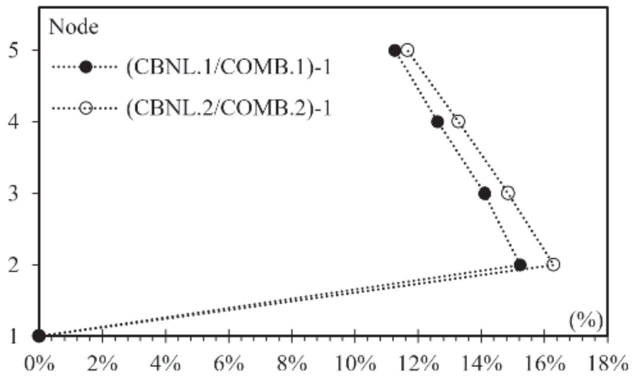
For α coefficient, it was obtained $0,64$ and in accordance with NBR 6118 [2], 2nd order effects must be considered, because $\alpha > \alpha_1$.

Regarding γ_z coefficient, two values were obtained, one for each one of the actions' combinations. For the first combination, equation **(11)**, which has wind as main accidental action, the obtained value was $\gamma_z = 1{,}10$. For the second combination, equation **(12)**, with vertical load as main accidental action, the obtained value was $\gamma_z = 1{,}11$. NBR 6118 [2] recommends that 2nd order effects must be considered if $\gamma_z > 1{,}10$. Therefore, with α parameter and γ_z coefficient is possible to verify that it is necessary to consider 2nd order effects. The next section deals with this subject.

6.2 Second order analysis

Figure 2 shows results of horizontal displacements of nodes 1 to 5, for 1st order analysis, COMB.1 equation **(9)** and COMB.2 equation **(10)**, and nonlinear analysis (2nd order) arising out of the two previous combinations, and CBNL.1 CBNL.2, respectively.

Figure 3 – Horizontal displacements, difference (%)



It is found that the larger displacement amplitudes are obtained from combination 1, which uses equation **(9)**, which has wind action as main accidental. However, to the same combination, COMB.1, there was obtained the lowest value to γ_z coefficient. This is because 2nd order effects are due to the product of vertical loads by respective horizontal displacements. While in COMB.1 it was verified the greatest horizontal displacements, COMB.2 has the largest vertical loads and greater 2nd order effect, in this case. Displacements' difference between 1st and 2nd order analysis, for each of the combinations, is featured in Figure 3.

It is verified that to the node 5 (top of the frame), with the first actions' combination there is an increase in displacements of 11,26%, when performing 2nd order effects analyses. For the second combination, the increase was 11,66%. In both cases, the biggest difference is obtained for node 2, with maximum value of 16,29% in the second combination.

Bending moments at pillars' base (bars 1, 5 and 9), obtained for all combinations (1st and 2nd order) are presented in Figure 4 and Figure 5. In Figure 4, wind action is the main accidental action, and Figure 5 has wind action as a secondary accidental action. In both figures, it is noted that the portion due only to 2nd order efforts is greater than 10% in all pillars and combinations (right vertical axis in the figures), in which "$[2^a/1^a]-1$" represents the difference in percentage of geometric nonlinear analysis (2nd) over linear analysis (1st).

Figure 4 – Relations between bending moments in 1st and 2nd order analysis: Combination 1

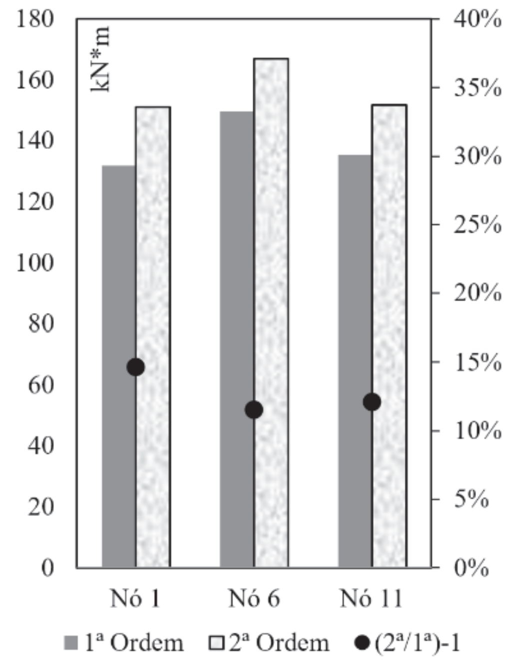
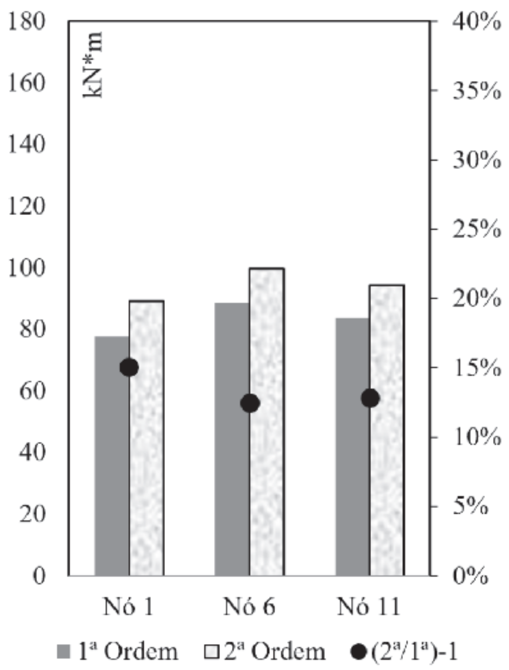


Figure 5 – Relations between bending moments in 1st and 2nd order analysis: Combination 2



6.3 *Approximate procedure (simplified)*

To differentiate the results, at the figures' legend, results obtained by simplified or approximate analysis (described in 4.2) are indicated by “γ_z” and results obtained by nonlinear geometric analysis are indicated by “ 2ª ”.

Results of horizontal displacement from nodes 1, 2, 3, 4 and 5 are presented in Figure 6 and the difference between the two procedures is reported in Figure 7. Bending moment values at the pillars' base, with their respective comparing results, are featured in Figure 8.

Figure 6 – Horizontal displacements (cm)

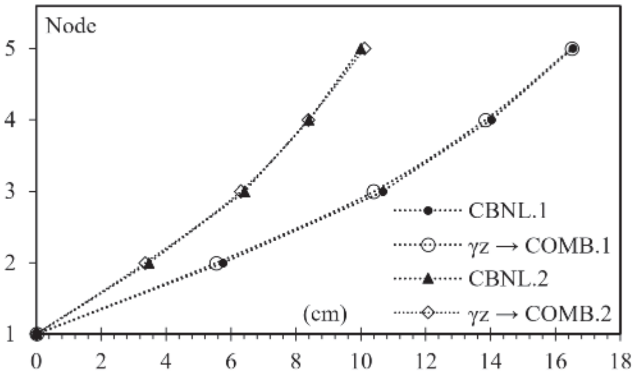


Table 1 – Horizontal displacements (cm)

Node	$dif = \frac{CNBL.1}{\gamma_{z\ COMB.1}} - 1$	$dif = \frac{CNBL.2}{\gamma_{z\ COMB.2}} - 1$
1	0	0
2	3,79%	3,14%
3	2,69%	1,90%
4	1,30%	0,30%
5	0,18%	-1,25%

Table 2 – Bending moment at pillars' base (kN·m)

Node	$dif = \frac{CNBL.1}{\gamma_{z\ COMB.1}} - 1$	$dif = \frac{CNBL.2}{\gamma_{z\ COMB.2}} - 1$
1	2,92%	1,86%
6	0,22%	-0,22%
11	1,32%	1,18%

These results prove that approximate procedure achieved an excellent performance compared to refined method, which uses geometric stiffness matrix. In Table 1 and Table 2, it is possible to better visualize the difference between procedures for displacements and bending moments, respectively. It is noted that for displacements at the top of the frame (node 5), relative difference is only 0,18% for combination 1, and only -1,25% for combination 2, and in the latter case, approximate procedure is in favor of safety.

7. Conclusion

The study presented in this article reports the importance of checking 2nd order effects in order to guarantee the structure's safety.

Figure 7 – Horizontal displacements: difference (%)

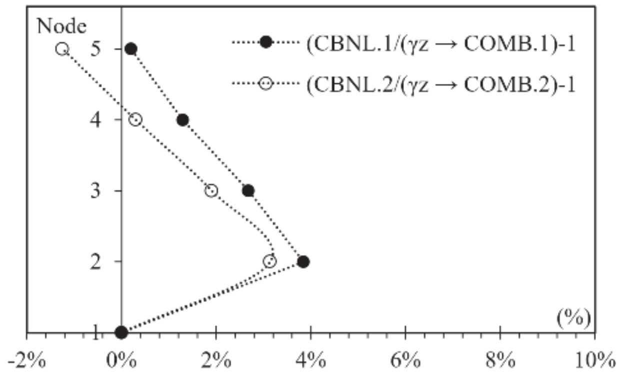
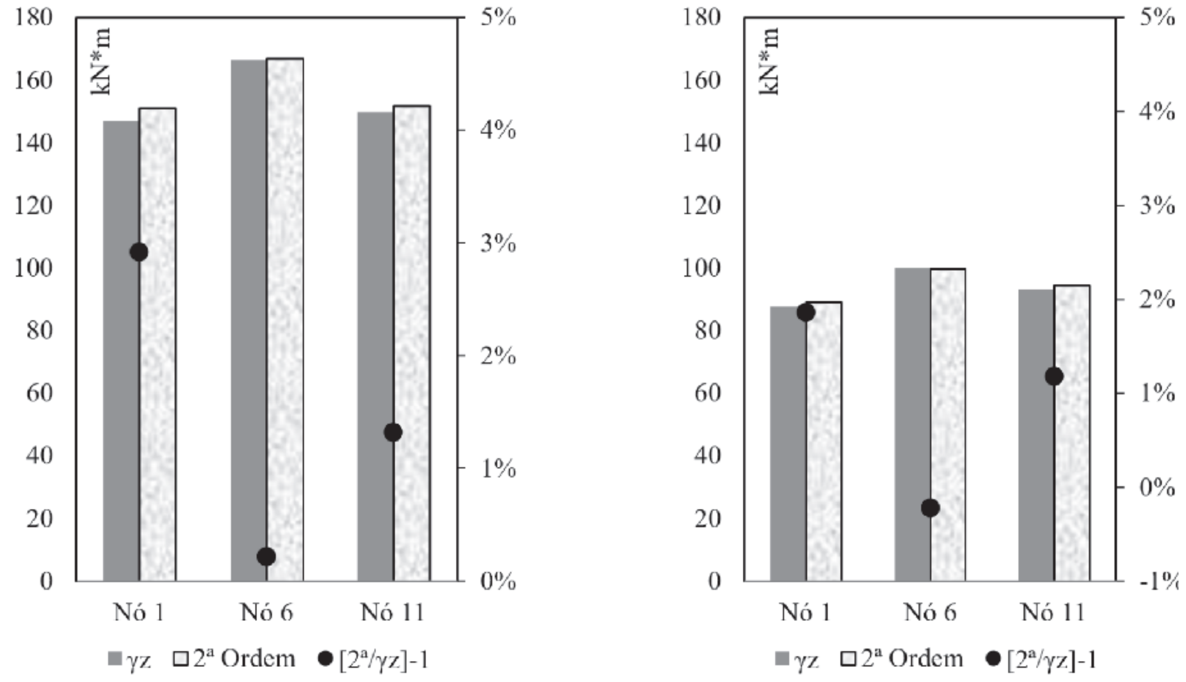


Figure 8 – Relations between bending moments: geometric nonlinear analysis and approximate procedure



A Wind as main accidental load

B Wind as secondary accidental load

It was found that the α parameter and the γ_z coefficient were effective to demonstrate the need of evaluation of these effects.

Geometric nonlinear analysis, using geometric stiffness matrix, was satisfactory to obtain efforts and displacements due to 2nd order effects. These effects have shown to be greater than 10% of the 1st order effects. Fact that the simplified procedures α and γ_z already indicated.

The approximate procedure, which consists in multiplying horizontal forces by the γ_z coefficient, proved to be suitable to obtain the desired 2nd order effects of the studied plane frame, both to the displacements and bending moments. It was found that the approximate procedure application is simple and does not require advanced knowledge on nonlinear geometric analysis, as it is required in the refined method. However, the results are valid to structural characteristics simulated in this article and this verification should not be extrapolated for other structures.

8. Acknowledgements

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