IBRACON Structures and Materials Journal
Revista IBRACON de Estruturas e Materiais
ISSN 1983-4195 ismj.org

ORIGINAL ARTICLE

Reliability of codes provisions for punching shear design

Confiabilidade das prescrições normativas para dimensionamento à punção

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Received 19 September 2022
Revised 29 April 2023
Accepted 03 July 2023
Corrected 27 March 2024

Abstract: This paper presents a reliability analysis of the punching shear of flat slabs without shear reinforcement. The evaluation is performed for the codes ACI 318, Eurocode 2, Model Code 2010, and ABNT NBR 6118:2014. Six models were used for predicting the punching strength, the design models, the Critical Shear Crack Theory (CSCT), and a Nonlinear Finite Element Analysis (NLFEA) model. Reliability analysis was performed to evaluate the safety level of code provisions. Design code provisions were evaluated in terms of sufficient reliability criteria based on target reliability β=3.8. This target reliability was based in the Model Code 2010 criteria. This criterion represents an acceptable value level of safety in design of structures. The results showed that the reliability indexes β presented satisfactory for all slabs designed by the Model Code 2010. However, this paper also shows some of the tested slabs presented reliability index below 3.0, being the ACI 318 the code with the lowest reliability index.

Keywords: structural reliability, punching shea strength, resistance error model, flat slabs, design codes.

Resumo: Este artigo apresenta a análise de confiabilidade do dimensionamento à punção de lajes lisas sem armadura de cisalhamento. A pesquisa avaliou a confiabilidade obtida pela norma ACI 318, Eurocode 2, Código Modelo 2010 e pela norma brasileira ABNT NBR 6118:2014. Para a estimativa da resistência à punção seis modelos foram estudados, os modelos das normas, a Teoria da Fissura Crítica de Cisalhamento (TFCC) e um modelo não-linear em elementos finitos. A avaliação da confiabilidade foi realizada para avaliar o nível de segurança das prescrições normativas. Para isso, o critério utilizado é baseado na comparação dos índices de confiabilidade β com o índice de confiabilidade alvo, estipulado em 3,8, conforme critério do Código Modelo 2010. Esse critério representa um valor aceitável no nível de segurança do dimensionamento da estrutura. Os resultados obtidos a partir dessa metodologia mostrou que o Código Modelo 2010 é a norma que apresenta os índices mais elevados de confiabilidade. Por outro lado, a norma ACI 318 apresentou os menores índices de confiabilidade, alcançando índices de confiabilidade menores que 3,0.

Palavras-chave: confiabilidade estrutural, resistência à punção, incerteza de modelos de resistência, lajes lisas, códigos de dimensionamento.

How to cite: G. R. Silva, A. Campos Filho, and M. V. Real, "Reliability of codes provisions for punching shear design," *Rev. IBRACON Estrut. Mater.*, vol. 17, no. 4, e17401, 2024, https://doi.org/10.1590/S1983-41952024000400001

1 INTRODUCTION

The market demands of civil construction have required more significant slab spans and, simultaneously, smaller beam heights. These demands have led many designers to adopt the solution of flat slab in reinforced or prestressed concrete. The floor without beams allows a smaller height between slabs and a more extensive range of choices of internal division in residential or commercial buildings, savings on formwork and concrete, simplified in complementary projects, ease of reinforcing and concreting, smaller execution deadlines, smaller loads in the foundations due to the reduction of the weight of the structure, better airing and lighting, and more extensive architectural freedom.

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Financial support: Coordenação de Aperfeiçoamento de Pessoal de Nível Superior (CAPES).
Conflict of interest: Nothing to declare.
Data Availability: the data that support the findings of this study are available from the corresponding author, [SILVA, G. R.], upon reasonable request.
This document has an erratum: https://doi.org/10.1590/S1983-41952024000400011

Because flat slabs are supported directly by the columns, there is an elevated shear stress concentration on the connecting regions between the slab and the columns. The main consequence of these elevated shear stresses is the failure characterized by the absence of reinforcement yielding, generating a brittle failure. These elevated shear stresses characterize the punching shear phenomenon, an ultimate limit state by shearing in the surroundings of concentrated forces [1]. Due to this characteristic, it is necessary to design the structure for, in case of failures happen; it does not happen by punching shear, but by bending. When not adequately measured, the failure by punching shear can cause serious accidents such as the collapse of a slab or even the total ruin of the structure.

While different formulations for the design of flat slabs are observed, a gap in the knowledge about the reliability of these methods is noticed. Therefore, a better understanding of the reliability obtained by each method is of fundamental importance for a safe and precise design of these elements. With the knowledge of the reliability of each method, the normative prescriptions might vary to make designs more economical and, at the same time, with an acceptable failure probability. Therefore, this paper presents a study about the reliability of punching shear of flat slabs, more specifically of the internal column-slab connection without shear reinforcement, design by the provisions of ABNT NBR 6118:2014 [2], ACI 318 [3], EUROCODE 2 [4] and Model Code fib 2010 [5].

2 RESISTANCE MODELS FOR PUNCHING SHEAR IN FLAT SLABS

Six models were used to predict the punching shear resistance using mean material strengths and unitary partial resistance factors. Therefore, was possible compared the estimate resistance with the experimental results taken from the literature.

2.1 Eurocode 2

The model used by Eurocode 2 is based on the code MC 90 [6]. The verification of the punching strength is performed on two control perimeters. In the first control perimeters, the diagonal compressive strain of concrete is indirectly verified, and the estimation of the resistance is given by Equation 1.

$$V_R = 0.30 f_c \left(1 - \frac{f_c}{250}\right) u_0 d \tag{1}$$

Where d is the effective depth of the slab, u_0 is the perimeter at column periphery and f_c is the average concrete compressive strength.

In the second control perimeter, moved 2d from the column edge or concentrated load area, the punching shear strength is verified through Equation 2.

$$V_R = 0.18 u_1 d \xi (100 \rho f_c)^{\frac{1}{3}} \tag{2}$$

Where ρ is reinforcement ratio, $\rho = \sqrt{\rho_x \rho_y}$, limited in 0.02, u_1 is the control perimeter moved 2d from the column edge and ξ is a factor accounting for size effect defined by Equation 3.

$$\xi = \left(1 + \sqrt{\frac{200}{d}}\right) \leq 2.0 \tag{3}$$

With d in mm.

2.2 ACI 318

For the ACI 318 [3], the verification is performed in only one control perimeter. In this model, for predicting punching strength, the effect of flexural reinforcement is not taken into consideration. The punching resistance of slab-column connections without shear reinforcement is given by the smallest value among the verification of Equations 4, 5 and 6.

$$V_R = \frac{1}{3} b_0 d \sqrt{fc} \quad (4)$$

$$V_R = \frac{1}{6} b_0 d \left(1 + \frac{2}{\beta}\right) \lambda \sqrt{f_c} \quad (5)$$

$$V_R = \frac{1}{12} b_0 d \left(2 + \frac{\alpha_s d}{b_0}\right) \lambda \sqrt{f_c} \quad (6)$$

Being λ the factor of modification of concrete, β the ratio of the maximum and minimum column dimension of the column, α_s a constant which depends on the position of the column and λ_s the size effect modification factor determined by the Equation 7:

$$\lambda_s = \sqrt{\frac{2}{1+0.004d}} \leq 1 \quad (7)$$

With d in mm.

2.3 ABNT NBR 6118:2014

The ABNT NBR 6118:2014 [2] is also based on the model of MC 90, but differently from EC2 [4], there is no explicit limitation in the flexural reinforcement ratio, nor in the value of size effect. In addition, the partial safety factor for concrete resistance ($\gamma_c = 1{,}4$) in the verification of the control perimeter u_1 is not explicit in the Brazilian code. However, it must be suppressed in order to obtain an adequate estimate of punching shear resistance using this model. In this way, the models of resistance are equal to the Equations 1 and 2 without the limitation of flexural reinforcement ratio and the parameter of size effect.

2.4 Critical Shear Crack Theory (CSCT)

The CSCT was proposed by Muttoni [7] and presents a mechanical explanation of the punching phenomenon. According to Muttoni and Schwartz [8]. As stated by Muttoni and Schwartz [8], the opening of the critical shear crack reduces the resistance of the compressive strut, being this one proportional to the product ψd. Beyond the reduction of the punching shear resistance with the increase of the slab rotation, the shear transference depends on the crack roughness, which is a function of the maximum aggregate size. Supported by these concepts, Muttoni [9] proposed a new formulation for the evaluation of punching capacity V_R . This strength can be estimated as shown in Equation 8.

$$\frac{V_R}{b_0 d \sqrt{f_c}} = \left(\frac{3/4}{1+\frac{15\psi d}{d_{go}+d_g}}\right) \quad (8)$$

Where b_0 is the control perimeter located 2d from the face of the column, d_{go} is the diameter of reference of the aggregated admitted as 16 mm, d_g is the maximum diameter of the aggregated and ψ is the slab rotation.

To determine the final punching strength, the relation between the applied load V and the slab rotation ψ have to be known. In the majority of the cases the relation between load-rotation can be obtained through a non-linear analysis of the slab bending behavior. However, simplified expressions can be used with good precision, as shown in Equation 9 [7].

$$\psi = 1.5 \left(\frac{r_s f_y}{d E_s}\right) \left(\frac{V}{V_{flex}}\right)^{\frac{3}{2}} \quad (9)$$

Where V is the shear force, V_{flex} is the shear force associated with flexural capacity of the slab, r_s is distance from the center of support to the surrounding line of radial contraflexure, f_y is the yielding strength of tensioned flexural reinforcement and E_s is the modulus of elasticity of the tensioned flexural reinforcement.

2.5 Model Code 2010

The model used for estimating the punching strength used by MC 2010 [5] is based on the CSCT proposed by Muttoni [7]. In this model, an adjustment of failure criterion of CSCT was made to obtain characteristic values for the punching shear. The value of the punching resistance, without shear reinforcement, is calculated through the Equation 10:

$$V_R = k_\psi \sqrt{f_c} b_0 d \tag{10}$$

Where k_ψ is a factor accounting for opening and roughness of cracks, calculated by the Equation 11:

$$k_\psi = \frac{1}{1.5+0.9\psi d k_{dg}} \leq 0.6 \tag{11}$$

where k_{dg} is parameter related to the aggregated size, calculated by the Equation 12:

$$k_{dg} = \frac{32}{16+d_g} \tag{12}$$

The MC 2010 [5] presents four Levels of Approximation (LoA) for calculating ψ, the higher the level, the better is the slab rotation estimation, and the more complex is the assessment. In this study, the LoA II was used and can be determined according to Equation 13:

$$\psi = 1.5 \left(\frac{r_s f_y}{d E_s}\right) \left(\frac{m_s}{m_R}\right)^{\frac{3}{2}} \tag{13}$$

Where m_S is the average moment per unit length for calculating flexural reinforcement in the support strip and m_R is the average flexural strength per unit length in the support strip.

2.6 Numerical Model

A nonlinear model in finite elements was elaborated using the ANSYS APDL software to simulate the punching strength of flat slabs without shear reinforcement.

The element SOLID186 was adopted for the concrete modeling, being a three-dimensional quadratic element with 20 nodes and having three degrees of freedom each node (translation according to the axes X, Y and Z). The element REINF264 was used in an incorporated model for the steel bars [10].

The constitutive model of concrete is part of the inner library of ANSYS software, and it is called Drucker-Prager Concrete. This model presents a failure surface for the tension behavior and in tension-compression and a yield surface for compression behavior. The Rankine surface was used for the tension behavior and in tension-compression, on the other hand, for the yield surface in compression behavior, a Drucker-Prager surface was used [10].

In the DP-concrete model the material is admitted as linear elastic until the initial yield surface is reached. After this point, the hardening law, which determines how the yield surface moves, becomes effective. The hardening laws depend on the HSD (hardening, softening, dilatation) model used.

With regard to the tensile concrete behavior, it is admitted that the material has an elastic linear behavior until the failure surface is reached, when cracking happens. This phenomenon is introduced through the tensile softening law, which also depends on the HSD model used. After the state of stress of a point reaching the failure surface, this surface starts to move according to the law adopted, which is applied in relation to the effective stress by effective plastic deformation. In this way, it is possible to simulate the tension stiffening effect [10].

The ANSYS software provides four HSD models, being: exponential, steel reinforcement, fracture energy and linear [10]. After some tests with the models mentioned, it was concluded that the best model for this study was HSD6, which represents the linear model. This model, despite being the simplest, has presented excellent results in the analyzed slabs in the study. The Figure 1a shows the hardening function in compression and the Figure. 1b the softening function in tension of the HSD model.

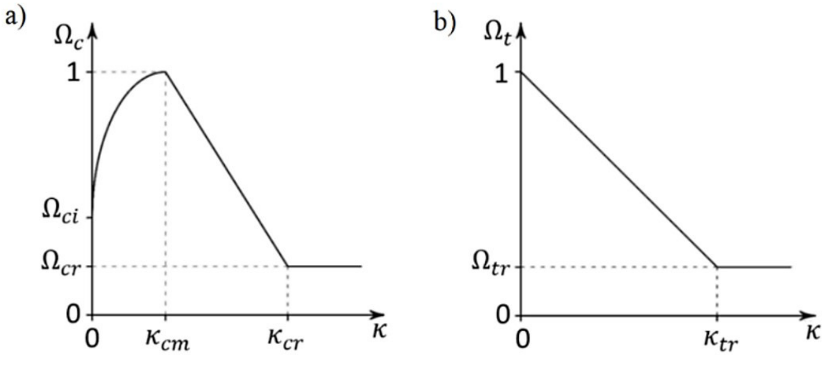

Figure 1. Hardening and softening functions in compression and tension [10]

The hardening law function of this model is show in the Equation 14.

$$\Omega_c = \begin{cases} \Omega_{ci} + (1-\Omega_{ci})\sqrt{2\frac{k}{k_{cm}} - \frac{k^2}{k_{cm}^2}} \ for\ 0 < k \le k_{cm} \\ 1 - \frac{1-\Omega_{cr}}{k_{cr}-k_{cm}}(k - k_{cm})\ for\ k_{cm} < k < k_{cr} \\ \Omega_{cr}\ for\ k \ge k_{cr} \end{cases} \tag{14}$$

The softening law adopted by this study was the linear law, with the function presented in Equation 15.

$$\Omega_t = \begin{cases} 1 - (1-\Omega_{tr})\frac{k}{k_{tr}}\ for\ 0 < k \le k_{tr} \\ \Omega_{tr}\ for\ k \ge k_{tr} \end{cases} \tag{15}$$

In all models considered in this study, the value of Ω_{ci}, Ω_{cr}, $\Omega_{tr}$and k_{tr} was 40%, 10%, 2%. For k_{cm} the Equation 16 was adopted:

$$k_{cm} = \varepsilon_{c1} + \frac{f_c}{E_c} \tag{16}$$

With ε_{c1} is the strain at maximum compressive stress defined by MC 2010 [5].

For k_{cr} the Equation 17 was adopted:

$$k_{cr} = \varepsilon_{c,lim} - \Omega_{cr}\frac{f_c}{E_c} \tag{17}$$

With $\varepsilon_{c,lim}$ is the ultimate strain defined by MC 2010 [5].

For the representation of the steel behavior the perfect elastic-plastic model was adopted. The material can be represented by the ANSYS model, called BISO (Bilinear Isotropic Hardening), using two entry parameters: the initial yield strength (f_y) and the hardening module (E_t) [10]. To represent the perfect elastic-plastic behavior, E_t was defined as 1% of the steel elastic modulus.

In order to save computational time significantly, a quarter of full flat slabs have been modeled (considering plane of symmetry). In the nonlinear incremental analyzes, the external load was applied by increasing the displacement.

In the analysis, a finite element mesh which presents a good behavior for different geometries in the analyzed slabs was used. The adopted mesh was approximately 100 mm. This mesh has shown itself as adequate to all the studied slabs. The geometry of the numerical model, with the boundary conditions, the point of application of the displacements are presented in Figure 2. The validation of the model was performed through the comparison of the numerical results with experimental values, as presented in the following items.

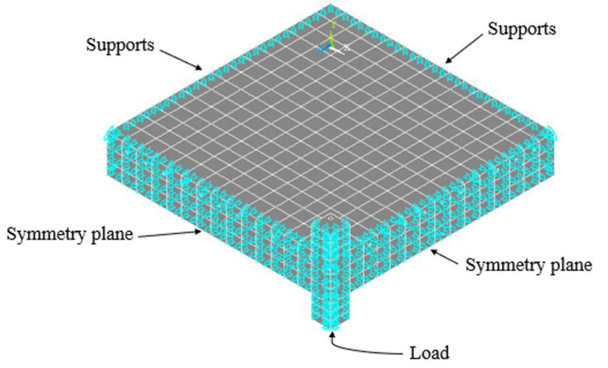

Figure 2. Geometry and boundary conditions

3. MODEL ERROR

Aiming to evaluate the results of the resistance models with experimental results, the random variable resistance model error ($E_{m,R}$) was studied. In this work, the experimental failure loads are admitted as exact, thus, the failure loads obtained by the models, predicting the ultimate strength. In none of the models the safety factors were used. The samples of the random variable E_R are obtained through the Equation 18.

$$E_{m,R} = \frac{V_{exp}}{V_R} \tag{18}$$

Where V_{exp} is the failure load obtained by experimental tests and V_R is the punching capacity obtained by the models for predicting punching shear capacity.

3.1 Experimental results from the literature

A review of the primary research developed until the present moment was made. With the results of this review, it was possible to verify the accuracy of the punching resistance evaluation methods and to perform a study about the error associated with the models analyzed.

A database of 65 flat slabs without shear reinforcement from the literature was studied. Squared columns support all the slabs. Beyond that, all slabs failed by punching and no standardization was made in the flexural reinforcement ratio (ρ). Table 1 presents the studied researchers, the experimental failure load, and the variable value $E_{m,R}$ of each model in the analyzed database.

Table 1. Test series considered in present study and resistance model error statistics

Ref.	Slabs	V_{exp} (kN)	V_{exp}/V_R NBR	EC2	ACI 318	MC 2010	CSCT	NLFEA
[11]	A-1b	365	0.96	1.10	1.24	1.18	1.03	1.02
	A-1c	356	0.89	1.03	1.13	1.10	0.96	0.98
	A-1d	351	0.81	0.94	0.99	1.00	0.87	0.90
	A-1e	356	1.01	1.16	1.35	1.24	1.08	1.00
	A-2b	400	0.94	1.17	1.62	1.21	1.06	1.02
	A-2c	467	0.88	1.10	1.37	1.09	0.95	0.87
	A-7b	512	1.06	1.33	1.73	1.34	1.17	1.11
	A-3b	445	0.87	1.24	1.67	1.17	1.03	0.94
	A-3c	534	0.99	1.41	1.85	1.31	1.15	1.03
	A-3d	547	0.93	1.33	1.66	1.19	1.05	0.93
	A-4	400	0.89	1.03	1.05	1.07	0.93	0.96
	A-5	534	0.95	1.19	1.42	1.15	1.00	0.86
	B-9	505	0.97	1.13	1.36	1.18	0.97	0.95
	B-14	578	0.93	1.24	1.45	1.13	0.95	1.03

Table 1. Continued…

Ref.	Slabs	V_{exp} (kN)	V_{exp}/V_R NBR	EC2	ACI 318	MC 2010	CSCT	NLFEA
[12]	I/2	176	0.91	1.18	1.28	1.40	1.22	1.09
	II/1	825	1.00	1.00	1.16	1.26	1.10	0.92
	II/2	390	1.04	1.17	1.37	1.32	1.15	1.03
	II/3	365	0.96	1.08	1.27	1.33	1.16	0.95
	II/4	117	1.01	1.40	1.65	1.34	1.18	1.16
	II/5	105	0.90	1.25	1.46	1.28	1.12	1.03
	II/6	105	0.89	1.23	1.42	1.33	1.16	1.01
[13]	HS2	249	0.91	1.11	0.96	1.22	1.06	1.02
	HS3	356	1.09	1.33	1.38	1.39	1.21	0.90
	HS4	418	1.16	1.46	1.79	1.48	1.30	1.05
	HS7	356	1.17	1.43	1.33	1.53	1.33	1.03
	HS8	436	1.02	1.17	1.22	1.25	1.09	0.98
	HS9	543	1.08	1.24	1.46	1.31	1.15	0.98
	HS10	645	1.12	1.30	1.67	1.36	1.19	1.10
	HS12	258	1.15	1.54	1.45	1.60	1.39	1.13
	HS13	267	1.12	1.50	1.58	1.52	1.33	0.89
	HS14	498	1.30	1.59	1.47	1.60	1.40	0.97
	HS15	560	1.27	1.55	1.33	1.54	1.34	1.11
	NS1	320	1.15	1.41	1.59	1.47	1.28	0.98
[14]	ND65-2-1	1200	1.09	1.09	1.53	1.38	1.21	1.00
	ND95-1-1	2250	1.15	1.15	1.45	1.40	1.22	0.99
	ND95-1-3	2400	1.01	1.09	1.49	1.23	1.07	0.87
	ND95-2-1	1100	0.92	0.92	1.25	1.17	1.03	0.86
	ND95-2-1D	1300	1.10	1.10	1.50	1.39	1.22	1.01
	ND95-2-3	1450	1.05	1.15	1.64	1.34	1.18	0.97
	ND95-2-3D	1250	0.94	1.03	1.49	1.21	1.06	0.86
	ND95-2-3D+	1450	1.02	1.11	1.57	1.30	1.14	0.94
	ND95-3-1	330	1.03	1.29	1.62	1.30	1.14	1.10
	ND115-1-1	2450	1.14	1.14	1.36	1.39	1.21	1.01
	ND115-2-1	1400	1.06	1.06	1.38	1.35	1.18	0.97
	ND115-2-3	1550	1.06	1.15	1.60	1.34	1.17	0.97
[15]	PG-1	1023	1.08	1.08	1.48	1.41	1.23	1.11
	PG-11	763	0.97	0.97	1.03	1.31	1.14	0.98
	PG-6	238	0.87	1.07	1.40	1.19	1.04	0.84
	PG-7	241	1.05	1.27	1.33	1.43	1.24	1.11
[16]	OC11	423	0.87	0.99	1.30	1.19	1.04	1.12
[17]	1	560	0.84	0.89	1.16	1.14	1.00	0.91
	1A	587	0.87	0.93	1.20	1.19	1.04	0.95
[18]	L1	316	1.15	1.35	1.77	1.48	1.30	1.08
[19]	M1	441	0.99	1.09	1.48	1.28	1.12	1.15
[20]	L1	309	1.11	1.38	1.63	1.47	1.29	1.00
[21]	SB1	253	0.98	1.23	1.35	1.32	1.06	0.99
[22]	PT31	1433	1.09	1.09	1.32	1.42	1.27	0.97
[23]	LR	232.3	0.84	1.04	1.28	1.11	0.97	0.89
[24]	LR_A	249.9	0.89	1.12	1.37	1.24	1.08	1.01
	LR_B	216.4	0.78	0.98	1.20	1.09	0.95	0.85
	LR_C	259.2	0.92	1.15	1.40	1.28	1.11	0.98
[25]	L0-01	571	0.84	0.92	1.02	1.00	0.87	1.12
[26]	LS-05	779	1.07	1.17	1.30	1.34	1.17	1.05
[27]	PG19	860	0.97	0.97	0.99	1.30	1.13	0.97
	PG20	1094	0.98	0.98	1.23	1.24	1.08	0.96
		Mean	1.00	1.17	1.40	1.29	1.13	0.99
		CoV	0.11	0.14	0.15	0.11	0.11	0.08

3.2 Resistance model error

The Figure 3 presents the relation between V_{exp} and the V_R obtained with the six models studied. It is possible to observe an excellent approach between the experimental results and the theoretical models CSCT, ABNT NBR 6118:2014, EC2 and NLFEA.

A bivariate correlation analysis was performed in order to verify the dependence of the resistance model error in relation to the main variables of the resistance model. The adopted variables were the concrete compressive strength (f_c), the flexural reinforcement ratio (ρ), the steel yield strength (f_y), the column dimension (c), the effective depth of the slab (d) and the relation between the column dimension and the effective depth of the slab (c/d).

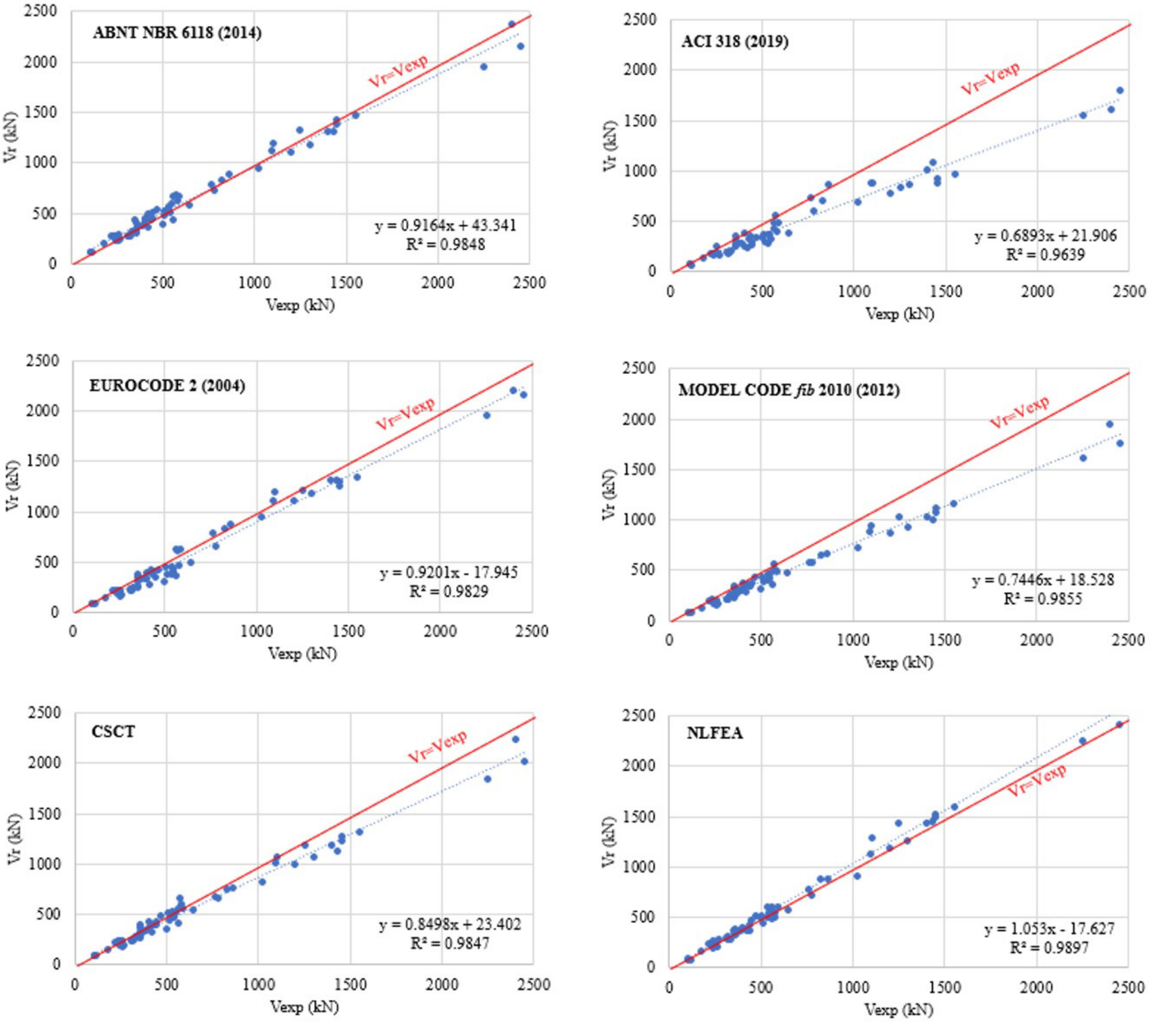

Figure 3. Relation between experimental and predicted value of punching shear strength

The evaluation of the correlation for all the models is made considering a level of significance α=5%, and the results of the significant variables are different for each model studied. From the observation of the Figure 4, it is possible to notice that samples presented correlations which are not much significant. In this sense, for the present study, two methods were used to characterize the variable $E_{m,R}$. The first of them is the adjustment of the distribution of probability for the data obtained and the second is based on generalized linear models (GLM) with distribution Gama and logarithmic link function.

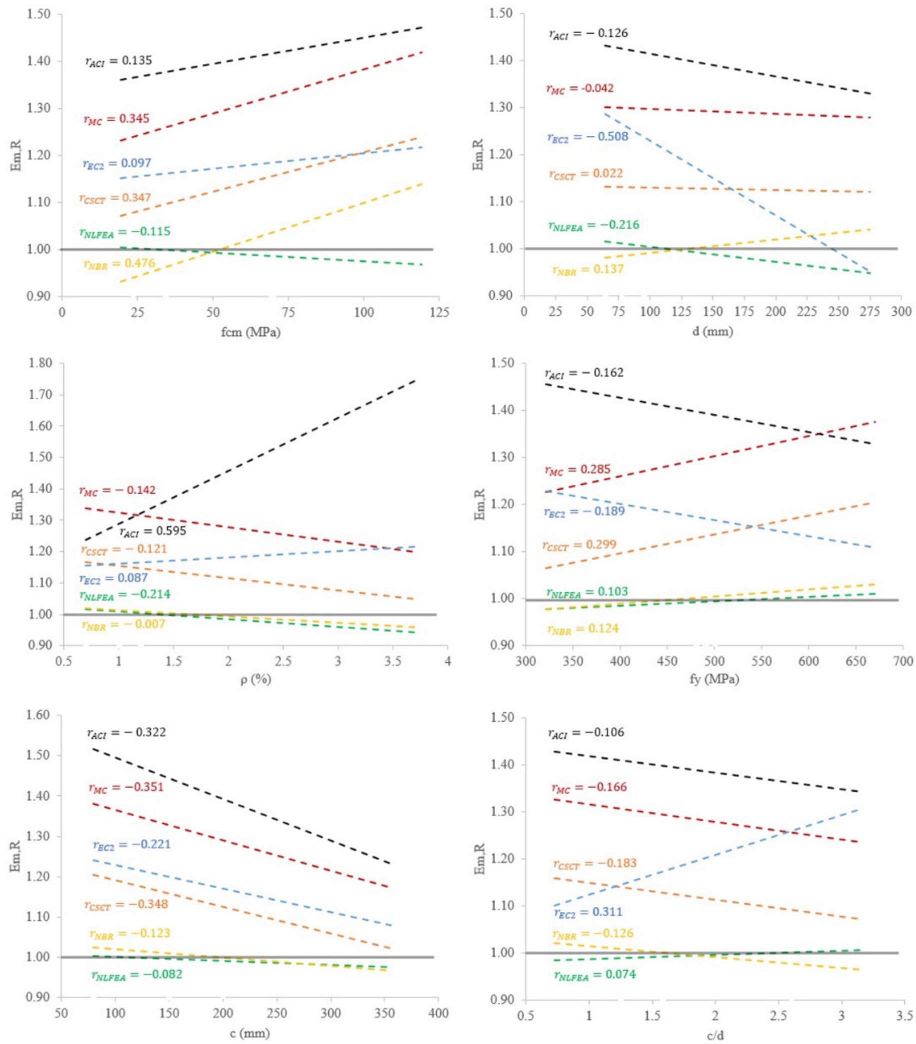

Figure 4. Correlations analysis for the predictive models

Only the significant variables for the elaboration of the GLM were considered in each resistance predictive model. In this way, the GLM in its general format is given by the Equation 19:

$$E_{m,R} = \exp\left(\beta_0 + \beta_1 f_{cm} + \beta_2 \rho + \beta_3 c + \beta_4 d + \beta_5 \frac{c}{d} + \beta_6 f_y\right) + \varepsilon \tag{19}$$

In Equation 19, β_0 to β_6 are constants and ε describes the random part of the resistance model error. The residuals were fitted to normal distributions. The results of the parameters are summarized in the Table 2.

Table 2. Resistance model error parameters.

Model	β_0	β_1	β_2	β_3	β_4	β_5	β_6	$\varepsilon \sim N(\mu,\sigma)$	
								μ	σ
ABNT NBR 6118	-0.1086	0.0021	-	-	-	-	-	0.0	0.0976
EUROCODE 2	0.3255	-	-	-	-0.0013	0.0045	-	0.0	0.1456
ACI 318	0.3162	-	0.1444	-0.0012	-	-	-	0.0	0.1394
CSCT	0.0350	0.0010	-	-0.0003	-	-	0.0002	0.0	0.1111
MC 2010	0.1892	0.0010	-	-0.0004	-	-	0.0002	0.0	0.1256
NLFEA	0.0658	-	-0.0214	-	-0.0003	-	-	0.0	0.0777

4 RELIABILITY ANALYSIS

4.1. Resistance and Load variables

The random variables adopted for this model were the concrete compressive strength, yield strength of steel, slab thickness, effective cover of reinforcement and the resistance model error. The random variables adopted to determine the load in structure were the dead load (D_n), live load (L_n), and load model error. The probability distribution, mean value, and coefficient of variation of these variables are shown in Table 3.

Table 3. Random variable data for reliability analysis.

Random variable	Distribution	Mean	Cov.	Reference
Concrete compressive strength (f_c)	Normal	$1.22f_{ck}$ *(MPa)*	0.15	[28]
Clear cover of reinforcement (d')	Normal	d' (mm)	0.125	[29]
Slab thickness (h)	Normal	h (mm)	0.4/h+0.006	[30]
Steel yield strength (fy)	Normal	$1.12f_{yk}$ *(MPa)*	0.05	[30]
Residual of the resistance model error (ε)	Normal	Following Table 2		
Dead load (D)	Normal	$1.06D_n$	0.12	[31]
Live load (L)	Gumbel	$0.92L_n$	0.25	[31]
Load model error ($E_{m,S}$)	Lognormal	1.00	0.10	[30]

4.2. Description of the studied slabs

A squared slabs (17 x 17 m) with thickness of 180 mm were designed, being the internal column the interest region of study, as shown in Figure 5. The characteristic concrete compressive strength adopted was 30 MPa. The concrete cover of reinforcement adopted was 15 mm. The dimensions of the columns were determined in a way in which the verification of the punching shear without shear reinforcement was attended in its limit state for each design code ($V_{Sd} = V_{Rd}$). This verification was made taking into consideration the partial load and resistance factors recommended in each design code (Table 4). The reliability study of the punching shear of flat slabs without shear reinforcement was made through the definition of a group of 48 slabs. The following parameters were evaluated: ρ= 0.8, 1.3, 1.8 and 2.3%; L_n/D_n= 0.20, 0.40 and 0.60; flat slab design by ABNT NBR 6118:2014 [2], ACI 318 [3], EC2 [4] and MC 2010 [5].

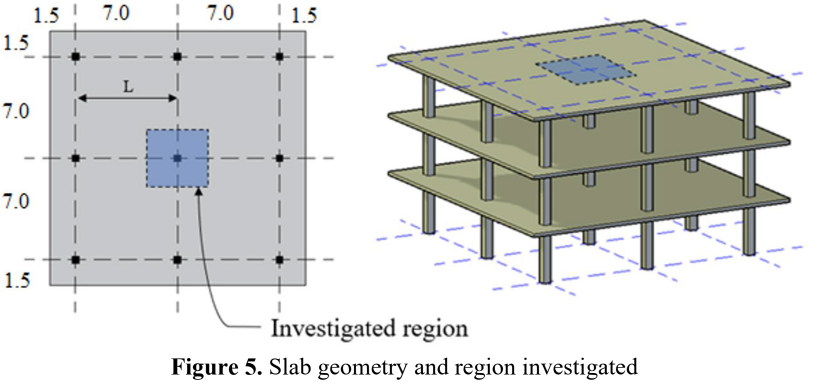

Figure 5. Slab geometry and region investigated

Table 4. Load combinations

Design code	Load combinations
ABNT NBR 6118:2014	$F_{sd} = 1.4D_n + 1.4L_n$
ACI 318	$F_{sd} \geq \begin{cases} 1.4D_n \\ 1.2D_n + 1.6L_n \end{cases}$
EC 2	$F_{sd} \geq \begin{cases} 1.15D_n + 1.50L_n \\ 1.35D_n + 1.05L_n \end{cases}$
MC 2010	$F_{sd} = 1.15D_n + 1.50L_n$

4.3. Limit state function

To assess the punching failure mode in flat slabs, the following equation of limit state was used:

$$g(\boldsymbol{X}) = E_{m,R} V_R(\boldsymbol{R}) - E_{m,S} V_S(D, L) \quad (20)$$

Where $\boldsymbol{X}$ is the vector of the random variables of the problem, V_R is the function of random variable that represents the resistance model, $\boldsymbol{R}$ is the vector of the random variables of resistance, V_S is the function of random variable that represents the load model, D is the random variable related to the dead load, L is the random variable related to the live load, $E_{m,R}$ is the resistance model error variable, $E_{m,S}$ is the load model error variable.

The punching shear strength is evaluated by the ABNT NBR 6118:2014 [2] resistance model. For this resistance model the slab was considerate an isolated element, delimited by the line of contraflexure radial moments which are zero at a distance rs ≈ 0.22L. The load is evaluated through the determination of the reaction of the column in the slab using a linear model of finite elements developed in ANSYS software. The reliability index is calculated using limit state Equation 20. Reliability indexes are evaluated using the First Order Reliability Method (FORM). The reliability analysis is made through routines developed in the programming language Python by the authors.

5. RELIABILITY ANALYSIS RESULTS

The reliability results were evaluated considering the reliability index β in terms of sufficient and uniform reliability criteria. The target reliability index adopted was 3.8. This index was based in the MC 2010 [5] criteria and corresponds to the value for ultimate limit state verification in case of medium consequence of failure and reference period of 50 years. The reliability index β for flat slabs without shear reinforcement oscillated between a minimum of 2.42 for the slab design by the ACI 318 [3] and a maximum of 5.58 for the slab design by the MC 2010 [5]. The results for each code are summarized in Table 5.

Table 5. Summary of reliability index results

Design code	βmin	βmax	βrange
ACI 318	2.42	4.19	1.77
NBR 6118:2014	3.23	3.50	0.27
MC 2010	4.23	5.58	1.35
EC2	3.51	3.85	0.34

The range of reliability indexes obtained for each code in function of the load ratio L_n/D_n is presented in Figure 6. It was observed that the increase in the load ratio contributes to the decrease of the reliability index in three of design codes. This occurs because the live load had a high Cov in comparison to the dead load. For large load ratios, just the slabs designed by MC 2010 [5] achieve the target reliability index. Also, it was observed that the load ratio has no significant impact in reliability of slabs design by ACI 318 [3] and EC 2 [4]. This result can be related to the load combination used by design code, where for large load ratios the variability of live load is equalize by the partial load factors of combination.

In Figure 7, the range of reliability indexes is shown in terms of reinforcement ratio. It was observed that reinforcement ratio has significant impact in the reliability indexes of the slabs design by ACI 318 [3]. The reliability indexes for small reinforcement ratio are well below target reliability indexes. This result can be related with the non-consideration of flexural reinforcement in the punching shear resistance. Research such as Muttoni [7] indicate that for low values of the reinforcement ratio, the ACI 318 [3] presents estimated resistance values greater than those observed in experimental tests, corroborating with the results presented here. In addition, the low values of the ACI 318 [3] codes may be associated with the target reliability index used in the calibration. According to Beck et al. [32] the American code was calibrated for a target reliability index equal to 3.0.

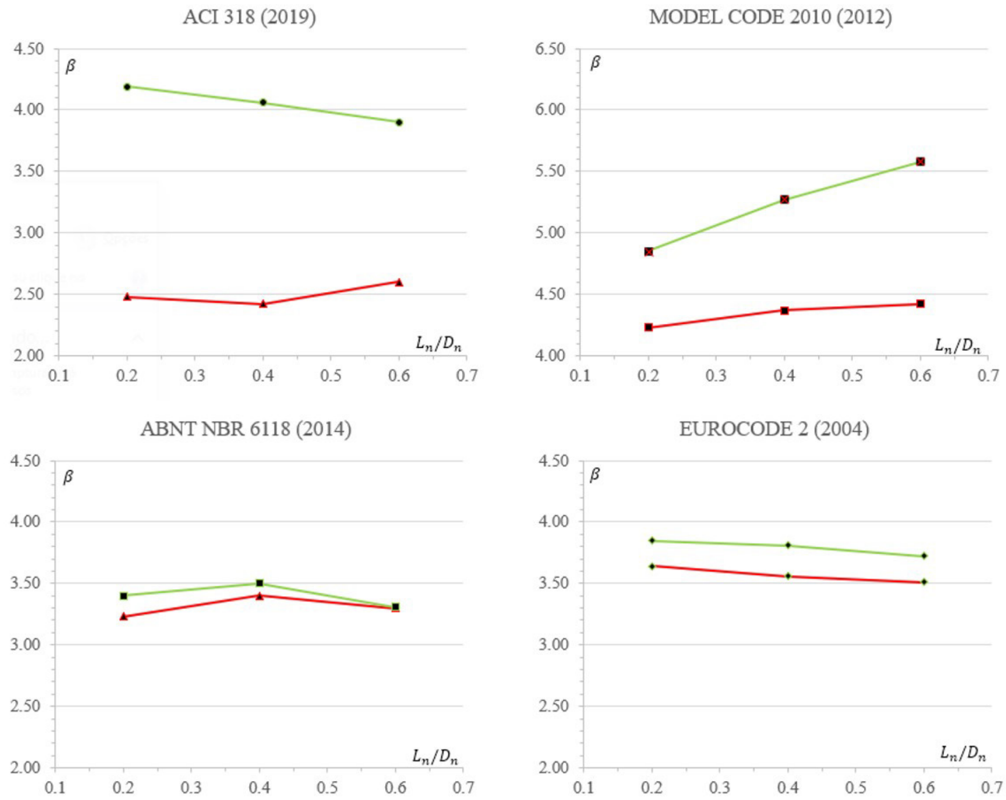

Figure 6. Reliability index bounds of code designed as function of load ratio

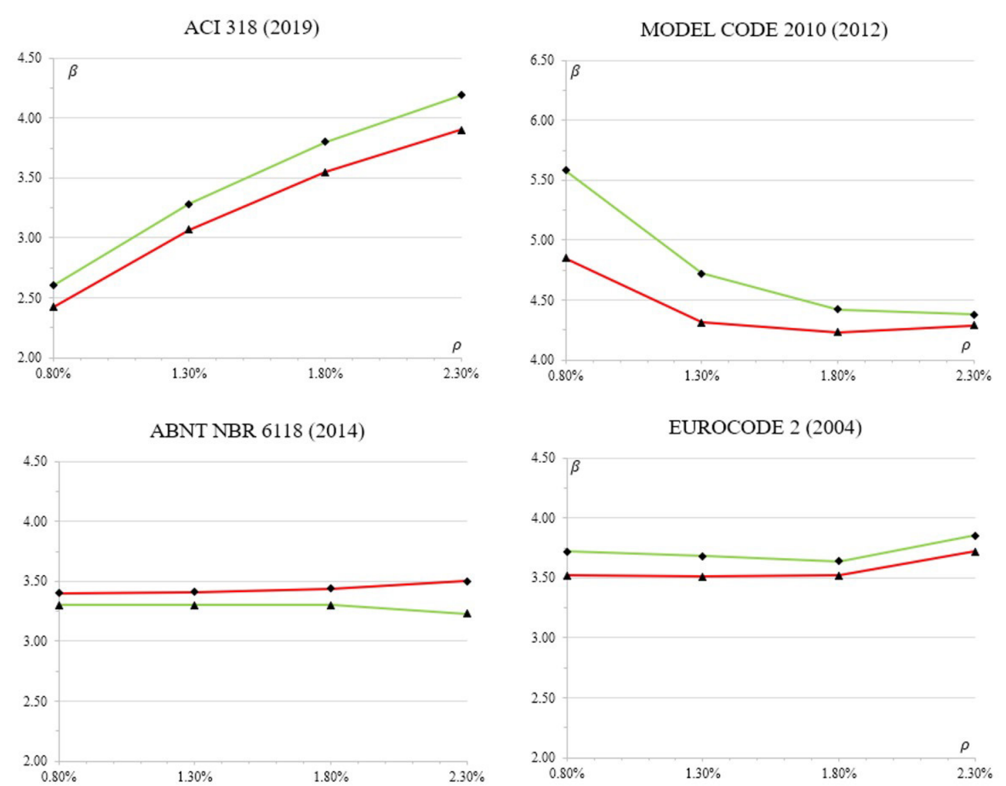

Figure 7. Reliability index bounds of code designed as function of reinforcement ratio

6 CONCLUSIONS

This study evaluated the reliability of the internal column-slab connection of flat slabs without shear reinforcement. Six models of predicting punching shear were analyzed, and the resistance error model was determined using a GLM based on 65 experimental tests. In this paper, the NLFA model best predicted the punching shear strength.

For the study of reliability, 48 flat slabs were designed. The analyzes obtained in this study pointed out that, in general, the reliability indexes decreased with the increase of the load ratio, and the reinforcement ratio has significant impact in the reliability indexes of the slabs design by ACI 318 [3].

The target reliability index adopted was 3.8. All slabs designed by MC 2010 [5] reach the target reliability index provided the best results in terms of sufficient reliability with a reliability index higher than 4.23 and smaller than 5.58. The ABNT NBR 6118:2014 [2] provided the best results regarding the uniform reliability with a range of 0.27.

The slabs designed by ACI 318 [3] achieved the smallest target reliability index adopted in this study. The reason for that is that the American codes were calibrated for a target reliability index $\beta T= 3.0$ [32], and for the EC2 [4] and MC 2010 [5], a target reliability index $\beta T= 3.8$. The authors found no studies on the calibration of ABNT NBR 6118:2014 [2].

This paper also showed the influence of the flexural reinforcement ratio on the estimation of punching shear resistance. For the ACI 318 [3] that does not consider this parameter in the determination of resistance, the reliability values obtained are below 3.0, indicating a safety level below the appropriate level defined for this code. This tendency to decrease the level of security, occurs for low flexural reinforcement ratios. The American code provides a minimum bending reinforcement in order to adjust the punching shear resistance estimate obtained by the code. However, the observed reliability results for the flexural reinforcement ratio equal 0.8% was not achieved the minimum expected with the code calibration.

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Author contributions: GRS: conceptualization, formal analysis, methodology, writing; ACF: conceptualization, methodology, supervision; MVR: conceptualization, methodology, supervision.

Editors: Leandro Trautwein, Mario Pimentel