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An improved framework for volume change of shrink/swell soils subjected to time-varying climatic effects

Austin H. Olaiz[1#], Mohammad Mosawi[1], Claudia E. Zapata[1]

Article

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Abstract

The ability to estimate soil volume change as a function of time is a valuable tool in the design or forensic analysis of shallow foundations and pavement structures. This paper presents an improved framework for estimating the volume change of shrink/swell soils due to time-varying climatic effects using the Lytton et al. (2005) approach with the suction envelope models created by Vann & Houston (2021) and updated considerations of short-term varying climate. The procedure can be easily implemented in any country due to its mechanistic-empirical nature. The authors present an example calculation of the proposed framework using the data from an American Association of State Highway and Transportation Officials (AASHTO) Long-Term Pavement Performance (LTPP) Seasonal Monitoring Program (SMP) section, located approximately 80 miles northeast of Dallas, Texas. The volume change estimated from the proposed framework was compared to 70 measured data points from sections from the SMP study and the results look promising. The models are universal and can be used in any part of the world provided measured data is available to calibrate for local conditions. Ongoing calibration effort with the remaining LTPP SMP sections will allow obtaining calibration factors for the proposed framework that will improve the estimation of the volume change predictions under pavements and facilitate the implementation into current design procedures.

1. Introduction

The ability to estimate soil volume change as a function of time is a valuable tool in the design of shallow foundations of pavement structures. Specifically pertaining to pavement design, estimating soil volume change as a function of time allows for the prediction of the potential cumulative International Roughness Index (IRI). The time-varying volume change can also be a valuable tool in the forensic analysis of existing foundation movement of a lightly loaded structure on shallow footings.

This paper presents an improved framework for estimating the volume change of shrink/swell soils due to time-varying climatic effects using the Lytton et al. (2005) approach with the suction envelope models created by Vann & Houston (2021). The proposed framework for estimating soil volume change of shrink/swell soils as a function of time is presented as an example calculation with data from an AASHTO Long-Term Pavement Performance (LTPP) Seasonal Monitoring Program (SMP) section TX 48-1068 (FHWA, 1995), which is located approximately 80 miles northeast of Dallas, Texas. The SMP study includes measured data from 1/1994 to 9/1997 for the TX 48-1068 section. The construction date of the TX48-1068 section is 3/1987. As such, the example calculation will use the time frame of 3/1987 to 9/1997 so that a comparison of predicted and measured volume change can be performed.

2. Volume change of shrink/swell soils

The determination of the magnitude of potential soil volume change is a key focus of geotechnical engineering as it causes significant infrastructure damage each year. Studies have been published, which empirically relate soil index properties (Atterberg limits, gradation, mineralogy, etc.), along with soil engineering properties (density, moisture content, swell pressure, etc.), to volume change.

Direct laboratory measurements of the volume change potential of a soil helps improve the estimation of potential volume change in the field. The 1-D oedometer "Response to Wetting Test" as described in ASTM D4546 (ASTM, 2021) is the common type of laboratory test for volume

[#]Corresponding author. E-mail address: Austin.Olaiz@asu.edu
[1]Arizona State University, School of Sustainable Engineering and The Built Environment, Tempe, AZ, USA.

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change determination. One key difference from the laboratory oedometer test compared to the field conditions the soil will experience is the final degree of saturation. The response to wetting test inundates the sample, driving to almost full saturation. However, it is the probability that the soil will reach this moisture level over the period of the structure/pavements design life is very low (Houston & Houston, 2017).

A common method for volume change estimation is the Potential Vertical Rise published by the Texas Department of Transportation (TxDOT, 1978), which includes both empirical-based relationships and result from an oedometer test. In 2005, the Texas DOT updated the approach to determining the volume change of expansive soils using the work by Lytton et al. (2005), which encompassed a suction-based approach. The study concluded that the previous empirical-based approach significantly over-estimated the soil heave and did not account for the shrinkage of the soil during dry climatic periods.

A thorough literature review of volume change estimates of unsaturated soil (oedometer-based, or suction-based) was performed by Vann (2019). The authors of this paper have carefully reviewed this relative literature summary as part of the research leading up to this paper.

The suction-based approach by Lytton et al. (2005) for estimating the volume change of shrink/swell soils which was adopted by the Texas DOT and the Post-Tensioning Institute for the design of slabs on ground (PTI, 2004, 2008), was the compilation of efforts of several related studies including: Lytton (1977), McKeen & Hamberg (1981), Holtz & Gibbs (1956), Covar & Lytton (2001), Lytton et al. (2004). The approach encompasses the volumetric strain caused by changes in both stress states of the soil (matric suction and net normal stress) and uses the closed-form solution of the Richard's unsaturated moisture flow equation (Bear, 1972) developed by Mitchell (1979) to estimate the soil volume-change as a function of time when no groundwater table is present. The relationship between the change in each stress state and the volumetric strain, referred to as the compression indices, must be directly measured or empirically determined.

One limitation of the Mitchell equation (1979) used in the Lytton et al. (2005) framework is that the climate boundary condition is assumed and modelled to be a sinusoidal pattern. Aubeny & Long (2007) proposed an improvement to this limitation by fitting a Fourier equation to the climate data. The Fourier fit to the climate data allows for the irregularities of climate data to be captured and allows for the development of asymmetrical suction profiles. Aubeny & Long (2007) also studied the uncertainty of the key variables required in the Mitchell's equation (1979) for the change in soil suction with time. The study concluded that the diffusion coefficient (α) had significant ranges for a given soil, low reproducibility, and discrepancies between lab and field measurements, and was dependent upon the number of climatic cycles per year (n) when performing a back-calculation from the depth of equilibrium suction.

Vann & Houston (2021) developed correlations between the 30-year Thornthwaite Moisture Index (Thornthwaite, 1948; Witczak et al., 2006) and soil suction envelopes using measured data from over 40 geotechnical studies (Vann, 2019). The suction envelope correlations to the *TMI* allow for key aspects of the Aubeny & Long (2007) approach to estimating volume change of shrink/swell soils as function of time to be determined without the need to measure or estimate the diffusion coefficient or number of climatic cycles per year.

This paper presents an improved framework for estimating the volume change of shrink/swell soils due to time-varying climatic effects. This framework builds upon the work presented by Lytton et al. (2005) and incorporates the latest suction envelope models proposed by Vann & Houston (2021). The framework presented is applicable to uncovered sites where the groundwater table effects are negligible, but it has been calibrated to account for covered areas and for the spatial variation between the pavement center and edges.

3. Framework outline

The following outline summarizes the steps of the improved framework for estimating the volume change of shrink/swell soils due to time-varying (monthly) climatic effects:

1. Weather station identification and data extraction
2. 30-year and monthly Thornthwaite Moisture Index per Witczak et al. (2006)
3. Determination of suction envelope parameters per Vann & Houston (2021)
    a. Depth to stable suction
    b. Magnitude of stable suction
    c. Limits of suction variation at the surface
    d. Climatic parameter (r)
4. Back-calculation of variables for Mitchell (1979) equation
5. Development of long-term wet and dry suction profiles
6. Initial estimation of monthly changes in suction at the surface per Perera (2003)
7. Adjustment to estimation of monthly changes in suction using limits of suction variation at the surface from Vann & Houston (2021)
8. Fourier equation fit to the monthly suction change at the surface per Aubeny & Long (2007)
9. Generation of monthly suction profile per Aubeny & Long (2007)
10. Suction profile adjustments for varying surface boundary conditions
11. Determination of net normal stress profile
12. Estimation of suction compression index (assuming value is not directly measured)
13. Calculation of strain monthly
14. Calculation of volume change monthly

3.1 Step 1: climate data

A Season Monitoring Program (SMP) pavement section approximately 80 miles northwest of Dallas, Texas (TX 48-1068) is used to provide an example for the proposed framework (FHWA, 2021). For the purposes of this example calculation, the climate data was gathered from the weather station nearest to the site and identified using the open-access Thornthwaite Moisture Index (*TMI*) GIS map developed by Olaiz (2017), which uses the National Oceanic and Atmospheric Administration's (NOAA) 30-year climate normal database for the United States. Figure 1 presents an excerpt for the GIS map, which has the Paris, TX weather station selected.

The "Station ID" shown in the pop-up window in Figure 1 (USC00416794) is the only data needed for the purposes of this study. However, the remaining data shown may be helpful to get an understanding of the general climatic conditions at the site.

The NOAA climate data associated with each station in the country can be extracted from the online NOAA FTP site. It is recommended that the extracted weather data be filtered to contain the necessary variables for computation of the Thornthwaite Moisture Index (Witczak et al., 2006):

- Year
- Month
- Monthly Precipitation (cm)
- Monthly Average Temperature (Celsius)

Note that the Vann & Houston (2021) models used in the proposed analysis correlate the suction envelope parameters to a 30-year *TMI* value. As such, the climate data from the NOAA database for station USC00416794 was extracted for the date range of 9/1967 to 9/1997 (the last date of measured data from the SMP study for the TX 48-1068 section).

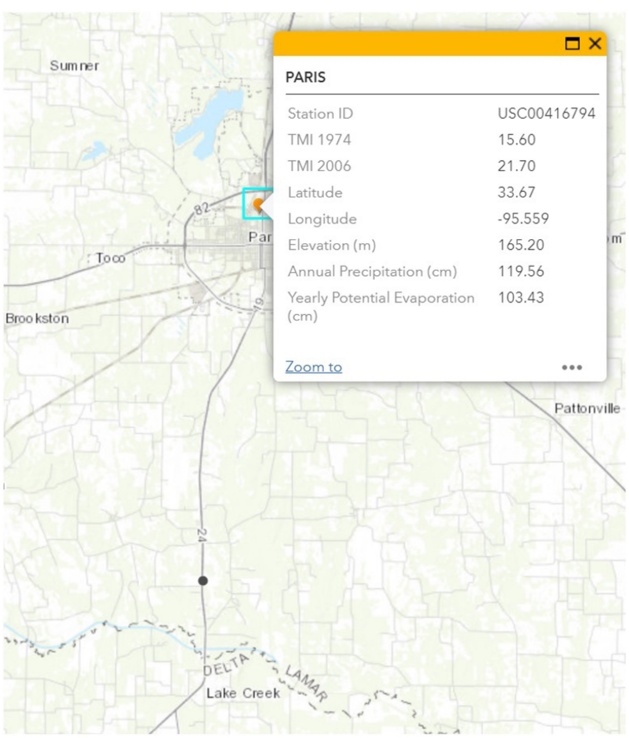

Figure 1. Paris (Texas) weather station (NOAA ID USC00416794) data from online *TMI* GIS map (Olaiz, 2017).

3.2 Step 2: monthly and 30-year Thornthwaite Moisture Index (TMI) (Witczak et al., 2006)

To determine yearly *TMI* for each month, first the potential evapotranspiration (*PET*) for each month must be calculated:

$$PET(cm) = f_1 f_2 1.6\left(\frac{10t}{I}\right)^a \tag{1}$$

where, f_1 is the fraction of the number of days in month divided by the average number of days in month, 30; f_2 is the fraction of the number of hours in a day divided by the base of 12 h in a day; t is the mean monthly temperature in degrees Celsius; I is the annual heat index; and a is a coefficient.

$$I = \sum_{i=1}^{12}\left(\frac{t_i}{5}\right)^{1.514} \tag{2}$$

where, t_i is the mean temperature for the ith month, and

$$\begin{aligned} &a = 6.75I^3x10^{-7} - 7.71I^3x10^{-5} \\ &1.792Ix10^{-2} + 0.49239 \end{aligned} \tag{3}$$

The *TMI* (Witczak et al., 2006) can now be determined by:

$$TMI = 75\left(\frac{P}{PET} - 1\right) + 10 \tag{4}$$

where, P is the precipitation for the given month.

To visualize the climate data over time, the monthly average temperature, monthly rainfall, and the calculated *TMI* can be plotted (Figure 2). For the example calculation at the TX 48-1068 SMP section, the 30-year weather data was analyzed (9/1967 to 9/1997). For the comparison of measured *versus* predicted data, measured elevation change data was also extracted from the LTPP-SMP database between 3/1987 and 9/1997.

The 30-year *TMI* value (Witczak et al., 2006) calculated from the NOAA data set for the USC00416794 station is +29.6. This value does differ slightly from the +21.7 value previously shown on the *TMI* GIS map (Figure 1) due to the difference in date ranges used in the Olaiz (2017) study.

3.3 Step 3: suction envelope parameters (Vann & Houston, 2021)

The suction envelope defines the maximum and minimum suction values at the surface and within the subsurface to a depth of stable suction. The suction envelopes are established using the following parameters: (1) equilibrium suction, ψ_e,

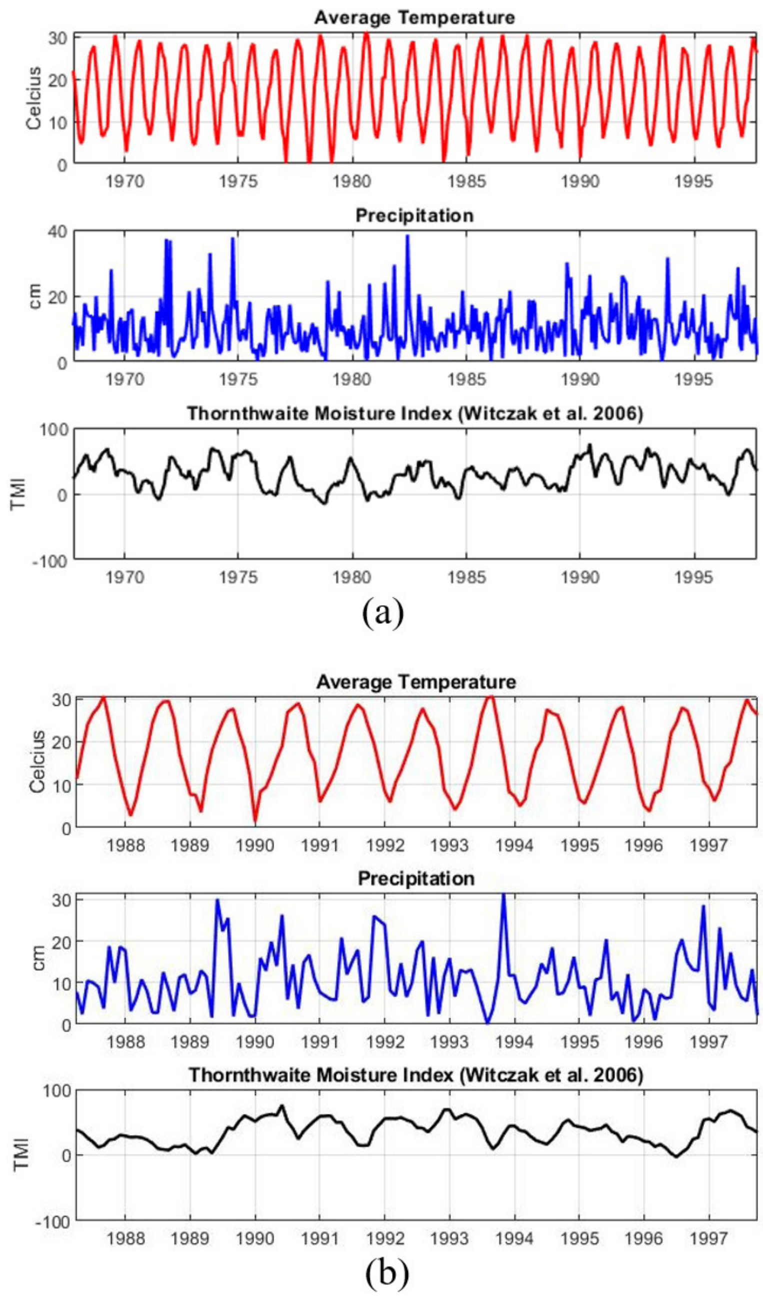

Figure 2. Monthly average temperature and rainfall data for NOAA weather station USC00416794 with the calculated yearly *TMI* (Witczak et al., 2006) between (a) 9/1967 and 9/1997 and (b) 3/1987 and 9/1997.

(2) depth to equilibrium suction, D_{ψ_e}, (3) change in suction at the ground surface, $\Delta\psi$, and (4) climate parameter, r.

The depth at which the climate-driven fluctuations in soil suction begins to stabilize, or equilibrate is determined using the Vann & Houston (2021) model (Figure 3) relatively flat uncovered sites subjected to natural climate surface flux conditions:

The equation for the Depth to Equilibrium Suction (D_{ψ_e}) *versus TMI* regression shown above is:

$$D_{\psi_e} = 1.617 + \frac{2.617}{1 + e^{(2.36 + 0.1612TMI)}} \quad (5)$$

With an $R^2 = 0.9045$ and standard error = 0.3147 m.

The stable, or equilibrium, suction value is determined using the Vann & Houston (2021) model (Figure 4). The soil suction unit of pF (log to the base 10 of soil suction in centimeters of water) was used in the Vann & Houston (2021) study due to its extensive use in the geotechnical practice, with regards to unsaturated soils. Note that log of suction in kPa units is approximately equal to suction in pF units, minus 1 (i.e., 4.0 pF = 3.0 log (suction (kPa))).

The equation for the Equilibrium Suction (ψ_e) as a function of *TMI* is:

$$\psi_e\left(pF\right) = 0.0002TMI^2 - 0.0053TMI + 3.9771 \qquad (6)$$

With an $R^2 = 0.6539$ and a standard error = 0.1959 pF.

The limits of the potential surface flux, or potential change in suction at the surface, is determined using the Vann & Houston (2021) model (Figure 5).

The equation for the potential change in suction at the surface ($\Delta\psi$) as a function of *TMI*, as shown above is:

$$\Delta\psi\left(pF\right) = 1.2109e^{\left(-0.005TMI\right)} \qquad (7)$$

With an $R^2 = 0.9184$ and a standard error = 0.1835 pF.

Aubeny & Long (2007) presented illustrative suction envelopes, developed from unsaturated flow analyses (Mitchell, 1980), to demonstrate that asymmetrical soil suction envelopes are expected, depending on the climate

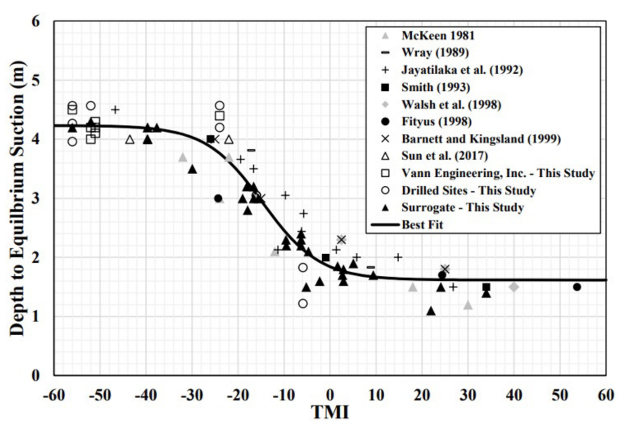

Figure 3. Relationship between *TMI* and the depth to constant soil suction for uncovered and non-irrigated sites (Vann & Houston, 2021).

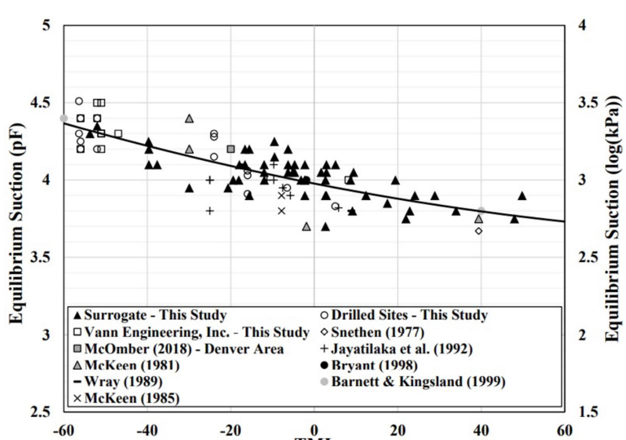

Figure 4. Equilibrium Suction vs. *TMI* with Literature Values (Vann & Houston, 2021).

(*TMI*). Aubeny & Long introduced a climate parameter, *r*, that is the percentage of the total anticipated change in soil suction at the surface, ($\Delta\psi$), comprising the wet side of the suction envelope. The climatic parameter can be expressed in terms of the equilibrium suction (Ψ_e) and the minimum (Ψ_{wet}) and maximum (Ψ_{dry}) suction at the surface ($z = 0$):

$$r = \frac{\left(\psi_e - \psi_{wet_{z=0}}\right)}{\left(\psi_{dry_{z=0}} - \psi_{wet_{z=0}}\right)} = \frac{\left(\psi_e - \psi_{wet_{z=0}}\right)}{\Delta\psi} \qquad (8)$$

Houston & Vann created a relationship between the climatic parameter and *TMI* (Figure 6).

The equation for the climatic parameter (*r*) as a function of *TMI* as shown above is:

$$r = 0.3725e^{\left(-0.009TMI\right)} \qquad (9)$$

With an $R^2 = 0.7998$ and a standard error = 0.1132.

Table 1 presents the suction envelope parameters for the SMP TX 48-1068 section which had 30-year *TMI* of 29.6.

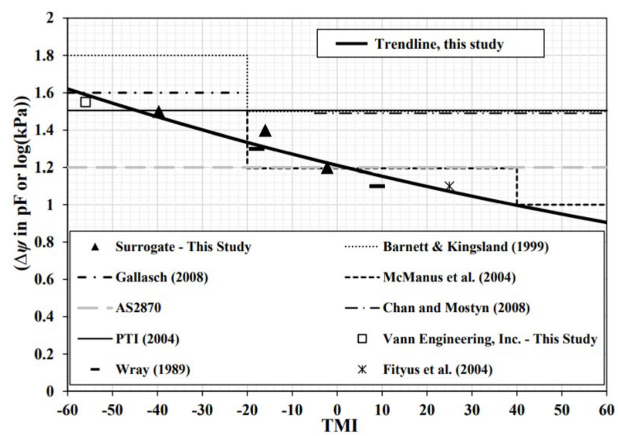

Figure 5. Limits of the potential change in suction at the surface vs. *TMI* with literature values (Vann & Houston, 2021).

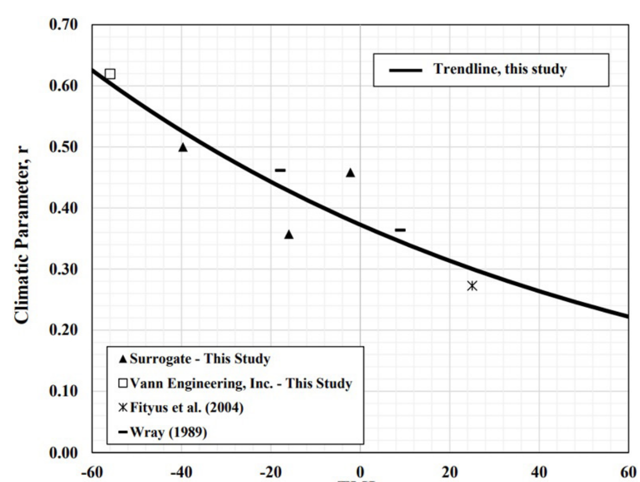

Figure 6. Relationship between the Climate Parameter, *r*, and *TMI* (Vann & Houston, 2021).

Table 1. Suction envelope parameters for the SMP TX 48-1068 per Vann & Houston (2021) with a *TMI* = 29.6.

Suction Envelope Parameter	Value
Depth to Equilibrium Suction ($D_{\O_e}$)	1.62 m
Equilibrium Suction (Ψ_e)	3.84 pF
Change in suction at the surface ($\Delta\psi$)	1.044 pF
Climatic parameter (r)	0.2854

3.4 Step 4: back calculation of variables for Mitchell's equation (1980)

The suction envelope can now be generated using the simplified unsaturated flow equation derived by Mitchell in 1980. The adjustment to the equation by Aubeny & Long (2007) for asymmetrical suction envelopes has also been incorporated into this study.

The Mitchell (1979) equation for change in suction with depth and time, simplified by (Naiser, 1997) to consider only the extreme suction cases (wet and dry), by taking the time variable to infinity, is used to obtain the shape of the envelopes.

$$\psi(z) = \psi_e + \Delta\psi e^{\left(-z\sqrt{\frac{n\pi}{\alpha}}\right)} \tag{10}$$

where, ψ is units of pF and z, n and α are in consistent units, ψ (z) is the suction value at any depth *z, n* is the frequency of suction cycles, and α is the diffusion coefficient.

The suction change with depth is a function of change in suction at the surface ($\Delta\psi$ *in pF units*) and the equilibrium suction (ψ_e).

Once the key parameters of the profiles are established for any given *TMI*, all information required for the Mitchell (1980) flow computations (such as the ratio of the diffusion coefficient to the number of seasonal cycles per year) can be back-calculated (Vann, 2019). The *n*, α, π terms in the Mitchell (1980) equation, are back-calculated using the known equilibrium depth, D_{ψ_e} , change in suction at surface, $\Delta\psi$, and the 0.2 pF difference (Lytton et al., 2005; Vann, 2019), wet to dry, at the depth of equilibrium.

$$\frac{n\pi}{\alpha} = \left(\frac{\ln\left(\frac{0.2\,pF}{\Delta\psi}\right)}{-D_{\psi_e}}\right)^2 \tag{11}$$

The suction profile can now be generated using Equations 10 and 11 and the previously computed components of the surrogate suction, where suction is in pF units and depth is in meters.

3.5 Step 5: development of the wet and dry suction envelope

The suction envelope defines the boundary conditions for the suction value at any depth within the soil profile. At the ground surface, the minimum (wet) and maximum (dry) suction values can be determined using the following expressions:

$$\psi_{wet_{z=0}} = \psi_e - r\Delta\psi \tag{12}$$

$$\psi_{dry_{z=0}} = \psi_{wet_{z=0}} + \Delta\psi \tag{13}$$

The minimum (wet) and maximum (dry) suctions for the TX 48-1068 section are 3.54 and 4.58, respectively.

The step size, or thickness of depth intervals (*dz*) must be determined. A sensitivity analysis should be performed to determine the number of steps (n_s) needed for the analysis; however, a value of 20 steps has been found to be sufficient for the volume change calculation performed in this study. The step size is computed by:

$$dz = \frac{D_{\psi_e}}{(n_z - 1)} \tag{14}$$

The step size for the SMP TX 48-1068 section is 8.526 cm using 20 steps with a depth of equilibrium suction of 1.62 m.

The wet and dry limit suction curves are iteratively calculated as the depth (*z*) is increased from 0 (ground surface) to the depth of equilibrium suction.

$$\psi(z_i)_{wet} = \psi_e - r\Delta\psi e^{\left(-z_i\sqrt{\frac{n\pi}{\alpha}}\right)} \tag{15}$$

$$\psi(z_i)_{dry} = \psi_e + (1-r)\Delta\psi e^{\left(-z_i\sqrt{\frac{n\pi}{\alpha}}\right)} \tag{16}$$

The suction for the SMP TX 48-1068 section is shown in Figure 7.

3.6 Step 6: initial estimate of monthly changes in suction at the surface (Perera, 2003)

It is important to note that the following steps for determining the suction at the surface over time are specific to a deterministic approach for estimating historic ground movements. Such approach can be used for a case study, forensic analysis, or calibration efforts based on measured data. The suction at the surface over time can also be modeled using a stochastic analysis with randomly generated monthly *TMI* values based on the historic averages and standard deviations. The second type of analysis can be used for designs of future structures; however, an example of such analysis is not presented in this paper.

In 2003, Perera studied the relationship between in-situ moisture content, suction, *TMI*, and index soil properties. He developed correlations for two models: the *TMI-*P_{200} model, which is valid for granular base materials; and the *TMI-*P_{200}*/wPI* model, used to estimate the equilibrium suction of subbase and subgrade materials (Rosenbalm, 2011). The two models are briefly explained in the following paragraphs.

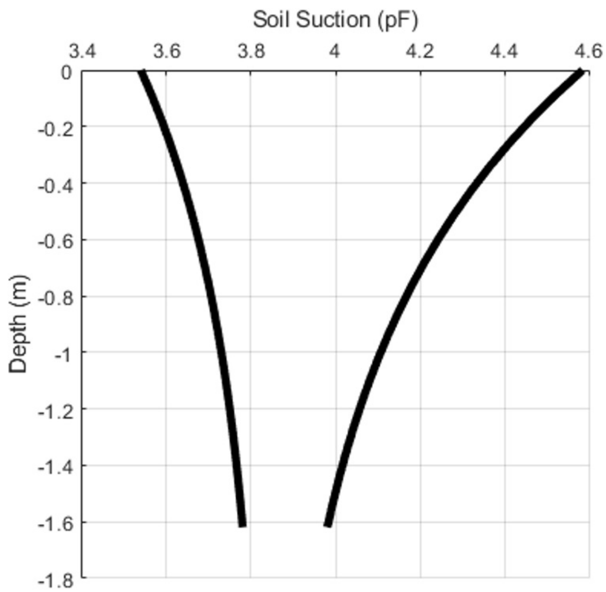

Figure 7. Suction envelope for the SMP TX 48-1068 section using the Vann & Houston (2021) models.

The *TMI-P200/wPI* model is of interest to this study. This model was developed for fine-grained material, which makes it suitable for expansive soils. For such materials, in addition to P_{200}, the weighted plasticity index, *wPI*, property was added, where:

$$wPI = \left(\frac{P_{200}}{100}\right) PI \tag{17}$$

The *wPI* for the example in Paris, TX site is 18.4 based on the percent passing the #200 sieve of 74% and a *PI* of 20.

The following equation is used to calculate suction based *TMI*, P_{200}, and *wPI* (Perera 2003).

$$\Psi = 0.3\left[e^{\left[\frac{\beta}{TMI+\gamma}\right]} + \delta\right] \tag{18}$$

where, Ψ is the matric suction of the soil; and $\beta, \gamma, and\, \delta$ are regression constants.

Rosenbalm developed equations for each regression constant. These equations are used when *wPI* is less than 0.5 (Rosenbalm, 2011):

$$\beta = 2.56075\left(P_{200}\right) + 393.4625 \tag{19}$$

$$\gamma = 0.09625\left(P_{200}\right) + 132.4875 \tag{20}$$

$$\delta = 0.025\left(P_{200}\right) + 14.75 \tag{21}$$

The following equations are used when *wPI* $\geq$ 0.5:

$$\beta = 0.006236\left(wPI\right)^3 - 0.7798334\left(wPI\right)^2 + 36.786486\left(wPI\right) + 501.9512 \tag{22}$$

$$\gamma = 0.00395\left(wPI\right)^3 - 0.04042\left(wPI\right)^2 + 1.454066\left(wPI\right) + 136.4775 \tag{23}$$

$$\delta = -0.01988\left(wPI\right)^2 + 1.27358\left(wPI\right) + 13.91244 \tag{24}$$

3.7 Step 7: adjustment to the estimation of monthly changes in suction at the surface (Vann & Houston, 2021)

It has been observed by the authors that the suction at the surface calculated using the Perera (2003) model typically will not reach the long-term minimum and maximum suction values observed by Vann & Houston (2021). This may not cause a significant issue if the analysis period is relatively short (e.g., less than ten years), however; for the purpose of pavement design, which typically incorporates a design life of 20+ years, it is recommended that the surface suction values determined from the Perera model be adjusted so that they will reach the limits observed by Vann & Houston (2021). This can be conducted by normalizing the maximum and minimum suction values from the Perera model to the previously computed potential change in suction at the surface ($\Delta\psi$).

$$\left(\psi_i\right)_{norm} = \left(\psi_{wet}\right)_{z=0} + \Delta\psi\left(\frac{\left(\psi_i\right)_{z=o} - \left(\psi_{Perra}\right)_{\min}}{\left(\psi_{Perra}\right)_{\max} - \left(\psi_{Perra}\right)_{\min}}\right) \tag{25}$$

After iterating the process for each month, the adjusted surface suction values can be plotted to help visualize adjustment (Figure 8).

3.8 Step 8: fourier equation fit to the monthly surface suction (Aubeny & Long, 2007)

In order to model the suction changes as a function of time and depth, an equation must be developed to represent the variation of suction at the surface. Typically, a simple sinusoidal fit has been used to represent the surface suction variation with time. However, Aubeny & Long (2007), proposed that a Fourier transform can be used to improve the goodness of fit. As such, an 8th degree Fourier series is used in this analysis to fit an equation to the highly variable surface suction data.

In general, the Fourier series is a sum of sine and cosine functions that describes a periodic signal. It is represented in either the trigonometric form or the exponential form.

$$y = a_0 + \sum_{i=1}^{n} a_i \cos(iwx) + b_i \sin\left(iwx\right) \tag{26}$$

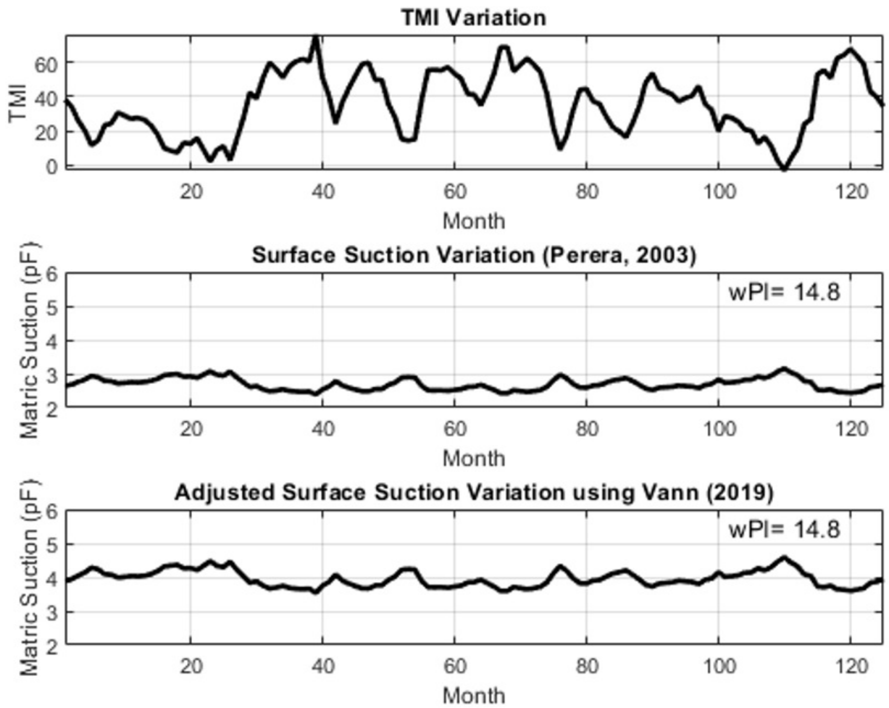

Figure 8. Monthly *TMI*, Perera (2003) surface suction, and the Vann (2019) adjusted surface suction for the TX 48-1068 section for the date range 3/1987 to 9/1997.

where, x represents time (for this analysis), a_0 models a constant (intercept) term in the data and is associated with the $i = 0$ cosine term, w is the fundamental frequency of the signal, n is the number of terms (harmonics) in the series, and $1 \leq n \leq 8$, and a_i and b_i are the fitting parameters. Additional background information on the Fourier series can be found on MathWorks help center (MathWorks, 2021).

Figure 9 presents the 1st and 8th order Fourier fit to the Vann & Houston (2021) adjusted surface suction for the TX 48-1068. The 1st order fit closely represents the original approach to modeling the surface suction flux by Lytton et al. (2005) using Mitchell (1979) equation. The adjusted R^2 for the 1st order Fourier fit to the suction data is 0.2903, while the 8th order Fourier fit increases the adjusted R^2 to 0.7056.

One limitation of requiring an equation to represent the surface suction, is that generally the equation fit will not be able to encompass the maximum and minimum values of the individual monthly data. For purposes of the shrink/swell volume change analysis, the inclusion of the peaks of the surface suction can provide more accurate and conservative representation of extreme events. As such, the Fourier surface suction equation can be normalized between the maximum and minimum values of the surface suction; however, this additional step was not performed as part of the example analysis presented in this paper.

Note that the initial suction is a function of the *TMI* value for that month. The initial suction (time = 0) can be adjusted using a phase shift of the Fourier equation. Lytton et al. (2005), provided values of phase shifts for different initial conditions of the soil (wet, dry, and equilibrium). The example calculation in this report does not include the phase shift for the initial conditions.

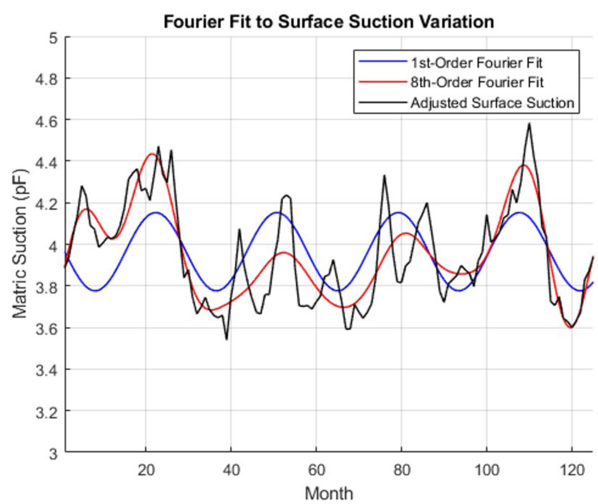

Figure 9. 1st and 8th order Fourier fit to the surface suction data for the TX 48-1068 section for the date range 3/1987 to 9/1997.

3.9 Step 9: monthly suction profile (Aubeny & Long, 2007)

The next step is to model the suction profile change with time using Aubeny & Long's (2007) adjusted 1979 Mitchell equation.

$$u(y,t) = U_e - \left(U_{dry} - U_{wet}\right) \sum_{k=1}^{\infty} \alpha_k \exp\left(-\sqrt{\lambda k}\right) \cos\left(\tau k - \sqrt{\lambda k}\right) \tag{27}$$

where, y is the depth, $\lambda = \pi y^2 n_\lambda / \alpha_k$, $\tau = 2\pi t n_\lambda$, n_λ = lowest frequency of cyclic suction variation, and $\alpha_k = (2/k\pi) sin(k\pi r)$ with k = 1,2,3…

For a given point in time, the suction profile can be estimated using the Aubeny & Long approach. Figure 10 presents the estimated suction at time = 1 month for the TX 48-1068 SMP site. Note that the long-term or extreme boundaries of the suction profile are known from the Vann (2019) correlations with the 30-year *TMI* value previously presented herein.

The monthly change in suction at each depth can be determined by calculating the following months suction profile and taking the difference of the two values at each depth. It is this ongoing change in soil suction with time that drives the volume change of shrink/swell soils. Figure 11 shows the suction profiles for t = 1 month, and t = 2 months, for the TX 48-1068 SMP site. Figure 12 shows the suction profiles for month 1 through month 12.

Figure 13 presents the monthly suction profiles over the date range 3/1987 to 9/1997 life for TX 48-1068 SMP section. From the figure, the significant swings of the suction profile from wet to dry, over the date range 3/1987 to 9/1997, can be observed. The monthly change in suction at each depth in the soil profile can be determined from this model.

To account for hysteresis effects associated with the wetting and drying of soil, it is important to record if the

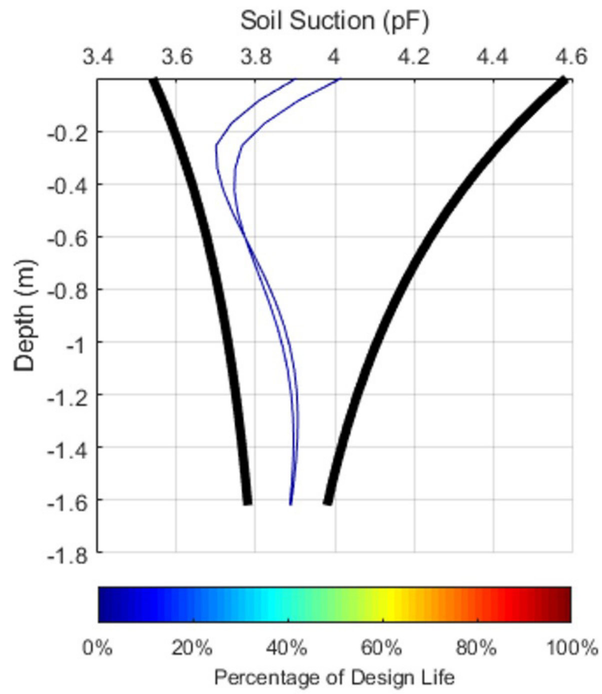

Figure 11. The estimated initial suction profiles (time = 2 months) for TX 48-1068 section.

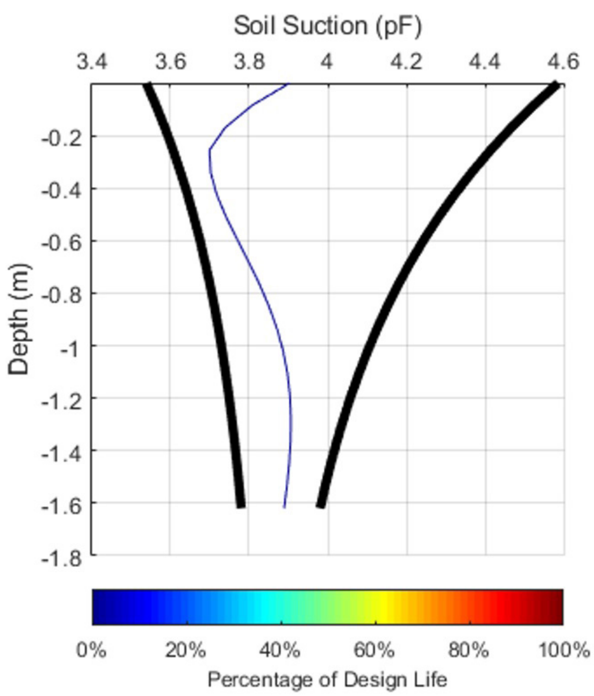

Figure 10. The estimated initial suction profile (time = 1 month) for TX 48-1068 section.

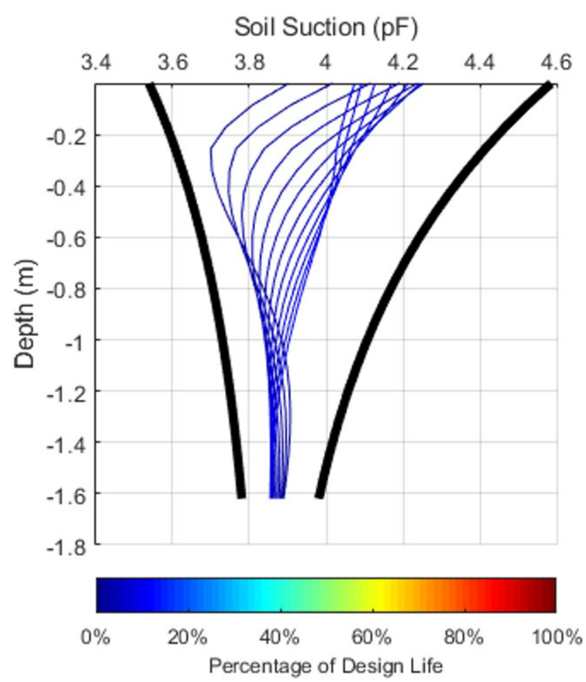

Figure 12. The estimated initial suction profiles (time = 12 months) for TX 48-1068 SMP section.

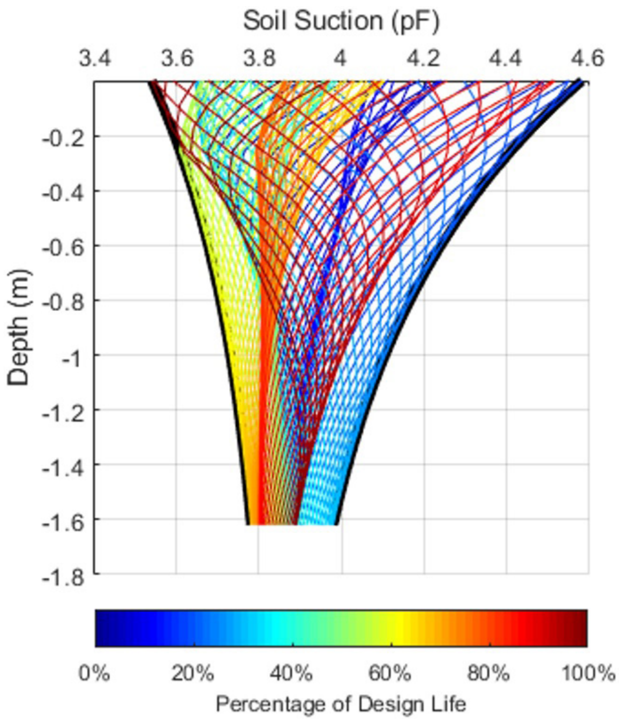

Figure 13. Monthly suction profiles over the date range 3/1987 to 9/1997.

soil is wetting or drying at each depth and time during the iterative analysis. This information will be used during the strain calculation discussed later in this report.

3.10 Step 10: suction profile adjustments due to varying boundary conditions

It is possible that the project site has a variable groundwater table, nearby vegetation, or is constructed using a moisture barrier. If any of these boundary conditions are present, the suction envelope and profile must be adjusted. (e.g. the suction profile will reach osmotic suction at the groundwater table depth). No variable boundary conditions were included in this example calculation.

3.11 Step 11: net normal stress profile

The net normal, or overburden stress, is a key component of shrink/swell volume change determination as it will help reduce potential soil swelling and can increase soil shrinkage. The overburden stress profile is determined using the conventional total stress approach.

$$\sigma_z = \sum \left(\gamma_{soil} z\right) \tag{28}$$

Note that the water content is subject to change over time as the soil suction changes, which will affect the magnitude of the net normal stress. However, for purposes of this example calculation, the effect of the changing water content on the net normal stress is negligible and is not included in the analysis.

If there are foundation loads or increases overburden stresses due to pavement layers above the subject soil profile, an increase of stress will be applied throughout the net normal stress profile.

3.12 Step 12: suction compression index (Covar & Lytton, 2001)

The most widely accepted method for estimating volumetric strain is the one developed for the Texas DOT and the Federal Highway Administration, FHWA, by Lytton et al. (2005), which is as follows:

$$\frac{\Delta V}{V} = -\gamma_h \log\left(\frac{h_f}{h_i}\right) - \gamma_\sigma \log\left(\frac{\sigma_f}{\sigma_i}\right) - \gamma_\pi \log\left(\frac{\pi_f}{\pi_i}\right) \tag{29}$$

where, $\frac{\Delta V}{V}$ is the volumetric strain (volume change with respect to initial volume); γ_h is the the matric suction compression index; γ_σ is the mean principal stress compression index; γ_π is the osmotic suction compression index; h_i is the initial matric suction; h_f is the final matric suction; σ_i is the initial mean principal stress; σ_f is the final mean principal stress; π_i is the initial osmotic suction; and π_f is the final osmotic suction.

Although, total suction is the sum of matric suction and osmotic suction, Fredlund wrote "Matric suction in a soil mass change is a result of moisture infiltration and evaporation at the ground surface. Osmotic suction in the soil does not appear to be highly sensitive to modest changes in the water content of the soil. As a result, a change in the total suction is quite representative of a change in the matric suction." (Fredlund et al., 2012). Also, Lytton wrote: "It is the change of matric suction that generates the heave and shrinkage, while osmotic suction rarely changes appreciably." (Lytton et al., 2005). Thus, the change in matric suction is responsible to shrinkage and heave and osmotic suction does not affect enough to be concerned (Lytton et al., 2005; Fredlund et al., 2012). Thus, the equation can be rewritten as:

$$\frac{\Delta V}{V} = -\gamma_h \log\left(\frac{h_f}{h_i}\right) - \gamma_\sigma \log\left(\frac{\sigma_f}{\sigma_i}\right) \tag{30}$$

Note that the net normal stress portion of the equation is added if the soil is wetting (swelling) and subtracted if the soil is drying (shrinking).

The Suction compression index, γ_h, is a parameter used to relate total suction to volume change to predict heave or shrinkage in expansive soils. This value can either be measured or estimated using soil index properties (Atterberg limits and gradation) as described in Covar & Lytton (2001).

First, the mineralogical zone is determined using Figure 14 with soils plasticity index (*PI*) and liquid limit (*LL*).

The zone for the TX 48-1068 SMP section site is Zone 2, using a *LL* = 60% and a *PI* = 40%.

The percent fine clay *(%fc)* is then calculated using the percent passing #200 sieve (P_{200}) and the percent clay (*%clay*) obtained via hydrometer testing.

$$\%fc = \left(\frac{\%clay}{P_{200}} \right) \tag{31}$$

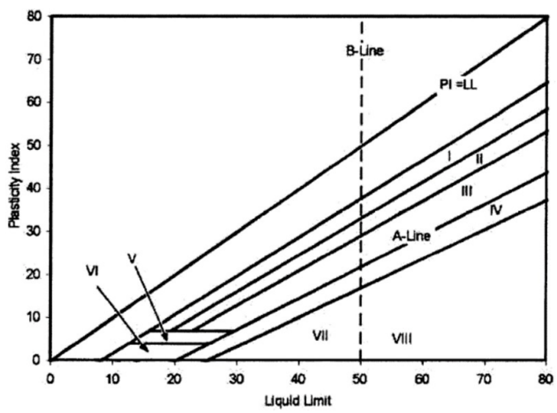

Figure 14. Mineralogical zones for soil (Covar & Lytton, 2001), units in %.

The *%fc* for the example site is 21.05%, using a *%clay* = 20% and P_{200} = 95%.

The average suction compression index (γ_0) can now be determined using the charts developed by Covar & Lytton (2001), which are separated by mineralogical zones (Figure 15).

The average suction compression index for the TX 48-1098 SMP section is 0.051 using Zone 1, *%fc* = 43.78%, *LL* = 38%, and *PI* = 20%.

The adjusted suction compression index (γ_h) is now determined by:

$$\gamma_h = \gamma_0 \left(\%fc \right) \tag{32}$$

The adjusted suction compression index for TX 48-1098 SMP section site is 0.0223.

The hysteresis effects of the soil are now accounted for using the equations from the PTI (2008) Manual. The wetting and dying suction compression indices must be calculated for each depth and time of the analysis using the recorded wetting/drying information from Step 9.

$$\gamma_{swell} = \gamma_h e^{(\gamma_h)} \tag{33}$$

$$\gamma_{shrink} = \gamma_h e^{(-\gamma_h)} \tag{34}$$

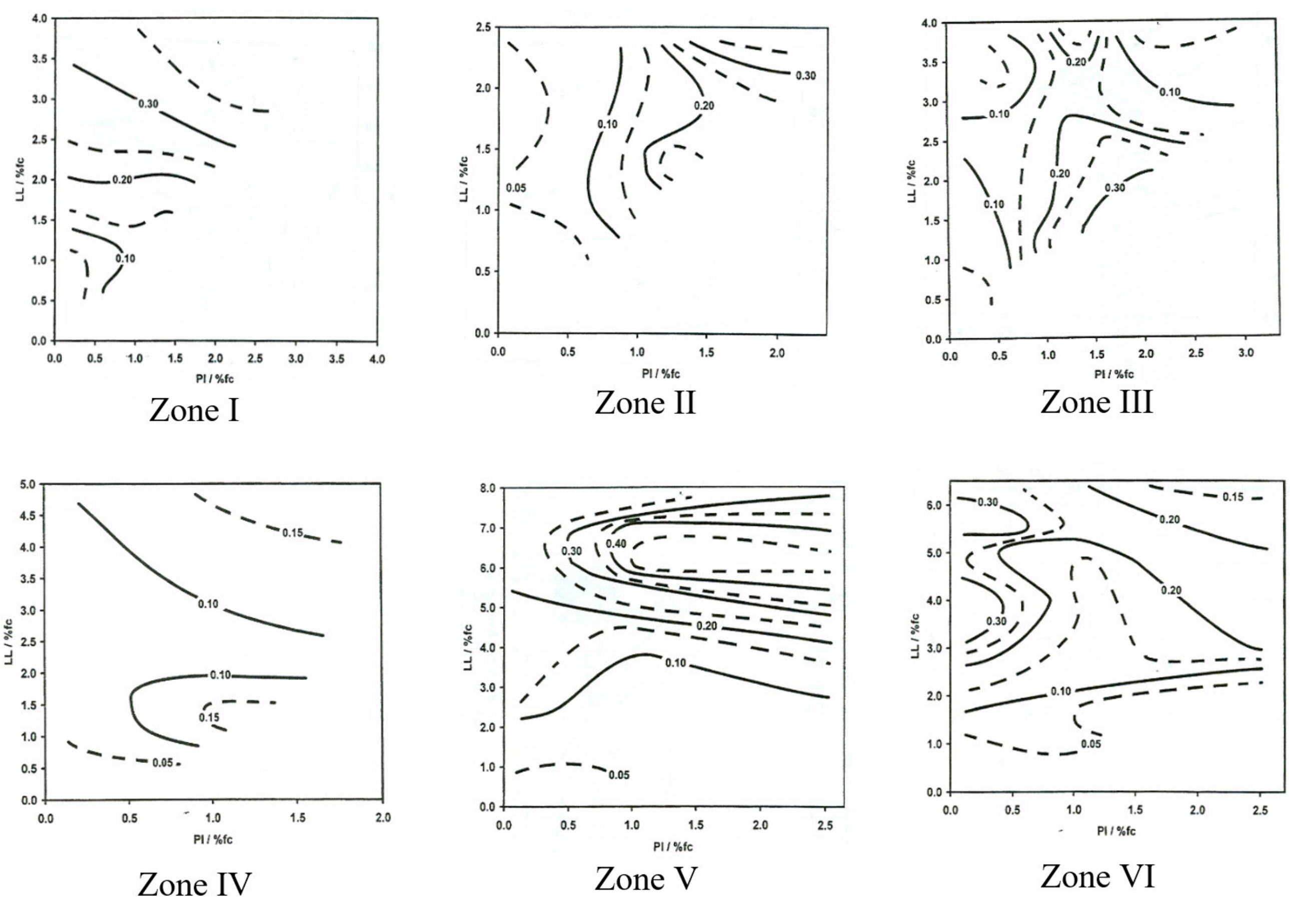

Figure 15. Suction Compression Index based on Mineralogical Classification and Soil Index Properties (Covar & Lytton, 2001).

3.13 Step 13: monthly strain calculation

The mean principal stress compression index, γ_σ, can be calculated using its relation to the compression index, C_c, and void ratio, e, as follows (Lytton et al., 2005):

$$\gamma_\sigma = \frac{C_c}{1+e_0} \tag{35}$$

where, C_c is the compression index; and e_0 is the void ratio. For purposes of this example calculation, the mean principal stress compression index was assumed to be 10% of the suction compression index as recommended by Lytton et al. (2005).

The mean principal stress must be iteratively determined at each depth and time step, as it is a function of the net normal stress and the wetting/drying condition.

$$\sigma = \frac{1+2K_0}{3}\sigma_z \tag{36}$$

where, σ_z is the previously calculated vertical stress at a point below the surface in the soil mass; and K_0 is the 1-D at-rest lateral earth pressure coefficient.

$$K_0 = e\left(\frac{1-\sin(\phi')}{1+\sin(\phi')}\right)\left(\frac{1+d\sin(\phi')}{1-k\sin(\phi')}\right)^n \tag{37}$$

Values of coefficients *e, d, k, and n* for different soil conditions are given in Table 2.

The angle of internal friction, ϕ', can be estimated from its empirical correlation with plasticity index, PI, based on triaxial compression tests.

$$\phi' = 0.0016PI^2 - 0.3021PI + 36.208 \tag{38}$$

The internal angle of friction for the TX 48-1068 SMP site is 30.8° using a *PI* = 20.

Using the data developed from the iterative steps discussed above, and the suction-overburden-strain relationships, the volume change over time can be estimated. Figure 16 presents the volume change estimation for the Paris, TX site.

4. Estimated volume change comparison to measured data

The estimated volume change from the proposed framework for the TX 48-1068 site was compared to the measured data gathered from the LTPP SMP database. The estimated data was normalized to the value of the first measurement. Figure 17 presents the measured and estimated volume change for the TX 48-1068 SMP section.

Table 2. Lateral earth pressure parameter coefficients.

Conditions	K_0	*e*	*d*	*k*	*n*
Cracked	0	0	0	0	1
Drying (Active)	1/3	1	0	0	1
Equilibrium (at rest)	1/2	1	1	0	1
Wetting (within movement active zone)	2/3	1	1	0.5	1
Wetting (below movement active zone)	1	1	1	1	1
Swelling near surface (passive earth pressure)	3	1	1	1	2

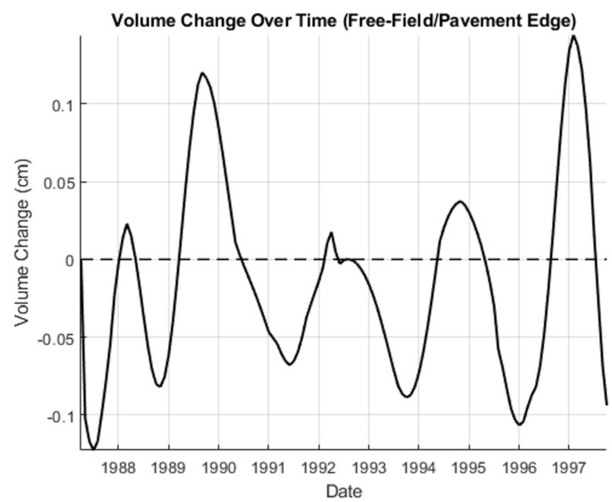

Figure 16. Volume change over time for the TX 48-1068 site.

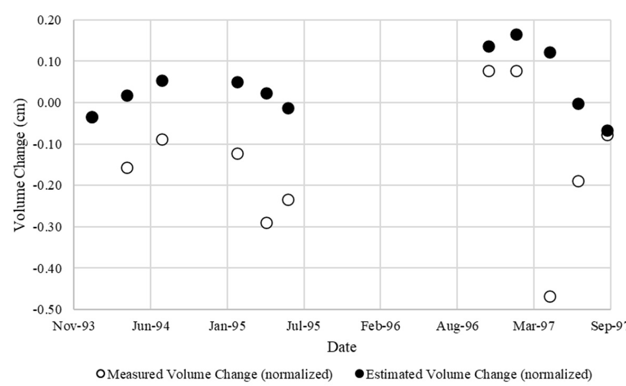

Figure 17. Measured *vs.* estimated volume change normalized to the initial measurement for the TX 48-1068 SMP Section.

A preliminary analysis of the proposed framework presented in this paper was performed on eight more sections from Alabama, Colorado, Montana, Nebraska, South Dakota and Texas from the LTPP SMP program. The resulting comparison, presented in Figure 18, of the estimated volume change and the measured field data for 70 data points yielded a coefficient of correlation of 0.45. This result is promising, and the data allows for a calibration of the model to the field conditions once all the parameters have been defined.

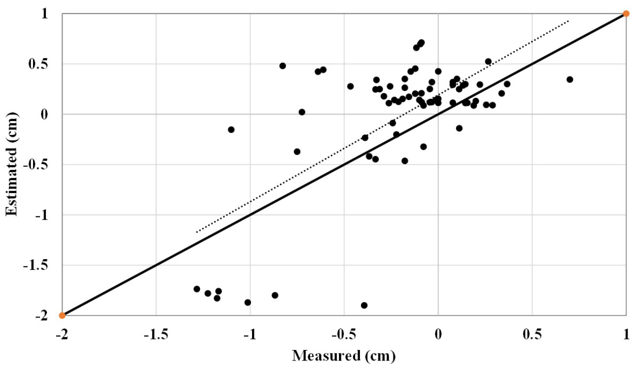

Figure 18. Measured *vs.* estimated volume change normalized to the initial measurement for the eight SMP sections.

5. Conclusions

This paper presented a preliminary mechanistic-empirical framework for the estimation of volume change. The models are universal and can be used in any part of the world provided measured data is available to calibrate for local conditions. Ongoing calibration effort with the remaining LTPP SMP sections will allow obtaining calibration factors for the proposed framework that will improve the estimation of the volume change predictions under pavements and facilitate the implementation into current design procedures.

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Declaration of interest

The authors have no conflict of interests pertaining to the material included in this paper.

Authors' contributions

Austin Olaiz: conceptualization, formal analysis, methodology, writing – original draft. Mohammad Mosawi: data curation, formal analysis, writing – original draft. Claudia Zapata: conceptualization, funding acquisition, project administration, supervision, writing – review & editing.

List of symbols

a	a coefficient
a_0	constant (intercept) term, associated with the i=0 cosine term
a_i	fitting parameter
b_i	fitting parameter
C_c	compression index
D_{ψ_e}	depth to equilibrium suction
dz	thickness of depth intervals
e	void ratio
f_1	fraction of the number of days in month divided by the average number of days in month, 30
f_2	fraction of the number of hours in a day divided by the base of 12 h in a day
h_i	initial matric suction
h_f	final matric suction
I	annual heat index
K_0	lateral earth pressure coefficient
n	frequency of suction cycles
n_λ	lowest frequency of cyclic suction variation
n_z	number of steps needed for the analysis
P	precipitation for the given month
P_{200}	percent passing #200 sieve
PI	plasticity index
t	mean monthly temperature in degrees Celsius
t_i	mean temperature for the ith month
r	climate parameter
w	fundamental frequency of the signal
wPI	weighted plasticity index
x	time of the analysis
z	depth
α	diffusion coefficient
β,γ,δ	regression constants
$\Delta V / V$	volumetric strain
γ_h	matric suction compression index
γ_{soil}	unit weight of the soil
γ_σ	mean principal stress compression index
γ_π	osmotic suction compression index
$\varnothing'$	angle of internal friction
π_i	initial osmotic suction
π_f	final osmotic suction
σ_f	final mean principal stress
σ_i	initial mean principal stress
σ_z	net normal, or overburden, stress
$\Delta\psi$	change in suction at the ground surface
Ψ	matric suction
ψ_e	equilibrium suction
$\psi_{wet_{z=0}}$	minimum (wet) suction at the surface (z = 0)
$\psi_{dry_{z=0}}$	maximum (dry) suction at the surface (z = 0)
$\psi\ (z)$	suction value at any depth z
$\%fc$	percent of fine clay

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