Soils and Rocks
An International Journal of Geotechnical and Geoenvironmental Engineering
www.soilsandrocks.com
ISSN 1980-9743
ISSN-e 2675-5475

Comparative analysis of non-recursive three-dimensional (3D) modifications of Hoek-Brown failure criterion

Nnamdi Emmanuel Ezendiokwere[1#], Ogbonna Friday Joel[1], Victor Joseph Aimikhe[2], Adewale Dosunmu[2]

Article

Keywords
Hoek & Brown failure criterion
Non-recursive failure criteria
Geomaterials
Intermediate principal stress
Rock strength

Abstract

A comparative analysis of three-dimensional (3D) modifications of Hoek & Brown (1988) rock failure criterion was carried out. The correlations between failure stress and the other principal stresses were first determined using polyaxial test data for five geomaterials including KTB amphibolites, Westerly granite, Dunham dolomite, Shirahama sandstone and Yuubari shale. The prediction accuracies of five non-recursive, three-dimensional modifications to Hoek-Brown failure criteria and the original two-dimensional Hoek-Brown criterion were later investigated using root mean square error and coefficient of determination as measures of misfit. The results reveal that the intermediate principal stress significantly affects strength in geomaterials like the Dunham dolomite. It also moderately affects the strength of geomaterials like the KTB amphibolites, the Westerly granite and the Yuubari shale. Moreover, the intermediate principal stress has mixed effects on strength in the Shirahama sandstone. In addition, the original Hoek-Brown failure criterion could still be used with reasonable accuracy for geomaterials whose strength shows low dependence on the intermediate principal stress. While a three-dimensional Hoek-Brown criterion must be used for geomaterials like the Dunham dolomite, whose strength shows a high dependence on the intermediate principal stress. The original Hoek-Brown failure criteria should be used with caution for geomaterials like the Shirahama sandstone, the KTB amphibolites, the Westerly granite, and the Yuubari shale, whose strength shows either mixed or intermediate dependence on the intermediate principal stress. Average prediction accuracies followed the order: simplified Priest (2012), Ma et al. (2020), and Jiang & Zhao (2015). Both original Hoek & Brown (1988) and Li et al. (2021) criteria were tied, while Liu et al. (2019) was the least.

1. Introduction

Geomaterials can be found either buried underground (occasionally with outcrops) or on the surface as remnants of weathering processes (Ranjith et al., 2017). Underground geomaterials are usually subjected to complex states of stress that are non-hydrostatic (Zhou et al., 2014). Hence, the principal stresses all have different magnitudes (Elyasi & Goshtasbi, 2015). This state of underground stress is also referred to as polyaxial or true triaxial, suggesting different values for each principal stress. The non-hydrostatic state of stress is usually quite common in regions of high tectonic and geologic activities (Jaeger et al., 2010). How geomaterials behave under various states of stress is considered highly important (Lorenzo et al., 2013), especially for underground and engineering constructions (Zuo et al., 2015). This is because geomaterials have failure stress thresholds beyond which they do tend to yield or fail (Yua et al., 2002).

The strength of materials, including geomaterials, is usually predicted using strength or failure criteria (Jiang, 2017). A failure criterion is a simple expression describing failure stress in terms of confining principal stresses and material properties (Singh & Singh, 2012). Its main use is for predicting the level of stress that a given material can withstand without failing. Numerous failure criteria have been proposed over the years by different researchers (Li et al., 2021). These failure criteria were mostly developed empirically by using best-fitting curves to describe experimental strength data of principal stresses (Ma et al., 2020). Among the numerous failure criteria developed so far, one of the most popular is

[#]Corresponding author. E-mail address: nnamdi_ezendiokwere@uniport.edu.ng
[1]University of Port Harcourt, World Bank Africa Centre of Excellence in Oilfield Chemicals Research, Department of Petroleum and Gas Engineering, Port Harcourt, Rivers State, Nigeria.
[2]University of Port Harcourt, Department of Petroleum and Gas Engineering, Port Harcourt, Rivers State, Nigeria.

Submitted on May 7, 2021; Final Acceptance on December 2, 2021; Discussion open until August 31, 2022.
https://doi.org/10.28927/SR.2022.070821

the Hoek-Brown failure criterion (Liu et al., 2019). It was empirically developed for jointed or intact geomaterials using a large number of triaxial test data.

For over thirty years, the Hoek & Brown failure criterion has been widely applied in rock mechanics and rock engineering design (Ma et al., 2020). A possible explanation for its wide adoption is that its parameters can easily be gotten from simple uniaxial compression tests, discontinuity characterizations and mineralogical investigations (Jiang & Zhao, 2015). Although the original Hoek & Brown criterion has satisfactorily predicted failure stress in numerous applications, it has also given unsatisfactory results in other cases (Zhang et al., 2013). The reason is that the Hoek & Brown failure criterion in its original form is two-dimensional (2-D). Consequently, the original Hoek & Brown failure criterion has two major shortcomings. First, the original Hoek & Brown criterion neglects the non-linearity of strength behavior in geomaterials. Also, the conventional Hoek & Brown failure criterion does not incorporate the influence of the intermediate stress on strength in geomaterials (Mogi, 2007).

The influence of the intermediate stress on strength and failure in intact rocks can be gauged by the difference in failure stress recorded in conventional triaxial extension and compression tests (Liu et al., 2019). Hence, Murrell (1965), by analyzing results of two leading rock mechanics experts of that time, deduced that the difference in strength under compressive and extensive stress conditions can be traced to the influence of the intermediate stress. Similarly, other researchers like Handin et al. (1967) and Mogi (1967) in separate experiments later confirmed Murrell's findings. The above findings later inspired numerous researchers to consider independently applying the principal stresses during experiments. More recent experiments have equally demonstrated that the influence of the intermediate stress cannot be ignored (Zuo et al., 2015).

In addition, available three-dimensional (3D) modifications of Hoek-Brown are either recursive or non-recursive. The recursive three-dimensional (3D) modifications of Hoek-Brown require recursive numerical strategy in computing the predicted failure stress (Li et al., 2021). As such, they are computationally complex, since they require advanced algorithms for determining strength of geomaterials. The non-recursive criteria on the other hand, do not pose any serious computational inconvenience. They are quite easy to use, as no iterative procedure is required for estimating strength when using them (Li et al., 2021). Following extensive literature search, most of the earlier three-dimensional modifications to Hoek-Brown failure criterion are recursive in nature and have benefitted from considerable research effort in the past. While most non-recursive three-dimensional modifications of Hoek-Brown were more recently proposed.

Hence, there appears a dearth of research publications that have conducted comparative performance study exclusively for non-recursive three-dimensional (3D) modifications of Hoek-Brown rock failure criterion. It is this research void that this study intends to fill using experimental polyaxial data of some geomaterials usually encountered in the engineering practice. So, a comparative analysis of non-recursive three-dimensional (3D) modifications of Hoek-Brown failure criterion was carried out. It is believed that anyone planning to use Hoek-Brown criterion in three dimensions would find this study useful. Especially, those who do not have available the necessary equipment and software able to handle the recursive 3D modifications. Results obtained from studies like this could also be useful in selecting failure criteria for inclusion in geomechanical software. Choosing the non-recursive criteria would reduce the computational power requirements and complexity of the software.

2. Hoek & Brown failure criterion

Using a set of wide-ranged experimental data, Hoek & Brown (1980) proposed the original Hoek & Brown failure criterion as follows.

$$\sigma_1 = \sigma_3 + \sigma_c \left(m \frac{\sigma_3}{\sigma_c} + s \right)^{0.5} \tag{1}$$

where σ_1 and σ_3 are maximum and minimum effective principal stresses at failure, σ_c is uniaxial compressive strength of intact rock, m and s are dimensionless parameters depending on rock properties. Parameter s equals 1 for intact rocks, while the values of m depend upon rock texture and mineralogy and are found in Hoek & Brown (1997). It was subsequently updated (Hoek & Brown, 1988) and modified (Hoek et al., 1992) to current generalized form:

$$\sigma_1 = \sigma_3 + \sigma_c \left(m_b \frac{\sigma_3}{\sigma_c} + s \right)^{a} \tag{2}$$

where m_b is the value of m for the rock-mass; s and a are constants which depend on the rock-mass properties. Parameters m_b, s and a are derivable from the Geological Strength Index (GSI) as follows (Hoek et al., 2002):

$$m_b = exp\left(\frac{GSI - 100}{28 - 14D} \right) m_i \tag{3}$$

$$s = exp\left(\frac{GSI - 100}{9 - 3D} \right) \tag{4}$$

$$a = 0.5 + \frac{1}{6}\left[\exp\left(-GSI/15\right) - \exp\left(-20/3\right) \right] \tag{5}$$

where the disturbance factor D, is a factor depending on the degree of disturbance from blast damage and stress relaxation, with values ranging range from 0 for undisturbed in situ rock-masses to 1 for very disturbed rock-masses. Criteria for selecting D are found in literature.

2.1. Some existing three-dimensional modifications of Hoek & Brown failure criterion

2.1.1. Pan-Hudson criterion

Pan & Hudson (1988) developed a three-dimensional version of Hoek & Brown strength criterion expressed as

$$\frac{3}{\sigma_c}J_2 + \frac{\sqrt{3}}{2}m\sqrt{J_2} - m\frac{I_1}{3} = s\sigma_c \tag{6}$$

where I_1 and J_2 are first stress invariant and second deviatoric stress invariant given by

$$I_1 = \sigma_1 + \sigma_2 + \sigma_3 \tag{7}$$

$$J_2 = \frac{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2}{6} \tag{8}$$

Where σ_1, σ_2, and σ_3 are major, intermediate and minor effective principal stresses.

2.1.2. Priest criterion

Priest (2005) proposed a three-dimensional version of generalized Hoek & Brown failure criterion by incorporating Drucker & Prager (1952) criterion. This criterion is expressed as follows.

$$J_2^{1/2} = AJ_1 + B \tag{9}$$

where A and B are empirical parameters; and J_1 is the mean effective stress ($I_1/3$). The idea is to calculate values of A and B parameters for Drucker & Prager failure surface intersecting Hoek & Brown failure point (σ_1, σ_2, σ_3). The process is similar to identifying Drucker & Prager parameters giving circumscribed fit for the Mohr-Coulomb strength criterion. Priest criterion was later simplified in 2012 to (Li et al., 2021):

$$\sigma_1 = 3\left[\omega\sigma_2 + (1-\omega)\sigma_3\right] + \sigma_c\left[m_b\frac{\omega\sigma_2 + (1-\omega)\sigma_3}{\sigma_c} + s\right]^a - (\sigma_2 + \sigma_3) \tag{10}$$

Where ω is the intermediate stress parameter.

2.1.3. Zhang & Zhu criterion

Zhang & Zhu (2007) equally developed a three-dimensional version of the original Hoek & Brown failure criterion. They presented their extension of Hoek & Brown criterion by combining the general Mogi (1971) criterion with the original Hoek & Brown criterion. The Zhang & Zhu criterion is published in Zhang (2008) as:

$$\frac{9}{2\sigma_c}\tau_{oct}^2 + \frac{3}{2\sqrt{2}}m\tau_{oct} - m\sigma_{m,2} = s\sigma_c \tag{11}$$

where τ_{oct} and $\sigma_{m,2}$ are respectively the octahedral shear stress and the effective mean stress given by

$$\tau_{oct} = \frac{1}{3}\sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2} \tag{12}$$

$$\sigma_{m,2} = \frac{\sigma_1 + \sigma_3}{3} \tag{13}$$

2.1.4. Jiang & Zhao criterion

Jiang & Zhao (2015) also developed a three-dimensional extension to the original Hoek & Brown criterion. They arrived to the new criterion substituting a group of parameters in the original Hoek & Brown criterion by the second deviatoric stress invariant function. The proposed criterion can be expressed as:

$$\frac{\sqrt{3J_2}}{\sigma_c} = \left(m\frac{\sigma_3}{\sigma_c} + s\right)^a \tag{14}$$

2.1.5. Liu et al. criterion

Liu et al. (2019) equally proposed a three-dimensional modification of the Hoek & Brown criterion. They generated the new failure criterion replacing the deviatoric stress component with the same second deviatoric stress invariant function used by Jiang & Zhao (2015). In addition, they also replaced the least principal stress component in the original Hoek & Brown criterion by a function of the new parameter ω. According to them, the parameter ω, is an additional rock property which quantifies the influence of the intermediate principal stress on the compressive stress failure of geomaterials. They also suggested that both the intermediate and least principal stresses have strengthening effects on the rock strength. The proposed failure criterion is expressed as follows.

$$\sqrt{3J_2} = \sigma_c \left(\frac{m_b}{\sigma_c} \omega \sigma_2 + (1-\omega)\sigma_3 + s \right)^a \tag{15}$$

2.1.6. Ma et al. criterion

Ma et al. (2020) developed a three-dimensional criterion modifying the generalized Hoek & Brown criterion and considering the strength enhancement due to high values of the intermediate principal stress. Mogi (1967) stated that the strength at failure increases with σ_2 by an amount proportional to σ_2. Therefore, Mogi suggested a general empirical function to correlate the maximum shear stress and the effective normal stress. The empirical function has the following expression, where n is a constant no bigger than 1.The monotonically increasing function f gives a failure envelope depending on the rock type.

$$\frac{\sigma_1 - \sigma_3}{\sigma_c} = \left(m \frac{\sigma_3 + n\sigma_2}{\sigma_c} + s \right)^a \tag{16}$$

where the term $n\,\sigma_2$ represents contribution of σ_2 to normal stress on the failure plane, with the parameter *n* ranging between 0 and 0.5. Considering the general form of Mogi (1967) criterion, the above modified Hoek & Brown criterion was developed by supplanting σ_3 in the generalized Hoek & Brown criterion. Clearly, the difference between the new criterion and the generalized Hoek & Brown criterion is the introduced term $n\,\sigma_2$. So, when *n* equals 0, the failure criterion reduces to the generalized Hoek & Brown criterion.

2.1.7. Li et al. criterion

Li et al. (2021) proposed another three-dimensional modification to the original Hoek & Brown criterion incorporating the influence of the intermediate principal stress. They did this through direct substitution of the maximum principal stress in the high intermediate stress dependence range and substituting the least principal stress in the low intermediate stress dependence range. According to them, the ultimate influence of the intermediate stress is to transform the straight line failure curves of the two-dimensional original Hoek & Brown criterion in the $\sigma_1 - \sigma_2$ space into parabolic curves. These curves can then be conveniently divided into two sections: a region of high σ_2 dependence and a region of low σ_2 dependence separated by a point of peak stress at σ_2^*. With this demarcation, they were able to arrive at two separate criteria characterizing the two regions of intermediate stress dependences.

$$\sigma_1 = \frac{b\sigma_2 + \sigma_3}{b+1} + \sigma_c \left(\frac{m}{\sigma_c} \frac{b\sigma_2 + \sigma_3}{b+1} + s \right)^a \quad \left(\sigma_2 \le \sigma_2^* \right) \tag{17}$$

$$\frac{b\sigma_2 + \sigma_1}{b+1} = \sigma_3 + \sigma_c \left(m \frac{\sigma_3}{\sigma_c} + s \right)^a \quad \left(\sigma_2^* \ge \sigma_2 \right) \tag{18}$$

Where b is the intermediate stress parameter

2.2. Classification of the three-dimensional (3D) modifications for the Hoek & Brown failure criterion

From the literature, it can be deduced that the methods generally employed by researchers in generating three-dimensional modifications of Hoek & Brown failure criterion can be categorized into three groups (Li et al., 2021). The first group corresponding to three dimensional modifications is derived by the incorporation of a deviatoric shape function into the Hoek & Brown criterion e.g. Zhang et al. (2013), Jiang & Zhao (2015), Jiang (2017), among others. A popular method of doing this is by introducing Lode dependence into the deviatoric plane, based on tensile and compressive meridian radii ratio. However, the failure criteria generated using the Lode dependency are generally complicated (Jiang & Zhao, 2015). The second group corresponds to the use of a three-dimensional versions of the Hoek & Brown failure criterion by combining the Hoek & Brown criterion with other three-dimensional criteria e.g. Priest (2005) (the comprehensive Priest criterion), and Zhang & Zhu (2007). The third group generate their own three-dimensional Hoek & Brown criterion by incorporating a weighted combination of the intermediate and least principal stresses; e.g. Priest (2012) (the simplified Priest criterion), Liu et al.(2019), Ma et al. (2020), Li et al. (2021) and so on.

In addition, these three groups can further be divided into recursive and non-recursive criteria, based on the computational method and requirements. The recursive criteria require some level of recursive numerical strategy in computing the predicted failure stress (Li et al., 2021). Apart from the simplified versions, most recursive criteria are found among the first and second categories described above. Examples of the recursive three-dimensional Hoek & Brown criteria include Jiang et al. (2011), Lee et al. (2012), Jiang & Xie (2012) etc. But the non-recursive criteria, on the other hand, do not pose serious computational inconvenience. Examples of the non-recursive three-dimensional Hoek & Brown criteria include the simplified Priest (2012) criterion, Jiang & Zhao (2015), Liu et al. (2019), Li et al. (2020) etc.

3. Materials and methods

The aim of this study was to conduct a comparative analysis of failure predictability of three-dimensional (3D) modifications of Hoek & Brown rock failure criterion. An extensive literature search was conducted to identify various three-dimensional (3D) modifications to Hoek & Brown failure criterion. The classification schemes discussed above also show a method of classification based on computational procedure, which divides modified three-dimensional Hoek & Brown criteria into two. However, from review of related literature, extensive studies seem to have been conducted on the recursive criteria. And this group have equally benefited by far from more research activities, as can be found in studies like Zhang (2008), Li et al. (2021), Ma et al. (2020) among others. But no study has been carried out exclusively on the non-recursive group of criteria.

Here, polyaxial test data from different geomaterials were obtained from literature, including polyaxial test data of KTB amphibolite, Westerly granite, Dunham dolomite, Shirahama sandstone and Yuubari shale. The sources of the polyaxial test data used are given in Table 1 and the Hoek & Brown criterion parameters for studied geomaterials are given in Table 2. Then, using the obtained data, the failure predictabilities of the selected non-recursive three-dimensional (3D) Hoek & Brown rock failure criteria were compared. The comparison was limited to the non-recursive group of modified three-dimensional criteria based on the calculations suited the obtained data. Based on the generated results, the failure stress prediction accuracies of the selected rock failure criteria were also determined, while identifying which criterion performed best for each type of the geomaterial studied.

Also, the measure of the intermediate principal stress dependency of the geomaterials was determined using both correlation coefficient and partial correlation coefficient. Partial correlation coefficient was introduced because partial correlation coefficient is known to measure the relationship between a pair of variables, under the influence of a third variable (Ma et al., 2020). So, measuring the relationship between the failure stress and the least principal stress without neglecting the effect of the least principal stress on the failure stress gives a more intuitive intermediate stress dependency. Consequently, the intermediate stress dependency parameter values for the studied geomaterials were also obtained for the selected three-dimensional failure criteria as captured in Table 3.

Table 1. Sources of polyaxial test data.

Geomaterial	Number of data points	Source
KTB Amphibolite	40	Chang & Haimson (2000) Colmenares & Zoback (2002) Al-Ajmi & Zimmerman (2005) Zhang (2008)
Westerly granite	45	Haimson & Chang (2000) Al-Ajmi & Zimmerman (2005) Zhang (2008)
Dunham dolomite	53	Mogi (1971) Al-Ajmi & Zimmerman (2005) Zhang (2008)
Shirahama sandstone	38	Takahashi & Koide (1989), Colmenares & Zoback (2002)
Yuubari shale	26	Takahashi & Koide (1989) Colmenares & Zoback (2002)

Table 2. Hoek & Brown criterion parameters for studied geomaterials (Li et al., 2021).

Geomaterial	σ_c	m_i
KTB Amphibolite	165	35.17
Westerly granite	201	38.62
Dunham dolomite	261.5	9.6
Shirahama sandstone	80	8.1
Yuubari shale	61.7	10.2

Table 3. Intermediate stress dependency parameter values for studied geomaterials.

Geomaterial	σ_2-dependence parameter value			
	Liu et al. (2019)	Li et al. (2021)	Ma et al. (2020)	Simplified Priest (2012)
KTB Amphibolite	0.05	0.410	0.10	0.24
Westerly granite	0.06	0.269	0.11	0.24
Dunham dolomite	0.10	0.606	0.30	0.29
Shirahama sandstone	0.05	0.421	0.04	0.29
Yuubari shale	0.05	0.325	0.15	0.28

The correlation coefficient between the failure stress and the intermediate principal stress is given by (Colmenares & Zoback, 2002):

$$r[\sigma_1,\sigma_2] = \frac{Cov[\sigma_1,\sigma_2]}{\sqrt{Var[\sigma_1]Var[\sigma_2]}} \tag{19}$$

Where $r[\sigma_1,\sigma_2]$ is the correlation coefficient between σ_1 and σ_2, $r[\sigma_i,\sigma_j]$ is the covariance of σ_1 and σ_2, $Var[\sigma_1]$ and $Var[\sigma_2]$ are the deviations of σ_1, and σ_2 respectively. And the partial correlation coefficient between failure stress and intermediate principal stress, given the least principal stress given is mathematically expressed as (Ma et al., 2020):

$$r[\sigma_1,\sigma_2:\sigma_3] = \frac{r[\sigma_1,\sigma_2]-r[\sigma_1,\sigma_3].r[\sigma_2,\sigma_3]}{\sqrt{\left(1-r^2[\sigma_1,\sigma_3]\right).\left(1-r^2[\sigma_2,\sigma_3]\right)}} \tag{20}$$

Where $r[\sigma_1,\sigma_2:\sigma_3]$ is the partial correlation coefficient between σ_1 and σ_2 given σ_3, $r[\sigma_i,\sigma_j]$ is the correlation coefficient between σ_i and σ_j, $r^2[\sigma_i,\sigma_j]$ is the square of the correlation coefficient between σ_i and σ_j. The comparative results between the correlation and the partial correlation coefficients for the various geomaterials considered are shown in Figure 1. One of the measure of the misfit between the calculated and test used is the root mean squared error (RMSE) given by (Li et al., 2021):

$$RMSE = \sqrt{\frac{1}{N}\sum_{i=1}^{N}\left(\sigma_{1i}^{cal}-\sigma_{1i}^{test}\right)^2} \tag{21}$$

where N is the number of test data pairs; and σ_{1i}^{cal} and σ_{1i}^{test} are the ith calculated and measured values of σ_1 respectively. Another measure of misfit that was equally used is the coefficient of determination (DC). Given the fact that some of the three modifications directly predicted failure stress, while the others predicted other combinations of rock properties (like the second deviatoric stress invariant, J_2) that still incorporated the failure stress, σ_1, the DC was introduced to remove any mathematical bias thereof. Mathematically, the coefficient of determination is given by (Jiang & Zhao, 2015):

$$DC = 1-\frac{\sum_{i=1}^{N}\left(\sigma_{1i}^{cal}-\sigma_{1i}^{test}\right)^2}{\sum_{i=1}^{N}\left(\sigma_{1i}^{cal}-\bar{\sigma}^{test}\right)^2} \tag{22}$$

Where,

$$\bar{\sigma}^{test} = \sum_{i=1}^{N}\frac{\sigma_{1i}^{test}}{N} \tag{23}$$

The root mean square error and the coefficient of determination were later used in ascertaining the failure stress prediction accuracies of the selected criteria for the geomaterials studied as depicted in Figures 2-6. Meanwhile, according to Ma et al. (2020), the statistical values of the correlation coefficients for the different geomaterials can also be utilized in classifying their principal stress interdependencies. For this purpose, we utilized the following range of r values to classify the principal stress interdependences into low, intermediate and high dependences as follows:

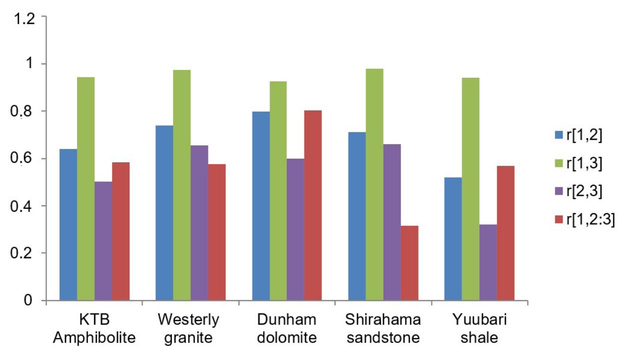

Figure 1. Correlation and partial correlation coefficients between the principal stress components.

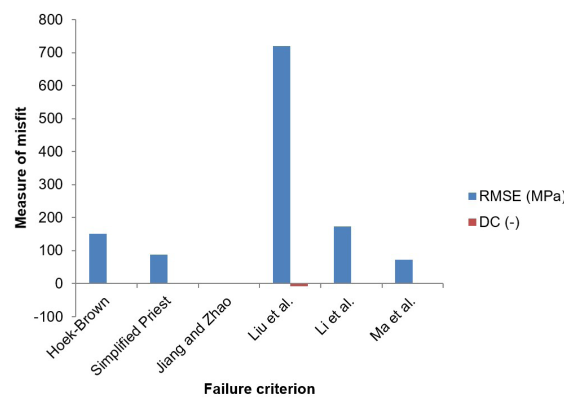

Figure 2. Measures of misfit for KTB Amphibolite.

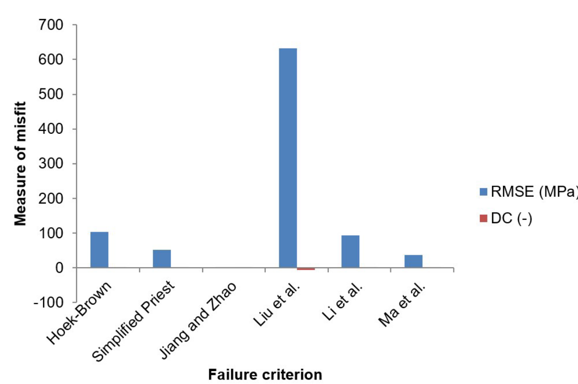

Figure 3. Measures of misfit for Westerly granite.

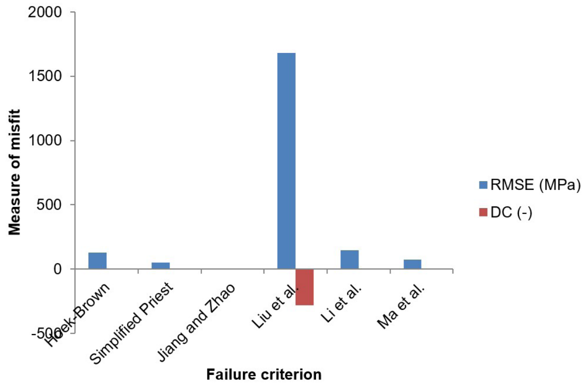

Figure 4. Measures of misfit for Dunham dolomite.

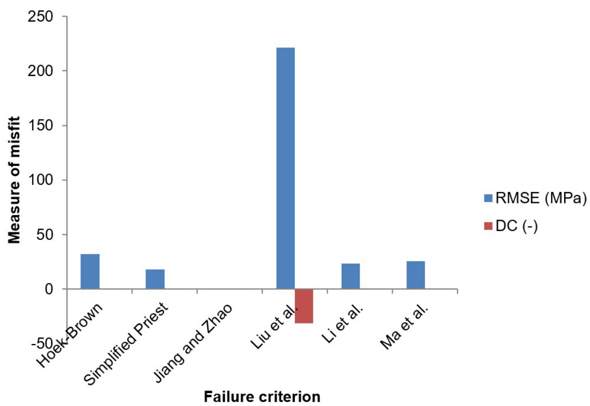

Figure 5. Measures of misfit for Shirahama sandstone.

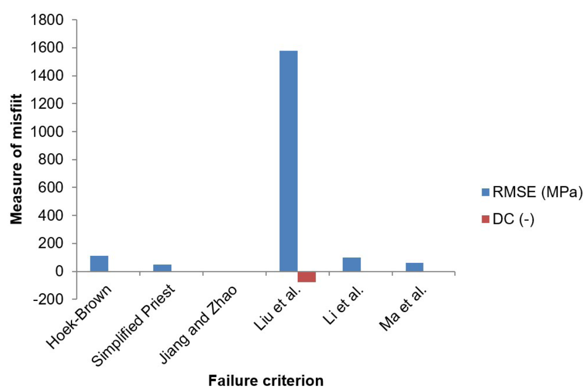

Figure 6. Measures of misfit for Yuubari shale.

(a) $0 < r < 0.4$ represents geomaterials with low dependence
(b) $0.4 \leq r < 0.7$ represents geomaterials with intermediate dependence
(c) $0.7 \leq r < 1$ represents geomaterials with high dependence.

4. Analysis and results

Table 4 shows the values of the correlation and partial correlation coefficients between the principal stress components. Following the classification scheme developed above, the correlation between the rock strength or failure stress and other principal stresses can be described as high, intermediate or low. The second column in Table 4 shows the correlation between the failure stress and the intermediate principal stress for the geomaterials studied. The result reveals that both KTB amphibolites and Yuubari shale had correlation coefficients which suggests an intermediate dependency of failure stress on the intermediate stress. While the Westerly granite, the Dunham dolomite and the Shirahama sandstone all showed correlation coefficients suggesting a high dependence of failure stress on the intermediate stress.

Table 4 also shows the correlation between the rock strength and the least principal stress in the third column. From the table, a very high correlation can be seen between the failure stress and the least principal stress for all the studied geomaterials. This high level of correlation is actually expected since classical rock mechanics demonstrates that the failure stress for any given geomaterial is largely influenced by the least principal stress (Jaeger et al., 2010). In addition, Table 4 equally shows the correlation between the intermediate stress and the least principal stress in the fourth column. The results obtained reveals that the KTB amphibolites, the Westerly granite, the Dunham dolomite, and the Shirahama sandstone all had correlation coefficients suggesting an intermediate correlation between the intermediate stress and the least principal stress. While the Yuubari shale results showed a low correlation between the intermediate stress and the least principal stress.

In order to give a better representation of the relationship between the failure stress and the intermediate stress, a partial correlation was also carried out. The partial correlation captures the dependence of strength on the intermediate stress in the presence of the least principal stress. Results from Table 4 and Table 5 show that the KTB amphibolite, the Westerly granite and the Yuubari shale all showed intermediate dependence on the intermediate stress. While the Dunham dolomite and the Shirahama sandstone displayed high and low dependence respectively.

Furthermore, the above results agree with the results of Colmenares & Zoback (2002), who also outlined these geomaterials (except Westerly granite, which they did not consider) as having intermediate to high strength dependence on the intermediate stress. However, Colmenares & Zoback (2002) reported mixed (high and low, depending on σ_3 values) σ_2 dependency results for the Shirahama sandstone. Which means that the Shirahama sandstone gave high correlation coefficient, but low partial correlation values between failure stress and intermediate stress in this study. Colmenares & Zoback (2002) also reported a strong strength dependency on σ_2 for confining stresses less than 100 MPa. But low

Table 4. Correlation and partial correlation coefficients between the principal stress components.

Geomaterial	r[1,2]	r[1,3]	r[2,3]	r[1,2:3]
KTB Amphibolite	0.640532	0.943167	0.500845	0.584602
Westerly granite	0.738096	0.972383	0.654320	0.577058
Dunham dolomite	0.797761	0.925552	0.599201	0.802214
Shirahama sandstone	0.710847	0.979049	0.660457	0.315420
Yuubari shale	0.518859	0.939148	0.321015	0.567598

Table 5. Categories of principal stress interdependencies for the studied geomaterials.

Geomaterial	r[1,2]	r[1,3]	r[2,3]	r[1,2:3]
KTB Amphibolite	intermediate	high	intermediate	Intermediate
Westerly granite	High	high	intermediate	Intermediate
Dunham dolomite	High	high	intermediate	High
Shirahama sandstone	High	high	intermediate	Low
Yuubari shale	intermediate	high	Low	Intermediate

Table 6. Measures of misfit for KTB Amphibolite.

Failure Criterion	RMSE (MPa)	DC (-)
Hoek & Brown (1980)	151.5995	0.7730
Simplified Priest (2012)	87.65936	0.9241
Jiang & Zhao (2015)	0.556811	0.8640
Liu et al. (2019)	719.3018	-7.3383
Li et al. (2021)	173.2040	0.7037
Ma et al. (2020)	73.11651	0.9472

Table 7. Measures of misfit for the Westerly granite.

Failure Criterion	RMSE (MPa)	DC (-)
Hoek & Brown (1980)	103.0620	0.8636
Simplified Priest (2012)	51.61271	0.9658
Jiang & Zhao (2015)	0.345612	0.9135
Liu et al. (2019)	632.5512	-6.1747
Li et al. (2021)	93.62682	0.8874
Ma et al. (2020)	38.04349	0.9814

strength dependency on σ_2 for confining stresses higher than 100 MPa, which agrees with the results of this study. Results showed in Table 5 completely agrees with the results by Ma et al. (2020), who reported exact categories of strength dependency on σ_2 for the geomaterials in this study. Figure 1 shows a pictorial presentation of the above results.

Figure 2 and Table 6 show the measures of misfit of failure stress predictions for KTB Amphibolite using root mean square error and coefficient of determination. From Figure 2, the highest RMSE was recorded by Liu et al. (2019) criterion, followed by Li et al. (2021), original Hoek & Brown (1980), simplified Priest (2012) and Ma et al. (2020) criteria, while the lowest was given by Jiang & Zhao (2015) criterion. But based on the coefficient of determination, the failure criterion with the highest coefficient of determination is Ma et al. (2020), followed by simplified Priest (2012), Jiang & Zhao (2015), Hoek & Brown (1980), and Li et al. (2021) criteria. While Liu et al. (2019) criterion showed a negative coefficient of determination. This result shows that the best three-dimensional modified Hoek & Brown criterion for KTB Amphibolite is the Ma et al. (2020) three-dimensional criterion as exemplified by the criterion having the highest coefficient of determination.

While the worst three-dimensional modified Hoek & Brown criterion for KTB Amphibolite is the Liu et al. (2019) criterion since the criterion had an unusually high root mean square error (RMSE) and negative coefficient of determination (DC). The negative DC also shows that the criterion does not describe the obtained polyaxial data for KTB Amphibolite. In addition, Table 6 shows a coefficient of determination of 0.7730 for the original Hoek & Brown (1980) criterion, which suggests that it could still be used for KTB Amphibolite with reasonable accuracy. But caution needs to be exercised due to the intermediate σ_2-dependence of strength for KTB Amphibolite (Ma et al., 2020). So, a two-dimensional criterion like the original Hoek & Brown (1980) criterion may not sufficiently predict failure stress for KTB Amphibolite.

Figure 3 and Table 7 show the measures of misfit of failure stress predictions for the Westerly granite using root mean square error and coefficient of determination. From Figure 3, the highest RMSE was recorded by Liu et al. (2019) criterion, followed by original Hoek & Brown (1980), Li et al. (2021), simplified Priest (2012), and Ma et al. (2020) criteria, while the lowest was given by Jiang & Zhao (2015) criterion. But based on the coefficient of determination, the failure criterion with the highest coefficient of determination is the one by Ma et al. (2020), followed by the simplified Priest (2012), Jiang & Zhao (2015), Li et al. (2021) and Hoek & Brown (1980) criteria. While Liu et al. (2019) had the least and also showed a negative coefficient of determination. This result shows that the best three-dimensional modified Hoek & Brown criterion for Westerly granite is the Ma et al. (2020) three-dimensional criterion as exemplified by the criterion having the highest coefficient of determination.

While the worst three-dimensional modified Hoek & Brown criterion for Westerly granite is the Liu et al. (2019) criterion since the criterion had an unusually high RMSE and negative DC. The negative DC also shows that the criterion does not describe the obtained polyaxial data for Westerly granite. In addition, Table 7 shows a coefficient of determination of 0.8636 for the original Hoek & Brown (1980) criterion, which suggests it could also be used for Westerly granite with reasonable accuracy. But caution needs to be exercised due to the intermediate σ_2-dependence of strength for Westerly granite (Ma et al., 2020). So, a two-dimensional criterion like the original Hoek & Brown (1980) criterion may not sufficiently predict failure stress for Westerly granite.

Figure 4 and Table 8 show the measures of misfit of failure stress predictions for Dunham dolomite using root mean square error and coefficient of determination. From Figure 4, the highest RMSE was recorded by Liu et al. (2019) criterion, followed by Li et al. (2021), original Hoek & Brown (1980), Ma et al. (2020), and simplified Priest (2012) criteria, while the lowest was given by Jiang & Zhao (2015) criterion. But based on the coefficient of determination, the failure criterion with the highest coefficient of determination was simplified Priest criterion, followed by Ma et al. (2020), Jiang & Zhao (2015), Hoek & Brown (1980) and Li et al. (2021) criteria. While Liu et al. (2019) criterion had the least and also showed a negative coefficient of determination. This result shows that the best three-dimensional modified Hoek & Brown criterion for Dunham dolomite is the simplified Priest (2012) three-dimensional criterion as exemplified by the criterion having the highest coefficient of determination.

While the worst three-dimensional modified Hoek & Brown criterion for Dunham dolomite is the Liu et al. (2019) criterion since the criterion had an unusually high RMSE and the least DC. The negative DC also shows that the criterion does not describe the obtained polyaxial data for Dunham dolomite. However, with a very low coefficient of determination of 0.264412 as depicted in Table 8, it means that the original Hoek & Brown criterion should not be used for Dunham dolomite. The reason for this can be explained by the high σ_2 -dependence of strength for Dunham dolomite (Colmenares & Zoback, 2002; Ma et al., 2020). So, a two-dimensional criterion like the original Hoek & Brown criterion cannot predict failure stress for Dunham dolomite.

Figure 5 and Table 9 show the measures of misfit of failure stress predictions for Shirahama sandstone using root mean square error and coefficient of determination. From Figure 5, the highest RMSE was recorded by Liu et al. (2019) criterion, followed by original Hoek & Brown (1980), Ma et al. (2020), Li et al. (2021), and simplified Priest (2012) criteria, while the lowest was given by Jiang & Zhao (2015) criterion. But based on the coefficient of determination, the failure criterion with the highest coefficient of determination was simplified Priest (2012) criterion, followed by Jiang & Zhao (2015), Li et al. (2021), Ma et al. (2020), and Hoek & Brown (1980) criteria. While Liu et al. (2019) criterion had the least and

Table 8. Measures of misfit for Dunham dolomite.

Failure Criterion	RMSE (MPa)	DC (-)
Hoek & Brown (1980)	126.8728	0.2644
Simplified Priest (2012)	53.00868	0.8736
Jiang & Zhao (2015)	0.259971	0.4307
Liu et al. (2019)	1682.973	-279.878
Li et al. (2021)	145.0804	0.1025
Ma et al. (2020)	74.29655	0.7372

Table 9. Measures of misfit for Shirahama sandstone.

Failure Criterion	RMSE (MPa)	DC (-)
Hoek & Brown (1980)	31.98625	0.6791
Simplified Priest (2012)	17.98074	0.8981
Jiang & Zhao (2015)	0.196489	0.8707
Liu et al. (2019)	221.4067	-31.0186
Li et al. (2021)	23.43394	0.8269
Ma et al. (2020)	25.74247	0.7911

also showed a negative coefficient of determination. This result shows that the best three-dimensional modified Hoek & Brown criterion for Shirahama sandstone is the simplified Priest (2012) three-dimensional criterion as exemplified by the criterion having the highest coefficient of determination.

While the worst three-dimensional modified Hoek & Brown criterion for Shirahama sandstone is the Liu et al. (2019) criterion since the criterion had an unusually high RMSE and the least DC. The negative DC also shows that the criterion does not describe the obtained polyaxial data for Shirahama sandstone. From Table 9, given a coefficient of determination of 0.6791, it means that the original Hoek & Brown criterion should be used with caution for Shirahama sandstone. The reason for this can be explained by the intermediate or mixed σ_2-dependence of strength for Shirahama sandstone (Colmenares & Zoback, 2002; Ma et al., 2020). So, a two-dimensional criterion like the original Hoek & Brown criterion may not sufficiently predict failure stress for Shirahama sandstone.

Figure 6 and Table 10 show the measures of misfit of failure stress predictions for the Yuubari shale using the root mean square error and the coefficient of determination. From Figure 6, the highest RMSE was recorded by Liu et al. (2019) criterion, followed by original Hoek & Brown (1980), Li et al. (2021), Ma et al. (2020), and simplified Priest (2012) criteria, while the lowest was given by Jiang & Zhao (2015) criterion. But based on the coefficient of determination, the failure criterion with the highest coefficient of determination was Ma et al. (2020), followed by simplified Priest (2012) criterion, Jiang & Zhao (2015), Hoek & Brown (1980) and Li et al. (2021) criteria. While Liu et al. (2019) criterion had the least and also showed a negative coefficient of determination. This result shows that the best three-dimensional modified Hoek & Brown criterion for Yuubari shale is the Ma et al. (2020) three-dimensional criterion as exemplified by the criterion having the highest coefficient of determination.

The worst three-dimensional modified Hoek & Brown criterion for the Yuubari shale is again the Liu criterion since the criterion had an unusually high RMSE and the least DC. The negative DC also shows that the criterion does not describe the obtained polyaxial data for Yuubari shale. Again, From Table 10, given a low coefficient of determination of 0.5939, it means that the original Hoek & Brown criterion should also be used with caution for the Yuubari shale. The reason for this can be explained by the σ_2-dependence of strength for Yuubari shale, which can be categorized as intermediate (Colmenares & Zoback, 2002; Ma et al., 2020). So, a two-dimensional criterion like the original Hoek & Brown criterion might not sufficiently predict failure stress for Yuubari shale.

Table 11 shows the ranking of failure criteria based on failure stress prediction accuracy. Ranking was done in such a way that 1 went to the criterion with the highest coefficient of determination for a given geomaterial and 5 went to the criterion with the least coefficient of determination for same geomaterial. The average rank was then generated by taking the arithmetic average of the ranks of the criteria for each of the studied geomaterials. This was done to ascertain the failure criterion which best predicted the failure stress for the studied geomaterials. And from the result obtained in this study, the failure criterion with best average prediction accuracy is the simplified Priest (2012) failure criterion, followed by Ma et al. (2020), and Jiang & Zhao (2015) criteria. Both the original Hoek & Brown and Li et al. (2021) criteria were tied, while the least failure criterion for all the studied geomaterials was Liu et al. (2019) failure criterion.

Table 10. Measures of misfit for Yuubari shale.

Failure Criterion	RMSE (MPa)	DC (-)
Hoek & Brown (1980)	110.4301	0.5939
Simplified Priest (2012)	48.53427	0.9183
Jiang & Zhao (2015)	0.291664	0.8458
Liu et al. (2019)	1576.939	-76.0232
Li et al. (2021)	96.84813	0.3895
Ma et al. (2020)	61.49562	0.9269

Table 11. Ranking of failure criteria based on the failure stress prediction accuracy.

Failure Criterion	KTB amphibolite	Westerly granite	Dunham dolomite	Shirahama sandstone	Yuubari shale	Average rank
Hoek & Brown (1980)	4	5	4	5	4	4
Simplified Priest (2012)	2	2	1	1	2	1
Jiang & Zhao (2015)	3	3	3	2	3	3
Liu et al. (2019)	6	6	6	6	6	6
Li et al. (2021)	5	4	5	3	5	4
Ma et al. (2020)	1	1	2	4	1	2

5. Conclusion

From this study, the following conclusions can be drawn.

(a) The intermediate principal stress significantly affects strength in geomaterials like the Dunham dolomite. It also moderately affects the strength of geomaterials like KTB amphibolites, Westerly granite and Yuubari shale. While the intermediate principal stress has mixed effects on strength in Shirahama sandstone.
(b) Based on results obtained here, it can be said that the original Hoek & Brown failure criteria could still be used with reasonable accuracy in predicting the failure stress for geomaterials whose strength shows low dependence on the intermediate principal stress.
(c) While a three-dimensional form of the Hoek & Brown criterion must be used in predicting failure stress for geomaterials like Dunham dolomite, whose strength shows a high dependence on the intermediate principal stress.
(d) But the original Hoek & Brown failure criteria should be used with caution in predicting failure stress for geomaterials like Shirahama sandstone, KTB amphibolites, Westerly granite, and Yuubari shale, whose strength shows either mixed or intermediate dependence on the intermediate principal stress.
(e) Based on the results of this study, the three-dimensional failure criterion with best average prediction accuracy was the simplified Priest (2012) failure criterion, followed by the Ma et al. (2020), and Jiang & Zhao (2015) criteria. Both the original Hoek & Brown and Li et al. (2021) criteria were tied, while the least failure criterion for all the studied geomaterials was Liu et al. (2019) failure criterion.

Acknowledgements

The authors wish to thank the Petroleum Technology Development Fund (PTDF), Nigeria, for sponsorship of this research with grant number PTDF-SPandD-AOGRG-V. III-117-011B.

Declaration of interest

The authors hereby declare no inherent conflict of interest in this study.

Authors' contributions

Nnamdi Emmanuel Ezendiokwere: Conceptualization, Data acquisition, Project administration, Visualization, Writing – original draft. Ogbonna Friday Joel: Funding acquisition, Project administration, Supervision. Victor Joseph Aimikhe: Formal Analysis, Investigation, Supervision. Adewale Dosunmu: Supervision, Validation, Project administration.

List of symbols

σ_1	maximum effective principal stresses at failure, MPa
σ_2	intermediate effective principal stresses at failure, MPa
σ_3	minimum effective principal stresses at failure, MPa
σ_c	uniaxial compressive strength of the intact rock, MPa
m	and s Hoek & Brown dimensionless parameters related to the characteristic of rock masses
m_b	value of Hoek & Brown constant, m for the rock mass in jointed rocks
a	Hoek & Brown constant that depend on the characteristics of the rock mass
GSI	Geological Strength Index
D	disturbance factor measuring disturbance due to blast damage and stress relaxation.
I_1	first stress invariant stress invariant, MPa
J_2	second deviatoric stress invariant, $(\text{MPa})^2$
A and B	Drucker-Prager empirical parameters
τ_{oct}	octahedral shear stress, MPa
$\sigma_{m,2}$	effective mean stress, MPa
n	Ma et al. σ_2 -dependency parameter, dimensionless
β	Mogi σ_2 -dependency parameter, dimensionless Li et al. σ_2 -dependency parameter, dimensionless
ω	Priest, Liu et al. σ_2 -dependency parameter, dimensionless
r [σ_1, σ_2]	coefficient of correlation between σ_1 and σ_2., dimensionless
$Cov[\sigma_1,\sigma_2]$	covariance between σ_1 and σ_2
$Var[\sigma_1]$	variance of σ_1
$r[\sigma_1,\sigma_2:\sigma_3]$	partial coefficient of correlation between σ_1 and σ_2, given σ_3, dimensionless
$RMSE$	root mean square error, MPa
DC	coefficient of determination, dimensionless

References

Al-Ajmi, A.M., & Zimmerman, R.W. (2005). Relation between the Mogi and the Coulombfailure criteria. *International Journal of Rock Mechanics and Mining Sciences*, 42(3), 431-439. http://dx.doi.org/10.1016/j.ijrmms.2004.11.004.

Chang, C., & Haimson, B.C. (2000). True Triaxial strength and deformability of the German continental deep drilling program (KTB) deep hole amphibolite. *Journal of Geophysical Research*, 105(B8), 8999-9013. http://dx.doi.org/10.1029/2000JB900184.

Colmenares, L.B., & Zoback, M.D. (2002). A statistical evaluation of intact rock failure criteria constrained by polyaxial test data for five different rocks. *International Journal of Rock Mechanics and Mining Sciences*, 39(6), 695-729. http://dx.doi.org/10.1016/S1365-1609(02)00048-5.

Drucker, D.C., & Prager, W. (1952). Soil mechanics and plastic analysis for limit design. *Quarterly of Applied Mathematics*, 10(2), 157-165.

Elyasi, A., & Goshtasbi, K. (2015). Using different rock failure criteria in wellbore stability analysis. *Geomechanics for Energy and the Environment*, 2, 15-21. http://dx.doi.org/10.1016/j.gete.2015.04.001.

Haimson, B.C., & Chang, C. (2000). A new true triaxial cell for testing mechanical properties of rock, and its use to determine rock strength and deformability of Westerly Granite. *International Journal of Rock Mechanics and Mining Sciences*, 37(1-2), 285-296. http://dx.doi.org/10.1016/S1365-1609(99)00106-9.

Handin, J., Heard, H.C., & Magouirk, J.N. (1967). Effects of the intermediate principal stress on the failure of limestone, dolomite, and glass at different temperatures and strain rates. *Journal of Geophysical Research*, 72(2), 611-640. http://dx.doi.org/10.1029/JZ072i002p00611.

Hoek, E., & Brown, E.T. (1980). Empirical strength criterion for rock masses. *Journal of Geotechnical Engineering*, 106(9), 1013-1035. http://dx.doi.org/10.1061/AJGEB6.0001029.

Hoek, E., & Brown, E.T. (1988). The Hoek-Brown failure criterion: a 1988 update. In J.H. Curran (Ed.), *15th Canadian Rock Mechanics Symposium: rock engineering for underground excavations* (pp. 31–38). Toronto, Canada: University of Toronto Press

Hoek, E., & Brown, E.T. (1997). Practical estimates of rock mass strength. *International Journal of Rock Mechanics and Mining Sciences & Geomechanics Abstracts*, 34(8), 1165-1186. http://dx.doi.org/10.1016/S1365-1609(97)80069-X.

Hoek, E., Carranza-Torres, C., & Corkum, B. (2002). Hoek-Brown failure criterion: 2002 edition. In R. Hammah (Ed.), *Proceedings of the 5th North American Rock Mechanics Symposium* (Vol. 1, pp. 267–273), Toronto, Canada: University of Toronto Press.

Hoek, E., Wood, D., & Hudson, S.S.J.A. (14-17 September, 1992). A modified Hoek-Brown criterion for jointed rock masses. In J. Hudson (Ed.), *Rock Characterization: ISRM Symposium* (pp. 209-214). Chester, UK: British Geotechnical Society.

Jaeger, J.C., Cook, N.G.W., & Zimmerman, R.W. (2010). *Fundamentals of rock mechanics* (5th ed.). Malden: Blackwell Publishing.

Jiang, H. (2017). A failure criterion for rocks and concrete based on the Hoek-Brown criterion. *International Journal of Rock Mechanics and Mining Sciences*, 95, 62-72. http://dx.doi.org/10.1016/j.ijrmms.2017.04.003.

Jiang, H., & Xie, Y. (2012). A new three-dimensional Hoek-Brown strength criterion. *Lixue Xuebao*, 28(2), 393-406. http://dx.doi.org/10.1007/s10409-012-0054-2.

Jiang, H., & Zhao, J. (2015). A simple three-dimensional failure criterion for rocks based on the Hoek–Brown criterion. *Rock Mechanics and Rock Engineering*, 48, 1807-1819. http://dx.doi.org/10.1007/s00603-014-0691-9.

Jiang, H., Wang, X.W., & Xie, Y.L. (2011). New strength criteria for rocks under polyaxial compression. *Canadian Geotechnical Journal*, 48(8), 1233-1245. http://dx.doi.org/10.1139/t11-034.

Lee, Y.K., Pietruszczak, S., & Choi, B.H. (2012). Failure criteria for rocks based on smooth approximations to Mohr-Coulomb and Hoek-Brown failure functions. *International Journal of Rock Mechanics and Mining Sciences*, 56, 146-160. http://dx.doi.org/10.1016/j.ijrmms.2012.07.032.

Li, H., Guo, T., Nan, Y., & Han, B. (2021). A simplified three-dimensional extension of Hoek-Brown strength criterion. *Journal of Rock Mechanics and Geotechnical Engineering*, 13(2), 568-578. http://dx.doi.org/10.1016/j.jrmge.2020.10.004.

Liu, C., Han, Y., Liu, H.-H., & Abousleiman, Y. (2019). Wellbore stability analysis by integrating a modified Hoek-Brown failure criterion with dual porochemoelectroelastic theory. *SPE Journal*, 24(5), 1957-1981. http://dx.doi.org/10.2118/195685-PA.

Lorenzo, R., Cunha, R.P., & Cordão-Neto, M.P. (18-20 September, 2013). Material point method for geotechnical problems involving large deformation. In M. Bischoff, E. Ramm, E. Oñate, R. Owen & P. Wriggers (Eds.), *III International Conference on Particle-Based Methods: Fundamentals and Applications* (pp. 510-521). Stuttgart: CINME.

Ma, L., Li, Z., Wang, M., Wu, J., & Li, G. (2020). Applicability of a new modified explicit three-dimensional Hoek-Brown failure criterion to eight rocks. *International Journal of Rock Mechanics and Mining Sciences*, 133, 104311. http://dx.doi.org/10.1016/j.ijrmms.2020.104311.

Mogi, K. (1967). Effect of the intermediate principal stress on rock failure. *Journal of Geophysical Research*, 72(20), 5117-5131. http://dx.doi.org/10.1029/JZ072i020p05117.

Mogi, K. (1971). Fracture and flow of rocks under high triaxial compression. *Journal of Geophysical Research*, 76, 1255-1269. http://dx.doi.org/10.1029/JB076i005p01255.

Mogi, K. (2007). *Experimental rock mechanics*. London: Taylor and Francis.

Murrell, S.A.F. (1965). The effect of triaxial stress systems on the strength of rocks at atmospheric temperatures. *Geophysical Journal International*, 10(3), 231-281. http://dx.doi.org/10.1111/j.1365-246X.1965.tb03155.x.

Pan, X.D., & Hudson, J.A. (1988). A simplified three-dimensional Hoek–Brown yield criterion. In M. Romana (Ed.), *Rock mechanics and power plants* (pp. 95–104). Madrid: Balkema.

Priest, S. (2005). Determination of shear strength and three-dimensional yield strength for the Hoek–Brown criterion. *Rock Mechanics and Rock Engineering*, 38(6), 299-327. http://dx.doi.org/10.1007/s00603-005-0056-5.

Priest, S. (2012). Three-dimensional failure criteria based on the Hoek–Brown criterion. *Rock Mechanics and Rock*

Engineering, 45(6), 989-993. http://dx.doi.org/10.1007/s00603-012-0277-3.

Ranjith, P.G., Zhao, J., Ju, M., Silva, R.V.S., Rathnaweera, T.D., & Bandara, A.K.M.S. (2017). Opportunities and challenges in deep mining: a brief review. *Engineering*, 3(4), 546-551. http://dx.doi.org/10.1016/J.ENG.2017.04.024.

Singh, M., & Singh, B. (2012). Modified Mohr–Coulomb criterion for non-linear triaxial and polyaxial strength of jointed rocks. *International Journal of Rock Mechanics and Mining Sciences*, 51, 43-52. http://dx.doi.org/10.1016/j.ijrmms.2011.12.007.

Takahashi, M., & Koide, H. (1989). Effect of the intermediate principal stress on strength and deformation behavior of sedimentary rocks at the depth shallower than 2000 m. In V. Maury (Ed.), *Rock at great depth: rock mechanics and rock physics at great depth (pp. 19-26)*. Rotterdam: Balkema.

Yua, M., Zana, Y., Zhaob, J., & Yoshimine, M. (2002). A unified strength criterion for rock material. *International Journal of Rock Mechanics and Mining Sciences*, 39, 975-989. http://dx.doi.org/10.1016/S1365-1609(02)00097-7.

Zhang, L., & Zhu, H. (2007). Three-dimensional Hoek–Brown strength criterion for rocks. *Journal of Geotechnical and Geoenvironmental Engineering*, 133(9), 1128-1135. http://dx.doi.org/10.1061/(ASCE)1090-0241(2007)133:9(1128).

Zhang, L.Y. (2008). A generalized three-dimensional Hock-Brown strength criterion. *Rock Mechanics and Rock Engineering*, 41(6), 893-915. http://dx.doi.org/10.1007/s00603-008-0169-8.

Zhang, Q., Zhu, H., & Zhang, L. (2013). Modification of a generalized three-dimensional Hoek–Brown strength criterion. *International Journal of Rock Mechanics and Mining Sciences*, 59, 80-96. http://dx.doi.org/10.1016/j.ijrmms.2012.12.009.

Zhou, X., Shou, Y., Qian, Q., & Yu, M. (2014). Three-dimensional non-linear strength criterion for rock-like materials based on the micromechanical method. *International Journal of Rock Mechanics and Mining Sciences*, 72, 54-60. http://dx.doi.org/10.1016/j.ijrmms.2014.08.013.

Zuo, J., Liu, H., & Li, H. (2015). A theoretical derivation of the Hoek-Brown failure criterion for rock materials. *Journal of Rock Mechanics and Geotechnical Engineering*, 7(4), 361-366. http://dx.doi.org/10.1016/j.jrmge.2015.03.008.