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An approach to robust condition monitoring in industrial processes using pythagorean membership grades

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Abstract: In this paper, a robust approach to improve the performance of a condition monitoring process in industrial plants by using Pythagorean membership grades is presented. The FCM algorithm is modified by using Pythagorean fuzzy sets, to obtain a new variant of it called Pythagorean Fuzzy C-Means (PyFCM). In addition, a kernel version of PyFCM (KPyFCM) is obtained in order to achieve greater separability among classes, and reduce classification errors. The approach proposed is validated using experimental datasets and the Tennessee Eastman (TE) process benchmark. The results are compared with the results obtained with other algorithms that use standard and non-standard membership grades. The highest performance obtained by the approach proposed indicate its feasibility.

Key words: Robust diagnostic approach, industrial plants, fuzzy algorithms, pythagorean fuzzy sets.

INTRODUCTION

Nowadays, there is a marked necessity in industrial plants to produce with higher quality in order to comply with environmental and industrial regulations (Hwang et al. 2010, Venkatasubramanian et al. 2003a). However, the faults in equipments can have an unfavorable impact on the availability of systems, environment and the safety of operators. For such reasons, the faults need to be detected and isolated using an effective condition monitoring system (Isermann 2011).

Within the condition monitoring methods are those based on models (Camps Echevarría et al. (2014a,b), Ding (2008), Patan (2008), Venkatasubramanian et al. (2003a,b)) and those based on historical data (Bernal de Lázaro et al. 2015, 2016, Pang et al. 2014, Sina et al. 2014). In the first approach, the use of models representing the operation of the processes is needed. The tools used in this approach are based on the generation of residuals obtained from the difference between the measurable signals from the real process and the values obtained from a model of this process. This implicates a high knowledge about the characteristics of the processes, their parameters, and operation zones. However, this knowledge is usually difficult to achieve due to the complexity of the industrial plants. On the other hand, the approaches based on historical data do not need a mathematical model, and they do not require much prior knowledge of the process parameters (Wang & Hu 2009, Cerrada et al. 2016, 2018). These characteristics constitute an advantage for complex systems, where relationships among variables are nonlinear, and not totally known. Therefore, it is difficult to obtain an analytical model that describes, efficiently, the dynamics of the process.

The use of techniques based on fuzzy tools has increased significantly in recent years in several scientific areas. In image processing the works Wang et al. (2020a,b) can be cited. In Gao et al. (2019) a new dendritic neuron model (DNM) taking into account the nonlinearity of synapses, not only for a better understanding of a biological neuronal system, but also for providing a more useful method for solving, is proposed. In the field of control theory, the authors of Tong et al. (2019) propose a data-based design approach for a Networked Tracking Control System which utilizes the input-output data of the controlled process to establish a predictive model with the help of Fuzzy Cluster Modelling technology. In Liu et al. (2015), a new robust dataset classification approach is presented based on neighbor searching and kernel fuzzy c-mean. The approach adopts the neighbor searching method with the dissimilarity matrix to normalize the dataset, and the number of clusters is determined by controlling clustering shape. In condition monitoring applications the works Rodríguez-Ramos et al. (2017, 2018b) can be cited.

A main aspect in the use of fuzzy sets is the provision of membership grades. In order to enhance the capability of fuzzy sets to capture and model user provided membership information, researchers have introduced non-standard second order fuzzy sets such as intuitionistic (Atanassov 1986, 2012) and interval type-2 fuzzy sets (Mendel et al. 2006, Mendel & Wu 2010). These non–standard fuzzy sets allow the inclusion of imprecision and uncertainty in the specification of membership grades.

Recently, Prof. Ronald R. Yager introduced another class of non-standard fuzzy subset named Pythagorean fuzzy subset (Yager 2014). In Yager (2014), it is shown that the space of Pythagorean membership grades is greater than the space of intuitionistic membership grades. This allows the use of the Pythagorean fuzzy sets in a greater set of applications than the intuitionistic fuzzy sets.

Normally, the data obtained from complex industrial processes are corrupted by noise. This introduces uncertainties in the observations which seriously affect the performance of the condition monitoring systems. On the one hand, this situation can cause false alarms when the fault diagnosis system confuses the Normal Operation Condition (NOC) with a fault. In another sense, the fault diagnosis system can present problems to correctly distinguish or classify a fault that is affecting the process. Both situations will lead to erroneous decision making, and may cause economic losses and affect industrial safety.

In order to overcome the problems mentioned above, and to obtain a robust condition monitoring scheme, an approach based on the use of Pythagorean membership grades is proposed which constitutes the main contribution of this paper. This approach allows to obtain a new variant of the known Fuzzy C-Means algorithm, called Pythagorean Fuzzy C-Mean algorithm (PyFCM), and it's kernel version (KPyFCM) in order to achieve greater separability among classes and reduce classification errors. Both algorithms also constitute, other contributions of the paper.

The organization of the paper is as follow: in Section of Material and Methods, the general characteristics of the tools used in the proposed methodology are presented. Also, the principal theoretical aspects of the Pythagorean memberships grades theory are presented. In Section Description of the proposal, a description of the classification methodology using fuzzy clustering techniques is presented. In Section Study Cases and Experimental Design the proposed methodology is evaluated with four synthetic datasets and with the benchmark Tennessee Eastman (TE) process.

Next, an analysis of the results obtained and a comparison with other computational tools is developed in Section Analysis of results. Finally, the conclusions are presented.

MATERIALS AND METHODS

In this section, a general description of Fuzzy C-Means (FCM), and Intuitionistic FCM (IFCM) algorithms are presented first. Later, the general characteristics of the Pythagorean membership grades, the new variant of the FCM algorithm, Pythagorean FCM algorithm (PyFCM), and its kernel version (KPyFCM) are also presented.

Fuzzy C-Means

Different methods have been proposed for fuzzy clustering. Among them, the most common are those based on distance. One of these methods, and the most popular, is the Fuzzy C-Means (FCM) algorithm (Bezdek et al. 1984) which uses the optimization criterion (1) to group the data according to the similarity between them.

$$J_{FCM} = \sum_{i=1}^{c} \sum_{k=1}^{N} (u_{ik})^m (d_{ik})^2 \tag{1}$$

The exponent $m > 1$ in (1), is an important factor that regulates the fuzziness of the resulting partition. The fuzzy clustering allows for obtaining the membership degrees matrix $U = [u_{ik}]_{c \times N}$ where u_{ik} represents the fuzzy membership degree of the sample k to the ith class, which satisfies:

$$\sum_{i=1}^{c} u_{ik} = 1, k = 1, 2, ..., N \tag{2}$$

where c is the number of classes and N is the number of samples. In this algorithm, the similarity is evaluated by using the distance function d_{ik}, represented by the Eq. (3). This function provides a measure of the distance between the data and the center of the classes $v = v_1, v_2, ..., v_c$, being $\mathbf{A} \in \mathfrak{R}^{n \times n}$ the norm induction matrix, where n is the quantity of measured variables.

$$d_{ik}^2 = (\mathbf{x}_k - \mathbf{v}_i)^T \mathbf{A} (\mathbf{x}_k - \mathbf{v}_i) \tag{3}$$

The measure of dissimilarity is the square distance between each data point and the clustering center $\mathbf{v}_i$. This distance is weighted by a power of the membership degree $(u_{ik})^m$. The value of the cost function J is a measure of the weighted total quadratic error and statistically, it can be seen as a measure of the total variance of $\mathbf{x}_k$ regarding $\mathbf{v}_i$.

The conditions for local extreme in Eq. (1) and Eq. (2) are derived using Lagrangian multipliers (Bezdek 1981):

$$u_{ik} = \frac{1}{\sum_{j=1}^{c} \left(d_{ik,\mathbf{A}} / d_{jk,\mathbf{A}} \right)^{2/(m-1)}} \tag{4}$$

$$\mathbf{v}_i = \frac{\sum_{k=1}^{N} \left(u_{ik}^m \mathbf{x}_k \right)}{\sum_{k=1}^{N} u_{ik}^m} \tag{5}$$

In Eq. (5), it should be noted that $\mathbf{v}_i$ is the weighted average of the data elements that belong to a cluster, i.e., it is the center of the cluster i. FCM algorithm is an iterative procedure where N data are grouped in c classes. Initially, the user should establish the number of classes (c). The centers of the c classes are initialized in a random form, and they are modified during the iterative process. In a similar way, the membership degrees matrix U is modified until the convergence, i.e. $\|U_t - U_{t-1}\| < \epsilon$, where ϵ is a tolerance limit prescribed a priori, and t is an iteration counter.

The stopping criteria used in this algorithm are:

1. Criterion 1: Maximum number of iterations (Itr_{max}).

2. Criterion 2: $\|U_t - U_{t-1}\| < \epsilon$

Intuitionistic Fuzzy C-Means algorithm

Intuitionistic fuzzy c-means clustering algorithm (Chaira 2011) is based upon intuitionistic fuzzy set theory. Fuzzy set generates only membership function $\mu(x), x \in X$, whereas intuitionistic fuzzy set (IFS) given by Atanassov 2012 considers both membership $\mu(x)$ and nonmembership $\upsilon(x)$. An intuitionistic fuzzy set $\mathbf{A}$ in X, is written as:

$$\mathbf{A} = \{x, \mu_{\mathbf{A}}(x), \upsilon_{\mathbf{A}}(x) \,|x \in X\} \tag{6}$$

where $\mu_{\mathbf{A}}(x) \longrightarrow [0,1]$, $\upsilon_{\mathbf{A}}(x) \longrightarrow [0,1]$ are the membership and non-membership degrees of an element in the set $\mathbf{A}$ with the condition: $0 \leq \mu_{\mathbf{A}}(x) + \upsilon_{\mathbf{A}}(x) \leq 1$.

When $\upsilon_{\mathbf{A}}(x) = 1 - \mu_{\mathbf{A}}(x)$ for every $x \in \mathbf{A}$, then the set $\mathbf{A}$ becomes a fuzzy set. For all intuitionistic fuzzy sets, a hesitation degree $\pi_{\mathbf{A}}(x)$ is also indicated (Atanassov 2012). It express the lack of knowledge in defining of whether x belongs to IFS or not and is given by:

$$\pi_{\mathbf{A}}(x) = 1 - \mu_{\mathbf{A}}(x) - \upsilon_{\mathbf{A}}(x); 0 \leq \pi_{\mathbf{A}}(x) \leq 1 \tag{7}$$

A graphical representation of the meaning of $\pi_{\mathbf{A}}(x)$ is shown in Figure 1.

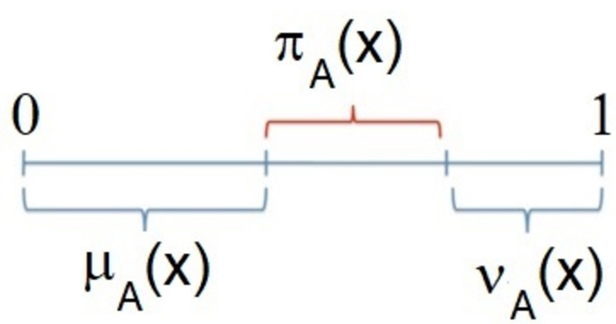

Figure 1. Graphical representation of the meaning of the hesitation degree $\pi_{\mathbf{A}}(\mathbf{x})$.

Intuitionistic fuzzy c-means objective function contains two terms: (i) modified objective function of conventional FCM using Intuitionistic fuzzy set and (ii) intuitionistic fuzzy entropy (IFE). IFCM minimizes the objective function as:

$$J_{IFCM} = \sum_{i=1}^{c} \sum_{k=1}^{N} u_{ik}^{*m} d_{ik}^{2} + \sum_{i=1}^{c} \pi_i^{*} e^{1-\pi^{*}} \tag{8}$$

$u^{*}_{ik} = u^{m}_{ik} + \pi_{ik}$, where u^{*}_{ik} denotes the intuitionistic fuzzy membership and u_{ik} denotes the conventional fuzzy membership of the *kth* data in the *ith* class. π_{ik} is the hesitation degree, which is defined as:

$$\pi_{ik} = 1 - u_{ik} - (1 - u^{\alpha}_{ik})^{1/\alpha}, \alpha > 0 \tag{9}$$

and it is calculated from Yager's intuitionistic fuzzy complement as

$$N(x) = (1 - x^{\alpha})^{1/\alpha}\ ,\ \alpha > 0 \tag{10}$$

thus, with the help of Yager's intuitionistic fuzzy complement, intuitionistic fuzzy set becomes:

$$A = \left\{x, \mu_A(x), (1 - \mu_A(x)^{\alpha})^{1/\alpha}\, |x \in X\right\} \tag{11}$$

and

$$\pi^{*}_{i} = \frac{1}{N}\sum_{k=1}^{N} \pi_{ik}, k \in [1, N] \tag{12}$$

The second term in the objective function is called intuitionistic fuzzy entropy (IFE). Initially, the idea of fuzzy entropy was given by Zadeh in 1968 (Zadeh 1968). It is the measure of fuzziness in a fuzzy set. Similarly in the case of IFS, intuitionistic fuzzy entropy gives the amount of vagueness or ambiguity in a set. For intuitionistic fuzzy cases, if $\mu_A(x_i)$, $v_A(x_i)$, $\pi_A(x_i)$ are the membership, non-membership, and hesitation degrees of the elements of the set $X = x_1, x_2, ..., x_n$, then intuitionistic fuzzy entropy, IFE that denotes the degree of intuitionism in fuzzy set, may be given as:

$$IFE(A) = \sum_{i=1}^{n} \pi_A(x_i) e^{[1-\pi_A(x_i)]} \tag{13}$$

where $\pi_A(x_i) = 1 - \mu_A(x_i) - v_A(x_i)$ IFE is introduced in the objective function to maximize the good points in the class. The goal is to minimize the entropy. Modified cluster centers are:

$$v^{*}_{i} = \frac{\sum_{k=1}^{n} u^{*}_{ik} x_k}{\sum_{k=1}^{n} u^{*}_{ik}} \tag{14}$$

Pythagorean Fuzzy C-Means algorithm (PyFCM)

In Yager (2014), a new class of nonstandard fuzzy sets called Pythagorean fuzzy sets (PFS) is introduced. Hereinafter, the membership grades associated with these sets will be named as Pythagorean membership grades. Following, the main characteristics of the Pythagorean membership grades are presented.

For expressing the Pythagorean membership grades a pair of values $r(x)$ and $d(x)$ for each $x \in X$ are assigned. Both values will be called as the strength of commitment at x in the case of $r(x) \in [0, 1]$ and the direction of commitment in the case of $d(x) \in [0, 1]$. The values $r(x)$ and $d(x)$ are associated with a pair of membership grades $A_Y(x)$ and $A_N(x)$. These memberships grades indicate the support for membership of x in A and the support against membership of x in A respectively. Next, it is shown that $A_Y(x)$ and $A_N(x)$ are related using the Pythagorean complement with respect to $r(x)$. More specially, the values of $A_Y(x)$ and $A_N(x)$ are defined from $r(x)$ and $d(x)$ as

$$A_Y(x) = r(x)cos(\theta(x)) \tag{15}$$

$$A_N(x) = r(x)sin(\theta(x)) \tag{16}$$

where

$$\theta(x) = (1 - d(x))\frac{\pi}{2} \tag{17}$$

and $\theta(x) \in [0, \frac{\pi}{2}]$ is expressed in radians.

First, it is shown that $A_Y(x)$ and $A_N(x)$ are Pythagorean complements with respect to *r(x)*. Squaring Eqs. (15) and (16)

$$A_Y^2(x) = r^2(x)cos^2(\theta(x)) \tag{18}$$

$$A_N^2(x) = r^2(x)sin^2(\theta(x)) \tag{19}$$

and, by adding both equations the following is obtained

$$A_Y^2(x) + A_N^2(x) = r^2(x)(cos^2(\theta) + sin^2(\theta)) \tag{20}$$

From the Pythagorean theorem, it is known that $cos^2(\theta) + sin^2(\theta) = 1$. Then

$$A_Y^2(x) + A_N^2(x) = r^2(x)\underbrace{(cos^2(\theta) + sin^2(\theta))}_{1} \tag{21}$$

and hence

$$A_Y^2(x) = r^2(x) - A_N^2(x) \tag{22}$$

Thus, it is evident that A_Y and A_N are Pythagorean complements with respect to $r(x)$.

The direction of the strength, $d(x)$, indicates on a scale of 1 to 0 how fully the strength *r(x)* is pointing to membership. From Eq. (17) $\theta(x) = (1 - d(x))\frac{\pi}{2}$. If $d(x) = 1$, then $\theta(x) = 0$, therefore, $cos(\theta(x)) = 1$ and $sin(\theta(x)) = 0$. From Eq. (16), $A_N(x) = 0$, and from Eq. (22) $A_Y(x) = r(x)$. This indicates that the direction of $r(x)$ is completely to membership. On the other hand, performing a similar analysis for $d(x) = 0$, it is obtained that $A_Y(x) = 0$ and $A_N(x) = r$. This indicates that the direction of the strength is completely to nonmembership. Intermediate values of $d(x)$ indicate partial support to membership and nonmembership.

In a general form, a Pythagorean membership grade is represented by a pair of values (a, b) such that $a, b \in [0, 1]$ and $a^2 + b^2 \leq 1$. In this case, $a = A_Y(x)$, indicates the degree of support for membership of x in A and, $b = A_N(x)$ indicates the degree of support against membership of x in A. Taking into account the pair (a, b), the equation (20) can be expressed as $a^2 + b^2 = r^2$. The latter indicates that a Pythagorean membership grade is a point of a circle of radius r.

An intuitionistic membership grade presented in Atanassov (2012) is also a pair (a, b) that satisfies $a, b \in [0, 1]$ and $a + b \leq 1$. In Yager (2014),it was demonstrated that the set of Pythagorean membership grades is greater than the set of intuitionistic membership grades. That result is clearly shown in Figure 2 taken from Yager (2014). Here, it is possible to observe that intuitionistic membership grades are all points under the line $x + y \leq 1$ and the Pythagorean membership grades are all points with $x^2 + y^2 \leq 1$.

Taking into account the theory of Pythagorean fuzzy sets, it can be said that the objective function on the Pythagorean Fuzzy C-Means algorithm (PyFCM) is similar to the one obtained for the IFCM algorithm according equation 8. In this case, a hesitation degree, $\pi_A(x)$, is given by:

$$\pi_A(x) = 1 - \mu_A^2(x) - v_A^2(x); 0 \leq \pi_A(x) \leq 1 \tag{23}$$

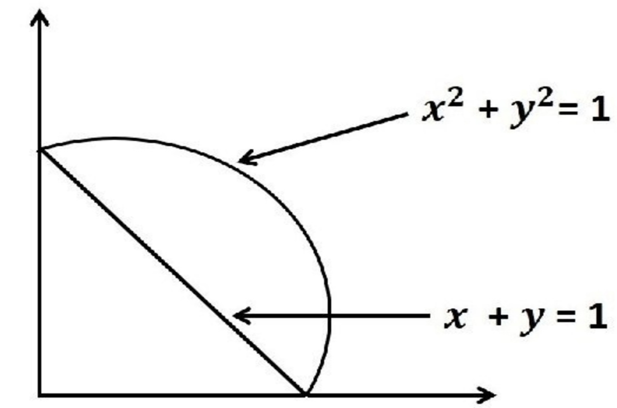

Figure 2. Comparison of space of Pythagorean and intuitionistic membership grades.

Therefore, in equation 8, π_{ik} is defined as:

$$\pi_{ik} = 1 - u_{ik}^2 - (1 - u_{ik}^{\alpha})^{2/\alpha}, \alpha > 0 \tag{24}$$

The most important implication of this result is the possibility of using the Pythagorean fuzzy sets in a larger set of situations than intuitionistic fuzzy sets. In the case of fault diagnosis, this result allows to improve the classification process.

Pythagorean Fuzzy C-Means algorithm based on a kernel approach

KFCM represents the kernel version of FCM. This algorithm uses a kernel function for mapping the data points from the input space to a high dimensional space, as shown in Figure 3.

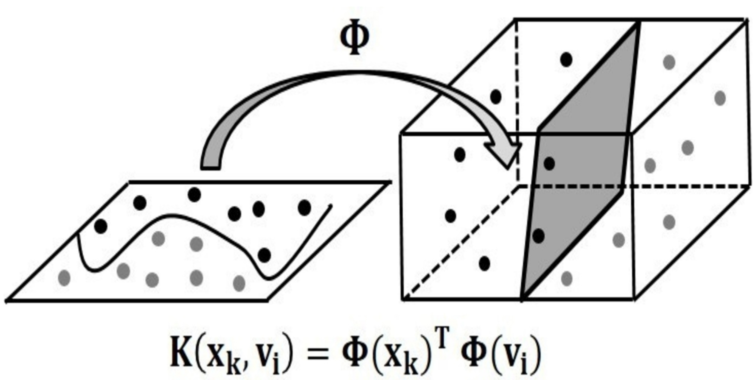

Figure 3. KFCM feature space and kernel space.

In this case, a kernel version of the PyFCM (KPyFCM) is obtained in order to achieve greater separability among classes, and reduce the classification errors. KPyFCM minimizes the objective function:

$$J_{KPyFCM} = \sum_{i=1}^{c} \sum_{k=1}^{N} u_{ik}^{*m} \left\| (\mathbf{x_k}) - (\mathbf{v_i}) \right\|^2 + \sum_{i=1}^{c} \pi_i^* e^{1-\pi^*} \tag{25}$$

where $u_{ik}^* = u_{ik}^m + \pi_{ik}$, π_{ik} hesitation degree, which is defined according to Eq. (24) and π_i^* is defined as the equation (12).

Also, $\|(\mathbf{x_k})-(\mathbf{v_i})\|^2$ is the square of the distance between $(\mathbf{x_k})$ and $(\mathbf{v_i})$. The distance in the feature space is calculated through the kernel in the input space as follows:

$$\|(\mathbf{x_k})-(\mathbf{v_i})\|^2 = \mathbf{K}(\mathbf{x_k},\mathbf{x_k}) - \mathbf{2K}(\mathbf{x_k},\mathbf{v_i}) + \mathbf{K}(\mathbf{v_i},\mathbf{v_i}) \tag{26}$$

If the Gaussian kernel is used, then $\mathbf{K}(\mathbf{x},\mathbf{x}) = \mathbf{1}$ and $\|(\mathbf{x_k})-(\mathbf{v_i})\|^2 = \mathbf{2}\left(\mathbf{1}-\mathbf{K}(\mathbf{x_k},\mathbf{v_i})\right)$. Thus, Eq. (25) can be written as:

$$J_{KPyFCM} = 2\sum_{i=1}^{c}\sum_{k=1}^{N} u_{ik}^{*m}\,\|1-\mathbf{K}(\mathbf{x_k},\mathbf{v_i})\|^2 + \sum_{i=1}^{c}\pi_i^{*}e^{1-\pi^{*}} \tag{27}$$

where,

$$\mathbf{K}(\mathbf{x_k},\mathbf{v_i}) = e^{-\|\mathbf{x}_k-\mathbf{v}_i\|^2/\sigma^2} \tag{28}$$

It is possible to find many different kernel functions in the scientific literature, and the Gaussian kernel is one of the most popular. In general, the selection of a kernel depends on the application (Bernal de Lázaro et al. 2015, Motai 2015, Nayak et al. 2015, 2016). In this paper, several experiments were performed using various kernel functions such as the Gaussian Kernel, the Polynomial Kernel and the Hyper-tangent Kernel. Considering the results that were obtained, the Gaussian Kernel was selected.

Minimizing Eq. (27) under the constraint shown in Eq. (2), yields:

$$u_{ik}^{*} = \frac{1}{\sum_{j=1}^{c}\left(\frac{1-\mathbf{K}(\mathbf{x_k},\mathbf{v_i})}{1-\mathbf{K}(\mathbf{x_k},\mathbf{v_j})}\right)^{1/(m-1)}} \tag{29}$$

$$\mathbf{v}_i = \frac{\sum_{k=1}^{N}\left(u_{ik}^{*m}\mathbf{K}(\mathbf{x_k},\mathbf{v_i})\mathbf{x_k}\right)}{\sum_{k=1}^{N}u_{ik}^{*m}\mathbf{K}(\mathbf{x_k},\mathbf{v_i})} \tag{30}$$

KPyFCM algorithm is presented in Algorithm 1.

Algorithm 1 Pythagorean Fuzzy C-Means algorithm based in a kernel approach (KPyFCM)

Input: data, c, $\epsilon > 0$, $m > 1$, σ, Itr_{max} (number of iterations)
Output: fuzzy partition $\mathbf{U}$, class centers $\mathbf{V}$
1. Initialize $\mathbf{U}$ to random fuzzy partition
2. $t \leftarrow 1$
3. **repeat**
4. Update the center of each class according to (30) for Gaussian kernels
5. Calculate the distances according to (26)
6. Update $\mathbf{U}$ according to (29).
7. $t \leftarrow t + 1$
8. **until** $\|U_t - U_{t-1}\| < \epsilon \wedge \mathrm{t} \geq Itr_{max}$
9. **return** fuzzy partition $\mathbf{U}$, class centers $\mathbf{V}$

DESCRIPTION OF THE PROPOSAL

The classification scheme proposed in this paper is shown in Figure 4. It presents an offline training stage and an online recognition stage. In the training stage, the historical data of the process are used to train (modeling the functional stages through the clusters) a fuzzy classifier. After the training, the classifier is used online (recognition) in order to classify every new sample taken from the process. The result intends to offer information about the system state in real-time for the operator .

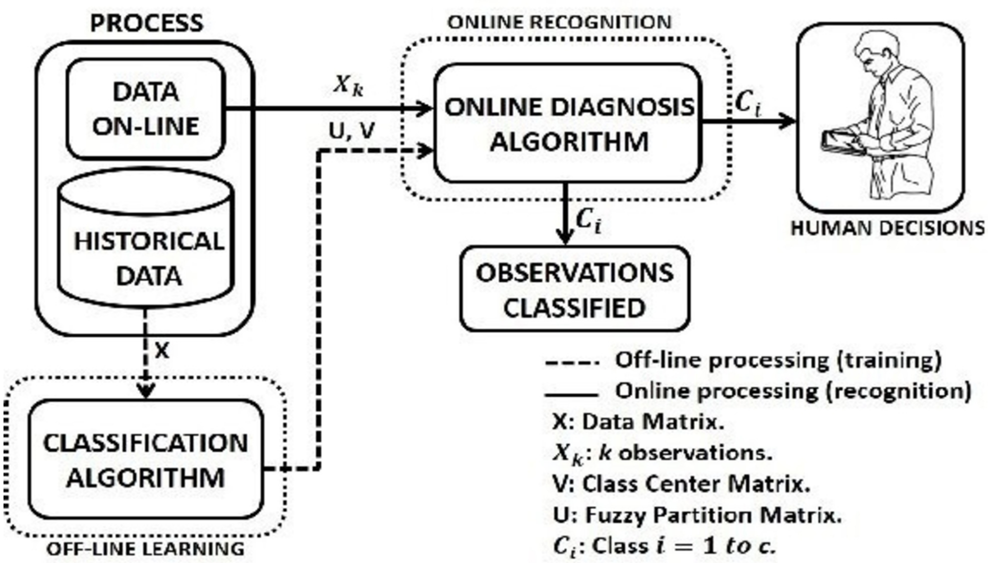

Figure 4. Classification scheme using fuzzy clustering.

The clustering methods group the data in different classes based on a measure of similitude. In the processes, the data are acquired by means of a SCADA (Supervisory Control and Data Acquisition) system, and the classes can be associated with functional states. In the case of statistical classifiers, each sample is compared with the center of each class by means of a measure of similitude to determine to which class the sample belongs. In the case of the fuzzy classifiers, the comparison is made to determine the membership degree of the sample to each class. In general, the higher membership degree determines the class to which the sample is assigned, as it is shown in (31).

$$C_i = \{i : max\{\mu_{ik}\}, \forall i, k\} \tag{31}$$

Off-line training

In this stage, a historical dataset representative of the different operation states of the process (classes) is used for training the fuzzy classifier, where the center of each class is determined.

Online recognition

In this stage, the observations obtained by the SCADA system are classified one by one. In the classification process, the distance between the received observation and each one of the class centers is calculated. Next, the fuzzy membership degree of the observation to each one of the c classes is obtained. The observation will be assigned to the class with highest membership degree (See Algorithm 2).

Algorithm 2 Recognition

Input: data X_k, class centers $\mathbf{V}$, m, σ.
Output: Current State.
for $k = 1$ to $k = N$ **do**
  Calculate the distances from the observation k to class centers according to Eq. (26).
  Calculate the membership degree of the observation k to the c classes according to Eq. (29).
  Determine to which class belongs the observation k using Eq. (31).
end for

The general condition monitoring scheme used for all performed experiments, is shown in Fig. 5. In this scheme, it can be seen that the on-line recognition algorithm determines the existence of a fault if j samples representative of it are received in a window of time. Later, an abnormal situation alarm of the process is executed. This is done with the aim of reducing false alarms in the presence of noise or outliers. The parameter j and the dimension of the window of time are selected by the expert operator of the plant in correspondence to the process.

STUDY CASES AND EXPERIMENTAL DESIGN

In this section the datasets shown in Figure 6 are presented to validate the performance of the novel condition monitoring scheme proposed in this paper. These datasets were created synthetically to have complex situations for classification despite having only two classes of two variables. The datasets Data A, Data B and Data D have 1000 observations each one and the dataset Data C has 700 observations. These datasets will be identified in the rest of the paper as Experimental Datasets (ED). In the training, 750 observations from the Data A, B and D were used and 250 observations were used in the recognition stage. In the case of the Data C, 525 observations were used in the training and 175 observation were used in the recognition stage.

Several experiments (σ=10,20,30,40,...,100) were performed and the value of σ that gave the best results in the classification was selected.

The values of the parameters used for the applied algorithms were: Number of iterations = 100, ϵ = 10^{-5}, m = 2, σ = 10 (only used for the version kernel of the algorithms).

The second case study is the Tennessee Eastman (TE) process benchmark which has been widely used to evaluate the performance of new control and monitoring strategies (Yin et al. 2012, Prieto-Moreno et al. 2015, Llanes-Santiago et al. 2019). The process consists of five major units interconnected as shown in Figure 7.

This benchmark contains 21 preprogrammed faults and one normal operating condition dataset. The datasets of the TE are generated for 48h with the inclusion of faults after 8 hours of simulation. The control objectives and general features of the process simulation are described in the paper Downs & Vogel (1993). All data sets used in this paper can be downloaded from http://web.mit.edu/braatzgroup/TE_process.zip. Table I shows the faults considered to evaluate the

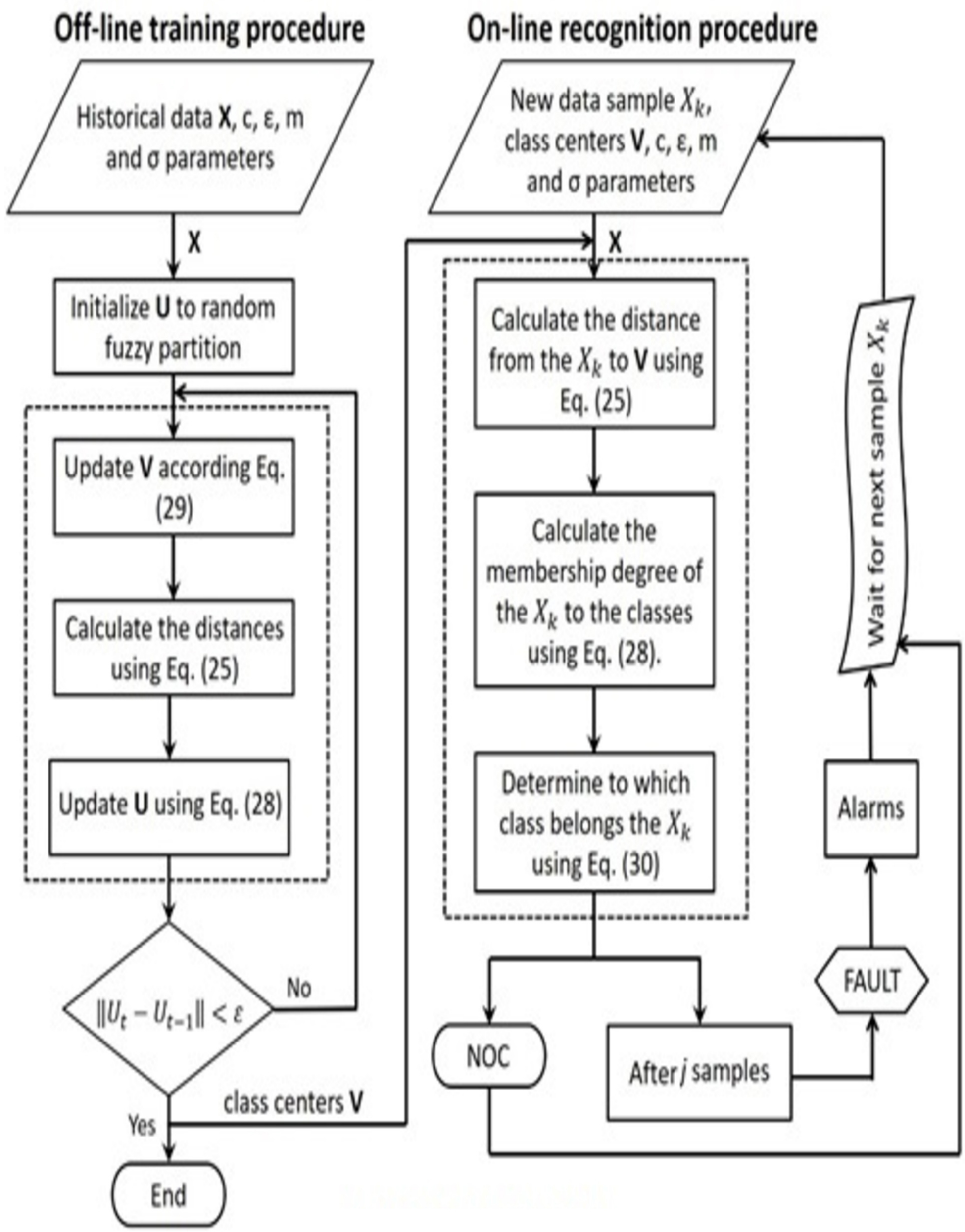

Figure 5. Flowchart of the condition monitoring process.

benefits of the proposal presented in this paper. For the training, 480 observations of each fault were used, and 960 observations in the online recognition.

Another problem of the current fuzzy clustering method is related to the correct selection of its parameters which is decisive in obtaining a high performance. Nowadays, these issues of crucial importance are open problems in the fault diagnosis applications and in others research fields (Wang et al. 2020a, Filho et al. 2015, 2016). The parameters Number of iterations, ϵ, m and σ were selected according to the experience in previous works (Rodríguez-Ramos et al. 2019, 2018a).

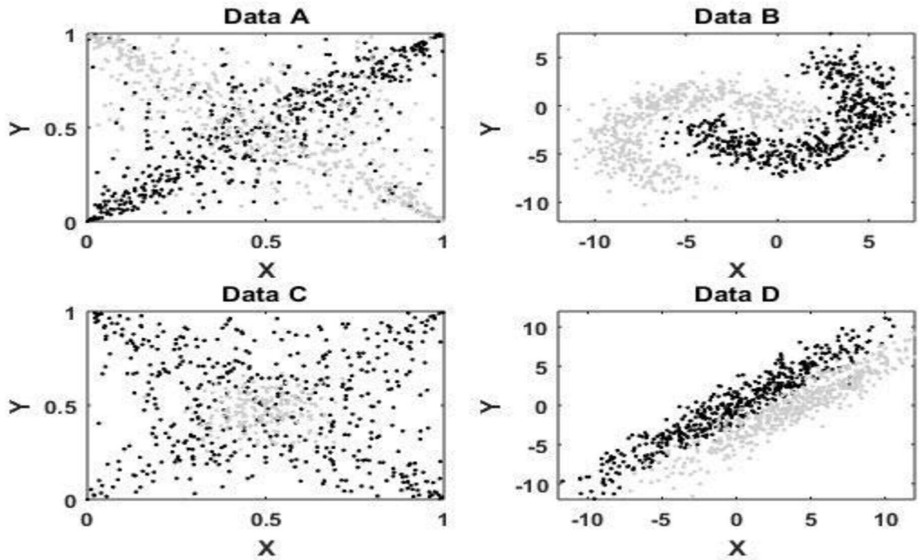

Figure 6. Datasets for experiments.

The values of the parameters used for the applied algorithms are: Number of iterations = 100, ϵ = 10^{-5}, m = 2, σ = 50 (only used for the version kernel of the algorithm).

Table I. Description of faults of the TE process.

Fault	Process variable	Type
F1	A/C feed ratio, B composition constant	step
F2	B composition, A/C ration constant	step
F6	A feed loss	step
F7	C header pressure loss-reduced availability	step

ANALYSIS OF RESULTS

A very important step in the design of fault diagnosis systems consist in verifying the quality of the performed task. The most used criterion for this analysis is the confusion matrix (CM). The confusion matrix is an indicator that allows for obtaining the performance of the classifier in the classification process. Each CM_{rs} element of a confusion matrix for $r \neq s$, indicates the number of times that the classifier confuses a state r with a state s in a set of L experiments. The results obtained from the application of the proposed methodology to fault diagnosis by using the experimental datasets (ED) and in the TE process are presented next. Figure 8 illustrates a confusion matrix and the respective interpretation.

Note that the main diagonal of the matrix CM_{rs} represents the number of correct samples detected/classified and the values out of this diagonal reflect the confusion between the classes or the operation states.

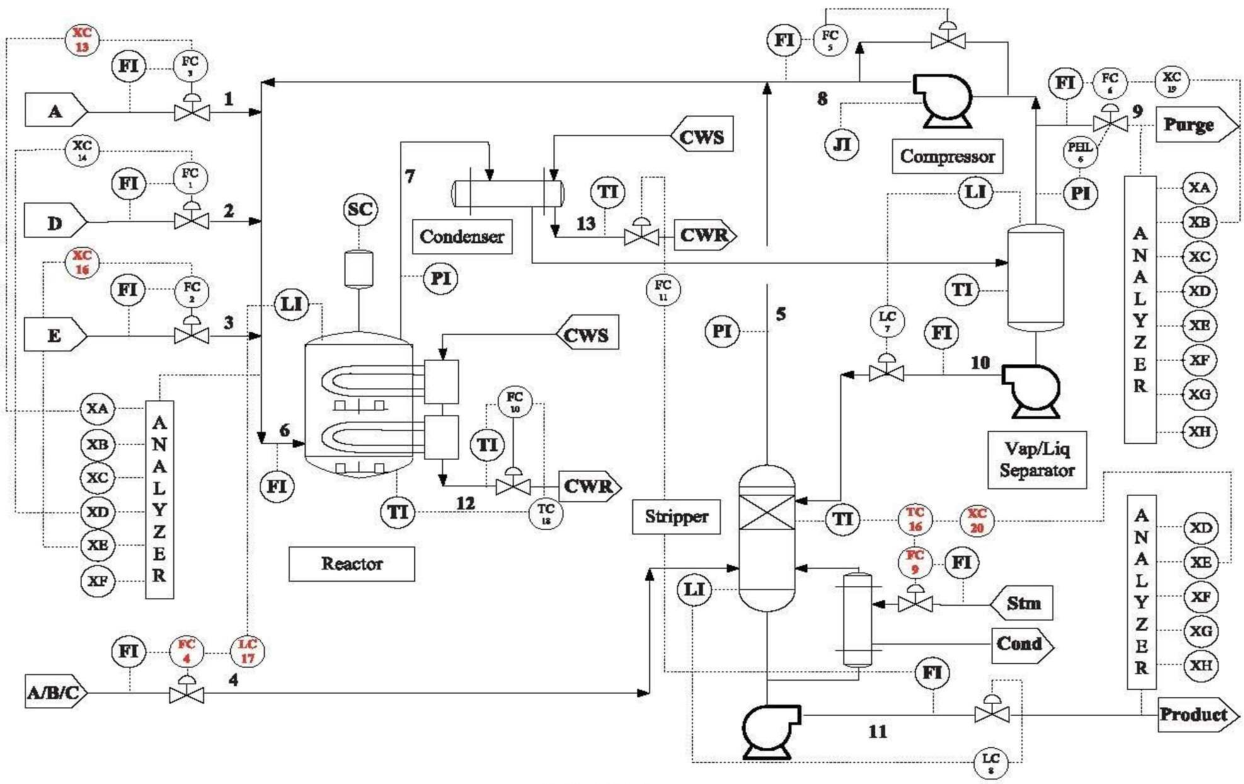

Figure 7. Piping diagram of the Tennessee Eastman process.

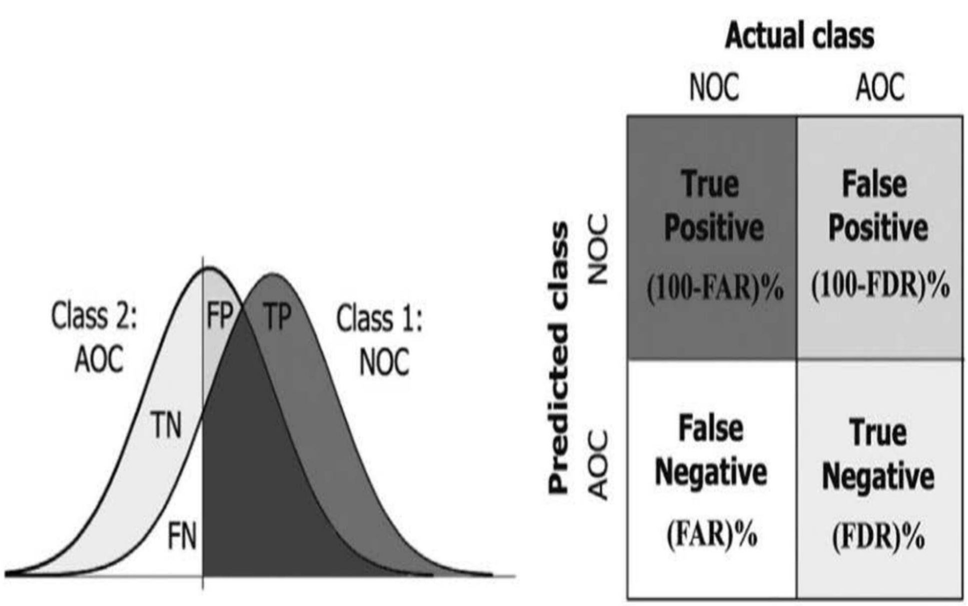

Figure 8. Confusion matrix interpretation for a binary classification process (Bernal de Lázaro et al. 2016).

Results for the experimental datasets

Tables II, III, IV and V show the confusion matrix for the experimental dataset (ED) where C1: Class 1 and C2: Class 2 (C1, C2: 250 (for Data A, Data B and Data D), C1, C2: 175 (for Data C)). The main diagonal is associated with the number of observations successfully classified. Since the total number of observations per class is known, the accuracy (TA=correctly classified observations/total observations) can also be computed. The last row shows the average (AVE) of TA.

Table II. **CM for the experimental data set (Data A).**

FCM					KFCM			
	C1	C2	TA (%)			C1	C2	TA (%)
C1	**115**	135	46.0		C1	**207**	43	82.8
C2	141	**109**	43.6		C2	37	**213**	85.2
AVE			**44.8**		**AVE**			**84.0**
IFCM					**KIFCM**			
	C1	C2	TA (%)			C1	C2	TA (%)
C1	**121**	129	48.4		C1	**219**	31	87.6
C2	132	**118**	47.2		C2	28	**222**	88.8
AVE			**47.8**		**AVE**			**88.2**
PyFCM					**KPyFCM**			
	C1	C2	TA (%)			C1	C2	TA (%)
C1	**129**	121	51.6		C1	**231**	19	92.4
C2	118	**132**	52.8		C2	13	**237**	94.8
AVE			**52.2**		**AVE**			**93.6**

Figure 9 shows the classification results by using the FCM, IFCM, PyFCM, KFCM, KIFCM and KPyFCM algorithms. That shows a global classification percentage obtained for each algorithm.

Results for TE process

Table VI shows the confusion matrix for experimental dataset where F1: Fault 1, F2: Fault 2, F6: Fault 6 and F7: Fault 7.

Figure 10 shows the classification results for the faults 1, 2, 6 and 7 by using the FCM, IFCM, PyFCM, KFCM, KIFCM and KPyFCM algorithms for TE process. A summary of the results can be seen in Figure 11, that shows a global classification percentage obtained for each algorithm.

All experiments were performed on a computer with the following characteristics: Intel Core i7-6500U 2.5 - 3.1GHz, memory: 8GB DDR3L. The Table VII shows that the average time delayed each

Table III. CM for the experimental data set (Data B).

FCM					KFCM			
	C1	C2	TA (%)			C1	C2	TA (%)
C1	**128**	122	51.2		C1	**199**	51	79.6
C2	135	**115**	46.0		C2	47	**203**	81.2
AVE			**48.6**		**AVE**			**80.4**
IFCM					**KIFCM**			
	C1	C2	TA (%)			C1	C2	TA (%)
C1	**134**	116	53.6		C1	**214**	36	85.6
C2	126	**124**	49.6		C2	34	**216**	86.4
AVE			**51.6**		**AVE**			**86.0**
PyFCM					**KPyFCM**			
	C1	C2	TA (%)			C1	C2	TA (%)
C1	**139**	111	55.6		C1	**233**	17	93.2
C2	124	**126**	50.4		C2	22	**228**	91.2
AVE			**53.0**		**AVE**			**92.2**

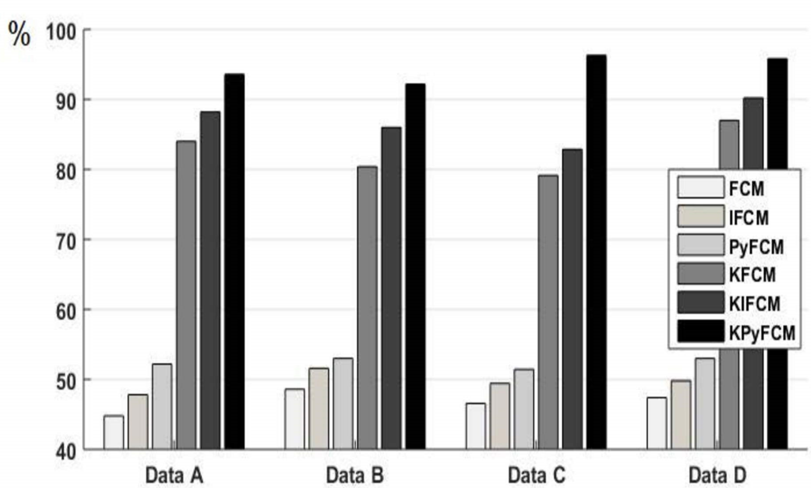

Figure 9. Classification for experimental datasets

algorithm to perform an execution. When comparing these execution times with the time constant of the TE process, it can be seen, that they are very small and therefore show the feasibility of applying these algorithms in the classification scheme.

Table IV. **CM for the experimental data set (Data C).**

FCM					KFCM			
	C1	C2	TA (%)			C1	C2	TA (%)
C1	**80**	95	45.71		C1	**135**	40	77.1
C2	92	**83**	47.4		C2	33	**142**	81.4
AVE			**46.6**		**AVE**			**79.1**
IFCM					**KIFCM**			
	C1	C2	TA (%)			C1	C2	TA (%)
C1	**84**	91	48.0		C1	**141**	34	80.6
C2	86	**89**	50.9		C2	26	**149**	85.1
AVE			**49.4**		**AVE**			**82.9**
PyFCM					**KPyFCM**			
	C1	C2	TA (%)			C1	C2	TA (%)
C1	**89**	86	50.9		C1	**169**	6	96.6
C2	84	**91**	52.0		C2	7	**168**	96.0
AVE			**51.4**		**AVE**			**96.3**

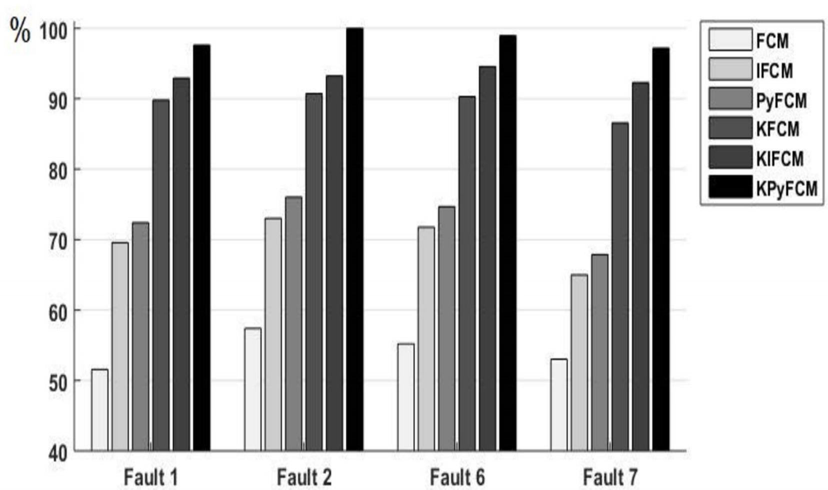

Figure 10. **Faults classification (%) for the TE process.**

As several algorithms are presented, it is necessary to analyze if there are significant differences among the results of them. To achieve this, it is necessary to apply statistical tests (García & Herrera 2008, García et al. 2009, Luengo et al. 2009).

Table V. CM for the experimental data set (Data D).

FCM					KFCM			
	C1	C2	TA (%)			C1	C2	TA (%)
C1	**116**	134	46.4		C1	**216**	34	86.4
C2	129	**121**	48.4		C2	31	**219**	87.6
AVE			**47.4**		**AVE**			**87.0**
IFCM					**KIFCM**			
	C1	C2	TA (%)			C1	C2	TA (%)
C1	**121**	129	48.4		C1	**223**	27	89.2
C2	122	**128**	51.2		C2	22	**228**	91.2
AVE			**49.8**		**AVE**			**90.2**
PyFCM					**KPyFCM**			
	C1	C2	TA (%)			C1	C2	TA (%)
C1	**131**	119	52.4		C1	**237**	13	94.8
C2	116	**134**	53.6		C2	8	**242**	96.8
AVE			**53.0**		**AVE**			**95.8**

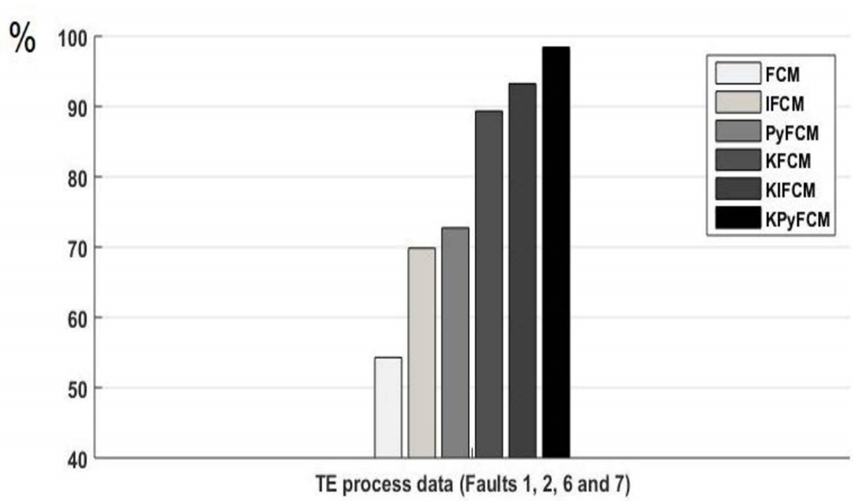

Figure 11. Global classification (%) obtained for each algorithm for the TE process.

Statistical tests

First, the non-parametric Friedman test is applied in order to demonstrate that there is at least one algorithm whose results have significant differences with respect to the results of the others.

Table VI. CM for the TE process (F1: 960, F2: 960, F6: 960, F7: 960).

FCM						**KFCM**					
	F1	F2	F6	F7	TA (%)		F1	F2	F6	F7	TA (%)
F1	**495**	211	109	145	51.56	F1	**862**	40	25	33	89.79
F2	153	**551**	77	179	57.40	F2	30	**871**	22	37	90.73
F6	225	68	**530**	137	55.21	F6	49	21	**867**	23	90.31
F7	244	115	92	**509**	53.02	F7	55	40	34	**831**	86.56
AVE					**54.30**	**AVE**					**89.35**
IFCM						**KIFCM**					
	F1	F2	F6	F7	TA (%)		F1	F2	F6	F7	TA (%)
F1	**668**	153	47	92	69.58	F1	**892**	28	17	23	92.92
F2	129	**701**	55	75	73.02	F2	19	**895**	14	32	93.23
F6	138	62	**689**	71	71.77	F6	21	12	**908**	19	94.58
F7	140	107	89	**624**	65.00	F7	37	16	21	**886**	92.29
AVE					**69.84**	**AVE**					**93.26**
PyFCM						**KPyFCM**					
	F1	F2	F6	F7	TA (%)		F1	F2	F6	F7	TA (%)
F1	**695**	163	56	100	72.40	F1	**937**	14	0	9	97.60
F2	134	**730**	69	85	76.04	F2	0	**100**	0	0	100.0
F6	148	72	**717**	79	74.69	F6	6	0	**950**	4	98.96
F7	145	127	101	**651**	67.89	F7	18	9	0	**933**	97.19
AVE					**72.74**	**AVE**					**98.44**

Table VII. Analysis of computational time.

Algorithm	**Time (seconds)**
FCM	0.1678
IFCM	0.2203
PyFCM	0.2758
KFCM	0.6082
KIFCM	0.6634
KPyFCM	0.7005

Afterward, if the null-hypothesis of the Friedman test is rejected, it is necessary to make a comparison in pairs to determine the best algorithm(s). For this, the non-parametric Wilcoxon test is applied.

Friedman Test

In our case, for six experiments ($k = 6$) and 10 datasets ($N = 10$), the value of statistical Friedman F_F = 340 was obtained. With $k = 6$ and $N = 10$, F_F is distributed according to the F distribution with $6-1 = 5$ and $(6-1) \times (10-1) = 45$ degrees of freedom. The critical value of $F(5{,}45)$ for $\alpha = 0.05$ is 2.4221, so the null-hypothesis is rejected ($F(5{,}45) < F_F$) which means that at least the average performance of at least one algorithm is significantly different from the average value of the performance of other algorithms.

Wilcoxon Test

Table VIII shows the results of the comparison in pairs of the algorithms (1: FCM, 2: IFCM, 3: PyFCM, 4: KFCM, 5: KIFCM, 6: KPyFCM) using the Wilcoxon test. The first two rows contain the values of the sum of the positive (R^+) and negative (R^-) rank for each comparison established. The next two rows show the statistical values T and the critical value of T for a level of significance $\alpha = 0.05$. The last row indicates which algorithm was the winner in each comparison. The summary in Table IX shows the number of times that each algorithm was the winner.

Table VIII. **Results of the Wilcoxon test.**

	1 vs 2	1 vs 3	1 vs 4	1 vs 5	1 vs 6	2 vs 3	2 vs 4	2 vs 5	2 vs 6	3 vs 4	3 vs 5	3 vs 6	4 vs 5	4 vs 6	5 vs 6
$\sum R^+$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$\sum R^-$	55	55	55	55	55	55	55	55	55	55	55	55	55	55	55
T	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$T_{\alpha=0.05}$	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8
Winner	2	3	4	5	6	3	4	5	6	4	5	6	5	6	6

Table IX. **Result of the comparison between the experiments.**

Algorithm	No.Wins	Ranking
FCM	0	6
IFCM	1	5
PyFCM	2	4
KFCM	3	3
KIFCM	4	2
KPyFCM	5	1

As can be seen, among the FCM, IFCM and PyFCM algorithms, the PyFCM algorithm obtains the better results. In the analysis with the Kernel algorithms, the KPyFCM algorithm obtains the better results. Taking into account all algorithms, it is shown that the KPyFCM algorithm obtains the best results.

CONCLUSIONS

The main contribution of this paper is the development of a robust scheme for condition monitoring in industrial systems by using Pythagorean membership grades. The fundamental motivation for this proposal is based on the fact that the space of Pythagorean membership grades is greater than the space of the standard and intuitionistic membership grades. In the classification process, an observation is assigned to the class in which it achieves the highest membership degree. Pythagorean membership functions allow for the use of a larger set of numeric values for assigning the membership degree to an observation; and that a subset of them have a numeric value greater than those allowed by institutionistic membership functions. In the classification process, the membership degree to a class is maximized. If there is a greater number of values in the search space and these values are greater than the institutionistic case, then this allows for the improvement of the classification process as can be seen in two case studies in the paper .

In the proposal, the FCM algorithm is modified by using Pythagorean fuzzy sets, and a new variant of that algorithm called Pythagorean Fuzzy C-Mean (PyFCM) algorithm is obtained. In addition, a kernel version of the PFCM algorithm (KPyFCM) is obtained in order to achieve greater separability among the classes, for reducing the classification errors. The approach proposed was validated using synthetic datasets and the TE process benchmark. The promising results obtained indicate the feasibility of the proposal.

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