https://doi.org/10.1590/jatm.v14.1284 ORIGINAL PAPER

Path Tracing Method to Evaluate the Signature Reliability Function of a Complex System

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ABSTRACT

This paper aims to compute the efficiency of a complex communication system (double-bridge). In this model, which applies the Path Tracing Method for evaluating the reliability function of the complex communication system, the process of analyzing the system's reliability has been used to lower the failure rate of the complex system in order to optimize its competence. The illustrative system is subdivided into nine minimal paths that are arranged in series-parallel combinations. Using the Path Tracing Method, the system's reliability, minimal signature, tail signature, signature, Barlow–Proschan index, expected time, expected cost, and sensitivity were determined. The results assist in the creation of a probabilistic approach to analyze and probabilistic method to evaluate the system signature and reliability in a way that is highly useful.

Keywords: Signature reliability; Complex system; Double-bridge structure; Barlow–Proschan index; Expected lifetime; Sensitivity.

INTRODUCTION

The essential issue in reliability theory from the start has been how to determine the reliability criteria of a system from the reliability criteria of its components. Reliability is the most important measurement tool for any system to evaluate the efficiency of its components. Reliability, sometimes known as *signature*, is based on the probability that one or more components in the system will fail. To date, there are no dynamical algorithms that have been devised to solve this problem exactly and scale to systems with more than 500 components. This is because, from a computing approach, nondeterministic polynomial-time (NP)-hard problems are among the most complex classes of problems. As a result, the time required to determine the system's reliability based on the reliability of its components typically grows exponentially with the n-size of the system. Thus, techniques for estimating the exact system reliability may require extensive calculation, even with the support of a computer software Even if enough memory is available, results may still be off if the necessary calculation takes too long to complete, leading to round problems.

Computationally efficient approximative algorithms for the system's reliability have been developed as a means of attacking the problem. It has been discovered that the bounding method is a particularly useful approximation technique. Finding a minimum and a maximum level of system reliability is part of this process.

Received: Aug 01, 2022 | **Accepted:** Sept 16, 2022
Peer Review History: Single-Blind Peer Review.
Section editor: Alison Moraes

This work provides a thorough review of these constraints for binary coherent systems and performs detailed numerical comparisons to determine the signature that best gives which assumptions on the reliabilities of the components. Specifically, in the case of mechanical systems, some reliability experts argue that it is sufficient to simply combine series and parallel arrangements to characterize the reliability networks of these systems (Mi, Li *et al.* 2018; Zhu and Zhang 2019; Malik, Chauhan *et al.* 2020; Malik, S., *et al.* 2020). However, the reliability networks of many basic engineering systems cannot be modeled using series-parallel networks. If a system cannot be reduced into a set of series and parallel arrangements, it is said to be complex. Typically, graphs are used to describe complex systems (Kuo and Zuo 2003; Todinov 2013). A graph G = (V, E), where V is an unfilled set of which elements are known as *vertices*, and E is a set of pairwise relationships between those vertices (nodes) with the probability of vertices/edges being operational (Gertsbakh and Shpungin 2011; Hassan and Mutar 2017a; Mutar 2017; Todinov 2013).

This study will highlight the fact that communication between two different nodes, source and sink nodes, must occur despite of node/edge failure. In this work, For the purposes of logic, every single node is perfectly reliable. Therefore, it will only be required to investigate the edge failure techniques. As a concept, two-terminal reliability can be regarded as the probability of data correctly being transmitted from the source to the sink. This network's reliability is defined as the probability that these two nodes are linked by minimal paths (or minimal cuts) of the operational and reliable edge (Kuo and Zuo 2003; Mutar 2020).

Both theoretical and practical perspectives on complex systems, networks, and reliability theory are explored in several books and articles in the Engineering literature (Beichelt 2012; Todinov 2013; Rausand 2014; Putcha, Dutta *et al.* 2021). The standard method for determining signature for system reliability uses minimal paths (Rodionov and Rodionova 2013). For a system to be functional, there must be some minimal subset, or path set, of which operation implies that the whole system is functioning properly (Mutar 2020). A minimal path is the smallest path set that does not contain any other path sets. For two-terminal reliability, a minimal path is the same as a path set, which is a route between the designated terminals that does not have any cycles. For example, components in most electronic systems have a complex structure. Jula and Costin (2012) offer two techniques for assessing the reliability of airplane electrical systems. Mutar and Hassan (2022) apply graph theory and diffeomorphism approaches to estimate the reliability evaluation of electrical aircraft systems with *Geometric Modeling of Reliability*. In addition, Horváth (2013) provided some significant observations of the properties of complex systems using the cxnet Complex Network Analyzer Program, as well as measurements he obtained through scientific research. Mi *et al.* (2018) provided a standard reliability test, such as the truth table approach, predicated on the supposition that instances are binary, i.e., either successful or unsuccessful. Hassan and Mutar (2017b) studied the geometry-based construction of spacecraft electrical device reliability models, namely the High-Pressure Oxygen Supply System (HPOSS). Sharma (2014) analyzed the reliability of a complex system in an uncertain and fuzzy context. Malik *et al.* (2020) evaluated the reliability of a seven-component complex system by analyzing Weibull failure principles. Mutar (2020) offered a method for generating the incidence matrix using minimal cut sets and minimal path sets, which was then contrasted to the system's truth table. Based on specific algebraic concepts, this comparison reveals the minimal cut sets for the complex system. Mutar (2022) illustrates the use of mathematical detection tests for HPOSS in order to determine the sensitivity of complex system reliability.

Signature reliability of complex systems has been studied by various authors in the past and present. Reliability statistics, optimization problem parameters, and best system design all rely on signatures. The importance of a system's signature, or defining characteristic, to the system's performance is explored and studied. The signature of a system is an effective tool for comparing different systems, and it has been related to other well-known reliability ideas. The signature of one system could be given to another's system simply by looking at it, and signature comparisons can be used to establish a variety of stochastic comparisons between systems. Samaniego (1985) was the first person to study the signature of coherent system components with independent and identically distributed (i.i.d.) lifespan components. Samaniego (2007) described the applications of system signature in the engineering and network domains, as well as the signature of a coherent system. Navarro and Rubio (2009) computed the signature of various coherent systems and evaluated their average lifetimes as well as their minimal paths. Marichal and Mathonet (2013) calculated the system's signature by applying the Boland's formula to the reliability function. By analyzing the system's structure–function, they discovered a method for analyzing the system's signature when the system was deemed to contain i.i.d. components. Eryilmaz (2014) used structural function to assess the signature of series and parallel coherent systems. He used a series and parallel combination of bridge systems to calculate signature and minimal signature.

Moreover, Barlow and Proschan (1975) discovered the significance of the coherent system component, which is the factor that decides the percentage of i.i.d. components. Costa Bueno (2011) defined the coherent system and signature forms linked with it. Owen (1972) suggested a method for analyzing the signature of the coherent system using tail signature. Bairamov and Arnold (2008) created the system structure function and evaluated the signature of the system utilizing the tail signature when elements have i.i.d. components exponentially random variables. Kumar *et al.* (2021) examined a bridge configuration and assessed the reliability function with the use of the Universal Generating Function. In addition, the authors have examined additional reliability indices, such as the Barlow–Proschan index, signature, and expected system lifetime, among others. Tyagi *et al.* (2021) described a renewable energy-based system in which the wind turbine and hydropower plant are coupled to create large quantities of electricity. The authors analyzed the tail signature, signature, the Barlow–Proschan index, expected lifetime and expected cost rate. Bisht and Singh (2022) determined the mean time to failure of all-digital protection systems using the minimal signature.

In this paper, a communication complex system applying the Path Tracing Method to determine the tail signature, signature, Barlow–Proschan index, expected time, expected cost, and sensitivity was studied. The next sections will present the literature review and signature analysis based on the reliability function; the analysis of certain numerical formulas for calculating the reliability function, whose main focus is on characterizing and deciphering a characteristic of a system known as its signature; the model setup and reliability calculation of a complex system; the numerical results for evaluating various parameters, including tail signature, signature, Barlow-Proschan index, expected time, expected cost, and sensitivity, using Owen's approach; followed by discussion and conclusions, respectively.

METHODS

Some Important Concepts

Reliability Theory

Coherent systems are represented as $G = (V, E)$, where V is an unfilled set of which elements are known as *vertices*, and E is a set of pairwise relationships between those vertices (nodes). In a graph, each element of E is referred to as an *edge*. There are just two conceivable outcomes for the system and any of its edges: operating or failing. Edges X_i have their respective states defined in terms of $X_i = 1$ when the edge is operational for this period and $X_i = 0$ when the edge is damaged (Aven and Jensen 1999; Gertsbakh and Shpungin 2011). The binary random variable X_i is defined in Eq. 1.

$$X_i = \begin{cases} 1, \text{ if edge i operational,} \\ 0, \text{ if edge i is damaged} \end{cases} \tag{1}$$

Consequently, the edge state vector is the vector $X = (X_1, X_2, \ldots, X_n)$, which defines each edge's state. The status of the system is monitored by the binary random variable $\varphi(X)$, is defined in Eq. 2.

$$\varphi(\boldsymbol{X}) = \begin{cases} 1, \text{ if the system is operational} \\ 0, \text{ if the network is damaged} \end{cases} \tag{2}$$

The structure function of the system is the function $\varphi(X)$. If $X_1, X_2, \ldots, X_n$ are i.i.d., then R_{system} is a function of the edge reliability values $R_1, R_2, \ldots, R_n$. If $R_{system} = (R_1, R_2, \ldots, R_n)$, then the system's reliability is written as given in Eq. 3.

$$R_{system} = R_s = H(R) \; if \; R = R_1 = R_2 = \cdots = R_n \tag{3}$$

Furthermore, this study analyzes the reliability between the source node S and the sink node t. It is referred to as the two-terminal network's reliability (Todinov 2013; Putcha, Dutta *et al.* 2021). This network's reliability is defined as the probability that these two nodes are connected by a path consisting of reliable and operational connections. In the next section, we will discuss analytical methods for evaluating the signature of a two-terminal reliability.

Path Tracing Method

This technique is a quick tool for creating a system reliability function to rely on in the formation of the signature reliability of complex systems and it is predicated mostly on a minimal path. A collection of all minimal paths can be used to create the incidence matrix of minimal paths. The rows and columns of the matrix will be generated by the minimal paths $P_1, P_2, \ldots, P_n$ define n minimal paths, with the paths representing rows and the components representing columns. The incidence matrix (IM) of all minimal paths is shown in Eq. 4.

$$IM = \begin{matrix} \\ P_1 \\ P_2 \\ \vdots \\ P_m \end{matrix} \begin{matrix} \begin{matrix} x_1 & x_2 & \cdots & x_n \end{matrix} \\ \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix} \end{matrix} \tag{4}$$

where $a_{ij} \in \{0,2\}$ with $i = 1, 2, \ldots, m, j = 1; 2, \ldots, n$ and $a_{ij} = 1$ if $X_j \in P$, otherwise $a_{ij} = 0$. The complex system reliability is calculated as a parallel configuration of all minimal path included in these rows can be estimated using the Eq. 5.

$$R_s = 1 - \prod_{i=1}^{n}(1 - P_i) \tag{5}$$

According to Bernoulli's random variables ($x_i^n = x_i$), the structural function of a system can only contains values of 1 or 0 (because $1^n = 1$ and $0^n = 0$). A replacement rule can be used to express this relationship (Aggarwal 1993; Mutar 2020).

Method for Obtaining the Signature of a Complex System

- Step 1: Assume that the i-th minimal lifespan n components of a complex system are used to calculate the signature $\mathcal{S}_i$ as Eq. 6:

$$\mathcal{S}_i = \frac{1}{\binom{n}{n-i+1}} \sum_{\substack{H \subseteq [n] \\ |H| = n-i+1}} \varphi(H) - \frac{1}{\binom{n}{n-1}} \sum_{\substack{H \subseteq [n] \\ |H| = n-1}} \varphi(H) \tag{6}$$

The reliability polynomial may be written in a different manner to demonstrate the link between the dominance and signature vectors (Bisht and Singh 2019; Kumar and Singh 2021). The polynomial will be referred to as Eq. 7:

$$H(R) = \sum_{j=1}^{n} \alpha_j \binom{n}{j} R^j Q^{n-j} \tag{7}$$

where $a_j = \sum_{i=n-j+1}^{n} \mathcal{S}_i$ for $j = 1, 2, 3, \ldots, n$.

- Step 2: Assuming the tail signature is calculated for the considered complex system, which is (n + 1)-tuple $= \bar{\mathcal{S}}(\bar{\mathcal{S}}_0, \bar{\mathcal{S}}_1, \ldots, \bar{\mathcal{S}}_n)$ by Eq. 8.

$$\bar{\mathcal{S}}_i = \sum_{k=i+1} \mathcal{S}_i = \frac{1}{\binom{n}{n-i}} \sum_{|H|=n-i} \varphi(H) \tag{8}$$

- Step 3: Compute the polynomial reliability function with Taylor expansion centered on $R = 1$ by Eq. 9:

$$P(R) = R^n H\left(\frac{1}{R}\right) \tag{9}$$

- Step 4: Determine the tail signature of the complex system using Eq. 8 as in Eq. 10:

$$\bar{\mathcal{S}}_i = \frac{(n-i)!}{n!} D^i P(1), i = 1,2, \ldots, n \tag{10}$$

- Step 5: Determine the signature of a complex system using Eq. 10 as in Eq. 11:

$$\mathcal{S}_i = \bar{\mathcal{S}}_{i-1} - \bar{\mathcal{S}}_i \,, i = 1,2, \ldots, n \tag{11}$$

Barlow–Proschan of the Complex System

Equation 12 calculates the Barlow–Proschan index of the i.i.d. components provided by its reliability function.

$$I_{BP}^{i} = \int_0^1 (\partial_i H)(R)\, dR, i = 1,2, \ldots, n \tag{12}$$

where H is the reliability functions of complex system (Ram *et al.* 2021).

Estimate the Expected Time of the Complex System

Equation 13 evaluates the expected lifetime $E(T)$ of a complex system with i.i.d. components with a mean ($\mu = 1$) by:

$$E(T) = \mu \sum_{i=1}^{n} \frac{M_i}{i} \tag{13}$$

where M is a minimal signature (Blokus 2020).

Expected X of the Complex System

The expected value of the system can be calculated using Eq. 1 for signature reliability (Eq. 14).

$$E(X) = \sum_{i=1}^{n} i\, \mathcal{S}_i \;, i = 1,2, \ldots, n \tag{14}$$

Finally, Eq. 15 calculates the expected cost rate of a complex system with expected lifetime (Kumar *et al.* 2022; Negi *et al.* 2022).

$$Expected\ cost\ rate = \frac{E(x)}{E(T)} \tag{15}$$

Sensitivity of the Complex System

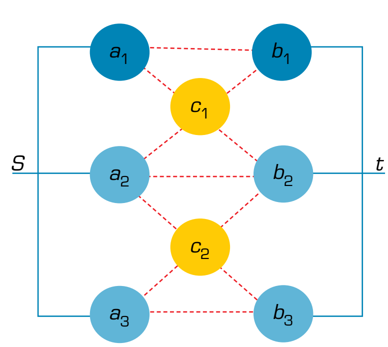
The formula for calculating the sensitivity $\mathbb{S}$ of a parameter β to its value is shown in Eq. 16.

$$\mathbb{S} = \frac{\partial R_s}{\partial \beta} \tag{16}$$

System Description

A Diagram of the Communication System

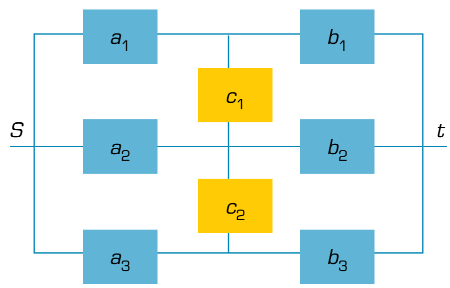
Consider a communication system in which a message is sent from three identical sources a_1, a_2 and a_3 to three identical receivers b_1, b_2 and b_3 using two transmitters c_1 and c_2 as shown in Fig. 1. Dashed lines denote the potential connections through which the message can be transmitted from the sources to the receivers and between the transmitters (Todinov 2013).



Source: Adapted from Todinov (2013).

Figure 1. A logical diagram of a communication system.

The system will be operational whenever the reliability network depicted in Fig. 2 has a two-terminal path across working components. The reliability of a communication system (double-bridge) cannot be reduced to series, parallel, or series—parallel configurations. It also has extremely complex reliability systems that cannot be described by series-parallel configurations (Kuo and Zuo 2003; Todinov 2013).

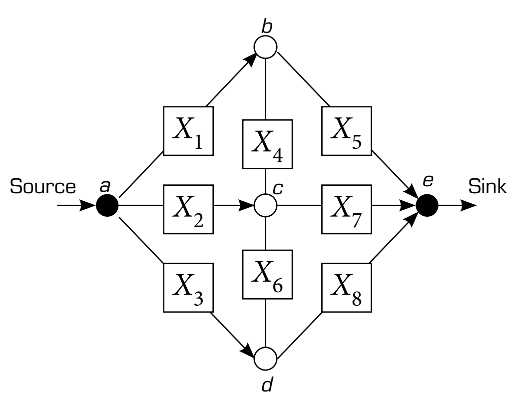


Source: Adapted from Todinov (2013).

Figure 2. A block diagram of a communication system.

Calculate all minimal path

The reliability of the two-terminal calculates the graph's probability of a successful data transfer from source to sink, i.e., to determine how likely it is that data will be successfully sent (a path of nonfailed edges joins source and sink). As instance, the complex system seen in Fig. 2 can be described using the mathematical notation graph $G = (V, E)$, where $V = \{a, b, ..., e\}$ and $E = \{1, 2, ..., 8\}$, are the two-terminal graph (Fig. 3). The graph G is simple, connected, and has just one direction for its edges in the same direction. Using the matrix-based method, the incidence matrix of graph G will be calculated for all minimal path sets from source node a to sink node e.



Source: Elaborated by the author.

Figure 3. A graph G of a communication system.

The incidence matrix represents all potential states of the system, with each component either a good or a fault condition. The minimal paths: $P_1, P_2, ..., P_9$. Connecting two terminal systems. To indicate the probability of a certain system state. Given that the incidence matrix encompasses all the possible minimal paths (for more details, see Aggarwal 1993; Kuo and Zuo 2003). By using Eq. 4, the incidence matrix of A complex system depicted in Fig. 3 is calculated in the following form (Eq. 17).

$$\boldsymbol{IM} = \begin{matrix} & X_1 & X_2 & X_3 & X_4 & X_5 & X_6 & X_7 & X_8 \\ P_1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ P_2 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ P_3 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ P_4 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ P_5 & 0 & 1 & 0 & 1 & 1 & 0 & 0 & 0 \\ P_6 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 1 \\ P_7 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 0 \\ P_8 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 1 \\ P_9 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 \end{matrix} \tag{17}$$

As a result, the n-component system has a mapping $\varphi : \{0,1\}^n \rightarrow \{0,1\}$, of potential states termed order of the system (in this example, $2^8 = 256$). We only evaluated at the minimal paths because they indicate the true status of the system's operation and the absence of energy. There are nine minimal paths can be obtained:

$P_1 = \{X_1 X_5\}, P_2 = \{X_2X_7\}, P_3 = \{X_3X_8\},$

$P_4 = \{X_1X_4X_7\}, P_5 = \{X_2X_4X_5\}, P_6 = \{X_2X_6X_8\}$

$P_7 = \{X_3X_6X_7\}, P_8 = \{X_1X_4X_6X_8\}, P_9 = \{X_3X_4X_5X_6\}$

The Reliability of Complex System

Using the Path Tracing Method, a source node to a sink node's minimal path is evaluated. As consequence, the structural function with multivariate polynomial reliability can be constructed utilizing the Eq. 5 and the above minimal paths.

$$R_s = 1 - [(1-P_1)(1-P_2)(1-P_3)(1-P_4)(1-P_5) \times (1-P_6)(1-P_7)(1-P_8)(1-P_9)] \tag{18}$$

Consequently, the system reliability is given by Eq. 19:

$$R_s = 1 - [(1-R_1R_5)(1-R_2R_7)(1-R_3R_8)(1-R_1R_4R_7)(1-R_2R_4R_5)(1-R_2R_6R_8)(1-R_3R_6R_7) \times (1-R_1R_4R_6R_8)(1-R_3R_4R_5R_6)] \tag{19}$$

Hence, the multivariate polynomial reliability becomes Eq. 20:

$$\begin{aligned}
R_s = {} & R_1R_5 + R_2R_4R_5 - R_1R_2R_4R_5 + R_3R_4R_5R_6 - R_1R_3R_4R_5R_6 - R_2R_3R_4R_5R_6 + \\
& R_1R_2R_3R_4R_5R_6 + R_2R_7 + R_1R_4R_7 - R_1R_2R_4R_7 - R_1R_2R_5R_7 - R_1R_4R_5R_7 - \\
& R_2R_4R_5R_7 + 2R_1R_2R_4R_5R_7 + R_1R_2R_3R_5R_6R_8 - R_2R_3R_6R_7 - R_1R_3R_4R_6R_7 + \\
& R_1R_2R_3R_4R_6R_7 - R_1R_3R_5R_6R_7 + R_1R_2R_3R_5R_6R_7 + 2R_1R_3R_4R_5R_6R_7 + R_2R_6R_8 - \\
& R_2R_3R_6R_8 + R_1R_4R_6R_8 - R_1R_2R_4R_6R_8 - R_1R_3R_4R_6R_8 + R_1R_2R_3R_4R_6R_8 - \\
& R_1R_2R_5R_6R_8 + R_2R_3R_4R_5R_6R_7 - 2R_1R_2R_3R_4R_5R_6R_7 + R_3R_8 - R_1R_3R_5R_8 + \\
& 2R_1R_2R_4R_5R_6R_8 + R_1R_2R_3R_4R_5R_8 - R_2R_3R_7R_8 - R_1R_3R_4R_7R_8 + \\
& R_1R_2R_3R_4R_7R_8 + R_1R_2R_3R_5R_7R_8 + R_2R_3R_4R_5R_7R_8 - R_2R_3R_4R_5R_8 - \\
& 2R_1R_2R_3R_4R_5R_7R_8 - R_2R_6R_7R_8 - R_3R_6R_7R_8 + 2R_2R_3R_6R_7R_8 - R_1R_4R_6R_7R_8 + \\
& R_1R_2R_4R_6R_7R_8 + 2R_1R_3R_4R_6R_7R_8 - 2R_1R_2R_3R_4R_6R_7R_8 + R_1R_2R_5R_6R_7R_8 + \\
& R_1R_3R_5R_6R_7R_8 - 2R_1R_2R_3R_5R_6R_7R_8 + R_1R_4R_5R_6R_7R_8 + R_2R_4R_5R_6R_7R_8 - \\
& 2R_1R_2R_4R_5R_6R_7R_8 + R_3R_4R_5R_6R_7R_8 - 3R_1R_3R_4R_5R_6R_7R_8 - R_1R_4R_5R_6R_8 - \\
& R_2R_4R_5R_6R_8 - R_3R_4R_5R_6R_8 + 2R_1R_3R_4R_5R_6R_8 - R_3R_4R_5R_6R_7 + \\
& R_1R_3R_4R_5R_7R_8 + R_3R_6R_7 + 2R_2R_3R_4R_5R_6R_8 - 3R_1R_2R_3R_4R_5R_6R_8 - \\
& 2R_2R_3R_4R_5R_6R_7R_8 + 4R_1R_2R_3R_4R_5R_6R_7R_8
\end{aligned} \tag{20}$$

The system's reliability can be deduced from Eq. 20 as $R_i = R$ for identical components (Eq. 21).

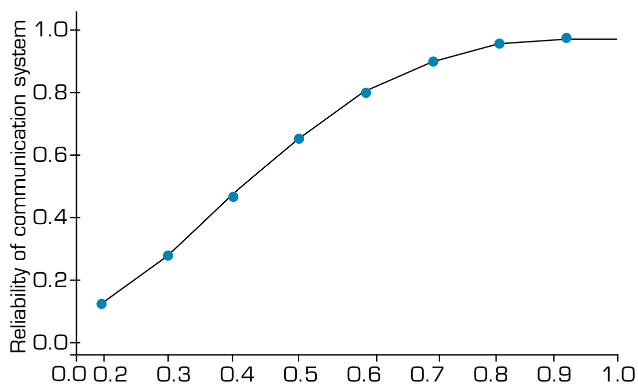
$$R_s = 3R^2 + 4R^3 - 9R^4 - 10R^5 + 27R^6 - 18R^7 + 4R^8 \tag{21}$$

Taking into consideration the reliabilities of the different components, one can calculate the reliability of structural system as shown in Table 1 and Fig. 4 (Bisht and Singh 2020; 2021).

Table 1. Values of reliability of communication system.

Component reliability	Reliability of communication system
0.2	0.135907
0.3	0.296808
0.4	0.486922
0.5	0.671875
0.6	0.823011
0.7	0.925137
0.8	0.978903
0.9	0.997631
1.0	1.000000

Source: Elaborated by the author.



Source: Elaborated by the author.

Figure 4. Component reliability versus reliability of a communication system.

Signature of the Complex System

Signature of a Communication System

The Owen's technique is used to determine the reliability function of a communication system, and the reliability will be expressed in the form R as shown in Eq. 22.

$$H(R) = 3R^2 + 4R^3 - 9R^4 - 10R^5 + 27R^6 - 18R^7 + 4R^8 \tag{22}$$

Now, from Eqs. 9 and 22, the polynomial function of the above reliability function is as follows:

$$P(R) = R^8 H\left(\frac{1}{R}\right) = R^8\left(\frac{4}{R^8} - \frac{18}{R^7} + \frac{27}{R^6} - \frac{10}{R^5} - \frac{9}{R^4} + \frac{4}{R^3} + \frac{3}{R^2}\right)$$

Then, the polynomial of reliability is given by Eq. 23.

$$P(R) = 4 - 18R + 27R^2 - 10R^3 - 9R^4 + 4R^5 + 3R^6 \tag{23}$$

Consider the tail signature $\overline{S}$ of the system, which is (n + 1)-tuple tuple = $\overline{S}$ ($\overline{S}_0$, $\overline{S}_1$, ..., $\overline{S}_n$). Using Eq. 10, the tail signature of a complex system can be determined as in Eq. 24.

$$\bar{S} = \left(1,1,1,\frac{27}{28},\frac{4}{5},\frac{11}{28},\frac{3}{28},0,0\right) \tag{24}$$

Consequently, using the preceding values of the tail signature, the signature S of the complex system is determined by the previously described procedure and Eq. 11, as in Eq. 25:

$$S = \left(0,0,\frac{1}{28},\frac{23}{140},\frac{57}{140},\frac{2}{7},\frac{3}{28},0\right) \tag{25}$$

Barlow–Proschan Index of a Communication System

Now, using Eq. 12, it is possible to determine the Barlow–Proschan index for the complex structure under consideration as follows:

$$I_{BP}^{1} = \int_{0}^{1} R + R^2 - 4R^3 - 7R^4 + 21R^5 - 16R^6 + 4R^7 \, dR = \frac{31}{210}$$

Similarly, the Barlow–Proschan index I_{BP}^{k} for $K = (1, 2, ..., 8)$ of all elements is determined as shown in Eq. 26:

$$I_{BP} = \left(\frac{31}{210},\frac{11}{70},\frac{31}{210},\frac{1}{21},\frac{31}{210},\frac{1}{21},\frac{11}{70},\frac{31}{210}\right) \tag{26}$$

Lifetime Expected for a Communication System

Now, using Eq. 22, the minimal signature M of the system is obtained in Eq. 27.

$$M = (0,3,4,-9,-10,27,-18,4\,) \tag{27}$$

The expected for a communication system with minimal signature is obtain in Eq. 28.

$$E(T) = 1.01190476 \tag{28}$$

Expected Cost Rate of a Communication System

The expected system value is computed using Eq. 14 as in Eq. 29:

$$E(X) = \ 5.264285 \tag{29}$$

Using the calculation in Eq. 15, the expected system cost rate is shown in Eq. 30.

$$ExRected\ cost\ rate\ = 5.202352 \tag{30}$$

Sensitivity of a Communication System

To determine the sensitivity of a communication complex system, let's assume probability values as follows: using Eq. 15, Information about the system's sensitivity is obtained as $\mathbb{S}_1 = 0.038$, $\mathbb{S}_2 = 0.064$, $\mathbb{S}_3 = 0.038$, $\mathbb{S}_4 = 0.006$, $\mathbb{S}_5 = 0.017$, $\mathbb{S}_6 = 0.006$, $\mathbb{S}_7 = 0.012$, $\mathbb{S}_8 = 0.017$. The system is also supposed to have i.i.d. components for the analysis of several sensitivity measurements are calculated as shown in Table 2.

Table 2. Sensitivity measurement of i.i.d. components of a communication complex system.

Component reliability	$\mathbb{S}_1$	$\mathbb{S}_2$	$\mathbb{S}_3$	$\mathbb{S}_4$	$\mathbb{S}_5$	$\mathbb{S}_6$	$\mathbb{S}_7$	$\mathbb{S}_8$
0.5	0.25	0.265625	0.25	0.09375	0.25	0.09375	0.265625	0.25
0.6	0.187238	0.196454	0.187238	0.068198	0.187238	0.068198	0.196454	0.187238
0.7	0.113803	0.117772	0.113803	0.037573	0.113803	0.037573	0.117772	0.113803
0.8	0.050636	0.051660	0.050636	0.013516	0.050636	0.013516	0.051660	0.050636
0.9	0.011721	0.011802	0.011721	0.001911	0.011721	0.001911	0.011802	0.011721

Source: Elaborated by the author.

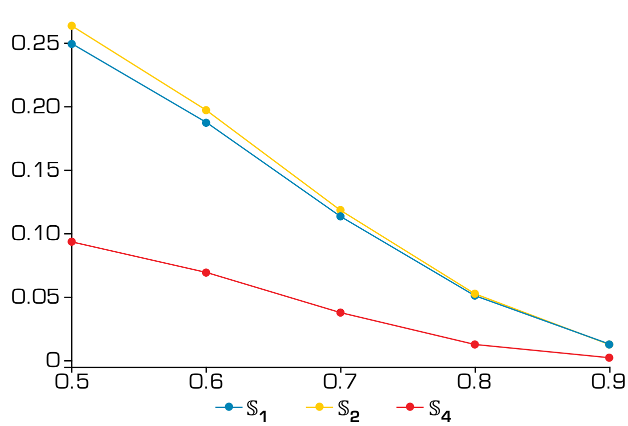
DISCUSSION

In this study, a signature is conducted to compare the probability and failure probability of working components of a system (Table 3). Cost analysis and component failure probability show that the system is in good condition. In addition, the major of the components are in good condition and will be able to function well. A great system can be built using these results. A thorough analysis of Table 2 and Fig. 5 demonstrates that the sensitivity of the double-bridge network corresponding to the parameters β_4 and β_6 initially drops and subsequently rises with time. B_2 and β_7 are discovered to be the most sensitive characteristics for the bridge network.

Table 3. Minimal signature, tail signature, signature and Barlow–Proschan index of different elements.

S. No.	0	1	2	3	4	5	6	7	8
Minimal signature	–	0	3	4	-9	-10	27	-18	4
Tail signature	1	1	1	27/28	4/5	11/28	3/28	0	0
Signature	–	0	0	1/28	23/140	57/140	2/7	3/28	0
Barlow–Proschan index	–	31/210	11/70	31/210	1/21	31/210	1/21	11/70	31/270

Source: Elaborated by the author.



Source: Elaborated by the author.

Figure 5. Effect of different edges on sensitivity of a communication system.

CONCLUSION

In this paper, the Path Tracing Method is used to examine the reliability characteristics of a communication complex system with eight ordered linearly multistate components, thereby expanding the concept of the general law of probability addition. Additionally, the authors determine the reliability function using the general law of addition of probabilities and examine the signature of system components, including system signature, expected lifetime, Barlow–Proschan index, and sensitivity.

Signature analysis enables reliability engineers to optimize network efficiency based on component value by revealing the failure probability of each component (edge). The impact of the failure probabilities of network edges is frequently utilized to analyze it. The analysis reveals that the suggested algorithm can be an effective method for evaluating the reliability characteristics of communication systems. The analysis showed that the component's signature is increasing in terms of expected cost and expected lifetime. The minima signature, tail signature, signature and the Barlow–Proschan index and signature of the considered system are is calculated as follows (as shown in Table 3). Using Owen's technique, the expected cost was calculated to be 5.264285.

The sensitivity analysis of the investigated networks will aid reliability engineers and systems engineers in constructing more reliable networks. The complex communication system (double-bridge) is shown to be most sensitive regarding parameter β_2 and β_7 has the highest sensitivity, while parameters β_4 and β_6 have the lowest sensitivity (as shown in Table 2). This work allows for the expansion of the highway communication complex system. Through this method, the reliability function and signature for a system with a complex engineering design and a variety of different measures of reliability can be determined.

DATA AVAILABILITY STATEMENT

All data sets were generated or analyzed in the current study.

FUNDING

Not applicable.

ACKNOWLEDGMENTS

Not applicable.

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