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Study of Head-Pursuit Cooperative Guidance Law for Near-space Hypersonic Interceptor

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ABSTRACT

In order to intercept hypersonic vehicles in near-space, a head-pursuit cooperative guidance law is proposed in this paper. Firstly, interceptors are regarded as multi-agents, and the communication relationship between them is represented by graph theory. Based on the time consistency theory of multi-agent system and sliding mode theory, a guidance law is designed along line-of-sight (LOS) to ensure the time cooperation of interceptors. Secondly, considering the requirement of the head-pursuit theory to the lead angle, a finite-time guidance law is designed perpendicular to LOS to ensure that each interceptor can complete head-pursuit interception. For the purpose of improving the intercept precision, extended state observers are used to estimate the system disturbances. The correctness of the guidance law is analyzed by Lyapunov stability theory. Finally, numerical simulations are presented and the results further verify the correctness of the guidance law.

Keywords: Multiagent system; Time consistency theory; Extended state observers; Finite-time guidance law.

INTRODUCTION

With the maturity of near-space technology, the advantage of hypersonic vehicle is becoming more and more obvious. Characteristics such as long flight distance, fast flight speed and strong maneuvering ability make the hypersonic vehicle have a strong ability of penetration. However, the traditional methods such as tail-chase interception and head-on interception have different deficiencies in the process of intercepting such targets. At the same time, with the development of antimissile system, it is difficult for a single interceptor to complete the combat tasks independently in the complex battlefield. In order to improve the interception probability for hypersonic vehicle, the cooperative interception of multiple interceptors is paid more attention to in the military field. As an important basic theory of multiagent cooperative guidance, the multiagent consistency theory has achieved good results in the cooperative control of pilotless aircraft, autonomous vehicle, robot and other fields, which provides a theoretical basis for the study of head-pursuit cooperative guidance against hypersonic vehicles in near-space. Therefore, the research on head-pursuit cooperative guidance law based on multiagent consistency theory for near-space interceptor has great strategic value.

For the purpose of intercepting hypersonic vehicle effectively, a new interception method was proposed by Golan and Shima (2004). This method required the interceptor to fly in the same direction as the target at a low speed in front of the target

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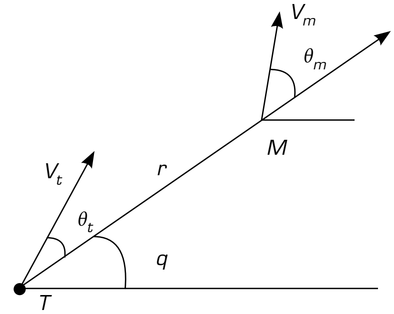
trajectory, so that the target can hit the interceptor from behind. Because the interceptor is in front of the target, this method is called head-pursuit interception. They used sliding mode control theory to design the guidance law and completed the head-pursuit interception of the target. In order to attenuate the chattering phenomena caused by sliding mode control theory, Liu K *et al*. (2015) improved the head-pursuit guidance law based on double-power reaching law. For the purpose of improving the convergence rate of the guidance system, Si and Song (2017) proposed a head-pursuit guidance law based on fast double-power reaching law, which not only attenuated the system chattering, but also accelerated the convergence rate of the system. Since target maneuvering is unknown, Zhang *et al.* (2018) used an adaptive algorithm to estimate the system disturbance and presented a head-pursuit guidance law based on time-scale separation which can improve system intercept accuracy and have strong robustness. Taking into account the autopilot dynamics, Zhu and Guo (2019) proposed a head-pursuit guidance law based on back-stepping sliding mode, which further improved the interception accuracy. Based on this, Zhu (2021a) combined fractional order theory with sliding mode theory and proposed a head-pursuit guidance law based on fractional order sliding mode theory, which further weakened the chattering of the system. At the same time, Zhu (2021b) introduced the finite time disturbance observer into the design of head-pursuit guidance law, which ensured the system converge in finite time and improved the accuracy of the system.

Cooperative guidance law for multiple missiles means that interceptors cooperate with each other in time and space and complete the combat mission. Lee *et al.* (2007) proposed a guidance law that constrained both the attack time and angle with given values, which can intercept targets with low speed. Based on the optimal control theory, Sun and Xia (2012) proposed an optimal cooperative guidance law. L Feng *et al.* (2014) combined sliding mode theory with target strategy switching and proposed a cooperative guidance law based on two-layer design scheme, which carried out the cooperative control of attack time and angle. Cho *et al.* (2015) proposed an adaptive cooperative guidance law with a wide range of expected collision angles which can deal with the unknown target maneuver. Zhao *et al.* (2016) proposed a cooperative guidance law based on the finite-time consistency theory to ensure that multiple interceptors hit the target simultaneously. Based on this, Shi *et al.* (2018) introduced the second-order sliding mode theory into the design of cooperative guidance law which weakened the chattering of the system effectively. Considering the communication between interceptors, Liu X and Liang (2019) designed a cooperative guidance law based on multiagent consistency principle to control the collision time and line-of-sight (LOS) angle. Up to now, most of the studies on cooperative guidance law focus on tail-chase interception and head-on interception, which is not suitable for the near-space interceptor, so that the interception rate against hypersonic vehicles in near-space is not enough. Therefore, this paper studies this field and puts forward a head-pursuit cooperative guidance law for near-space interceptor.

BACKGROUND INFORMATION AND PRELIMINARIES

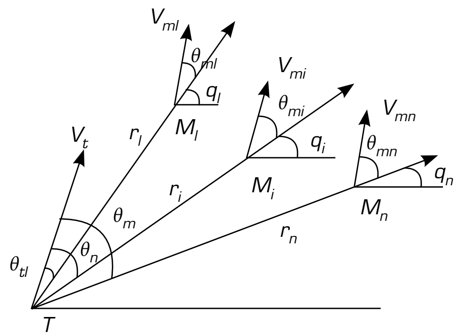
Figure 1 is the relative motion schematic of head-pursuit interception in longitudinal plane, where points T and M are the target and the missile, r is the missile-target range, q is the LOS angle, θ_t and θ_m the lead angles, V_t and V_m velocities, the subscripts t and m denote the target and missile.



Source: Retrieved from Zhu (2021a).

Figure 1. Relative motion schematic.

Figure 2 is the relative motion schematic between multimissiles and target, where points T and M_i are the target and the missile i (i = 1, 2, ¼ n), where n is the number of missiles. r_i is the distance between missile i and the target, q_i is the LOS angle of missile i, θ_t and θ_{mi} are the lead angles of target and the missile i, V_t and V_{mi} are the velocities of target and the missile i.



Source: Elaborated by the authors.

Figure 2. Relative motion schematic.

The control input can be projected along the LOS coordinate system. The projection along LOS is u_1, and the projection perpendicular to LOS is u_2. The control input can also be projected along the velocity coordinate system, a_{m1} along velocity direction, and a_{m2} perpendicular to velocity direction.

According to Fig. 1, the relative equations of head-pursuit interception can be obtained in Eqs. 1–4:

$$\dot{r} = V_m \cos\theta_m - V_t \cos\theta_t \tag{1}$$

$$\dot{q} = \left(V_m \sin\theta_m - V_t \sin\theta_t\right) / r \tag{2}$$

$$\dot{\theta}_t = \frac{a_t}{V_t} - \dot{q} \tag{3}$$

$$\dot{\theta}_m = \frac{a_{m2}}{V_m} - \dot{q} \tag{4}$$

where a_t is the target acceleration.

Differentiating Eq. 1 with respect to time and combining with equation Eq. 2, Eq. 5 is given:

$$\ddot{r} - r\dot{q}^2 = (\dot{V}_m \cos\theta_m - a_{m2} \sin\theta_m) - (\dot{V}_t \cos\theta_t - a_t \sin\theta_t) \tag{5}$$

Let $u_1 = \dot{V}_m \cos\theta_m - a_{m2} \sin\theta_m$, and it is the projection of the control input along LOS.

The time-to-go of missile i is equal to the ratio of r_i to $\dot{r}_i$, where $\dot{r}_i$ represents the relative distance between missile i and the target, and $\dot{r}_i$ represents the change rate of the relative distance. Therefore, the time-to-go of missile i can be denoted by t_{goi} as Eq. 6.

$$t_{goi} = -\frac{r_i}{\dot{r}_i} \qquad (i = 1, 2, \cdots n) \tag{6}$$

where r_i can be measured and $\dot{r}_i$ satisfies $\dot{r}_i = V_{mi} \cos\theta_{mi} - V_t \sin\theta_n$

Because of the need for multimissiles to collide with the target at the same time, the predicted collision moment can be denoted by t_{fi} as in Eq. 7.

$$t_{fi} = t - \frac{r_i}{\dot{r}_i} \qquad (i = 1, 2, \cdots n) \tag{7}$$

Differentiating t_{fi} with respect to time, Eq. 8 is given.

$$\dot{t}_{fi}=\frac{r_i^2\dot{q}_i^2}{\dot{r}_i^2}+\frac{r_iu_{1i}}{\dot{r}_i^2}+\frac{r_i(a_t\sin\theta_{it}-\dot{V}_t\cos\theta_{it})}{\dot{r}_i^2}\qquad(i=1,2,\cdots n)\tag{8}$$

Let $d_i = (r_i\,(a_t \sin\theta_{it} - \dot{V}_t \cos\theta_{it}))/(\dot{r}_i^2)$ be the unknown system disturbance, then Eq. 8 can be written as in Eq. 9.

$$\dot{t}_{fi}=\frac{r_i^2\dot{q}_i^2}{\dot{r}_i^2}+\frac{r_iu_{1i}}{\dot{r}_i^2}+d_i\qquad(i=1,2,\cdots n)\tag{9}$$

Then, the relative motion equation of missile I to the target can be obtained as in Eqs. 10 and 11:

$$\begin{cases}\dot{r}_i=V_{mi}\cos\theta_{mi}-V_t\cos\theta_{ti}\\ \dot{q}_i=\left(V_{mi}\sin\theta_{mi}-V_t\sin\theta_{ti}\right)/r_i\\ \dot{\theta}_{ti}=\dfrac{a_t}{V_t}-\dot{q}_i\\ \dot{\theta}_{mi}=\dfrac{a_{m2i}}{V_{mi}}-\dot{q}_i\end{cases}\tag{10}$$

$$\dot{t}_{fi}=\frac{r_i^2\dot{q}_i^2}{\dot{r}_i^2}+\frac{r_iu_{1i}}{\dot{r}_i^2}+d_i\tag{11}$$

According to the requirement of cooperative guidance law, each missile should satisfy the following formula (Eq. 12):

$$\lim_{r_i\to 0}(t_{fi}-t_{fj})=0;\qquad(i=1,2,\cdots n;\ j=1,2,\cdots n)\tag{12}$$

According to the requirement of head-pursuit interception proposed by Golan and Shima (2004), each missile should satisfy the following formula (Eq. 13):

$$\lim_{r_i\to 0}\theta_{mi}=0;\quad \lim_{r_i\to 0}\theta_{ti}=0;\quad \theta_{mi}=\kappa\theta_{ti}\qquad(i=1,2,\cdots n)\tag{13}$$

where κ is a lead factor and satisfies $\kappa > V_t/V_{mi}$.

In order to satisfy Eq. 12, the missiles can be seen as multiagents, the communication network of them can be represented by an undirected graph $G = (v, \zeta, \mathbf{C})$, where v represents a set of all nodes in G, z represents the lines that exist between all nodes in G, and ζ is the weight matrix of G. The entry of C is represented by C_{ij}, if the information can be exchanged directly between agent i and agent j, the value of c_{ij} is 1, otherwise the value of c_{ij} is 0. In particular, $c_{ii} = 0$. If there is a line for information exchange between any agent, the undirected graph is defined as connected. Before giving the cooperative guidance law, some lemmas related to the finite-time consistency of multiagent are introduced.

Lemma 1 (X Feng and Long 2007): For the following first-order multiagent system (Eq. 14):

$$\dot{x}_i=\upsilon_i\qquad(i=1,2,\cdots n)\tag{14}$$

where x_i and u_i are the state and control input of agent i respectively. When the undirected graph G is connected, and if the control input u_i satisfies $\upsilon_i = \mathrm{sig}^{\delta_i}\,[\sum_{j=1}^{n} c_{ij}\,(x_j - x_i)]$, where $0 < \zeta_i < 1$. Then there exists a finite time T^*, such that when the time satisfies $t > T^*$, the state of any agent j satisfies $x_j(\mathrm{t}) = x^*$, where x^* is a real number. The control input v_i is called the finite-time consistency protocol for multiple agents.

Lemma 2 (Yu *et al.* 2005): For the following nonlinear time-varying system (Eq. 15):

$$\dot{z}(t)=f(z,t),z\in\mathrm{R}\tag{15}$$

z is the system state and t is the time. Supposing that there is a continuous positive-definite function $V(x)$, and this function satisfies the following differential inequality (Eq. 16):

$$\dot{V}(z) \le -\mu V(z) - \lambda V^{\alpha}(z) \tag{16}$$

where $\mu,\lambda > 0, 0 < \alpha < 1$ are constants, and $z(0) = z_0$. The convergence time T of the system satisfies the following inequality (Eq. 17):

$$T \le \frac{1}{\mu(1-\alpha)} \ln \frac{V^{(1-\alpha)}(z_0) + \lambda}{\lambda} \tag{17}$$

Note 1: The target and missile are point masses, and the target velocity is a constant.

Note 2: The interceptor is in free flight if the distance satisfies $r \le 100$.

Note 3: If the distance satisfies $0.1 \le r \le 0.25$, it is considered that the missile has collided with the target.

Note 4: The function $sig^{\delta i}(\bullet)$ is defined as $sig^{\delta i}(\bullet) = |\bullet|^{\delta i}\, sig(\bullet)$.

DESIGN OF HEAD-PURSUIT COOPERATIVE GUIDANCE LAW

In this part, a head-pursuit cooperative guidance law is proposed based on time consistency theory of multiagent system and sliding mode theory, and the stability is analyzed by Lyapunov stability theory.

Design of guidance law along LOS

In this section, the guidance law along LOS is designed to ensure that the collision time of each missile tends to be consistent. At the same time, the disturbance is estimated by the extended state observer to improve the accuracy of the guidance law. Firstly, an extended state observer is designed as follows to estimate the disturbance d_i in Eq. 11 (Eq. 18):

$$\begin{cases} e_{ti} = z_{ti} - t_{fi}\,; \quad e_{di} = z_{di} - d_i \\ \dot{z}_{ti} = z_{di}\beta_{1i}e_{ti} + \dfrac{r_i^2\dot{q}_i^2}{\dot{r}_i^2} + \dfrac{r_i u_{1i}}{\dot{r}_i^2} \\ \dot{z}_{di} = -\beta_{2i} sig^{\gamma_{1i}}(e_{ti}) \end{cases} \tag{18}$$

where e_{ti} and e_{di} are the estimation errors, z_{ti} and z_{di} the estimated values of t_{ft} and d_i, β_{1i},β_{2i} and $0 < \gamma_{1i} < 1$ the observer parameters to be determined. According to Z Zhu *et al.* (2013), the disturbance d_i in Eq. 11 can be well estimated by Eq. 18.

According to the time consistency theory of multiagent system, the integral sliding mode surface is designed in Eq. 19:

$$s_{1i} = t_{fi}(\mathrm{t}) - t_{fi}(0) + \int_0^t -\mathrm{sig}^{\alpha_i}\left[\sum_{j=1}^{n} c_{ij}(x_j - x_i)\right] d\tau \tag{19}$$

where $0 < \alpha_i < 1$.

Differentiating Eq. 19 with respect to time yields, Eq. 20 is given.

$$\dot{s}_{1i} = \dot{t}_{fi}(\mathrm{t}) - \mathrm{sig}^{\alpha_i}\left[\sum_{j=1}^{n} c_{ij}(x_j - x_i)\right] \tag{20}$$

Substituting Eqs. 11 into Eq. 20 yields, Eq. 21 is given.

$$\dot{s}_{1i} = \frac{r_i u_{1i}}{\dot{r}_i^2} + \frac{r_i^2\dot{q}_i^2}{\dot{r}_i^2} - \mathrm{sig}^{\alpha_i}\left[\sum_{j=1}^{n} c_{ij}(x_j - x_i)\right] + d_i \tag{21}$$

Define a fast power reaching law as Eq. 22:

$$\dot{s}_{1i} = -k_{1i}s_{1i} - k_{2i}sig^{\lambda_{1i}}(s_{1i}) \tag{22}$$

where k_{1l} and k_{2l} are positive constants, $0 < \alpha_{1i} < 1$.

Therefore, according to Eqs. 21 and 22, the control input can be obtained as shown in Eq. 23.

$$u_{1i} = \frac{\dot{r}_i^2\left\{-k_{1i}s_{1i} - k_{2i}sig^{\lambda_{1i}}(s_{1i}) + \mathrm{sig}^{\alpha_i}\left[\sum_{j=1}^{n} c_{ij}(x_j - x_i)\right] - z_{di}\right\}}{r_i} - \frac{r_i^2\dot{q}_i^2}{r_i} \tag{23}$$

Theorem 1

Consider Eq. 11, the guidance law Eq. 23 along LOS direction will make the estimated collision time converge to a same constant with in a finite time. The proof is shown below:

Consider a Lyapunov function in Eq. 24:

$$V_{1i} = \frac{1}{2}s_{1i}^2 \tag{24}$$

According to Eqs. 19 and 23, the derivative of Eq. 24 can be got by Eq. 25:

$$\begin{aligned}\dot{V}_{1i} &= s_{1i}\dot{s}_{1i} \\ &= s_{1i}\left\{\frac{r_i^2\dot{q}_i^2}{\dot{r}_i^2} + \frac{r_i u_{1i}}{\dot{r}_i^2} + d_i - \mathrm{sig}^{\alpha_i}\left[\sum_{j=1}^{n} c_{ij}(x_j - x_i)\right]\right\} \\ &= s_{1i}\left[-k_{1i}s_{1i} - k_{2i}sig^{\lambda_{1i}}(s_{1i}) - z_{di} + d_i\right] \\ &= -k_{1i}s_{1i}^2 - k_{2i}sig^{\lambda_{1i}+1}(s_{1i}) \\ &= -2k_{1i}V_{1i} - 2^{\frac{\lambda_{1i}+1}{2}}k_{2i}V_{1i}^{\frac{\lambda_{1i}+1}{2}}\end{aligned} \tag{25}$$

According to Lemma 2, the sliding variable s_{1i} and its derivative $\dot{s}_{1i}$ will converge to zero in finite-time T_{1i} which satisfies Eq. 26:

$$T_{1i} \le \frac{1}{2k_{1i}\left(1 - \frac{\lambda_{1i}+1}{2}\right)}\ln\frac{V^{(1-\frac{\lambda_{1i}+1}{2})}(s_{1i0}) + 2^{\frac{\lambda_{1i}+1}{2}}k_{2i}}{2^{\frac{\lambda_{1i}+1}{2}}k_{2i}} \tag{26}$$

Then, differentiating Eq. 19 with respect to time, Eq. 27 is given:

$$\dot{t}_{fi}(t) = \mathrm{sig}^{\alpha_i}\left[\sum_{j=1}^{n} c_{ij}(x_j - x_i)\right], \quad (t > T_{1i}) \tag{27}$$

According to Lemma 1, the estimated collision time of each missile will converge to a same constant in a finite time.

Design of guidance law perpendicular to LOS

In order to ensure that each missile can collide with the target in the way of head-pursuit interception, a guidance law perpendicular to LOS is designed in this part. According Eq. 13, a new variable x_i is introduced n Eq. 28:

$$x_i = \theta_{mi} - \kappa\theta_{ti} \tag{28}$$

Then, the following system can be got as Eq. 29:

$$\dot{x}_i = \frac{a_{m2i}}{V_{mi}} + (\kappa - 1)\dot{q}_i - \kappa \frac{a_t}{V_t} \tag{29}$$

Let $\omega_i = -\kappa\, a_t/V_t$ be the unknown disturbance of system, and estimate it with the following extended state observer (Eq. 30):

$$\begin{cases} e_{xi} = z_{xi} - x_i\,; \quad e_{\omega i} = z_{\omega i} - \omega_i \\ \dot{z}_{xi} = z_{\omega i} - \beta_{3i} e_{xi} + \frac{a_{m2i}}{V_{mi}} + (\kappa - 1)\dot{q}_i \\ \dot{z}_{\omega i} = -\beta_{4i}\, sig^{\gamma_{2i}}(e_{xi}) \end{cases} \tag{30}$$

where e_{xi} and e_{wi} are the estimation errors, z_{xi} and z_{wi} the estimated values of x_i and ω_i, β_{3i}, β_{4i} and $0 < \gamma_{2i} < 1$ the observer parameters to be determined.

Define the sliding surface as follows (Eq. 31):

$$s_{2i} = x_i \tag{31}$$

Differentiating Eq. 19 with respect to time yields, Eq. 32 is given:

$$\dot{s}_{2i} = \dot{x}_i \tag{32}$$

Substituting Eq. 29 into Eq. 20 yields, Eq. 33 is given:

$$\dot{s}_{2i} = \frac{a_{m2i}}{V_{mi}} + (\kappa - 1)\dot{q}_i - \kappa \frac{a_t}{V_t} \tag{33}$$

Define a fast power reaching law in Eq. 34:

$$\dot{s}_{2i} = -k_{3i} s_{1i} - k_{4i}\, sig^{\lambda_{2i}}(s_{2i}) \tag{34}$$

where k_{3i} and k_{4i} are positive constants, $0 < \lambda_{2i} < 1$. According to Eqs. 33 and 34, Eq. 35 can be got:

$$a_{m2i} = V_{mi}\left[-k_{3i} s_{1i} - k_{4i}\, sig^{\lambda_{2i}}(s_{2i}) - (\kappa - 1)\dot{q}_i - z_{\omega i}\right] \tag{35}$$

According to the relation between LOS coordinate system and velocity coordinate system, there exists the following transformation relationship (Eq. 36):

$$\begin{bmatrix} a_{m1i} \\ a_{m2i} \end{bmatrix} = \begin{bmatrix} \cos\theta_{mi} & \sin\theta_{mi} \\ -\sin\theta_{mi} & \cos\theta_{mi} \end{bmatrix} \begin{bmatrix} u_{1i} \\ u_{2i} \end{bmatrix} \tag{36}$$

Therefore, Eq. 37 can be obtained based on Eqs. 23, 35 and 36:

$$u_{2i} = \frac{V_{mi}\left[-k_{3i} s_{1i} - k_{4i}\, sig^{\lambda_{2i}}(s_{2i}) - (\kappa - 1)\dot{q}_i - z_{\omega i}\right] + u_{1i}\sin\theta_{mi}}{\cos\theta_{mi}} \tag{37}$$

Theorem 2

Consider Eq. (10), the guidance law Eq. (37) perpendicular to LOS will make the lead angles satisfy Eq. (13), which ensures that each missile collide with the target in the way of head-pursuit interception in finite time. The proof is shown below:

Consider a Lyapunov function in Eq. 38.

$$V_{2i} = \frac{1}{2} s_{2i}^2 \tag{38}$$

According to Eqs. 23, 31, 35 and 37, the derivative of Eq. 38 can be got (Eq. 39):

$$\begin{aligned} \dot{V}_{2i} &= s_{2i}\dot{s}_{2i} \\ &= s_{2i}\left[\frac{a_{m2i}}{V_{mi}} + (\kappa - 1)\dot{q}_i + \omega_i\right] \\ &= s_{2i}\left[-k_{3i}s_{2i} - k_{4i} sig^{\lambda_{2i}}(s_{2i}) - z_{\omega i} + \omega_i\right] \\ &= -k_{3i}s_{2i}^2 - k_{4i} sig^{\lambda_{2i}+1}(s_{2i}) \\ &= -2k_{3i}V_{2i} - 2^{\frac{\lambda_{2i}+1}{2}} k_{4i} V_{2i}^{\frac{\lambda_{2i}+1}{2}} \end{aligned} \tag{39}$$

According to Lemma 2, the sliding variable s_{2i} and its derivative $\dot{s}_{2i}$ will converge to zero in finite-time T_{2i}, and T_{2i} satisfies Eq. 40:

$$T_{2i} \le \frac{1}{2k_{3i}\left(1 - \frac{\lambda_{2i}+1}{2}\right)} \ln \frac{V^{(1-\frac{\lambda_{2i}+1}{2})}(s_{2i0}) + 2^{\frac{\lambda_{2i}+1}{2}} k_{4i}}{2^{\frac{\lambda_{2i}+1}{2}} k_{4i}} \tag{40}$$

Then, according to Golan and Shima (2004), the lead angle θ_{mi} will be consistent with $\kappa\theta_{ti}$ and converge to zero. Therefore, the interceptor will collide with the target in the way of head-pursuit interception. Theorem 2 is proved.

Note 5: Because u_{1i} and a_{2mi} are not orthogonal, if they are used as the control input of the system directly, there will be cancelation. Therefore, u_{1i} and u_{2i} as shown in Eqs. 23 and 37 are used as the control inputs of the system.

Note 6: Depending on the head-pursuit cooperative guidance law, the interceptor velocity can no longer be assumed to be constant.

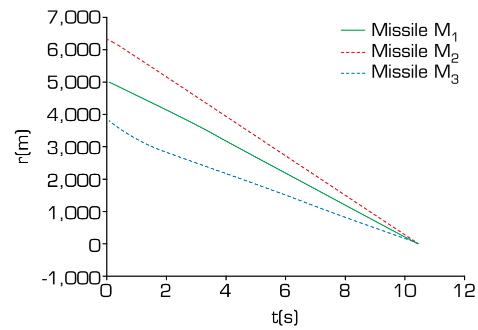
SIMULATIONS

In this section, the correctness of the head-pursuit cooperative guidance law is verified by mathematical simulation.

Suppose that there are three missiles to intercept a hypersonic vehicle in near-space, missile 1 and missile 2 can communicate directly, missile 2 and missile 3 can communicate directly also. However, missile 1 can communicate with missile 3 only through missile 2 indirectly. Therefore, based on graph theory, the weight matrix of communication network is shown in Eq. 41.

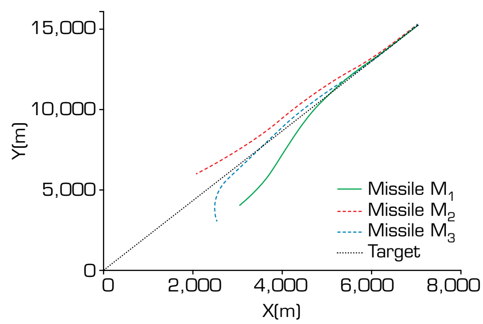
$$C = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \tag{41}$$

When the origin of the inertial coordinate system is shifted to the target, the information of the target is $X_t(0) = 0$, $Y_t(0) = 0$, $V_t = 1600$ m/s, $\theta_{t1}(0) = 10^\circ$, $a_t = 50\sin(0.25\pi t)$ m/s^2 , missile 1 is $X_{m1}(0) = 3000$, $Y_{m1}(0) = 4000$, $V_{m1}(0) = 1200$ m/s, $\theta m_1(0) = 10^\circ$, missile 2 is $X_{m2}(0) = 2000$, $Y_{m2}(0) = 6000$, $V_{m2}(0) = 1200$ m/s, $\theta_{m2}(0) = -20^\circ$,missile 3 is X_m3 (0)=2500 , $Y_{m3}(0) = 3000$, $V_{m3}(0) = 1200$ m/s, $\theta_{m3}(0) = 50^\circ$. The parameters of the head-pursuit cooperative guidance law are κ=2 , $\alpha_i = 0.95$, $\lambda_{1i} = 0.9$, $\lambda_{2i} = 0.35$, $k_{1i} = k_{3i} = 10$, $k_{2i} = k_{4i} = 0.4$, $\beta_{1i} = \beta_{3i} = 50$, $\beta_{2i} = \beta_{4i} = 150$, $\gamma_{1i} = \gamma_{2i} = 0.35$. Figures 3–10 are the simulation results.



Source: Elaborated by the authors.

Figure 3. Range between missile and target



Source: Elaborated by the authors.

Figure 4. Relative motion orbit.

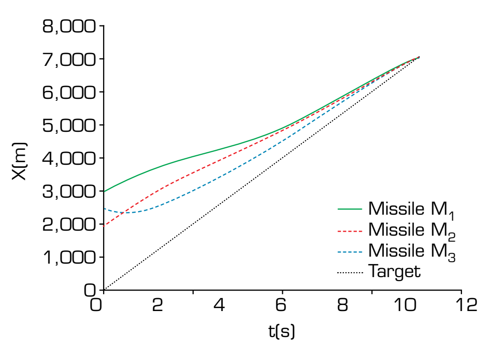


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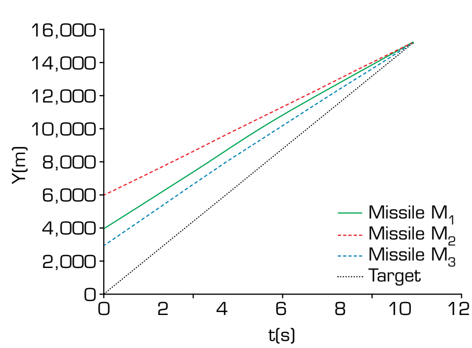
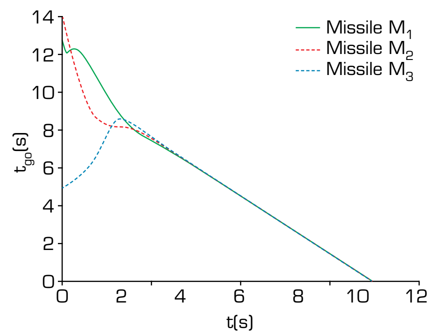
Figure 5. Change in the X-coordinate.



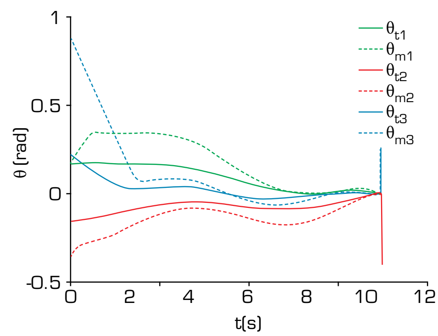
Source: Elaborated by the authors.

Figure 6. Change in the Y-coordinate.



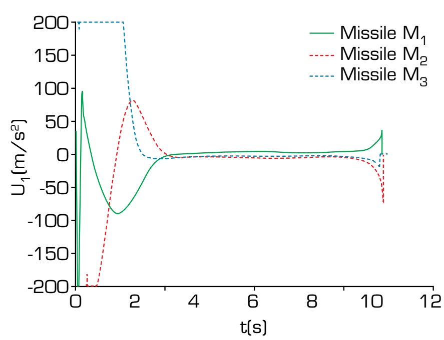
Source: Elaborated by the authors.

Figure 7. Time-to-go.



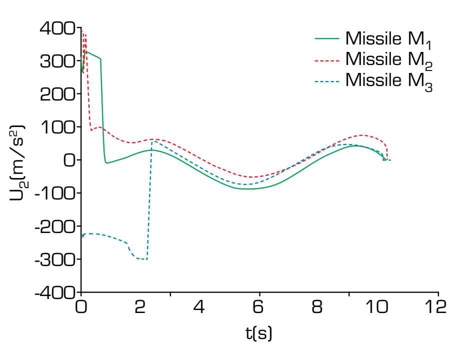
Source: Elaborated by the authors.

Figure 8. Lead angle θ.



Source: Elaborated by the authors.

Figure 9. Acceleration command u_1.



Source: Elaborated by the authors.

Figure 10. Acceleration command u_2.

Figure 3 shows the distances between the three missiles and the target, from which although the initial ranges of three missiles are different from each other greatly, they can all decrease gradually and converge to 0 at the same time. Figure 4 shows the relative motion orbit of the three missiles and the target, it can be seen that the three missiles colliding with the target at the same time. Figures 5 and 6 show the changes of missiles and target in X-coordinate and Y-coordinate. Although it can be seen from Fig. 4 that the missiles and target pass through the same coordinate position before the collision, they will not collide with each other in advance, which can be seen from Figs. 5 and 6. Because they pass through the same location at different times, the missiles do not collide with each other or with the target until the final point of encounter. Figure 7 shows the time-to-go of the three missiles, from which it can be seen that even though the time-to-go have a maximum gap of nearly 9 s at the initial moment, they reach a cooperative state after about 3 s. Figure 8 shows the lead angles of missiles and target. It can be seen from this figure that when the missile lead angle is similar to the target, it will increase to twice the target firstly, and then converge to zero in line with the target, such as missile 1. When the missile lead angle is equal to twice the target basically, it will reach twice the target lead angle quickly, and converge to zero consistent with the target, such as missile 2. When the initial missile lead angle is very different from the target, it will need to be adjusted for a long time by the guidance law to reach twice the target lead angle, and then it will converge to zero with a similar trend to the target, such as missile 3. Figure 9 shows the acceleration command along LOS. As can be seen from the figure, in the initial period of terminal guidance, the acceleration command of each missile varies greatly due to the difference of time-to-go at the initial time, but it will not change dramatically after about 3 s. That is because the time-to-go of each missile basically tends to be the same after 3 s, which can also be confirmed by Fig. 7. Figure 10 shows the acceleration command perpendicular to LOS, which indicates that each missile basically keeps the same trend after the initial adjustment, and there is no obvious chattering. At the same time, it also can be seen that the adjustment time of missile 2 is the shortest, and missile 3 is the longest, that is because the difference of initial lead angle, which also corresponds to Fig. 8. In addition, it can be seen from Figs. 9 and 10 that there is discontinuity in the commands u_1 and u_2, which is not acceptable in the real guidance and control system. However, the guidance law studied in this paper is based on the ideal hypothesis and is used to lay the groundwork for subsequent research, so it is acceptable here. Besides, the author will explore the problem of head-pursuit cooperative guidance law with dynamic characteristics in the further study.

In addition, the guidance law proposed in this paper is represented by G_1, and the head-pursuit guidance law of single missile system for comparison is represented by G_2, which has the form as Eq. 42.

$$G_2 = V_m\left[-k_1 s - k_2 sig^{\lambda}(s) - (\kappa - 1)\dot{q}_i\right] \tag{42}$$

where V_m = 1200 m/s , k_1 = 10, k = 0.4, λ_1 = 0.9.

The statistical results of miss distance under different times of Monte Carlo simulation are shown in Table 1.

Table 1. Results of Monte Carlo simulation.

Times of Monte Carlo simulation	30	50	100
Miss distance_G_1 (m)	0.051	0.058	0.061
Miss distance_G_2 (m)	0.147	0.158	0.164

Source: Elaborated by the authors.

It can be seen from the data in the table that the miss distance of the guidance law proposed in this paper is superior to that of the comparative guidance law under different times of Monte Carlo simulation.

In conclusion, the simulation results show that the guidance law proposed in this paper can make multiple missiles that have lower speed than the target carry out the head-pursuit cooperative interception against hypersonic vehicle in near space, which can provide effective theoretical guidance for the research on cooperative guidance law for near space interceptor.

CONCLUSION

For the purpose of intercepting hypersonic vehicle in near-space, a head-pursuit cooperative guidance law based on multiagent consistency theory and sliding mode control theory is proposed in this paper, which not only intercepts the hypersonic vehicle, but also implements the coordination between missiles, thus improving the interception rate. At the same time, the selection of the reaching law and the introduction of the extended state observer not only ensure the convergence speed, but also weaken the chattering. The correctness of the guidance law is verified by Lyapunov stability theory and numerical simulations. Compared with the traditional cooperative guidance law, the head-pursuit cooperative guidance law proposed in this paper can reduce the requirement on the interceptor velocity when intercepting hypersonic targets, avoid the influence of aerodynamic thermal corrosion on the guidance accuracy, and thus achieve effective interception of hypersonic vehicle in near-space.

However, the head-pursuit cooperative guidance law is only studied in the two-dimensional plane, and its applicability in the three-dimensional case has not been verified yet. Meanwhile, the research in this paper is still at the theoretical level. Therefore, the focus of the next research will be the research on the applicability in three-dimensional environment, and the validity of the guidance law will be further verified through the hardware-in-loop simulation.

CONFLIT OF INTEREST

Nothing to declare.

AUTHORS' CONTRIBUTION

Conceptualization: Chenqi Zhu; **Methodology:** Chenqi Zhu; **Validation:** Xiang Liu; **Writing - Original Draft:** Chenqi Zhu; **Writing - Review & Editing:** Chenqi Zhu and Xiang Liu.

DATA AVAILABILITY STATEMENT

All data sets were generated or analyzed in the current study.

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