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MATHEMATICAL SCIENCES

Visual Field Coordinate Systems in Visual Neurophysiology

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Abstract: In this work, algorithms have been developed to: 1. Compare visual field coordinate data, presented in different representation systems; 2. Determine the distance in degrees between any two points in the visual field; 3. Predict new coordinates of a given point in the visual field after the rotation of the head, around axes that pass through the nodal point of the eye. Formulas are proposed for the transformation of Polar coordinates into Zenithal Equatorial coordinates and vice versa; of Polar coordinates into Gnomic Equatorial of double meridians and vice versa; and projections of double meridians system into Zenithal Equatorial and vice versa. Using the transformation of polar coordinates into Cartesian coordinates, we can also propose algorithms for rotating the head or the visual field representation system around the dorsoventral, lateral-lateral and anterior-posterior axes, in the mediolateral, dorsoventral and clockwise directions, respectively. In addition, using the concept of the scalar product in linear algebra, we propose new algorithms for calculating the distance between two points and to determine the area of receptive fields in the visual field.

Key words: Coordinate systems, receptive-field size, transformation algorithm, visual system, visual field.

INTRODUCTION

Coordinates of the visual field face the same challenging problems that cartographers faced to represent points and areas on the earth's surface. The flat representation of the surface of a sphere has been the focus of proposals and attempts to accurately represent meridians and parallels on maps with minimal graphic distortion. The invention of a geographic coordinate system is generally credited to Eratosthenes of Cyrene, who composed his Geography at the Library of Alexandria in the 3rd century BC (Fraser 1970). To establish the position of a geographic location on a map, a map projection is used to convert geodetic coordinates to plane coordinates on a map. For the representations of the earth, cartographers prefer the zenith equatorial representation because it has a flat representation that preserves the surface area with aesthetically acceptable distortions.

Studies of the visual system require that the relationship between visual field and its representation in neural centers is accurately established. Topographic representation of the visual field is an essential feature of the organization of the visual system, at several stages of sensory processing. The methods most frequently used are based on the classical techniques developed for human perimetry, i.e., the use of campimeters or tangent screens. As a result, three types of representation of the visual field (sphere) are used in vision neurophysiology. The polar coordinate system, the zenithal equatorial system, and the tangent representation.

Perimetry using Landolt type campimeters enables the exploration of a large extent of the visual field without requiring realignment of the apparatus. When using this method, polar coordinate systems are commonly employed to define a location in visual space. This method has been widely used in the study of the topography of visual projections in both cortical and sub-cortical structures in which displacement of the stimulus is manually controlled.

Alternatively, the use of a hemisphere allows direct drawing of receptive field or scotomas during the mapping procedure. Different types of coordinates can be drawn on the surface of the sphere allowing the use of polar or equatorial zenithal coordinates, which uses meridians and parallels.

The use of the tangent screen, on the other hand, greatly facilitates the construction of automatic systems for controlling stimulus position and displacement. However, with the tangent screen, it is not possible to explore a large extent of the visual field without re-aligning the animal or the screen-projector system (see Bishop et al. 1962).

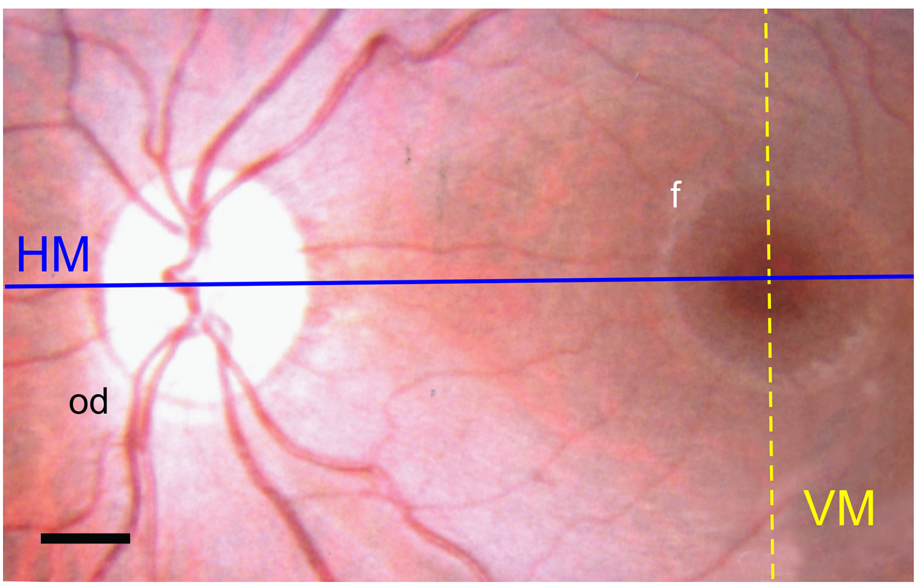


Figure 1. Fundus photograph of the left eye of a capuchin monkey showing the horizontal (blue line) and vertical meridians (yellow dashed line). The horizontal meridian (HM) passes through the center of the optic disc (od) and fovea (f). The vertical meridian (VM) is orthogonal to the horizontal meridian, passing through the fovea. Scale bar=5 degrees.

For the study of the visual system of primates, we used the following strategy: the nodal point of the eye contralateral to the recording sites was placed at the center of a 114 cm-diameter translucent plastic hemisphere. The cornea was covered with a contact lens selected by retinoscopy to focus the eye at 57 cm. The locations of the fovea and the center of the optic disk were projected onto the hemisphere. The horizontal meridian was defined as a line passing through both of these points and the vertical meridian as an orthogonal line passing through the fovea (Figure 1). The representation of the visual field in different visual areas was described in reference to these meridians.

The motivation for deducting the algorithms presented here was motivated by the work of visual projections in the primary visual cortex of the opossum. In that study, the authors utilized a Landolt' campimeter to map the opossum's receptor fields. They positioned the animals head in the center of the campimeter oriented in the horizontal plane through a line that passed through the auditory canal and the roof of the mouth behind the upper canines. When they first analyzed the data, the map

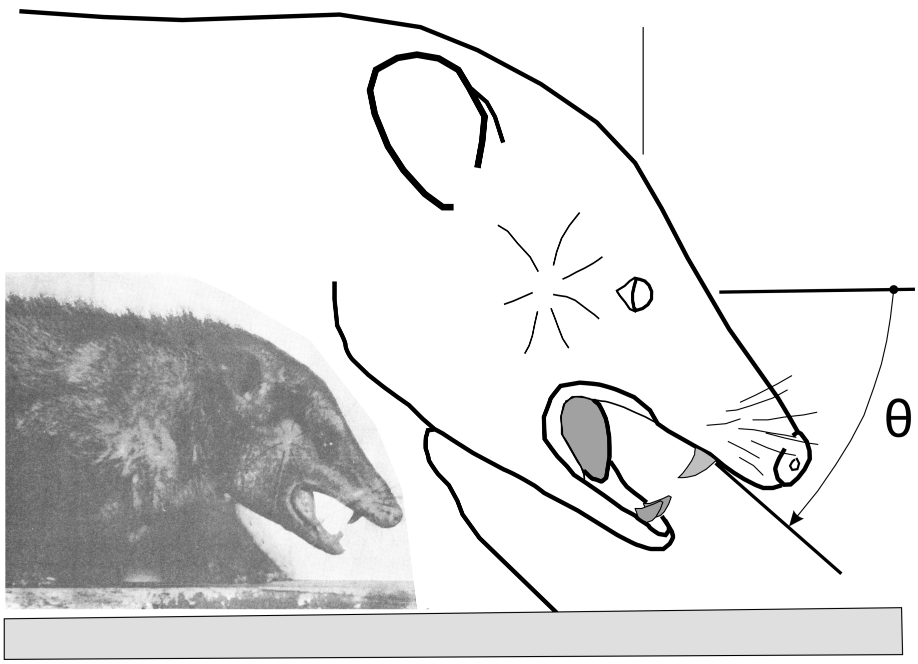


Figure 2. Opossum posture during locomotion. A Didelphis marsupilis aurita was induced to walk over a wood board in the dark. The gray rectangle is the lateral view of the marching board. The photography was synchronized to examine the position of the head during the march. The angle of the head was approximately 40 degrees.

of the primary visual area showed a substantial invasion of the ipsilateral visual field and the vertical meridian did not coincide with the edges of the primary visual area. This finding was different from those of all mammals (and primates) studied, which always had the projection of the contralateral hemifield in the primary visual cortex and the representation of the vertical meridian always coincided with the outer border of V1 (Daniel & Whitteridge 1961). When evaluating the opossum's posture of the head in locomotion, we found that the animal lowered the head at an angle of about 40 degrees, expanding the bilateral visual field (Figure 2). In this position, the vertical meridian changed its position and its projection coincided with the outer edge of the primary visual area. With this position the visual field projection in the primary visual area was restricted to the contralateral visual hemifield, similar to what was observed in all other mammal's species. For this reason, we deduct algorithms for the transformations of the coordinates with the changes in the position of the head and we applied it to the study published in 1978 (Sousa et al. 1978).

In this paper, we describe three types of projection (representation of a sphere) and we deduce equations to transform different coordinates of the visual field. We propose a series of algorithms to transform points in the visual field with the movement of the head and to calculate the receptive field area. A preliminary version of these algorithms was presents in abstract form previously (Gattass & Gatttass 1975).

METHODS

Visual Coordinate Systems

The topographic representation of the visual field is an important feature of the organization of the visual system. The type of representation is related to the apparatus used to map the receptive field at each point of the area of the cerebral cortex.

The initial studies of the visual projection in the visual areas used a campimeter of Landolt, which consists of an arc that rotate around the visual axis. It is used to map the visual field in polar coordinates and the resulting chart is referred as a polar zenithal equidistant projection. The polar coordinate system is a two-dimensional coordinate system in which each point on a plane is determined by a distance from a reference point and an angle from a reference direction. The reference point is called the pole, and the ray from the pole in the reference direction is the polar axis. The radial coordinate is called eccentricity and is often denoted by β, and the angular coordinate called polar angle is often denoted by α. Angles in polar notation are generally expressed in either degrees or radians. The angle alpha is defined to start at 0 degree from a reference direction, and to increase for rotations in counterclockwise orientation.

For the study of receptive field area, neurophysiologists used the same representation usually used to map the Earth's surface. It is referred as the equatorial zenithal of equal areas because it represents the globe as seen from a zenith (space) point, facing the equator. This representation shows field areas of the sphere with smaller distortion.

A geographic meridian (λ or longitude) is the half of an imaginary great circle on the Earth's surface, terminated by the North Pole and the South Pole, connecting points of equal longitude, as measured in angular degrees, east or west, of the Prime Meridian. The longitude meridians (μ) are intersected by parallel lines that start at the Equator. The position of a point along the meridian is given by that longitude and its latitude, measured in angular degrees, north or south, of the Equator. Each meridian is perpendicular to all circles of latitude.

The use of computer-controlled stimulus introduced the use of a tangent screen which is usually represented by a gnomic equatorial system of double meridians. The tangent screen is easy to implement on a projection screen or in a flat computer monitor. It is very useful to study restricted portions of the visual field but is inadequate for the study of large areas (larger than 40 degrees). It is usually referenced to the horizontal (ϵ) and vertical meridians (γ).

Figure 5 shows examples of visual field representation systems used in neurophysiology: the polar zenith of equidistant projections, the equatorial zenithal of Lambert's equal areas, and the equatorial gnomic of double meridians. Each representation uses different coordinate labels to representation of portions of a sphere. As an example, a table of variables are shown in the lower right. The table in Figure 5 shows the values of six points in each coordinate system. Details of each representational system will be presented after an example of the development of an algorithm.

Figure 6 show examples of visual field representation systems used in neurophysiology: the polar zenith of equidistant projections, the equatorial zenithal of Lambert's equal areas and the equatorial gnomic of double meridians. It shows the relationship of the visual axis and of the axis of sight for each coordinate system.

Definitions and References

When we started to develop these algorithms in 1975 (Gattass & Gattass 1975) we were confronted with the problem that physicists and mathematicians used the same symbols to represent different angles that defined a point in a spherical coordinate system. To aggravate the dilemma of our decision, the same spherical coordinate system was used by the visual field mapping devices (Landolt's C) in ophthalmology used in humans, but with angles defined by different symbols. As our initial problem applied to the opossum (Didelphis marsupialis), it was agreed that the visual axis would be represented by the Y axis, while the dorsoventral axis would be represented by the X axis. The Z axis would be the medial lateral axis.

In mathematics, a spherical coordinate system (r, θ, ϕ) is a coordinate system for three-dimensional space, where the position of a point is specified by three numbers: the radial distance of that point from a fixed origin, its polar angle measured from a fixed zenith direction (θ), and the azimuthal angle (ϕ) of its orthogonal projection on a reference plane that passes through the origin and is orthogonal to the zenith, measured from a fixed reference direction on that plane. It can be seen as the three-dimensional version of the polar coordinate system. The polar angle (α) was called colatitude, zenith angle, or normal angle.

The polar coordinate system used by ophthalmologists is a spherical coordinate system defined as (r, α, β). The radial distance by definition equals 1.

According to the conventions of geographical coordinate systems (r, lambda, mu), positions are measured by latitude (λ), longitude (μ). Here also, r is by convention equal 1. Neurophysiologists uses the Equatorial Zenithal system with positions defined latitude (lambda), longitude (mu).

Modern electrophysiologists that use a tangent screen or a computer monitor to generate visual stimuli uses a coordinate system named Equatorial Gnomic of Double Meridians. In this projection system, the meridians [Azimuth (ϵ), Elevation (γ)] are plotted in equal space, although they are progressively larger at increasing eccentricities on the tangent screen.

Figure 6a shows a Polar Zenith of Equidistant Projections. The projection is most commonly used in polar aspects for topographic maps of the polar regions. It is an azimuthal projection drawn to show Antarctic areas of Earth. It is based on a plane perpendicular to the Earth's axis in contact with the North or South Pole. This projection generally represents only one hemisphere. It was the earliest representation system used to map the visual field in the brain (Daniel & Whitteridge 1961, Allman & Kaas 1970). In this projection the visual (Y) axis coincides with the polar pole α = 0, β = 0.

Figure 6b shows an Equatorial Zenithal of Lambert's Equal Areas. The need to map receptive fields onto a plastic globe, corresponding to a visual hemifield introduced several apparatuses that could read meridians and parallel coordinates in a globe. Since preserving the receptive-field area was important in the projection, we used the Equatorial Zenithal of Lambert's Equal Areas (Gattass et al. 1978). In this projection, the visual (Y) axis coincides with the intersection of the horizontal (equator) with the vertical meridian λ = 0, μ = 0.

Figure 6c shows an Equatorial Gnomic of Double Meridians. It is the most used in visual physiology because a tangent screen is easy to implement on a projection screen or in a flat computer monitor. It is a flat representation with orthogonal meridians plotted at equal spacing. In this projection, the visual (Y) axis coincides with the intersection of the horizontal with the vertical meridian ϵ = 0, γ = 0.

Anatomical Planes

An anatomical plane is a hypothetical plane used to transect the body, in order to determine the location of structures. In animal anatomy, three principal planes are used: The sagittal plane, or lateral plane, is a plane parallel to the sagittal suture. It divides the head into left and right. The coronal, or frontal plane, divides the head into anterior and posterior portions. The horizontal or axial plane divides the body into cranial and caudal portions.

Rotation in the Sagittal Plane: Dorsal-ventral head rotation

In this section, we deduct the algorithms to transform locations in the visual field in different spherical coordinate systems. To establish notation, we use Figure 3. The right part of Figure 3 shows a spherical coordinate system where Y is the visual axis, X is the dorsal-ventral axis, and Z is the medial-lateral axis. Angles α and β are coordinates in the visual system. In the left part of the figure, we can see these angles represented in a polar coordinate system.

The spherical coordinate system also yields the following classical equations:

$$x = R\sin\alpha\sin\beta \tag{1}$$

$$y = R\cos\beta \tag{2}$$

$$z = R\sin\beta\cos\alpha \tag{3}$$

$$\alpha = \arctan\left(\frac{x}{z}\right) \tag{4}$$

$$\beta = \arccos\left(\frac{y}{R}\right) \tag{5}$$

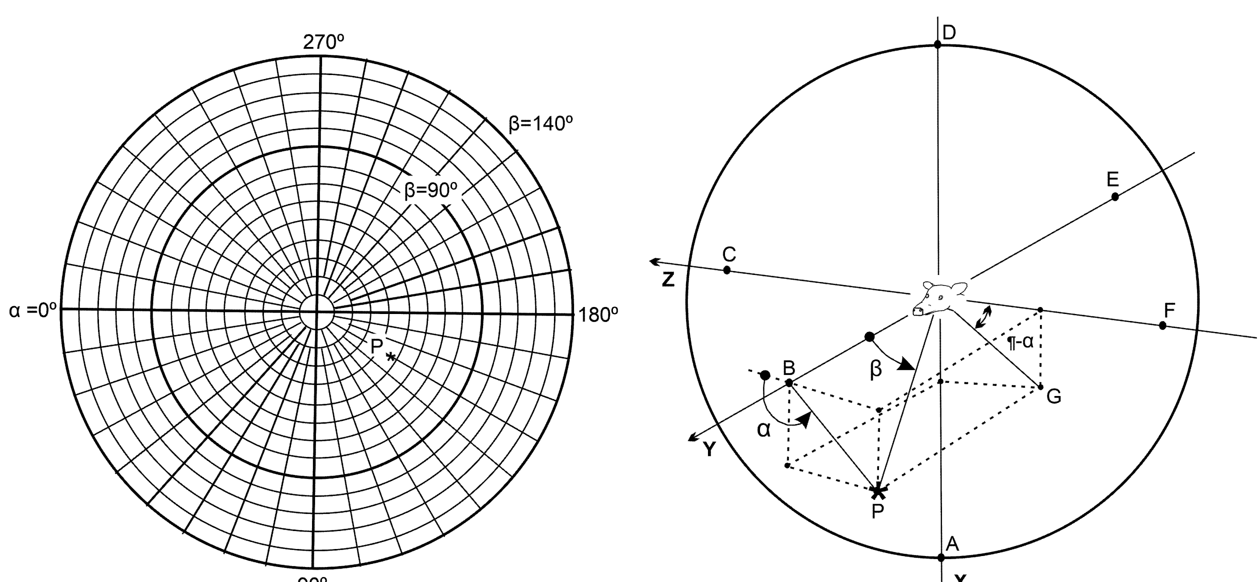


Figure 3. 3D representation of the visual axis. Left, representation of the visual field in polar coordinates. The origin of the polar coordinate system is aligned with the visual axis. Right, 3D drawing of the opossum's head and its relationship to the visual axes. Visual axis (Y), dorsal ventral axis (X) and medial lateral axis (Z)

Figure 4 shows the relationship of the field axes for three planes of rotations. In the left scheme (a), the head moves in the sagittal plane; in the middle scheme (b), the head moves in the horizontal plane; and in the right scheme (c), the head moves in the frontal plane. We now compute the new angles for each of these rotations.

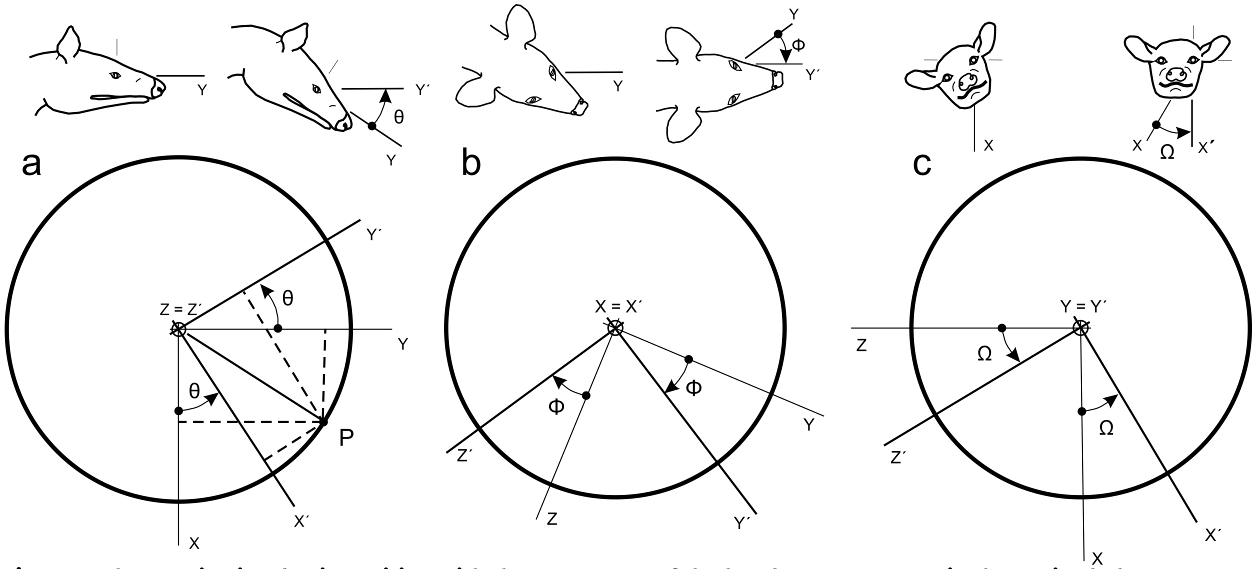


Figure 4. Changes in visual axis position with the movement of the head. A - movement in the sagittal plane, B - movement in the horizontal plane, and C - movement in the frontal plane.

When the head is rotated by an angle θ in the sagittal plane

$$x' = x\cos\theta + y\sin\theta \tag{6}$$

$$y' = y\cos\theta - x\sin\theta \tag{7}$$

$$z' = z \tag{8}$$

in this case, we have:

$$\alpha' = \arctan\left(\frac{x'}{z'}\right) \tag{9}$$

$$\beta' = \arccos\left(\frac{y'}{R'}\right) \tag{10}$$

replacing the values of x', y' and z', we have:

$$\alpha' = \arctan\left(\frac{x\cos\theta + y\sin\theta}{z}\right) \tag{11}$$

$$\beta' = \arccos\left(\frac{-x\sin\theta + y\cos\theta}{R}\right) \tag{12}$$

replacing the values of x, y and z as a function of α and β, we arrive at the desired solution:

$$\alpha' = \arctan\left(\frac{\sin\beta\sin\alpha\cos\theta + \cos\beta\sin\theta}{\sin\beta\cos\theta}\right) \tag{13}$$

$$\beta' = \arccos\left(\cos\beta\cos\theta - \sin\alpha\sin\beta\sin\theta\right) \tag{14}$$

where: α varies from 0 to 360 degrees, and β varies from 0 to 180 degrees.

RESULTS

Head Rotations

Rotation in the horizontal plane: Lateral-medial head movement

When the head rotates in the horizontal plane the field angles are transformed by:

$$\alpha' = \arctan\left(\frac{\sin\alpha\sin\beta}{\sin\beta\cos\alpha\cos\phi - \cos\beta\sin\varphi}\right) \tag{15}$$

$$\beta' = \arccos\left(\cos\beta\cos\phi - \cos\alpha\sin\beta\sin\phi\right) \tag{16}$$

Rotation in the frontal plane: Lateral-lateral head movement

When the head rotates in the frontal plane the filed angles are simply transformed by:

$$\alpha' = (\alpha - \omega) \tag{17}$$

$$\beta' = \beta \tag{18}$$

The following algorithms were deducted to be used for the following conversions:

Conversion of Polar coordinate to Equatorial Zenithal coordinates

$$\lambda = \arctan\left(\frac{\sin\beta\cos\alpha}{\cos\beta}\right) \tag{19}$$

$$\mu = \arccos\left(\sin\beta\sin\alpha\right) - 90^o \tag{20}$$

Conversion of Equatorial Zenithal coordinates to Polar coordinate

$$\alpha = \arctan\left(\frac{\sin\lambda\cos\mu}{-\sin\mu}\right) + 90^o \tag{21}$$

$$\beta = \arccos\left(-\cos\mu\cos\lambda\right) \tag{22}$$

Conversion of Polar coordinate into Equatorial Gnomics of Double Meridians coordinates

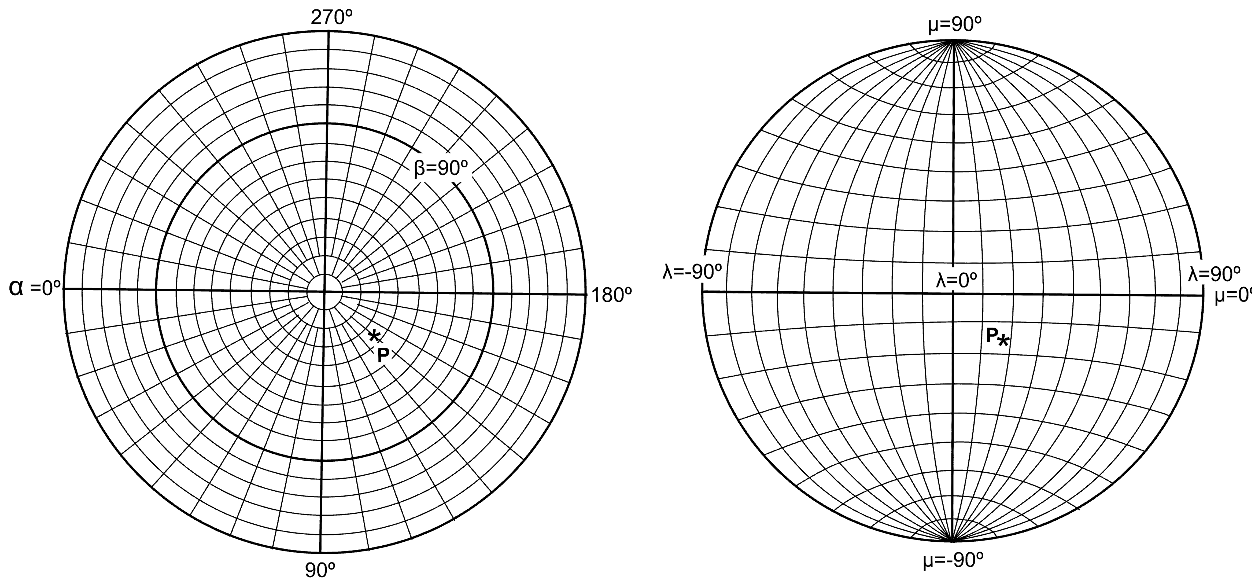
$$\epsilon = \arctan\left(\frac{\sin\beta\sin\alpha}{\cos\beta}\right) \tag{23}$$

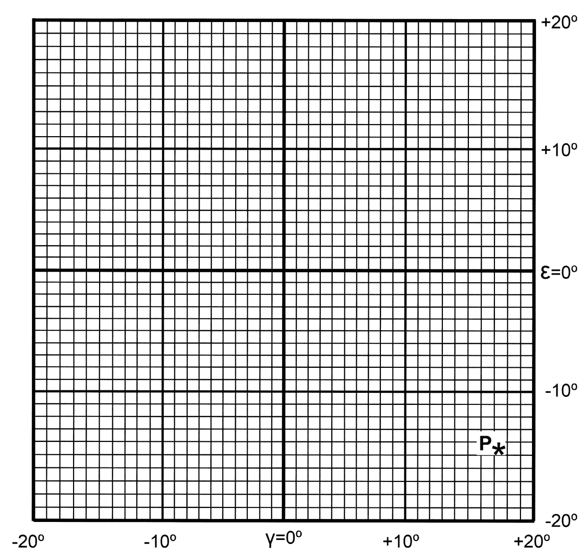
$$\gamma = \arctan\left(\frac{\sin\beta\cos\alpha}{\cos\beta}\right) \tag{24}$$

Conversion of Equatorial Gnomics of Double Meridians coordinates into Polar coordinates

$$\alpha = \arctan\left(\frac{\tan\epsilon}{\tan\gamma}\right) \tag{25}$$

$$\beta = \arctan\left(\frac{\tan\epsilon}{\sin\alpha}\right) \tag{26}$$





	α	β	γ	ε	λ	μ
A	90°	90°	-	-90°	-	90°
B	-	0°	0°	0°	0°	0°
C	0°	90°	-90°	-	-90°	0°
D	270°	90°	-	90°	-	90°
E	-	+/-180°	+/-180°	+/-180°	+/-180°	0°
F	180°	90°	90°	-	90°	-
P	140°	22°	17.2°	-14.5°	17.2°	-14°

Figure 5. Examples of visual field representation systems used in neurophysiology. Upper-left, the polar zenith of equidistant projections; Upper-right, the equatorial zenithal of Lambert's equal areas; and Lower-Left, the equatorial gnomic of double meridians. A table with the parameters of seven points and their values are shown in lower-right. The location of point "P" is shown in each representation system. Points A-F are reference points in each representation system.

Conversion of Equatorial Zenithal coordinates to Gnomic Equatorial coordinates of Double Meridians

$$\lambda = \gamma \tag{27}$$

$$\mu = \arctan\left(\tan\epsilon\cos\gamma\right) \tag{28}$$

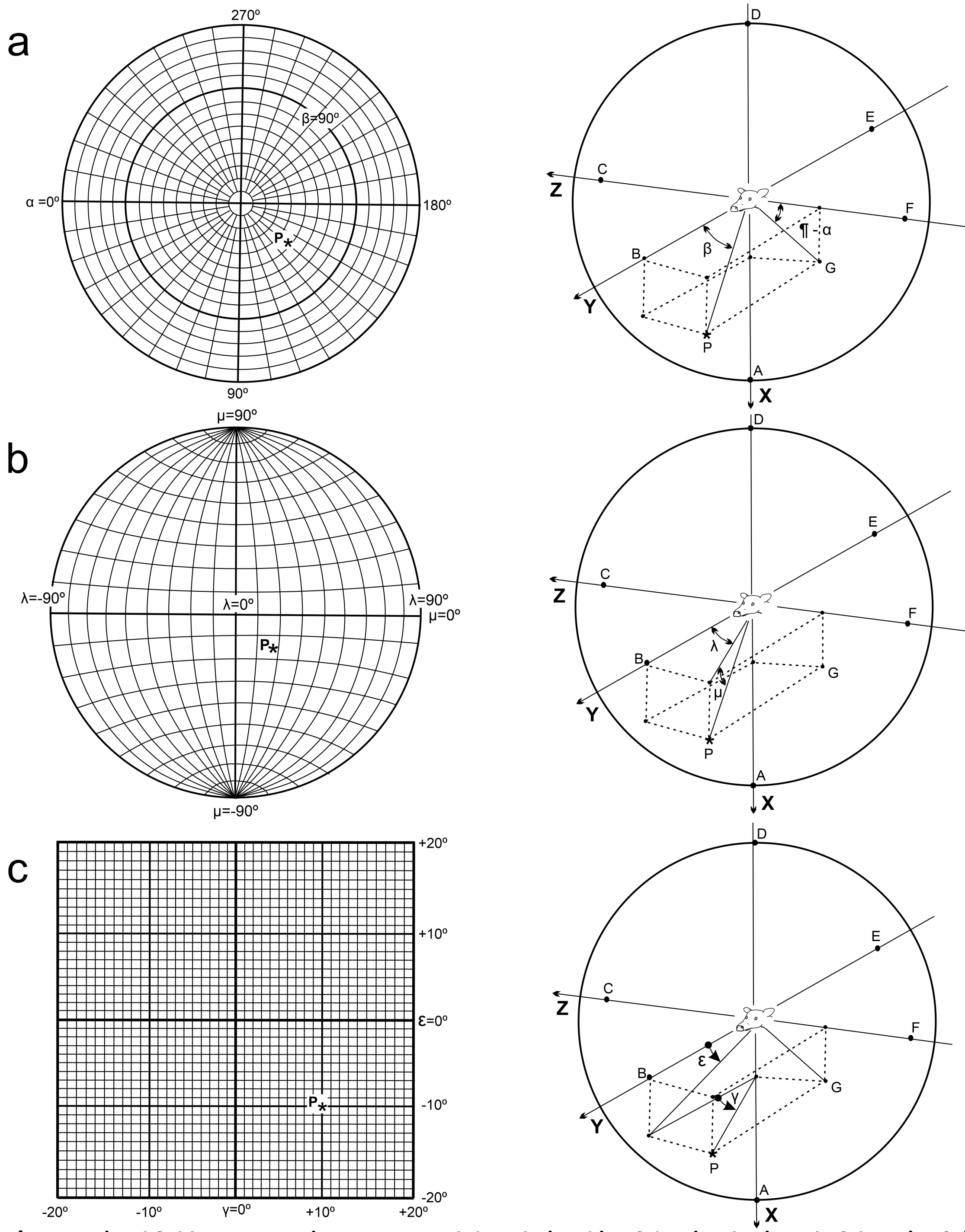


Figure 6. **Visual field representation systems and the relationship of the visual axis and of the axis of sight for each coordinate system. a) Polar; b) Equatorial of double meridians; and c) Equatorial zenithal systems.**

Conversion of Gnomic Equatorial coordinates of Double Meridians into Equatorial Zenithal coordinates

$$\gamma = \lambda \tag{29}$$

$$\epsilon = \arctan\left(\frac{\tan\mu}{\cos\lambda}\right) \tag{30}$$

Figure 7 shows examples of coordinate transformations by rotation of the head and eye.

DISTANCE AND RECEPTIVE FIELD AREA

In visual mapping studies the determination of the receptive field area and eccentricity is crucial to the description of the visual topography. In addition, the distance between two points in the visual field is also important. The algorithms outlined in the subsequent sections deducted for these purposes.

Algorithm for the determination of the surface area of a receptive field mapped with four corners.

Figure 8 shows the steps for the determination of the spherical angle α and the spherical angles. The procedure to compute the area of a polygon (P1-P4) in the visual field (sphere) has 4 steps:

1. Define P_1, P_2, P_3 and P_4 as a function of λ and μ
2. Compute the segments: $S_{12}, S_{23}, S_{34}, S_{41}, S_{24}$ and S_{13}
3. Compute the angles P_1, P_2, P_3 and P_4
4. Compute the area, A

For each two pints in the sphere defined by μ and λ we have:

$$p_1 = \pi/2 - \mu_1 \tag{31}$$

$$p_2 = \pi/2 - \mu_2 \tag{32}$$

and

$$N = \lambda_1 - \lambda_2 \tag{33}$$

and based on the spherical Law of cosine, we have for the four corners:

$$s_{12} = \arccos\left(\cos(p_1)\cos(p_2) + \sin(p_1)\sin(p_2)\cos N\right) \tag{34}$$

$$s_{23} = \arccos\left(\cos(p_2)\cos(p_3) + \sin(p_2)\sin(p_3)\cos N\right) \tag{35}$$

$$s_{34} = \arccos\left(\cos(p_3)\cos(p_4) + \sin(p_3)\sin(p_4)\cos N\right) \tag{36}$$

$$s_{41} = \arccos\left(\cos(p_4)\cos(p_1) + \sin(p_4)\sin(p_1)\cos N\right) \tag{37}$$

Using spherical trigonometry, we compute the spherical angles P_1-P_4:

$$P_1 = \arccos\left(\frac{\cos(s_{24}) - \cos(s_{41})\cos(s_{12})}{\sin(s_{41})\sin(s_{12})}\right) \tag{38}$$

$$P_2 = \arccos\left(\frac{\cos(s_{13}) - \cos(s_{23})\cos(s_{12})}{\sin(s_{23})\sin(s_{12})}\right) \tag{39}$$

$$P_3 = \arccos\left(\frac{\cos(s_{24}) - \cos(s_{23})\cos(s_{34})}{\sin(s_{23})\sin(s_{34})}\right) \tag{40}$$

$$P_4 = \arccos\left(\frac{\cos(s_{13}) - \cos(s_{34})\cos(s_{41})}{\sin(s_{34})\sin(s_{41})}\right) \tag{41}$$

and the receptive field area is computed by the following formula using spherical trigonometry, based on the spherical angles P_1-P_4:

$$RFarea = (P1 + P2 + P3 + P4 - 2\pi) \tag{42}$$

Algorithm for the determination of the eccentricity of a receptive field.

Figure 9 shows a graphic representation of the eccentricity β(Ecc). Using the two-vector scalar product concept, we arrive at the following algorithm:

$$\beta(Ecc) = |\arccos(\cos\mu\cos\lambda)| \tag{43}$$

Algorithm of the distance in degrees between any two points in the visual field.

Using the two-vector scalar product concept, we arrive at the following algorithm:

$$\delta = \arccos(\sin\beta_1 \sin\beta_2 \cos\alpha_1 - \alpha_2 + \cos beta_1 \cos\beta_2) \tag{44}$$

If the rotation algorithms and the distance between two points are correct, the distance from point a to point b must be equal to that of point a' to point b'.

$$\delta' = \delta = 20.89^o \tag{45}$$

DISCUSSION

The position of objects as viewing by the eye may have several mathematical representations. The conical nature of the eye system suggests spherical coordinates as a natural choice for the representation of the position of a point in the visual field. The study of vision, however, requires the correlation of the point representations as the head moves.

The visual field refers to the total area in which objects can be seen in the peripheral vision as you focus your eyes on a central point. By definition, it is the entire expanse of space visible at a given instant without moving the eyes. The visual field of the human eye spans approximately 120 degrees of arc. The visual field is not homogeneous. The human eye has much greater resolution in the macula, where there is a higher density of cone cells. A visual field test measures how far the eye sees in any direction without moving and how sensitive the vision is in different parts of the visual field. Scientists represent the perceptible space as a sphere surrounding the eye. Therefore, all the visual field representations used so far in visual physiology are based on a representation of a sphere, and they are based on methods developed to map the earth.

The representation of the earth on maps is made by four main types of map projections. These projections are: Azimuthal, Conic, Cylindrical, and Conventional or Mathematical projections. Azimuthal projections result from projecting a spherical surface onto a plane. It is a view from a spherical surface with an azimuthal point in space. Conic projections result from projecting a spherical surface onto a

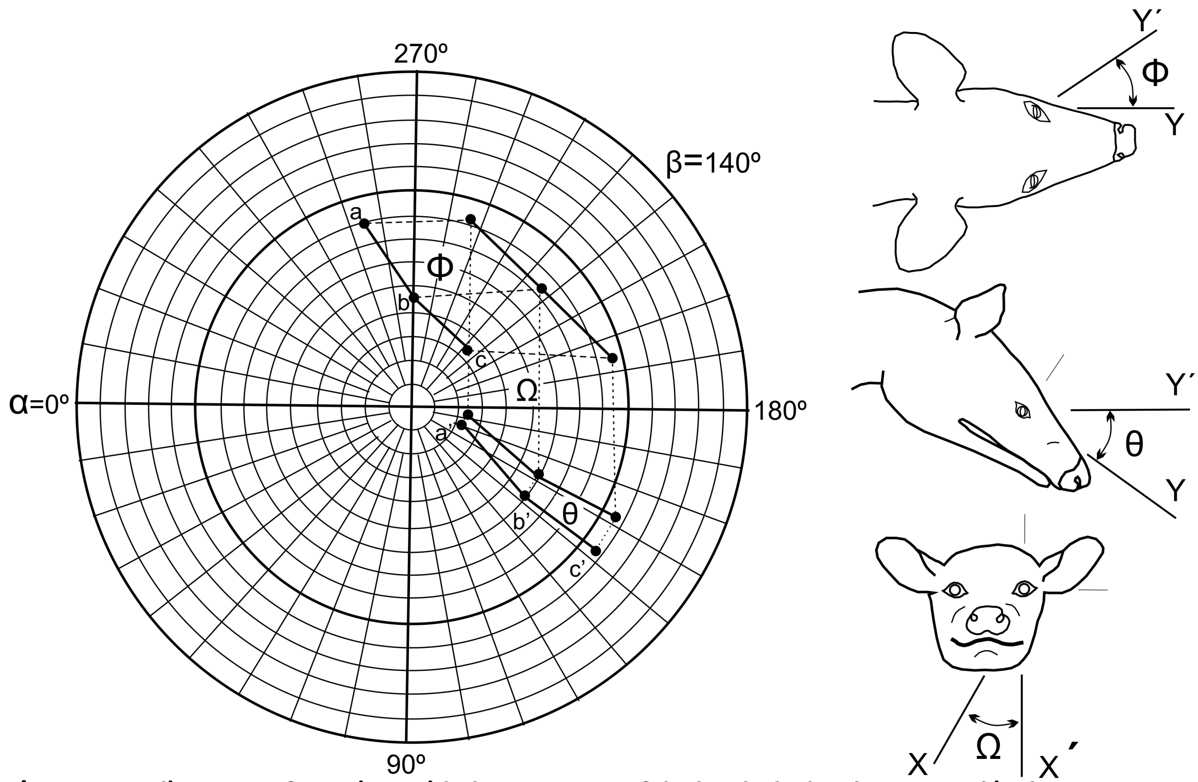


Figure 7. Coordinate transformations with the movement of the head. The head was moved in three steps (horizontal, sagittal and frontal planes) according to the figures in right. The position of three points (a,b and c) were plotted in polar coordinates for each movement, in left.

cone. It is a map projection in which the surface features of a globe are depicted as if projected onto a cone typically positioned to rest on the globe along a parallel (a line of equal latitude). A cylindrical projection, in cartography, is a map projection of the terrestrial sphere on the surface of a cylinder that is then unrolled as a plane. It is constructed by a systematic method of drawing the Earth's meridians and latitudes on the flat surface. Conventional, or mathematical, projection is a projection that is derived by mathematical computation and formulae and have few relations with the projected image. Azimuthal, Conic and Mathematical projections have been used in visual physiology.

There are several types of high detail vector maps of the world. The most recommended ones to map the earth are the Mercator and the Winkel-Tripel vector maps. The Mercator projection is a cylindrical conformal map projection that was originally created to display accurate compass bearings for sea travel. An additional feature of this projection is that all local shapes are accurate and correctly defined as an infinitesimal scale. It was made by Gerardus Mercator in 1569. The Mercator projection continues to be used since its proposal, in order to represent cities or regions. The calculations necessary to manage routes are relatively simple and proportional, that is, it distorts the space in both the north-south and east-west axes in order to maintain the forms. Google Maps mainly uses the Mercator projection because it allows to preserve the angles and therefore also shapes of small objects. For these reasons, it is useful for road navigation. It is the most commonly used projection, but it was never used in neurophysiology. The most popular and artistic projection is the vector Winkel-Tripel projection. It has been used to create a vector world map. It is a classic projection,

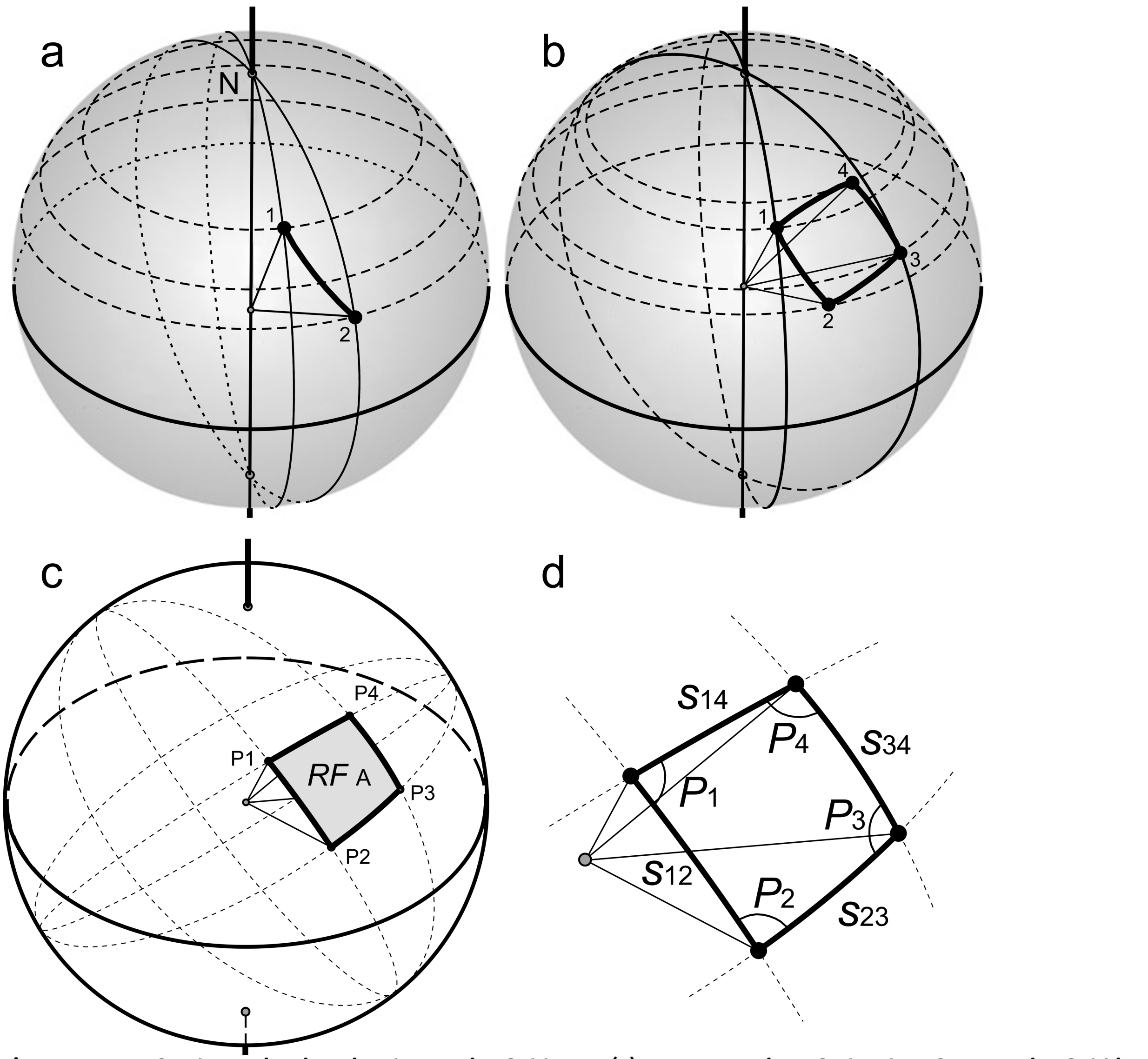


Figure 8. Steps for determination visual receptive field area. (a) Representation of a border of a receptive field in equatorial zenithal system. (b) Receptive field vertices (1-4). (c) definition of the spherical angles and (d) enlarged view showing the segments and angles for the calculation of the receptive field area.

ideal for creating all types of wall maps of the world. Its representation of a half sphere is used in vision neurophysiology as the equatorial zenithal of Lambert's equal areas (Johann Heinrich Lambert, 1772). It is a conformal conic projection.

The most accurate projection used in geographic maps is known as AuthaGraph. This is hands-down the most accurate map projection in existence. In fact, AuthaGraph World Map is so proportionally perfect, it magically folds it into a three-dimensional globe. It was invented by the Japanese architect Hajime Narukawa in 1999 by equally dividing a spherical surface into 96 triangles. It is drawn in segments (a representation with discontinuities) and it was never used in visual neurophysiology.

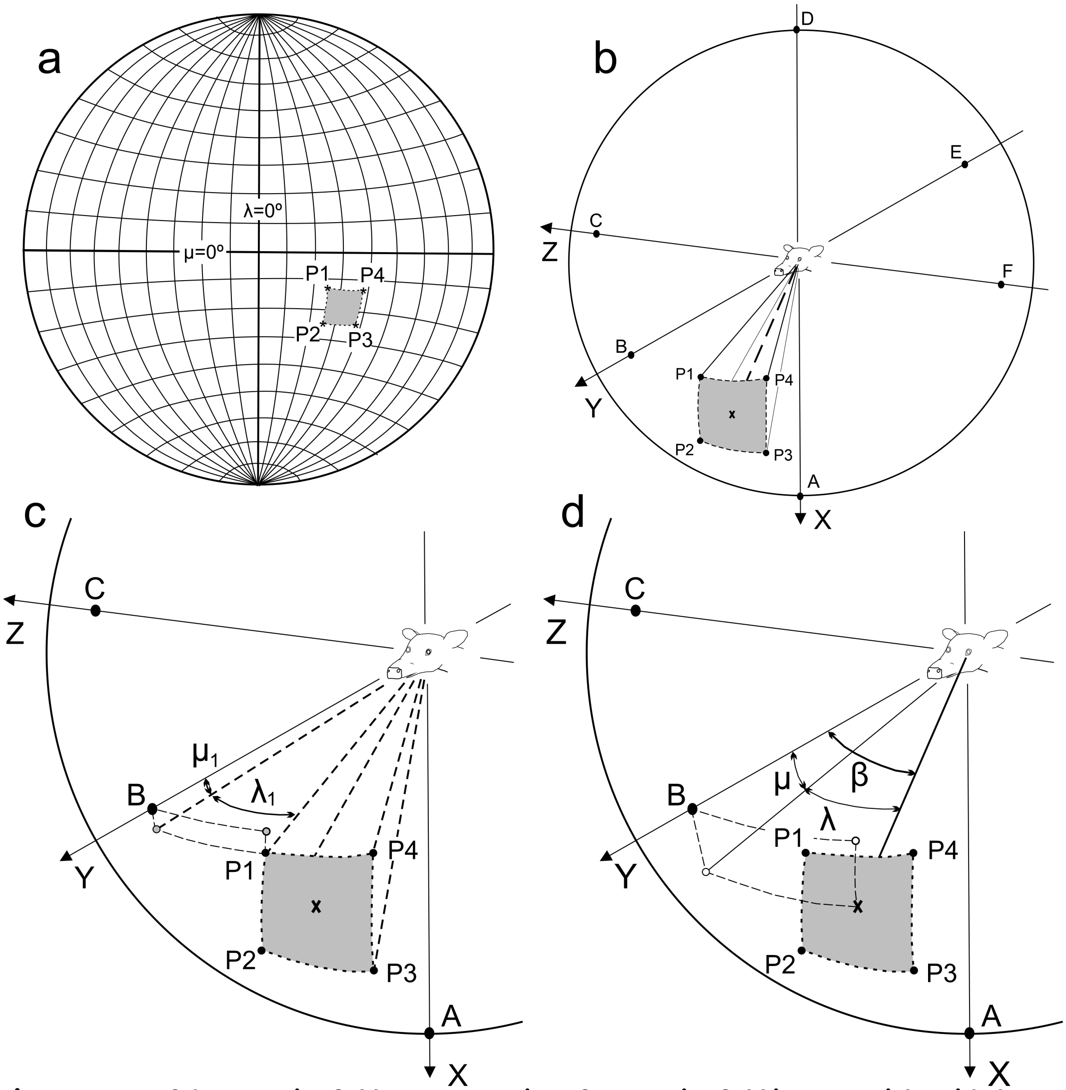


Figure 9. Area of the receptive field. Representation of a receptive field in equatorial zenithal system and the location of the receptive field vertices, P1-P4.

The Polar Zenith of Equidistant Projections was the first representation system used to map the visual field in the brain (Daniel & Whitteridge 1961, Allman & Kaas 1970). However, the need to map receptive field onto a plastic globe, corresponding to a visual hemifield introduced several apparatuses that could read meridians and parallel coordinates in a globe. Since it preserves the receptive-field area, it was considered an important projection by Gattass and collaborators (Gattass et al. 1978). They used the Equatorial Zenithal of Lambert's Equal Areas in their work since them.

The algorithms described here were used in several publications of the visual topography of cortical areas. The calculation of receptive field area and eccentricity of the receptive field centers was used to construct the representation of the topography in V3, V4, PO and POd (see Gattass et al. 2005).

The tangent screen is mostly used for the receptive field studies in visual neurophysiology. It allows the use of optical and computational systems to precisely map the receptive fields. Aiming to improve the precision and accuracy of the description of the neural response, we developed an apparatus for use with the opossum and other experimental animals (Oswaldo-Cruz & Gattass 1978). The apparatus was designed for use with a fixed tangent screen. It allows the movement of the head in all directions centered at the nodal point of the eye. We also developed new methods to automatically map visual receptive fields (Fiorani et al. 2014) to eliminate the subjective nature of the classical mapping.

In most studies of visual mapping, we represent the visual field as a sphere with the horizontal meridian considered the prime meridian at 0° longitude and the vertical meridian, 0° latitude (see Gattass et al. 2015). The vertical meridian divides the visual field into two visual hemispheres, the ipsilateral and the contralateral one. The representation of the visual field has a parallel with the cartographic maps of the Earth. Zero degrees latitude is the line designating the Equator and divides the Earth into two equal hemispheres (north and south). For convention, the meridian passing through the Royal Observatory in Greenwich, England was chosen as the prime meridians on Earth. Greenwich Meridian became the international standard for the prime meridian. It is the IERS Reference Meridian - Longitude Zero ($\lambda = 0$). The prime meridian divides Earth into the Eastern Hemisphere and the Western Hemisphere. The prime meridian is at 0° longitude and the prime parallel is at the equator. These coordinates are used to navigation on Earth. In the early stages of human travel, the sailors used star references to navigate using a navigational compass or quadrant. Thus, in the 15th-16th centuries navigation was based on the stars and maps. Today, in the everyday life, we use a Global Positioning System (GPS) to navigate in open land, sea and cities. The GPS, originally Navstar GPS (Department of Defense of the United States of America 2008), is a satellite-based radionavigation system owned by the United States government and operated by the United States Space Force. It is one of the global navigation satellite systems (GNSS) that provide geolocation and time information to a GPS receiver anywhere on or near the Earth where there is an unobstructed line of sight to four or more GPS satellites. GPS provides critical positioning capabilities to military, civil, and commercial users around the world. The United States government built the system, maintains it, and makes it freely accessible to anyone with a GPS receiver. The GPS project was initiated by the U.S. Department of Defense in 1973, with the first prototype spacecraft launched in 1978 and the full constellation of 24 satellites operational in 1993. The coordinates used in GPS are the ones early described as latitude and longitude. In vision physiology, the prime meridian is the horizontal meridian that passes through the center of the optic disk and the fovea. The vertical meridian, 0° latitude is orthogonal to the horizontal meridian and passes through the fovea.

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M.G. and R.G. designed and performed the research. R.G. prepared the figures and wrote the first draft of the manuscript. M.G. reviewed the text and tested the algorithms.

APPENDIX

List of Symbols

This paper uses the visual physiology conventions:

α, polar angle in spherical coordinate system

β, eccentricity in spherical coordinate system

λ, latitude in geographic coordinate system

μ, longitude in geographic coordinate system

ϵ, azimuth in tangent screen coordinate system

γ, elevation in tangent screen coordinate system

For head movements:

ϕ, angle in the horizontal plane

φ, angle in the frontal or vertical plane, around the visual axis

ω, angle in the sagittal plane