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Volume equations for *Khaya ivorensis* A. Chev. plantations in Brazil

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ABSTRACT

African mahogany (*Khaya* spp.) plantations are in expansion in Brazil and in the world. This fact justifies the need for studies related to its growth and yield. This paper aimed to evaluate the performance of single-entry and double-entry models for estimating merchantable and total volume for *Khaya ivorensis* plantations before the first thinning (7 years) and expected final cut (15 years). Volume data was from 100 and 46 trees in Minas Gerais and Pará states, respectively, by using an electronic dendrometer (Criterion RD 1000). Observed volumes were calculated by Smalian's formula. To validate the optical dendrometer, 10 trees were felled and had their volume measured, and compared with the volumes measured indirectly. The results showed that observed and estimated volumes were statistically equal, and that double-entry models were more precise than single-entry models. Schumacher and Hall model was the best equation to estimate merchantable volume for first thinning and for final cut in Minas Gerais stands. Spurr logarithmized model was the best equation to estimate total volume for first thinning and Spurr model for final cut in Pará stands. All chosen equations can be used to quantify merchantable and total volumes of *Khaya ivorensis* grown under similar conditions.

Key words: African mahogany, Forest mensuration, Non-destructive measurements, Thinning.

INTRODUCTION

Brazilian planted forest area was 7.84 million hectares (ha) in 2016. From this total, 72.7% was composed by *Eucalyptus* spp., 20.4% by *Pinus* spp. and 6.9% by non-conventional species plantations, such as *Acacia mearnsii*, *Acacia mangium*, *Hevea brasiliensis*, *Schizolobium amazonicum*, *Tectona grandis*, *Araucaria angustifolia*, *Populus* spp. and others. In 2010, the total area with non-conventional species was 465,390 ha, increasing 27.5% in a 6-year interval (IBÁ 2017).

In Brazil, African mahogany plantations have been planted to grow high value added wood, and now it is estimated that 10 thousand hectares of plantations have been established (Ribeiro et al. 2017). The use of African mahogany to compose

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pure stands is preferred to the Brazilian mahogany (*Swietenia macrophylla* King) because of the latter's susceptibility to the attack of *Hypsipyla grandella* Zeller shoot-borer (Ribeiro et al. 2016). This caterpillar attacks the terminal shoots of seedlings and young trees, which causes loss of the apical dominance, impeding the formation of a straight stem for merchantable use (Opuni-Frimpong et al. 2008, Tremacoldi et al. 2013). Some Meliaceae species are resistant to *H. grandella* attacks, such as *Khaya ivorensis* (African mahogany), *Toona ciliata* M. J. Roem. (Australian cedar) and *Azadirachta indica* A. Juss. (nim) (Lunz et al. 2009).

Considering all the African mahogany species, *Khaya ivorensis* is one of the most planted in Brazil, due to its high wood quality and appearance, similar to *Swietenia macrophylla* (Pinheiro et al. 2011). It possesses red-brown wood, with high merchantable value and endurance. Besides, it possesses relatively fast growth (Castro et al. 2008, Scolforo and Ferraz Filho 2014). Since large scale plantations of this species are recent in Brazil, few studies describe growth and yield parameters for forest management purposes. Recent studies have evaluated the performance of plantations and wood quality in different Brazilian states (Carvalho et al. 2010, Andrade et al. 2016, Corcioli et al. 2016), but information is still scarce for forest management prescriptions, such as timber yield and rotation age.

Although the African mahogany plantation area has increased in Brazil, forest management practices have yet to be developed. The best thinning prescriptions for this species (time and intensity) to best control stem shape and growth competition is still unknown in Brazil. Besides, *Khaya ivorensis* wood value (values of up to 1500 dollars per cubic meter for veneer wood have been reported, (ITTO 2017)) stress the need for studies about correct quantification of merchantable and total tree volume, and accurate valuation.

This study aimed to evaluate the fit of single-entry and double-entry models for plantations of *Khaya ivorensis* in Minas Gerais and Pará state, Brazil, with ages close to the first thinning (7 years old) and expected final cut (15 years old) to predict merchantable and total wood volume from tree diameter and height. After selecting the best equations, they were compared with other ones published on literature to verify how tree volume behaves at different sites and under varying management practices. To our knowledge, no volume equations are available for this species in similar plantation conditions (wide spacing and close to the expected rotation age).

MATERIALS AND METHODS

STUDY AREA

Data were gathered from two *Khaya ivorensis* stands, one located in Piumhi, Minas Gerais state (20.42º S and 46.02º W) and another in Santo Antônio do Tauá, Pará state (1.18º S and 48.13º W). Piumhi's Köppen climatic classification is Cwa, classified as a humid subtropical climate, and Santo Antônio do Tauá is classified into the Af region, classified as a tropical rainforest climate (Alvares et al. 2014). The initial planting spacings are 6 x 5.5 and 12 x 12 meters, for Piumhi and Santo Antônio do Tauá, respectively.

DATA SET

Volume determination was made indirectly, without tree felling. Criterion RD coupled with the laser hypsometer TruPulse 200 were used to measure diameters at different heights along the tree stem. In the first two meters of the stem the diameters were measured using a caliper and in the rest of the stem they were measured using the optical dendrometer.

Given that the end use of these trees is for high value added wood production, we only selected single stemmed trees for scaling (which makes up the majority of the trees in the plantations). Thus, for a tree to have been selected for scaling, no fork should have been present up to the tree's height to the live crown base.

Because of the stem shape variation, diameters were measured in shorter intervals up to two meters height (0.1; 0.7 and 1.3 meters). Above that, diameter measurements were made in one-meter intervals, up to the merchantable height (hc), which corresponds to the height of insertion of the first main branch. Heights and diameters at breast height were collected using measuring tape and caliper, respectively.

The Smalian method was used to calculate volume of each log sections (Rodriguez et al. 2014). Merchantable volume (vm) was considered as the sum of each log section plus stump volume. Total volume (vt) was considered as the merchantable volume plus the stem volume of the top of the tree.

Values of diameter at breast height (dbh) in centimeters and total height (ht) in meters were used to guide the number of sample trees. These values were obtained from continuous forest inventories performed since 2010 in Minas Gerais and since 2014 in Pará. Both variables presented normal distribution and were stratified into five classes with amplitude equal to the mean value plus and minus one standard deviation. It is well known that diameter variation can help explain volume variation. Thus, in order to best represent volume variability, trees were sampled in all diameter classes.

To represent trees close to the first thinning age (Minas Gerais state), 100 seven-year-old trees had their volume determined in 2016. To represent trees at age close to expected final cut (Pará state) volume measurements were performed twice, in 2014 and in 2015, on 14 and 15-year-old trees, respectively, totaling 46 trees. According to Silva et al. (2016a) and Ribeiro et al. (2017), the final rotation age for African mahogany in Brazil is estimated to occur between 15 and 20 years, largely a result of one or two thinning operations. The plantations from the States Minas Gerais and Pará were chosen because they had ages that we believe are close to first thinning and final cut, since determining volume at these points is important for planning harvest and commercialization operations. Figure 1 and Table I present information on the scaled trees.

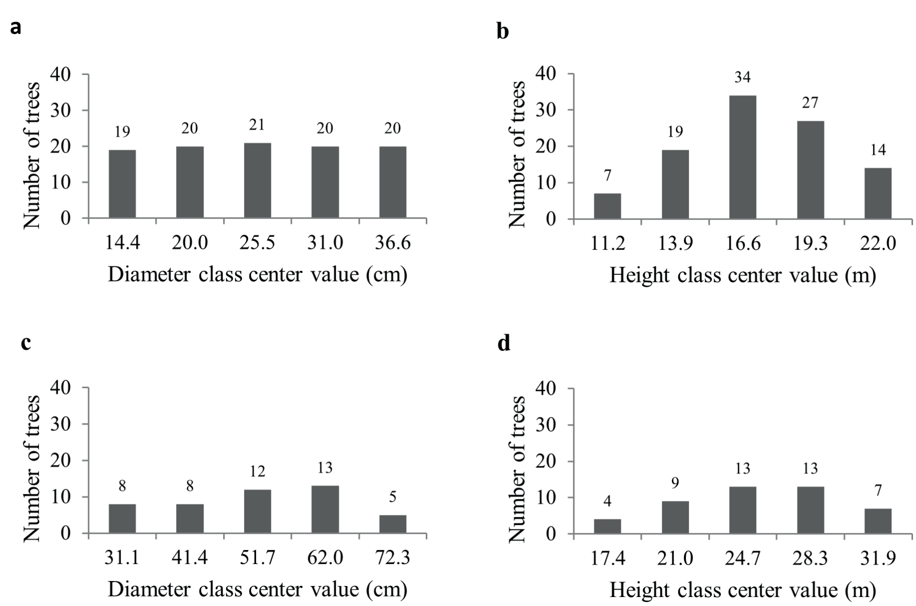

Figure 1 - Number of trees selected for volume determination per diameter at breast height and total height classes for *Khaya ivorensis* stands at the first thinning (**a, b**) and final cut (**c, d**) ages.

TABLE I
Summary statistics for the *Khaya ivorensis* trees measured with the optical dendrometer (First thinning and Final cut) and for the trees measured directly (Felled trees), where Min = minimum, Mean = mean, and Max = maximum values.

Variable	First thinning (n=100)			Final cut (n=46)			Felled trees (n=10)		
	Min	Mean	Max	Min	Mean	Max	Min	Mean	Max
Diameter at breast height (cm)	11.6	25.1	39.0	26.4	52.1	77.8	15.5	25.2	35.7
Total height (m)	9.8	17.1	23.0	15.5	25.5	33.7	12.1	15.9	19.6
Merchantable height (m)	2.7	7.7	14.3	3.3	8.2	14.1	4.3	6.1	9.3
Total volume (m^3)	0.07	0.44	0.96	0.55	2.22	5.16	0.13	0.41	0.78
Merchantable volume (m^3)	0.05	0.34	0.79	0.31	1.31	4.02	0.11	0.28	0.53

ACCURACY OF VOLUME DETERMINATION

The accuracy of indirect volume determination (i.e. using the optical dendrometer) was assessed by felling 10 trees (2 by diameter class) from the 100 trees which had its volume measured in the first thinning age (Table I).

To compare volumes obtained by the two methods (direct and indirect measurements) a statistical approach that evaluates the discrepancy between two values (X and Y) using a linear regression model was applied. According to the method, the values are equivalent if X=Y, yielding a model with intercept equal to zero ($\beta_0=0$) and angular coefficient equal to one ($\beta_1=1$). Thus, equivalence between data is rejected if the null hypothesis is rejected (H_0: $\beta_1=1$) (Borba and Nakano 2016). To guarantee consistency, we tested both the direct and indirect methods as X and Y, alternatively.

SELECTION OF THE TESTED VOLUME MODELS

The models were fitted using *nonlinear least squares* (*nls*) and *fitting linear models* (*lm*) functions from R statistical environment (R Core Team 2016) for nonlinear and linear models, respectively. Single-entry (also known as local equations, 1 to 3) and double-entry (also known as standard equations, 4 to 7) volume equations were selected from: Santos et al. (2012), Melo et al. (2013) and Miranda et al. (2014). Parameter significance was verified by the t-test considering a 5% significance level.

Kopezky-Gehrhardt (apud Assmann 1970):

$$v = \beta_0 + \beta_1 dbh^2 + \varepsilon_i \quad (1)$$

Meyer (apud van Laar and Akça 2007):

$$Ln(v) = \beta_0 + \beta_1 Ln(dbh) + Ln(\varepsilon_i) \quad (2)$$

Brenac (apud Possu et al. 2016):

$$Ln(v) = \beta_0 + \beta_1 Ln(dbh) + \beta_2(1/dbh) + Ln(\varepsilon_i) \quad (3)$$

Schumacher and Hall (1933):

$$v = \beta_0\, dbh^{\beta_1} h^{\beta_2} + \varepsilon_i \quad (4)$$

Schumacher and Hall logarithmized:

$$Ln(v) = \beta_0 + \beta_1 Ln(dbh) + \beta_2 Ln(h) + Ln(\varepsilon_i) \quad (5)$$

Spurr (1952):

$$v = \beta_0 + \beta_1 dbh^2 h + \varepsilon_i \quad (6)$$

Spurr logarithmized:

$$Ln(v) = \beta_0 + \beta_1 Ln(dbh^2 h) + Ln(\varepsilon_i) \quad (7)$$

Where: v = estimated merchantable or total volume (m^3); dbh = diameter at breast height (cm); h = merchantable or total height (m); β_1 = estimated parameters; Ln = natural logarithm; and ε_i = error.

Models were compared by the following statistics: root mean squared error (RMSE, equation 8) to measure precision (m^3), mean absolute bias (MAB, equation 9) to measure bias in m^3 and model efficiency (EF, equation 10), with values ranging from 0 to 1. These statistics are commonly used to assess model quality in studies dealing with volume modeling (e.g. Pelissari et al. 2011, Cecilia et al. 2014, Müller et al. 2014, Silva et al. 2016b). To permit comparison among all models, we transformed the volume estimates of models 2, 3, 5 and 7 back to cubic meters prior to the computations of the statistics presented in equations 8, 9 and 10.

$$\mathrm{RMSE} = \sqrt{\frac{\sum_{i=1}^{n} \left(v - \hat{v}\right)^2}{n-1}} \quad (8)$$

$$\mathrm{MAB} = \sqrt{\frac{\sum_{i=1}^{n} \left|v - \hat{v}\right|^2}{n}} \quad (9)$$

$$\mathrm{EF} = 1 - \frac{\sum_{i=1}^{n} \left(v - \hat{v}\right)^2}{\sum_{i=1}^{n} \left(v - \overline{v}\right)^2} \quad (10)$$

Where: n = number of trees; v = observed merchantable or total volume (m^3); $\hat{v}$ = estimated merchantable or total volume (m^3); and $\overline{v}$ = observed mean merchantable or total volumes (m^3).

We ranked each candidate model (models 1 to 7) by the goodness of fit statistics (formulas 8 to 10), and then generated a unique value for each model by summing the rank obtained in each of the three statistics. The model with the smallest sum was considered the best, as long as no problem was detected in the residual distribution. This was done for total and merchantable volume. In case of equal rank sum values, the best model was considered the one with the smallest standard deviation of residuals and best residual distribution.

The best selected models were submitted to a weighted regression to correct possible heteroscedasticity of the residuals, since the increase in diameters usually results in increasing the error variance. Thus different weighting techniques were used to correct heteroscedasticity in the models (Tomé et al. 2007, Mascaro et al. 2011). In the method used in this study, models were multiplied by a factor (w_i, equation 11) to describe diameter variability through the exponent c (Picard et al. 2012). To obtain the c value, diameter data were split into diameter classes and the volume standard deviation was calculated. This allowed to fit a linear regression between the log of volume standard deviation (dependent variable) and the central value of each diameter class (independent variable). The slope of the regression (β_1) corresponds to *c*.

$$w_i = \frac{1}{d^{2c}} \quad (11)$$

Where: w_i = weight; c = slope of the regression; d = diameter at breast height (cm).

ASSESSMENT OF PREDICTION ABILITY OF THE VOLUME MODELS

In order to verify how *Khaya ivorensis* tree volume behaves at different sites and under varying management practices, the best single and double entries merchantable and total volume models found in this study were compared with other models from literature. The double-entry (Akindele 2005) and single-entry (Henry et al. 2011) models for merchantable volume were used. In both cases, the *Khaya ivorensis* data was obtained from natural forests from Africa: Akindele's study area was in Nigeria and Henry's was in Gabon. This information was used to assess how trees growing under plantation conditions differ from trees grown under natural conditions.

We also compared the models fitted in this study to volume models from other *K. ivorensis* plantations in Figures 4 and 5. Silva et al. (2016b) studied a 2.5 to 4.9 year-old-African mahogany plantation in Pirapora, Minas Gerais state, Brazil, planted in 4 x 3 m spacing. The single-entry (Heryati 1) and double-entry (Heryati 2) volume models are from Heryati et al. (2011), from a 6-year-old stand planted in 4 x 3 m spacing in Segamat, Johor, Malaysia.

Merchantable volume:

$v = -0.0391 + 0.000107 dbh^{1.6115} h^{1.2689}$ (Akindele)

$v = 10.82 \, (dbh/100)^{1.89}$ (Henry)

Total volume:

$Ln(v) = -9.79216 + 2.022933 \, Ln(dbh) + 0.813326 \, Ln(h)$ (Silva)

$v = 0.00021 \, (dbh)^{2.26234}$ (Heryati1)

$v = 0.00014 \, (dbh^2 h)^{0.84469}$ (Heryati2)

The considered merchantable limit was the volume of the principle stem up to either the height of the live crown base or the height up to the first fork in the principal stem, whichever occurred first. Thus, merchantable volume excluded nonmerchantable aboveground compartments such tree tops, branches, twigs, foliage, stumps and roots.

RESULTS

INDIRECT VOLUME MEASUREMENT VALIDATION

The results of the discordance between the two forms of volume measurements (Criterion versus felled tree) was not significant (P value = 0.7016). Figure 2 illustrates the volume values obtained directly and indirectly, showing that values are close to each other.

SINGLE AND DOUBLE-ENTRY MODEL FITTING

Table II shows parameter values from the fit of the seven models to estimate merchantable and total volumes for trees at first thinning and final cut ages. Double-entry models had significant parameters in most of the cases. The single-entry models of Kopezky-Gehrhardt (1) and Brenac (3) presented some non-significant parameters, considering a 95% confidence level. Residual distribution plots presented no tendencies for most of the fitted models. Figure 3 shows the best fits.

The sum of the position ranking (obtained from equations 8, 9 and 10) is presented in Table III for all tested models. Double-entry models were better at describing volume variation than single single-entry models. The Schumacher and Hall model (4) was best to estimate merchantable volume at either the first thinning or final cut ages, and the Spurr model (6 and 7) was the best to estimate total volume at either the first thinning or final cut ages (Table III).

Although equation 3 presented the smallest rank sum among single-entry models, it was not selected to estimate merchantable volume at final cut because parameter β_2 was not significant (Table II). The best fitted models presented in Table IV were then submitted to a weighted regression to correct heteroscedasticity. The data was split into 5 diameter classes, resulting in c=1 for data from the first thinning and c=2 for final cut data. The selected models were then refitted using the weights provided by formula (11). The final equations are presented in Table IV.

ASSESSMENT OF THE PREDICTION ABILITY OF THE VOLUME MODELS

The volume equations for *K. ivorensis* in the studies estimating merchantable volume (Akindele 2005, Henry et al. 2011) and total volume (Heryati et al.

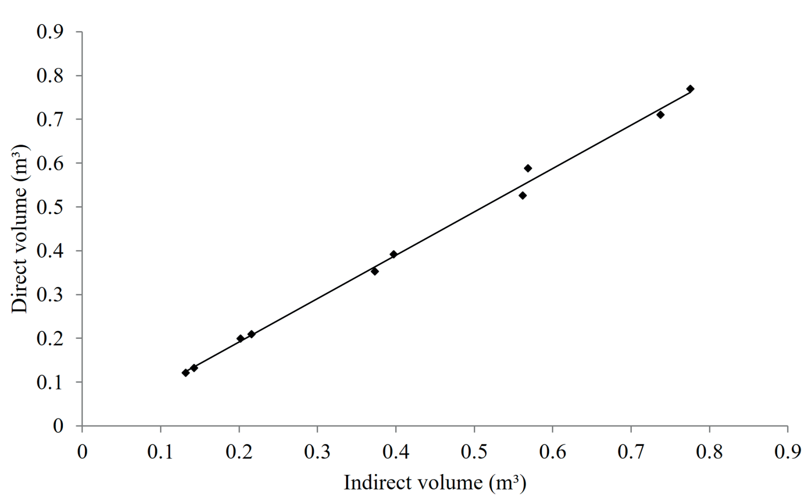

Figure 2 - Dispersion of volume values (m^3) from direct and indirect measurements.

TABLE II
Fitted coefficients for volume models for *Khaya ivorensis* trees at the first thinning and final cut ages (where * indicates that the model coefficients are significant considering a 5% significance level).

Age	Model	Coefficients	Merchantable volume		Total volume	
			Estimated value	P value	Estimated value	P value
First thinning	(1)	β_0	0.03959	0.005*	0.01103	0.3
		β_1	0.0004334	0*	0.000624	0*
	(2)	β_0	-7.64921	0*	-7.6961	0*
		β_1	2.0081	0*	2.102	0*
	(3)	β_0	-0.9847	0.4	-4.4749	0*
		β_1	0.3882	0.2	1.319	0*
		β_2	-34.3797	0*	-16.6172	0*
	(4)	β_0	0.0003299	0*	0.0001814	0*
		β_1	1.773	0*	1.594	0*
		β_2	0.5723	0*	0.9061	0*
	(5)	β_0	-8.43129	0*	-8.79555	0*
		β_1	1.89406	0*	1.78581	0*
		β_2	0.5747	0*	0.74577	0*
	(6)	β_0	0.04361	0*	0.06359	0*
		β_1	0.00005556	0*	0.0000301	0*
	(7)	β_0	-8.47268	0*	-8.95236	0*
		β_1	0.86313	0*	0.86475	0*
Final cut	(1)	β_0	0.09395	0.519	-0.11875	0.282
		β_1	0.000492	0*	0.000819	0*
	(2)	β_0	-7.8871	0*	-7.5855	0*
		β_1	2.0373	0*	2.10219	0*
	(3)	β_0	-16.536	0.003*	-11.9491	0*
		β_1	3.82	0.001*	3.0015	0*
		β_2	80.567	0.104	40.6467	0.122
	(4)	β_0	0.00003979	0.3723	0.000146	0.006*
		β_1	2.286	0*	1.694	0*
		β_2	0.6136	0.003*	0.883	0*
	(5)	β_0	-8.9092	0*	-8.89308	0*
		β_1	1.9952	0*	1.62042	0*
		β_2	0.82117	0*	0.99075	0*
	(6)	β_0	-0.02364	0.858	0.2915	0*
		β_1	0.00005709	0*	0.00002492	0*
	(7)	β_0	-8.7126	0*	-8.7589	0*
		β_1	0.8896	0*	0.85073	0*

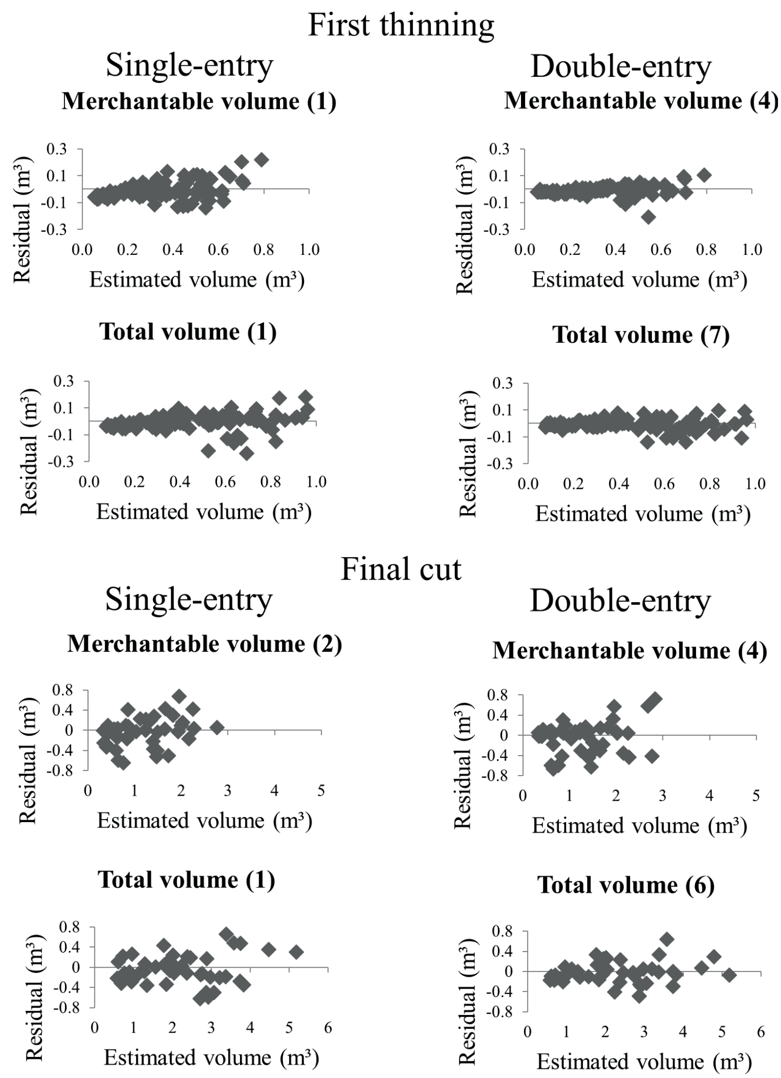

Figure 3 - Residual plots of the best models for estimating merchantable and total volumes for *Khaya ivorensis* at first thinning and final cut ages (numbers in brackets are the model number).

2011, Silva et al. 2016b) were used to compare the behavior of the models from this study in estimating merchantable and total volume for trees close to first thinning and final cut (Figure 4). The heights used to plot the behavior of the volume models of Figure 4 were the mean measured height for the trees divided into diameter classes. Thus, the center of the diameter classes used for the first thinning data were: 12.5, 17.5, 22.5, 27.5, 32.5 and 37.5 cm. The corresponding merchantable heights were: 8.2, 7.0, 8.8, 6.6, 7.6 and 7.6 m. The corresponding total heights were: 12.7, 14.8, 17.1, 18.2, 19.6 and 19.2 m. For the final cut data the center of the diameter classes used were: 30.5, 39.5, 49.5, 59.5, 69.5 and 79.5 cm. The corresponding merchantable heights were: 8.8, 6.8, 9.4, 8.4, 7.4 and 8.6 m. The corresponding total heights were: 21.2, 19.8, 24.1, 27.0, 28.7 and 28.7 m.

DISCUSSION

Single-entry and double-entry volume models were assessed and resulted in accurate equations to

TABLE III
Goodness-of-fit statistics and overall ranking for commercial and total volume models for *Khaya ivorensis* at first thinning and final cut ages. The best models are highlighted in bold. Numbers in brackets are the position in rank.

Age	Volume	Equation	RMSE (m^3)	MAB (m^3)	EF	Rank sum
First thinning	Merchantable volume	**(1)**	**0.0681 (6)**	**0.0000 (1)**	**0.8605 (6)**	**13**
		(2)	0.0718 (7)	0.0007 (3)	0.8448 (7)	17
		(3)	0.0628 (5)	0.0044 (6)	0.8827 (5)	16
		(4)	**0.0379 (1)**	**0.0015 (4)**	**0.9571 (1)**	**6**
		(5)	0.0392 (2)	0.0004 (2)	0.9543 (2)	6
		(6)	0.0503 (4)	0.0000 (1)	0.9240 (4)	9
		(7)	0.0477 (3)	0.0030 (5)	0.9315 (3)	11
	Total volume	**(1)**	**0.0630 (6)**	**0.0000 (1)**	**0.9372 (6)**	**13**
		(2)	0.0664 (7)	0.0003 (3)	0.9301 (7)	17
		(3)	0.0605 (5)	0.0027 (6)	0.9426 (5)	16
		(4)	0.0415 (1)	0.0018 (5)	0.9731 (1)	7
		(5)	0.0448 (3)	0.0004 (4)	0.9685 (3)	10
		(6)	0.0464 (4)	0.0000 (1)	0.9659 (4)	9
		(7)	**0.0433 (2)**	**0.0002 (2)**	**0.9704 (2)**	**6**
Final cut	Merchantable volume	(1)	0.4648 (7)	0.1879 (7)	0.6695 (7)	21
		(2)	**0.4333 (6)**	**0.0638 (6)**	**0.7128 (6)**	**18**
		(3)	0.4318 (5)	0.0480 (3)	0.7214 (4)	12
		(4)	**0.3812 (1)**	**0.0077 (2)**	**0.7828 (1)**	**4**
		(5)	0.3975 (2)	0.0514 (4)	0.7639 (2)	8
		(6)	0.4055 (3)	0.0000 (1)	0.7486 (3)	7
		(7)	0.4276 (4)	0.0623 (5)	0.7203 (5)	14
	Total volume	**(1)**	**0.3196 (5)**	**0.0000 (1)**	**0.9279 (5)**	**11**
		(2)	0.3210 (6)	0.0274 (7)	0.9273 (6)	19
		(3)	0.3291 (7)	0.0152 (6)	0.9253 (7)	20
		(4)	0.2164 (3)	0.0006 (3)	0.9677 (1)	7
		(5)	0.2187 (4)	0.0074 (4)	0.9670 (4)	12
		(6)	**0.2152 (2)**	**0.0002 (2)**	**0.9673 (3)**	**7**
		(7)	0.2143 (1)	0.0081 (5)	0.9676 (2)	8

TABLE IV
Final equations to estimate merchantable and total volume for *Khaya ivorensis* plantations at first thinning and final cut ages.

Age	Volume	Selected equation	Equation
First thinning	Merchantable volume	$vm = 0.01014 + 0.0004759\ dbh^2$	(1)
		$vm = 0.0002428\ dbh^{1.849} hm^{0.5952}$	(4)
	Total volume	$vt = -0.007635 + 0.0006507\ dbh^2$	(1)
		$Ln(vt) = -9.09353 + 0.88027\ Ln(dbh^2 ht)$	(7)
Final cut	Merchantable volume	$Ln(vm) = -6.4857 + 1.6628\ Ln(dbh)$	(2)
		$vm = 0.0001452\ dbh^{1.962}\ hm^{0.6165}$	(4)
	Total volume	$vt = -0.01487 + 0.0007744\ dbh^2$	(1)
		$vt = 0.2276 + 0.00002589\ dbh^2\ ht$	(6)

estimate merchantable and total volume for *Khaya ivorensis* trees. The data used did not correspond to exact ages of first thinning and final cut, since until the present, no merchantable thinning or final cut have been performed in any Brazilian stands in commercial scale. We believe that the chosen ages (7 and 14-15 years) are close to the planned ages for these silvicultural interventions to yield relevant information, especially considering the lack of information for this species. In addition, the information provided in this study is specific for Brazilian silviculture, which uses very wide spacing (resulting in densities lower than 300 trees per hectare) in the management of the plantations and present good climate and edaphic conditions. The application of our equations to plantations using more conventional spacing (e.g. 3x3 to 4x4 m spacing, as mentioned by Yahya et al. 1999 and Yeboah et al. 2014) must be done with care, since spacing can influence stem form (Nogueira et al. 2008, Ferreira et al. 2014). However, the information generated in this paper contributes to knowledge and establishment of *Khaya ivorensis* silviculture and are especially important considering the lack of information on this species.

Laser technology of the dendrometers used in this study gives support for indirect measurements (Nicoletti et al. 2012). Criterion RD 1000 have been used satisfactorily in recent studies (Yoon et al. 2013, Cushman et al. 2014, Uddin and Fujieda 2015). No meaningful difference between tree volume obtained from indirect and direct measurements from the 10 felled trees was found in our research. Other studies have also reached the same conclusion, considering the use of the dendrometer for measurement of diameters (Nicoletti et al. 2015) or for the models used to estimate volume (Rodriguez et al. 2014). Thus, indirect volume measurement is cheaper and

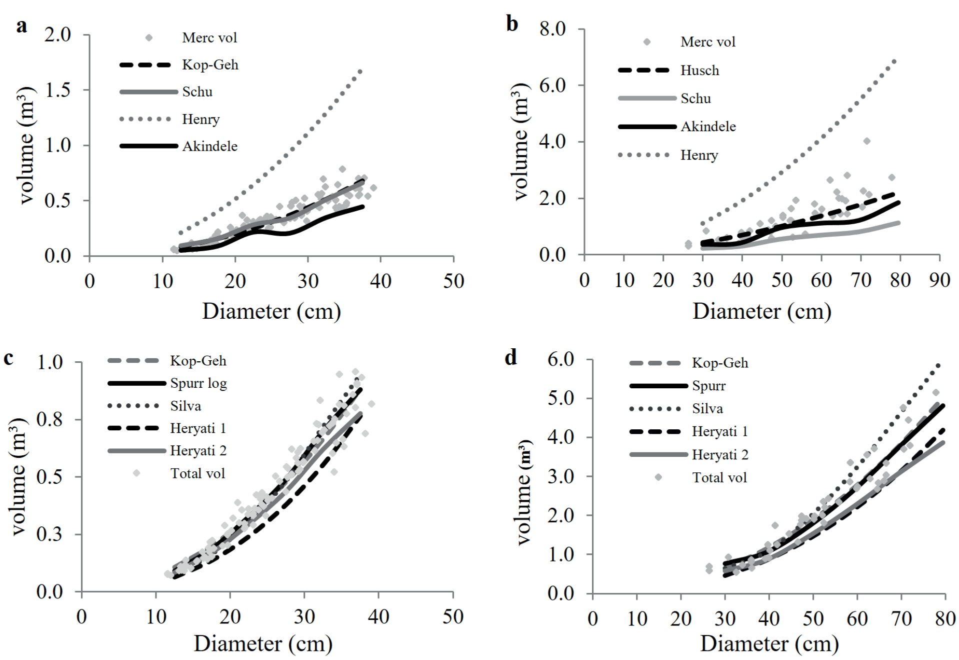

Figure 4 - Comparing selected equations to estimate merchantable volume (Merc vol) of trees at ages close to the first thinning (**a**) and final cut (**b**) and total volume (Total vol) of trees close to first thinning (**c**) and final cut (**d**), where: Kop-Geh = Kopezky-Gehrhardt's model (1); Schu = Schumacher and Hall's model (4); Meyer = Meyer's model (2); Spurr = Spurr's model (6); Spurr log = Spurr logarithmized' model (7). Henry, Akindele, Silva, Heriaty 1 and Heryati 2 were defined previously.

desirable (He et al. 2016), especially for plantations with high value trees managed at low densities, such as the African mahogany plantations of this study.

Double-entry equations were better to estimate volume variation then single entry equations, which was expected since two variables (diameter and height) have a greater capacity to detect the variation of the tree's stem form (Thiersch et al. 2006). The single-entry equations tested yielded satisfactory volume estimates, and have as advantage the optimization of data collection, since they rely on just one easily obtainable variable (i.e. diameter). However, its use is supported only in the cases that diameter have a high correlation with total height (Campos and Leite 2013). Hence, the single-entry equations generated in this study can be used with other *Khaya ivorensis* plantations with ages close to the first thinning and final cut as long as they present height distribution similar to the ones of this study (mean of 17.1 m and 25.5 m, respectively, Table I). An alternative to using double entry volume equations when only diameter values are known is to estimate tree height using a height x diameter equation, such as the ones presented in Silva et al. (2016b) and Ribeiro et al. (2018).

Considering the estimation of merchantable volume, we found that the equation proposed by Henry significantly over estimated the observed merchantable volume from the trees in our study, for younger and older plantations (Figure 4a, b). This large difference can be attributed to the fact that Henry's equation was developed for trees growing in native tropical rain forests in Gabon. Thus, given the different growth environment of trees grown under competition that is expected in tropical rain forests, it is expected that these trees will have larger merchantable heights than the trees measured in our study (around 8 m, Table I). For instance, Henry's equation gives an estimate of 1.1 m^3 for a tree with diameter of 30 cm. If we consider that this tree is a perfect cylinder, it would have to have a stem height of about 16 m. This illustrates the large errors that can arise from applying volume equations from the literature, especially single-entry equations.

On the other hand, Akindele's equation underestimated the merchantable volume of the trees found in this study, although at a much smaller scale than Henry's overestimation (Figure 4a, b). This equation better represent our data due to the fact that it is a double-entry equation. As for the equation's low estimated values compared to Henry's equation, we believe this is because the data were collected from the South of Nigeria, which is a drier site than Gabon, thus yielding trees with smaller merchantable heights.

The equations used to estimate the total volume (Figure 4c, d) presented results more agreeable to each other when compared to the ones used to estimate merchantable volume. This is expected since the trees used to build these models are from plantations. Silva's double-entry model yielded tree volume estimates that adequately portrayed the data found in our study for trees at the first thinning (Figure 4c). This was probably because the equation was fitted using data similar to ours, from a plantation in Minas Gerais, with age between 30 and 59 months and 4 x 3 m spacing. However, at the final cut (Figure 4d) the fit was not adequate, with large overestimation of volumes, especially for larger trees. This behavior supports the idea that descriptive equations have to be used on a database similar to the fitting data, since parameters are correlated to stand attributes.

The single-entry model Heryati 1 and the double-entry model Heryati 2 underestimated volume at both first thinning and final cut (Figure 4c, d). These models were fit using data form smaller spacing (4 x 3 m), which could explain the differences in volume estimates, since smaller spacing yields smaller trees with slower growth (Rocha et al. 2016).

From an economic point of view, it is interesting to know the stem quality of *Khaya ivorensis* (França et al. 2015) and how much wood corresponds to the more valuable merchantable timber. Figure 5 allow us to compare total volume and merchantable volume of trees in which ages are close to the first thinning and final cut using the equations presented in Table IV.

Considering a average tree at first thinning (d=25 cm), the difference between merchantable and total volumes is 0.0749 m^3 (Figure 5a). Considering a average tree at final cut (d = 52 cm), the difference is larger: 1.5940 m^3 (Figure 5b). Thus, information about merchantable volume is very important for an adequate portrayal of the economic viability of the plantation, since it can be valued at a much higher price than smaller diameter wood coming from the tree top and branches.

CONCLUSION

Indirect measurements provided volumes statistically equal to volumes generated by direct measurements, assuring precision for indirect measurements.

The fitted models presented good volume estimates. The Schumacher and Hall model is recommended for estimating merchantable volume for trees at ages close to first thinning (7 years) and final cut (14-15 years). Spurr logarithmized model was the best to estimate total volume of trees close to first thinning, and Spurr model was the best to estimate total volume of trees with ages close to final cut.

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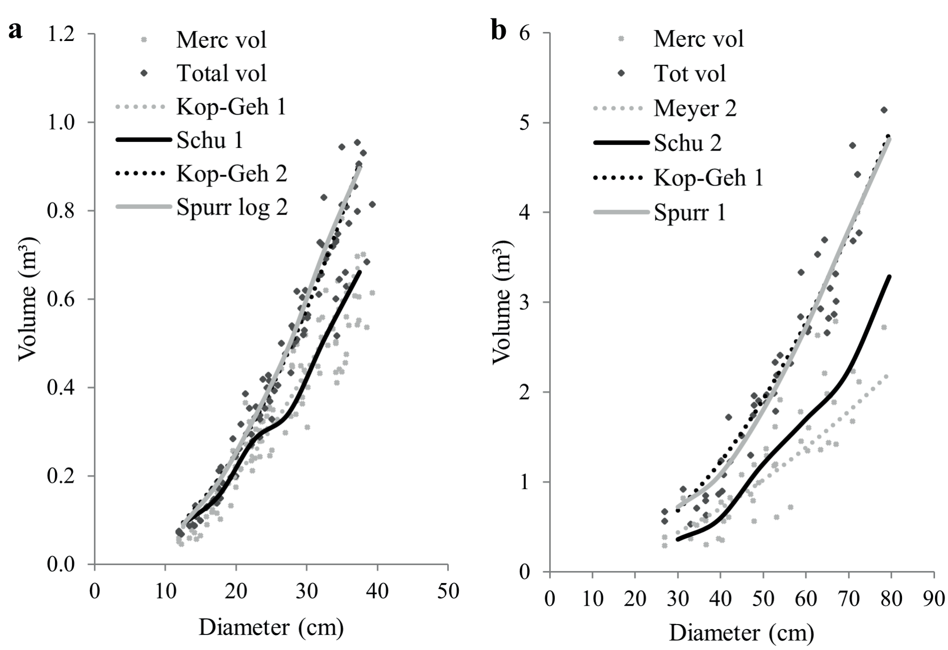

Figure 5 - Comparing selected equations to estimate merchantable (Merc vol) and total volume (Total vol) for trees at first thinning (**a**) and final cut (**b**) ages.

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