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The use of DEA as a tool to evaluate public expenditure on education: an analysis of the cities of the state of Rio de Janeiro

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Abstract: The aim of this article is to evaluate the efficiency of cities of the State of Rio de Janeiro in relation to municipal expenditures on education, as well as to identify the reasons that lead to inefficiency. In order to achieve the initial goal, this study uses DEA technique. The DMUs are 69 cities of the State of Rio de Janeiro. This paper uses following variables as inputs: municipal expenditure on elementary school, number of teachers of 9th grade of elementary school and number of students enrolled in 9th grade of elementary school. The following variables are used to compose output information: number of students who achieved advanced or proficient level in Portuguese and number of students who achieved advanced or proficient level in Mathematics. All data refer to the year 2013. In this study, output-oriented BCC model is used and, for its application, SIAD V3.0 software was used. GIS-DEA integration and some variables external to modelling were used to identify aspects that led to the inefficiency of some DMUs. This study concludes that cities that displayed a good performance are located in the least economically developed regions and that teachers' average salary is an important motivational factor.

Key words: Data Envelopment Analysis (DEA), education evaluation, efficiency, public expenditure.

INTRODUCTION

According to Goldemberg (1993), there are two main reasons why a country stimulates public policies in order to promote quality education for all. The first one is the need to prepare people for citizenship. The second one is related to the fact that the job market nowadays demands more qualified professionals. Thus, investing in education means investing in qualification, professional development, a worker's productivity and, therefore, promoting the development of a country.

Brazil presents some features that are typical of developing countries, such as the enormous deficiencies in the educational system. Saeb (Sistema de Avaliação da Educação Básica) indicators (BRASIL 2007b) for Portuguese and Mathematics in Elementary School show worrisome indexes. Data from educational systems show that some inequalities concerning different social layers still remain, which jeopardizes the universalization of elementary school (BRASIL 2006).

Ideb (Índice de Desenvolvimento da Educação Básica) is the main tool to evaluate basic education in Brazil. This index was created by Inep (Instituto Nacional de Estudos e Pesquisas Educacionais Anísio Teixeira) in 2007 and it stands as a pioneer initiative to bring together in one indicator two equally important concepts for the quality of education: school flow and the averages of the performances in the assessments. This indicator does not take into account the investment of public resources that each city does in its schools as a whole; therefore, Ideb does not measure the efficiency of public expenditure on education (BRASIL 2007a).

This article is intended to answer the following questions. Which cities of the State of Rio de Janeiro are efficient in relation to public spending on education? Which cities are inefficient? Is there a relationship between economic development and efficiency? Is there a relationship between geographical position and efficiency? Does better pay policy of the teachers improve performance?

The objective of this article is to identify the most and least efficient cities in the State of Rio de Janeiro, when it comes to municipal public expenditure on education. Furthermore, this article also attempts to identify the reasons that conduct to inefficiency in some cities. In order to help interpret the results, this study uses variables external to modelling and the Geographic Information Systems (GIS).

The tool used in the current study is Data Envelopment Analysis (DEA), which is a non-parametric method used to measure the efficiency of the productive units, knows as DMUs (Decision Making Units), which transform resources in results. DEA technique was primarily applied to educational field, thus its use to assess education is very common.

The great differential of this paper is to provide a tool for public managers of the State of Rio de Janeiro to better define public policies to guarantee the quality of basic education. This study also presents another differential. Unlike most studies on the subject, this paper does not use Ideb as a variable.

Apart from this introduction, this study displays 4 more sections. Section 2 presents the most used DEA models and then shows the concepts of Inverted Frontier and Composed Efficiency. Section 3 presents the case study used by displaying the 3 necessary stages to solve the problem and ultimately it presents the modelling that will be used. Section 4 shows the results, as well as the analysis and discussion of these results. Finally, Section 5 brings final considerations and conclusions.

MATERIALS AND METHODS

DEA is a non-parametric method used to measure the efficiency of the DMUs that transform resources (or inputs) in products (or outputs) (Soares de Mello et al. 2005).

DEA method considers that maximum production may be obtained by means of the observation of the most productive DMUs. This technique classifies the DMUs in efficient and inefficient based on the capacity to transform inputs in outputs (Charnes et al. 1978).

An inefficient DMU may become efficient by two ways. In this article, only the two most common ways are presented. The first one consists of maintaining the outputs and reducing the inputs (input-oriented). The second one involves maintaining the inputs and increasing the outputs (output-oriented) (Cooper et al. 2007).

DEA models

There are several DEA models that can be used, however, two of them are considered classic: CCR, proposed by Charnes et al. (1978), and BCC, proposed by Bunker et al. (1984).

CCR model

According to Charnes et al. (1978), the CCR model works with constant returns to scale, i.e., a variation in the inputs produces proportional variation in the outputs. This model is also known as the CRS model (Constant Returns to Scale).

This model calculates efficiency by maximizing the division of the weighted sum of outputs by the weighted sum of inputs. This model is extremely benevolent because the DMUs may choose the weights associated to each variable considering the only restriction that the chosen weights when applied to the other DMUs and to itself cannot be greater than 1 (Charnes et al. 1978).

Input-oriented CCR model

Formulation (1) presents the linearized CCR Multiplier Model, input-oriented. Weights v_i and u_j are the decision variables and are also known as multipliers, which originated the name of the model. Model (1) has multiple optimal solutions. (Charnes et al. 1978)

$$Max\ Eff_o = \sum_{j=1}^{s} u_j y_{jo}$$

subject to (1)

$$\sum_{i=1}^{r} v_i x_{io} = 1$$

$$\sum_{j=1}^{s} u_j y_{jk} - \sum_{i=1}^{r} v_i x_{ik} \leq 0, \forall\ k$$

$$v_i, u_j \geq 0, \forall\ i, j$$

The objective function represents efficiency, the first restriction represents normalization and the second restriction makes sure that the chosen weights applied to the other DMUs are not greater than 1.

Output-oriented CCR model

Likewise, one can develop an output-oriented CCR model, as formulation (2) displays. In this case, a new variable is necessary $h_o = 1/Eff_o$. This new variable h_o represents the number by which all the outputs must be multiplied keeping all the inputs constant in order that the DMU*o* reaches the efficient frontier. (Charnes et al. 1978)

Formulation (2) shows the CCR Multiplier Model, output-oriented. (Charnes et al. 1978)

$$Min\, h_o = \sum_{i=1}^{r} v_i x_{io}$$

subject to (2)

$$\sum_{j=1}^{s} u_j y_{jo} = 1$$

$$\sum_{j=1}^{s} u_j y_{jk} - \sum_{i=1}^{r} v_i x_{ik} \leq 0, \forall\ k$$

$$v_i, u_j \geq 0, \forall\ i, j$$

The objective function represents the number by which all the outputs must be multiplied in order to make DMU*o* efficient. The first restriction represents the normalization and the second restriction makes sure all the chosen weights applied to the other DMUs are greater than 1.

BCC model

According to Banker et al. (1984), the BCC model considers variable returns to scale, hence it is also known as VRS (Variable Returns to Scale). This model replaces CCR's proportionality with convexity. This model allows the DMUs that operate with low input levels to have increasing returns to scale, while those which operate with high levels to have decreasing returns to scale. It occurs due to convex frontier. BCC model allows the maximal productivity to vary according to the production scale.

Input-oriented BCC model

Formulation (3) presents the BCC Multiplier Model input-oriented.

$$Max\, Eff_o = \sum_{j=1}^{s} u_j y_{jo} + u_*$$

subject to (3)

$$\sum_{i=1}^{r} v_i x_{io} = 1$$

$$-\sum_{i=1}^{r} v_i x_{ik} + \sum_{j=1}^{s} u_j y_{jk} + u_* \leq 0,\ \forall k$$

$$v_i, u_j \geq 0,\ u_* \in R$$

The objective function represents efficiency, the first restriction represents normalization and the second restriction makes sure all the chosen weights applied to the other DMUs are not greater than 1. In this equation, u_* is interpreted as scale factor. The positive u_* indicates increasing returns to scale. The negative u_* indicates decreasing returns to scale. If u_* is null, it indicates constant returns to scale. Formulation (3) presents multiple optimal solutions. (Banker et al. 1984)

Output-oriented BCC model

Formulation 4 presents the BCC Multiplier Model, output-oriented.

$$Min\, Eff_o = \sum_{i=1}^{r} v_i x_{io} + v_*$$

subject to (4)

$$\sum_{j=1}^{s} u_j y_{jo} = 1$$

$$-\sum_{i=1}^{r} v_i x_{ik} + \sum_{j=1}^{s} u_j y_{jk} + v_* \leq 0,\ \forall k$$

$$v_i, u_j \geq 0,\ v_* \in R$$

The objective function represents efficiency, the first restriction represents normalization and the second restriction makes sure that the chosen weights applied to the other DMUs are not greater than 1. In this equation, v_* is interpreted as scale factor. The positive v_* indicates decreasing returns to scale. The negative v_* indicates increasing returns to scale. If v_* is null, it indicates constant return to scale. Formulation (4) presents multiple optimal solutions. (Banker et al. 1984)

Inverted frontier

DEA models present several advantages, however they make benevolent analysis with the DMUs, i.e., they highlight the strengths of each DMU. The DEA model shows a tie of various DMUs in the ranking of classic efficiency, i.e., various DMUs are considered efficient.

To resolve the issue of low discrimination, this article uses Inverted Frontier as Yamada et al. (1994) and Entani et al. (2002) introduced it. In order to produce Inverted Frontier, it is necessary to change inputs for outputs in the classic efficiency analysis and invert the orientation.

Inverted Frontier highlights each DMU's worst sides, i.e., it is a pessimistic analysis. Inverted Frontier stresses the negative features of the DMUs, therefore, it does not allow specialization, i.e., DMUs that show excellent results in few variables are not considered efficient (Leta et al. 2005).

Since Inverted Frontier emphasizes the weak points, the problem of having several productive units in a tie within the ranking remains. Therefore, Inverted Frontier alone cannot solve the problem of low discrimination that is inherent to the classic efficiency ranking.

Angulo Meza et al. (2002) and Leta et al. (2005) use the concepts of Inverted Frontier to produce a new aggregate efficiency index whose objective is resolving the low discrimination problem of the classic efficiencies of the productive units. This new index is based on the arithmetic mean of the optimistic index and the complement of the pessimistic index, which creates a fairer index as equation (5) shows:

$$\text{Composed Efficiency} = \frac{\text{Classic Efficiency} - \text{Inverted Efficiency} + 1}{2} \quad (5)$$

Case study

In order to choose the most appropriate models to achieve the proposed goal, this article follows the orientations of Golany & Roll (1989) according to this sequence: a) definition and selection of DMUs; b) selection of variables; c) selection and application of the model.

The DMUs of this article are the cities of the State of Rio de Janeiro. After defining the variables used, data are collected and the city that does not present relevant data is removed from the sample.

According to Dyson et al. (2001) and Cook et al. (2014), it is necessary to be attentive with the use of indexes, ratios and percentages as variables of a DEA model. This strategy works when all inputs and outputs are of the same kind, but mixing up indexes, ratios and percentages with absolute values makes the DEA tool misinterpret the results.

Due to this, the construction of the modelling must take into account only raw data and 3 (three) variables related to the municipal schools (elementary level), which compose the input information.

- Expenditure, which means municipal expenditure on the elementary school level;
- Teachers, which describes the number of 9th grade teachers of the elementary school;
- Students, which represents the number of students enrolled in the 9th grade of the elementary school.

In order to complete the modelling, and according to the scope of this study, two variables, which represent the results of the municipal school (elementary level), were used to compose the output information.

- Mathematics, which represents the number of students who achieved advanced or proficient level in this subject in Prova Brasil[1], according to Saeb's proficiency scale.
- Portuguese, which represents the number of students who achieved advanced or proficient level in this subject in Prova Brasil, according to Saeb's proficiency scale.

The information related to municipal expenditure was taken from FINBRA (Brazil's Finance)'s database (BRASIL 2013), which was made available by the Secretaria do Tesouro Nacional. This database provides information on expenditure and revenue. Thus, it was possible to discover the amount of investment in elementary schools of each city.

The student's data were taken from the database of Censo Escolar 2013 of Ministério da Educação e Cultura, which is carried out yearly by INEP (QEDU 2013a). The surveys are conducted in partnership with the state and city halls secretariats of Education.

The number of 9th grade students evaluated as well as the number of teachers were taken from

[1]Prova Brasil is a population-based assessment of public schools.

the database of Prova Brasil 2013 (QEDU 2013b). This exam is part of Saeb and was developed by INEP and its aim is to assess students from the 5th to the 9th grades of elementary schools within an urban scope, by means of Portuguese and Mathematics tests.

The information related to proficiency in Portuguese and Mathematics was taken from the database of Prova Brasil (QEDU 2013b). All data related to the outputs was found on QEdu platform (QEDU 2013c) which presents information about various educational data, such as Prova Brasil, Censo Escolar, IDEB and ENEM (Exame Nacional do Ensino Médio). All of them derive from official governmental sources.

There are 92 cities in the State of Rio de Janeiro, however the set of DMUs evaluated by this article is composed of 69 cities in the State of Rio de Janeiro. The data concerning the other 23 cities, necessary as it is for the modelling, was not provided by the aforementioned data sources.

In order to achieve the aim of this study, the model chosen was BCC. This model allows variable returns to scale. The cities analyzed are diverse when it comes to education, hence data presents different scales. Moreover, the variables Portuguese and Mathematics have a maximal value that equals the total number of students of each city. Thus, the proportionality between inputs and outputs demanded by the CCR model does not exist, which led to the use of the BCC model. (Soares de Mello et al. 2005)

This study will use the output-oriented BCC model, which maximizes the outputs, keeping the inputs unaltered. In public education studies, it is not common to decrease inputs to make the DMU efficient. An inefficient DMU must become efficient by keeping inputs and increasing outputs, i.e., the city must improve its results by using the same resources.

For application of DEA model, software SIAD V3.0 (developed by Angulo Meza et al. 2005) was used.

RESULTS AND DISCUSSION

In this section, the results found will be analyzed through Classic Efficiency, Inverted Frontier and Composed Efficiency rankings. After the analysis, some variables external to the modelling will be added, in order to help interpret the results.

GIS (Geographic Information System) will support the result analysis. According to Burrough et al. (2015), GIS is a set of tools that aids the collection, storage, manipulation and visualization of spatial data of the real world. Thus, the integration GIS-DEA may be a quite appropriate strategy for helping the analysis being studied, since the DMUs have a spatial quality.

Table I shows Classic Efficiency, Inverted Frontier and Composed Efficiency.

In order to facilitate the interpretation of results, a map with all the cities of the State of Rio de Janeiro (RJ) was built.

In Figure 1, the cities were divided into 4 groups. The cities which composed efficiency ranges from 0,69 to 1 are located in the first quartile. The cities which composed efficiency ranges 0,53 to 0,67 are located in the second quartile. The cities which composed efficiency ranges from 0,37 to 0,52 are located in the third quartile. Lastly, the cities which composed efficiency ranges from 0,07 to 0,37 are located in the fourth quartile.

Analyzing Figure 1, we may notice that cities from Região Metropolitana tend to present an unsatisfactory performance. One out of 18 best cities (first quartile) from composed efficiency ranking belongs to Região Metropolitana. Twelve out of 21 cities of Região Metropolitana are

Table I. **Classic Efficiency, Inverted Frontier and Composed Efficiency.**

CITY	CLASSIC EFFICIENCY	INVERTED FRONTIER	COMPOSED EFFICIENCY
Angra dos Reis	0,610646	0,624098	0,526641
Areal	0,773936	1	0,413144
Armação dos Búzios	1	0,408723	0,849457
Barra do Piraí	0,531983	0,411257	0,598267
Barra Mansa	0,790148	0,853844	0,49982
Belford Roxo	0,397414	1	0,212148
Bom Jesus do Itabapoana	1	1	0,533821
Cachoeiras de Macacu	0,507294	0,502679	0,536285
Campos dos Goytacazes	1	1	0,533821
Carapebus	0,266481	1	0,142253
Cardoso Moreira	0,512465	0,785075	0,388296
Casimiro de Abreu	0,4211	1	0,224792
Comendador Levy Gasparian	1	1	0,533821
Conceição de Macabu	0,38695	1	0,206562
Duas Barras	0,91204	0,262422	0,880601
Duque de Caxias	1	1	0,533821
Guapimirim	0,393452	0,666519	0,388053
Iguaba Grande	0,762615	0,476345	0,686639
Itaboraí	0,524981	0,878909	0,344887
Itaguaí	0,38869	1	0,207491
Italva	0,792182	0,920895	0,465112
Itaperuna	0,685262	0,50629	0,629361
Itatiaia	0,694848	0,492534	0,641821
Japeri	0,278821	1	0,14884
Macaé	1	0,813996	0,633115
Mangaratiba	0,765673	0,363974	0,748257
Maricá	0,735694	0,441371	0,690938
Mendes	1	0,540406	0,779162
Mesquita	0,28717	0,972783	0,167827
Miguel Pereira	1	0,126714	1
Miracema	0,631969	0,373702	0,67169
Natividade	1	1	0,533821
Nilópolis	0,33241	0,868236	0,247786
Niterói	0,540358	1	0,288455
Nova Friburgo	1	0,264765	0,926306
Nova Iguaçu	0,647786	0,893899	0,402441
Paracambi	0,622284	0,524735	0,585895
Paraíba do Sul	0,760174	0,465451	0,691151

Table I. (continuation).

CITY	CLASSIC EFFICIENCY	INVERTED FRONTIER	COMPOSED EFFICIENCY
Parati	0,260785	1	0,139212
Paty do Alferes	0,602182	0,321153	0,683841
Petropólis	1	1	0,533821
Pinheiral	0,875344	0,379171	0,798689
Piraí	0,807333	0,459173	0,719677
Porciúncula	1	0,436627	0,834562
Porto Real	0,570434	0,35656	0,647992
Quatis	0,132908	1	0,070949
Queimados	0,544521	0,847855	0,371895
Quissamã	0,492477	0,489235	0,535552
Resende	0,620582	0,448407	0,625732
Rio Bonito	0,476335	0,470285	0,537051
Rio Claro	0,484715	0,318937	0,622317
Rio das Flores	1	1	0,533821
Rio das Ostras	1	0,609161	0,74246
Rio de Janeiro	1	1	0,533821
Santo Antônio de Pádua	0,697992	0,540824	0,617721
São Fidélis	0,434024	0,606472	0,441765
São Gonçalo	0,609245	0,830078	0,415936
São João da Barra	0,579853	1	0,309538
São José do Vale do RioPreto	1	0,473419	0,814922
São Pedro da Aldeia	0,468116	0,582041	0,473006
Sapucaia	0,279113	0,858657	0,224449
Saquarema	0,792436	0,477321	0,702036
Seropédica	0,367839	0,903995	0,24761
Silva Jardim	0,322907	0,61254	0,379209
Sumidouro	1	0,386473	0,861335
Tanguá	0,225248	1	0,120242
Teresopólis	0,858818	0,68185	0,628291
Vassouras	1	0,717971	0,684375
Volta Redonda	1	1	0,533821

located in third and fourth quartiles. One out of 21 cities of Região Metropolitana is located in the first quartile. Seven out of 21 cities of Região Metropolitana are located in the second quartile. Two out of 21 cities of Região Metropolitana are out of the analysis.

It is also possible to observe in Figure 1 that Região Norte Fluminense presents similar tendencies of Região Metropolitana. None out of 18 best cities (first quartile) from composed efficiency ranking belongs to Região Norte Fluminense. Five out of 9 cities of Região Norte

Fluminense are located in third and fourth quartiles. None out of 9 cities of Região Norte Fluminense is located in the first quartile. Three out of 9 cities of Região Norte Fluminense are located in the second quartile and one out of 9 cities of Região Norte Fluminense is out of the analysis.

The features of these regions explain these results. According to Forti (2013) and Silva (2013), even though these regions account for the economic power of the State, basic needs of all population are not supplied. These regions are marked by social inequalities. According to Barros & Mendonça (2000), the lack of decent housing, high rates of criminality and the misdistribution of resources may influence these students' performance in a negative manner.

In order to better describe the results of the efficiency analysis, a set of 13 cities (7 best, 4 worst, 2 standard) were set apart. These cities will have some of their features scrutinized individually. To support such analysis, it is essential to use some variables external to modelling that are as following:

- Average Salary – it is an average of teachers' salaries of each city. This variable indicates teachers' dedication in each city.
- Number of School per Inhabitant – it is the total number of schools divided by the population of each city. This variable indicates schools' capillarity in each city.

The cities of Duque de Caxias, Rio de Janeiro, Quatis, Tangá, Miguel Pereira and Nova Friburgo will hereafter be analyzed.

- Duque de Caxias – it is efficient within the Classic Efficiency ranking, however it is inefficient in the Inverted Frontier, then it occupies an intermediate position within the Composed Efficiency ranking. The inefficiency in Inverted Frontier is explained by the rate of proficiency in Mathematics (6%), which is the second worst in the State. The Average Salary is R$ 3.451.22, the highest in the State. The investment in the teacher is high, but the rate of proficiency in Portuguese is close to the average (19%), which explains its intermediate position within the Composed Efficiency ranking.
- Rio de Janeiro – the State's capital is efficient within Classic Efficiency ranking, but it is inefficient in Inverted Frontier; thus, it occupies an intermediate position within Composed Efficiency ranking. The efficiency within Classic Efficiency ranking is explained by the fact that this city presents the greatest output values. It occurs due to the city has the highest population in the State. The Average Salary is R$ 3.090.17, i.e., there is a high investment in the teacher, yet the results are not satisfactory. The rate of proficiency in Portuguese is 32% and the rate of proficiency in Mathematics is 15%, which explains its intermediate position within the Composed Efficiency ranking.
- Quatis – It ranks the worst within Classic Efficiency and it is inefficient in Inverted Frontier. Hence, it occupies the last position within the Composed Efficiency ranking. The Average Salary is R$ 953.76, the worst in the State. The rate of proficiency in Portuguese is 6% and the rate of proficiency in Mathematics is 5%. The unsatisfactory performance within the Composed Efficiency ranking can be explained by the lack of teachers' motivation due to the low salary and the unsatisfactory results in the evaluations.
- Tanguá – It is the second worst city within the Classic Efficiency ranking and it is inefficient in Inverted Frontier, and

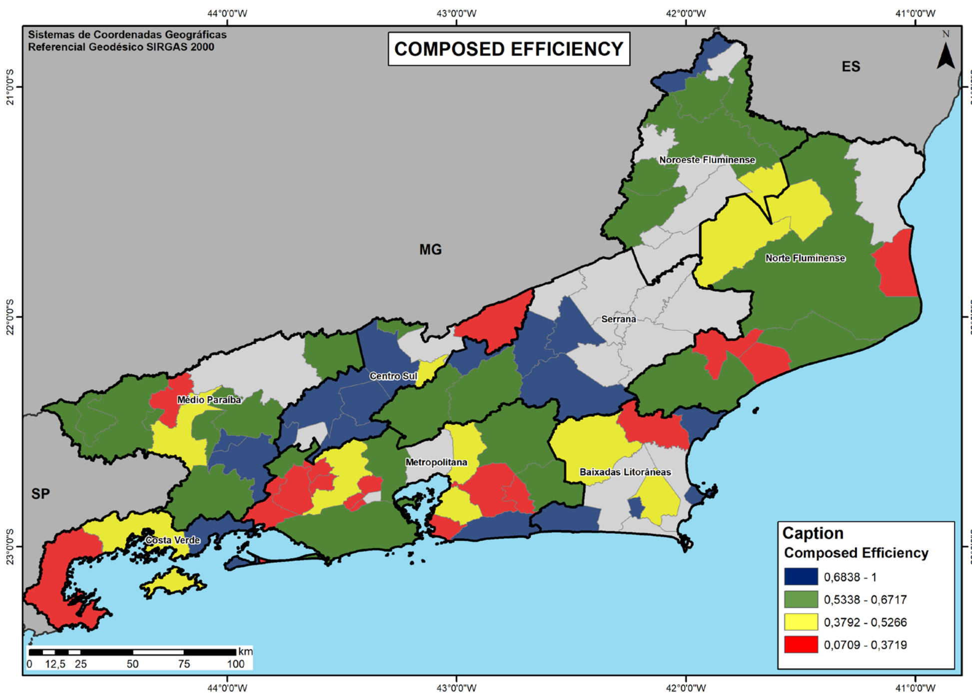

Figure 1. Composed Efficiency Map.

that is why it occupies the second worst position in the Composed Efficiency ranking. The Average Salary is R$ 1.348.89 and it is below the average of the State. It may discourage teachers. The rate of proficiency in Portuguese is 16% and the rate of proficiency in Mathematics is 5%. Excessive expenses, low salaries and unsatisfactory results may explain the unsatisfactory performance in the Composed Efficiency ranking.

- Nova Friburgo – It is efficient in the Classic Efficiency ranking and it occupies the third position in the Inverted Frontier ranking. That is why it occupies the second best position in the Composed Efficiency ranking. The Average Salary is R$ 1.265.78 and is below the average of the State. The city presents a high value for the auxiliary variable Number of Schools per Inhabitant, which may represent a motivational factor for the students. The rate of proficiency in Portuguese is 57% and the rate of proficiency in Mathematics is 29%. Low expenses and results above the average may explain excellent performance in the Composed Efficiency ranking.
- Miguel Pereira – It is efficient in the Classic Efficiency ranking and it occupies the best position in the Inverted Frontier ranking and that is why it occupies the best position in the Composed Efficiency ranking. The Average Salary is R$ 2.913.05, which is the fourth best salary in the State and may be a motivational factor for the teachers. The rate of proficiency in Portuguese is 78% and the rate of proficiency in Mathematics is 67%. Teachers' dedication may be a decisive factor to help produce the best results in the State. This explains its first position in the Composed Efficiency ranking.

The city of Comendador Levy Gaspaian has the lowest value for the input Expenditure. The city of Bom Jesus de Itabapoana has the lowest value for the input Students. The city of Natividade has the lowest value for the input Teachers. All these cities can be efficient by default or efficient from the start. Due to a mathematical distortion of the BCC model, these cities are considered efficient in the Classic Efficiency ranking. However, it is necessary to obtain more information on the cities in order to know if they are actually efficient or if they are efficient by default.

CONCLUSIONS

This study attempted to evaluate the efficiency of the public expenditure on education in the cities of the State of Rio de Janeiro, i.e., to assess municipal schools' capacity to transform resources in educational results. The non-parametric method DEA was chosen for it is able to compare DMUs that perform similar operations through the calculation of an efficiency index.

The output-oriented BCC model was chosen to help reach the objective of this work. Firstly, due to the fact that the outputs do not have a maximal value, so the proportionality between inputs and outputs does not exist, which rendered CCR model infeasible. Secondly, in studies in the field of public education, it makes no sense to decrease inputs to make a productive unit efficient, and that is why an output-orientation perspective was chosen.

This article presented a type of modelling that used raw data as its own variables, for ratios and percentages should not be used in a DEA modelling. Out of the 92 cities of the State, this study evaluated only 69, due to the unavailability of data for 23 of them.

The calculation of efficiencies allowed the formulation of three efficiency rankings known as Classic Efficiency, Inverted Frontier and Composed Efficiency. This analysis made it possible to classify cities as efficient and inefficient, through several different viewpoints.

After the application of the method and the analysis of the results, it was possible to check in general a strong relationship between the index of efficiency and the geographic localization. The regions marked by social inequalities presented low indexes of efficiency. Thus, one may conclude that the lack of worthy housing, the high rates of criminality and the misdistribution of services can influence students' performance in a negative way.

When it comes to teachers' average salary, the worst cities in the Composed Efficiency ranking presented below-the-average salaries. This may indicate that the average salary is an important factor to motivate teachers.

The DEA methodology used was important to reach the aim of the study, since it identified the level of proficiency in each city. The strategy of using factors external to the modelling and the GIS-DEA helped identify the causes of inefficiency in some cities.

The scope of this study may be further deepen by taking into consideration other variables that might be relevant for the efficiency analysis of the public municipal education. For instance, the number of families granted Bolsa Família (monthly cash allowances provided by the government to poor families as part of a social welfare program). The use of data clustering to increase the discrimination among the cities would be also very convenient.

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