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Intensity-Duration-Frequency equations for Rio Grande do Sul - Brazil, based on stationary rainfall series

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ABSTRACT

Heavy rainfall information is essential for environmental studies and water engineering. This study therefore aimed to adjust Intensity-Duration-Frequency (IDF) equations for 247 locations in the Rio Grande do Sul (RS) using stationary rainfall series. Mann-Kendall’s test was applied to identify the temporal trends in the Annual Maximum Daily Rainfall (AMDR) series of 271 rain gauges in RS. The Kappa, Generalized Extreme Value (GEV), Gumbel, two-parameters Log-Normal and three-parameters Log-Normal probabilistic distributions were adjusted to the AMDR series without significant temporal trend. The best distribution fit was given by Anderson-Darling’s test, so the AMDR was discretized up to 5 minutes. IDF equations coefficients were adjusted in *RStudio*, using Nash-Sutcliffe’s Coefficient and the Root-Mean-Square Error to evaluate them. In conclusion: the most suitable distributions for the AMDR were the multiparametric Kappa and GEV; the IDF equations coefficients adjustment was classified as “excellent”; coefficients *a* and *b* varied across the RS and are correlated with the AMDR and geographical positions; and the *c* and *d* coefficients were practically constant.

Keywords: goodness of fit test, Kappa probabilistic distribution, trends test.

Equações Intensidade-Duração-Frequência para o estado do Rio Grande do Sul – Brasil, baseadas em séries estacionárias de chuva

RESUMO

Informações sobre chuvas intensas são essenciais para estudos ambientais e engenharia de recursos hídricos. Assim, o estudo objetivou ajustar equações Intensidade-Duração-Frequências (IDF) de chuvas para 247 locais do Rio Grande do Sul (RS), utilizando séries estacionárias de chuva. Para identificar tendencias temporais significativas, o teste Mann-Kendall foi aplicado em 271 séries de Chuva Máxima Diária Annual (CMDA) do RS. As distribuições probabilisticas Kappa, GEV, Gumbel, Log-Normal 2 parâmetros e Log-Normal 2 parãmetros foram ajustadas às séries sem tendência de CMDA. Com as distribuições melhores ajustadas à cada série de CMDA, esta foi discretizada em intervalos de até 5 minutos. O ajuste dos coeficientes das equações IDF foi realizado no RStudio, utilizando o Coeficiente de Nash-Sutcliffe e a Raíz Quadrada do Erro Quadrático Médio para avaliá-lo. Em conclusão: as distribuições mais adequadas foram as multiparamétricas Kappa e GEV; o ajuste dos coeficientes da equação IDF foi classificado como “excelente”; os coeficientes *a* e *b* variam no RS e estão correlacionados com a magnitude da CMDA e sua localização; os coeficientes *c* e *d* foram praticamentes contantes.

Palavras-chave: distribuição probabilística Kappa, teste de aderência, teste de tendência.

1. INTRODUCTION

In hydrology, rainfall is of great interest, especially rainfalls of high intensity and in tropical regions (Oliveira, 2019). Information on heavy rainfall is essential for environmental studies, soil and water conservation, management of natural resources, and in water-resource engineering projects. Also, as a result of climate change, there has been an increase in extreme events on several continents, including heavy rainfall events (IPCC, 2014; Sarhadi and Soulis, 2017), which places socio economic pressure on governments, making studies on this topic quite relevant (Li *et al.*, 2018).

Thus, the adjustment of Intensity-Duration-Frequency (IDF) equations of rainfall characterize them from the point of view of their intensity, duration, and their probability of occurrence (or Returning Period (RP)). The adjustment of IDF equations may be preferentially done with the pluviographic data, as they provide better temporal discretization of rainfall (Caldeira *et al.*, 2015). However, it is common for this data not to be publicly available in developing countries, especially.

To overcome this obstacle, IDF equations can be adjusted using pluviometric data, employing the probabilistic modeling of the Annual Maximum Daily Rainfall (AMDR) (Peleg *et al.*, 2018; Alemaw and Chaoka, 2016; Coelho Filho *et al.*, 2017; Switzman *et al.*, 2017), associated to the technique of Daily Rainfall Disaggregation (DRD) (Coutinho *et al.*, 2019; Silva Cruz *et al.*, 2019).

One premise of hydrological data analysis based on probability distribution models is the null hypothesis of stationarity (Jakob, 2013), which states that the statistical characteristics of the variable series do not change significantly over time (Naghettini, 2017). Besides, the variability of rainfall is often pointed out as facilitating some environmental and socioeconomic problems, such as floods, landslides, material losses, etc. (Birara *et al.*, 2018). Therefore, investigating the temporal trends in hydrological data before estimating future scenarios is a scientific tool of great practical value when the aim is to get reliable estimates (Naghettini, 2017; Alemu and Bawoke, 2020).

That said, the objective of this study was to adjust rainfall Intensity-Duration-Frequency equations for 247 locations in the state of Rio Grande do Sul (RS) based on stationary rainfall series and multiparametric probabilistic models associated with the DRD.

2. MATERIALS AND METHODS

2.1. Study area

The state of Rio Grande do Sul (RS) is located in the southern region of Brazil, covering an area of about 282 thousand km^2, subdivided into 497 cities, with about 11 million people

(IBGE, 2019). The RS relief is composed of the geomorphological units *Planalto Meridional* (which has the highest altitudes), *Cuesta do Haedo*, Central Depression, Southeastern Mountain Ranges, and the Coastal Plain (Rio Grande do Sul, 2019), with a maximum altitude of 1385 m (Figure 1).

According to the Köppen climate classification, the climate of RS fits into the Cfa and Cfb types: Humid subtropical throughout the year, with hot and moderately hot summers, respectively (Kuinchtner and Buriol, 2001). The Cfa type is predominant in the state, and the Cfb is present only in the higher parts of the territory, as in the Southeast and Northeast Mountain Ranges.

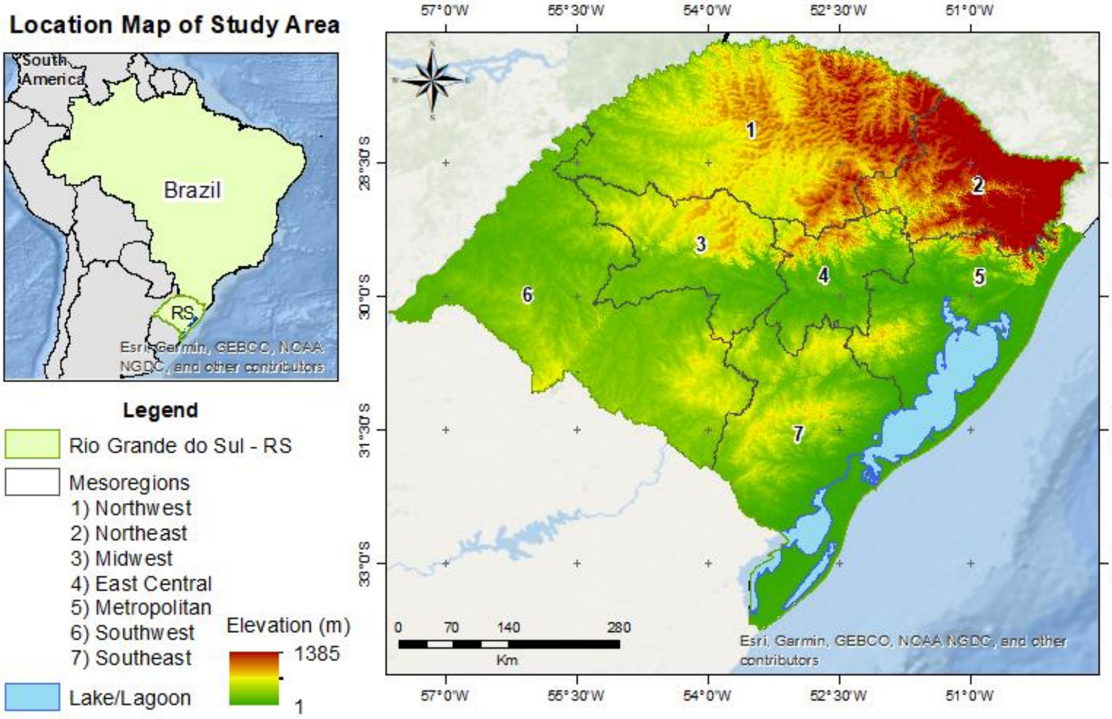

Figure 1. Location of the study area and Digital Elevation Model (DEM).

2.2. Rainfall data

The series was constituted using Total Daily Rainfall series, obtained from HidroWeb - Hydrological Information System of the National Water Agency (ANA), considering years with a maximum of 30 days of gap and only series with a minimum extension of 20 years. Using the Mann-Kendall's test (Mann, 1945; Kendall, 1975), this study investigated temporal trends in the AMDR series of 271 rain gauges located in RS and found no significant temporal trends ($\alpha = 0.05$) in 247 of them. Therefore, these 247 AMDR series were used to adjust the IDF equation coefficients in RS (Figure 2).

The rain gauges are well distributed in the territory but are more numerous in Meso Regions 1 and 2. The extent of the AMDR series varies between 20 and 73 years (1912 to 2018), and most are between 31 and 60 years (Figure 2A). The average AMDR (Figure 2B) varies between 61.26 mm y^{-1} and 127.84 mm y^{-1}, with the highest averages occurring in the Southwest (6), Northwest (1), and Midwest (3) mesoregions, while the lowest averages occur in the Metropolitan (5), Northeast (2) and Southeast (7) mesoregions. The highest AMDR value was observed in the Southwest mesoregion, with 358 mm.

2.3. Probability modeling of Annual Maximum Daily Rainfall (AMDR)

In Brazil, simpler probability distributions, such as 2 and 3 parameter Log-Normal, and Gumbel are commonly used in hydrological scope. However, studies have shown the superiority of multiparametric probabilistic distributions for hydrological modeling with extreme values, such as AMDR (Beskow *et al.*, 2015; Peleg *et al.*, 2018; Alemaw and Chaoka, 2016; Agilan and Umamahesh, 2017).

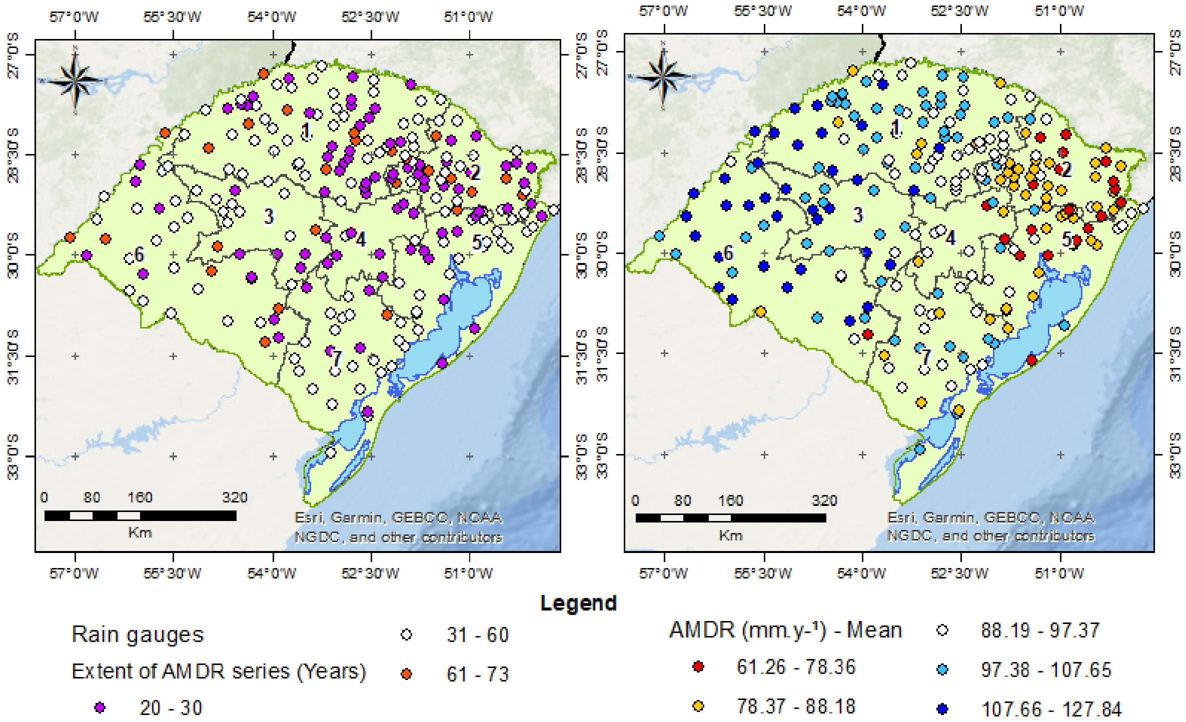

Figure 2. Spatial distribution of the rain gauges used: “A” shows the extent of the AMDR series, and “B” indicates the average AMDR of each series.

Thus, five probability distributions were adjusted for each AMDR series and referred for extreme value probability modeling (Naghettini, 2017):

- *two-parameter Log-Normal (2P-LN)*

The probability density function (PDF) of the two-parameter Log-Normal distribution, described by Naghettini (2017), is given by (Equation 1):

$$f(x) = \frac{1}{x \cdot \sigma_{\ln(X)} \cdot \sqrt{2\pi}} \cdot e^{\left\{-\frac{1}{2}\left[\frac{\ln(X) - \mu_{\ln(X)}}{\sigma_{\ln(X)}}\right]^2\right\}} \tag{1}$$

Where x is the AMDR, and $\mu_{\ln(X)}$ e $\sigma_{\ln(X)}$ are the distribution parameters estimated by the mean and standard deviation of the logarithmic data, respectively.

- *three-parameter Log-Normal (3P-LN)*

According to Naghettini (2017), the PDF of the three-parameter Log-Normal distribution is expressed by (Equation 2):

$$f(x) = \frac{1}{(x-\alpha) \cdot \sigma \cdot Y \cdot \sqrt{2\pi}} \cdot e^{\left\{-\frac{1}{2}\left[\frac{\ln(x-\alpha) - \mu \cdot Y}{\sigma \cdot Y}\right]^2\right\}} \tag{2}$$

In which x is the AMDR; Y is the given variable ln (x - α); μY is the parameter related to the 1st sample moment, estimated by the average logarithmic data; σY it is the parameter related to the 2nd sample moment, estimated by the standard deviation of the logarithmic data; and α is the parameter related to the 3rd sample moment, estimated based on the asymmetry coefficient.

- *Gumbel*

The PDF of the Gumbel distribution, presented by Naghettini (2017), is expressed by (Equation 3):

$$f(x) = \frac{1}{\alpha}\left[1 - k\left(\frac{x-\beta}{\alpha}\right)\right]^{\frac{1}{k-1}} \cdot exp\left\{-\left[1 - k\left(\frac{x-\beta}{\alpha}\right)\right]^{\frac{1}{k}}\right\} \tag{3}$$

Where X is the AMDR, and α and μ are the parameters of scale and location of the probability distribution.

- *Generalized Extreme Values (GEV)*

According to Naghettini (2017), the GEV distribution is given by the following PDF (Equation 4):

$$f(x) = \frac{1}{\alpha}\left[1 - k\left(\frac{x-\beta}{\alpha}\right)\right]^{\frac{1}{k-1}} \cdot exp\left\{-\left[1 - k\left(\frac{x-\beta}{\alpha}\right)\right]^{\frac{1}{k}}\right\} \tag{4}$$

Where α, k and β are parameters of scale, shape and location, respectively, and *x* is the AMDR.

- *Kappa*

According to Hosking (1994), the PDF of the Kappa distribution is given by (Naghettini, 2017) (Equation 5):

$$f(x) = \frac{1}{\alpha} \cdot \left[1 - \frac{k(x-\xi)}{\alpha}\right]^{\frac{1}{k}-1} \cdot [F_x(x)]^{1-h} \tag{5}$$

Where *x* is the AMDR, ξ and μ are parameters of scale and localization, k and h are shape parameters, and $F_X(x)$ is the cumulative distribution function (CDF), described in Equation (6):

$$F_x(x) = \left\{1 - h\left[1 - k\frac{(x-\xi)}{\alpha}\right]^{\frac{1}{k}}\right\}^{\frac{1}{h}} \tag{6}$$

The probability distribution parameters were adjusted using the Method of L-Moments. This method has been used to estimate parameters distribution in hydrological studies (Beskow *et al.*, 2015; Alemaw and Chaoka, 2016; Coelho Filho *et al.*, 2017) due to producing better estimates for small samples, which are the ones generally available for environmental studies, besides not being influenced by gaps in rainfall series (Parida, 1999; Ganora and Laio, 2015).

The suitability of the probability distribution models to the AMDR series was verified using the Anderson-Darling (AD) test (D'Agostino and Stephens, 1986), acting under the hypothesis that the dataset follows the tested probability distribution for a level of significance (LS), which in this study was 5% (α = 0.05). According to Beskow *et al.* (2015), the AD test has great potential in verifying the "goodness-of-fit" of probabilistic models to asymptotic series, as it gives more weight to the distribution tails, being more robust for the analysis of AMDR trends (Naghettini, 2017).

The probabilistic modeling of the AMDR and the goodness-of-fit of the distributions with the AD's test were carried out using the software *SYHDA – System of Hydrological Data*

Acquisition and Analysis.

2.4. Daily rainfall disaggregation and adjustment of the IDF equations

With the parameters of the probability distribution that best fit each AMDR series, the AMDR depths were estimated for the RP from 2 to 100 years. Based on these data, AMDR depths were calculated for durations of less than 1 day, employing the Daily Rainfall Disaggregation (DRD) technique using the Duration Relation Method (DRM).

The DRM consists of multiplying disaggregation constants by the 1-day AMDR depths, resulting in the AMDR depths associated with shorter durations than 1-day (from 24 hours to 5 minutes) (Tucci, 2009). In Brazil, this method is widely used because it is simple and provides satisfactory results when only pluviometric data are available (Penner and Lima, 2016).

Several groups of disaggregation constants can be applied to DRM. However, the results found by Caldeira *et al.* (2015) suggest the CETESB (1979) constants have a better performance in comparison to other groups of constants for the disaggregation of daily rainfall in RS. Thus, the disaggregation constants proposed by CETESB (1979) were employed in this study (Table 1).

Table 1. Values of the Daily Rain Disaggregation constants for the city of São Paulo, Brazil according to CETESB (1979).

Durations relations											
$\frac{\boldsymbol{h_{24h}}}{\boldsymbol{h_{1day}}}$	$\frac{h_{12h}}{h_{24h}}$	$\frac{h_{10h}}{h_{24h}}$	$\frac{h_{8h}}{h_{24h}}$	$\frac{h_{6h}}{h_{24h}}$	$\frac{h_{1h}}{h_{24h}}$	$\frac{h_{30'}}{h_{1h}}$	$\frac{h_{25'}}{h_{30'}}$	$\frac{h_{20'}}{h_{30'}}$	$\frac{h_{15'}}{h_{30'}}$	$\frac{h_{10'}}{h_{30'}}$	$\frac{h_{5'}}{h_{30'}}$
Disaggregation constants proposed by CETESB (1979)											
1.14	0.85	0.82	0.78	0.72	0.42	0.74	0.91	0.81	0.70	0.54	0.34

From the disaggregation of daily rainfall into shorter-duration depths, the rainfall's intensities associated with these durations were calculated, and this dataset of Intensity, Duration and RP was used to adjust the IDF equations coefficients. The mathematical model used to represent the relationship between this rainfall's characteristics was the one proposed by Chow (1962), as shown in Equation 7:

$$I = \frac{a \cdot TR^b}{(c+t)^d} \quad (7)$$

Where I is the maximum rainfall intensity (mm h^{-1}); RP is the Return Period (years); t is the rainfall duration (minutes); and *a*, *b*, *c* and *d* are the IDF coefficients adjusted for the location of each rain gauge.

Usually, the IDF equation coefficients are adjusted using the Least Squares Method (LSM) as the objective function. However, the LSM is more affected by extreme values (Bombardi *et al.*, 2017). Thus, the coefficients of the IDF equation were adjusted in the environment of the *RStudio*, by programming a routine for adjustment of a non-linear model that used the Nash-Sutcliffe Coefficient (C_{NS}) as the objective function and the dataset of Intensity, Duration and RP.

C_{NS} is widely used to assess predictions in the scope of hydrology (Nash and Sutcliffe, 1970), and it's described by the following Equation 8:

$$C_{NS} = 1 - \frac{\sum_{i=1}^{n}(I_{obs}-I_{est})^2}{\sum_{i=1}^{n}(I_{obs}-\overline{I_{obs}})^2} \quad (8)$$

Where n is the sample size; I_{obs} is the observed rainfall intensity when applying the probability distribution and rainfall disaggregation; I_{est} is the estimated rainfall intensity by the IDF equation considering the coefficients a, b, c and d adjusted in RStudio. CNS values can vary from -∞ to 1 and can be interpreted as: CNS = 1 suggests perfect suitability; 0.99 > CNS ≥ 0.75 suggests good suitability; 0.74 > CNS ≥ 0.36 suggests an acceptable adjustment; and CNS < 0.36 suggests an unsatisfactory adjustment (Motovilov *et al.*, 1999).

To assess the performance of the estimate of the IDF equation coefficients, the Square Root-Mean-Square Error (RMSE) was calculated, which expresses the average error of the estimate in the unit of the variable of interest (mm h-1), varies from 0 to +∞, and is "negatively oriented", that is, the lower the value, the better the estimate (Chai and Draxler, 2014) (Equation 9):

$$RMSE = \sqrt{\frac{1}{n}\sum_{j=1}^{n}(I_{est} - I_{obs})^2} \quad (9)$$

Where n is the sample size; I_{obs} is the observed rainfall intensity when applying the probability distribution and rainfall disaggregation; I_{est} is the estimated rainfall intensity by the IDF equation considering the coefficients *a*, *b*, *c* and *d* adjusted in RStudio. The RMSE has been widely used to assess estimates in environmental and climate studies because it gives more weight to the biggest errors, which is a useful feature when they are especially undesirable (Willmott and Matsuura, 2005).

3. RESULTS AND DISCUSSIONS

Analyzing the results of the Anderson-Darling test at a significance level of 5%, it was found that the best adjustment to the AMDR series (Figure 3) was given by the Kappa (72.1%), the GEV (27.1%), and the 2P-LN distributions, which was better for only 2 (0.8%) of the 247 series. These results corroborate the studies by Coelho Filho *et al.* (2017), Ye *et al.* (2018), and Back and Cadorin (2020), who also found that multiparametric distributions perform better when compared to commonly used ones, such as 2P-LN and Gumbel.

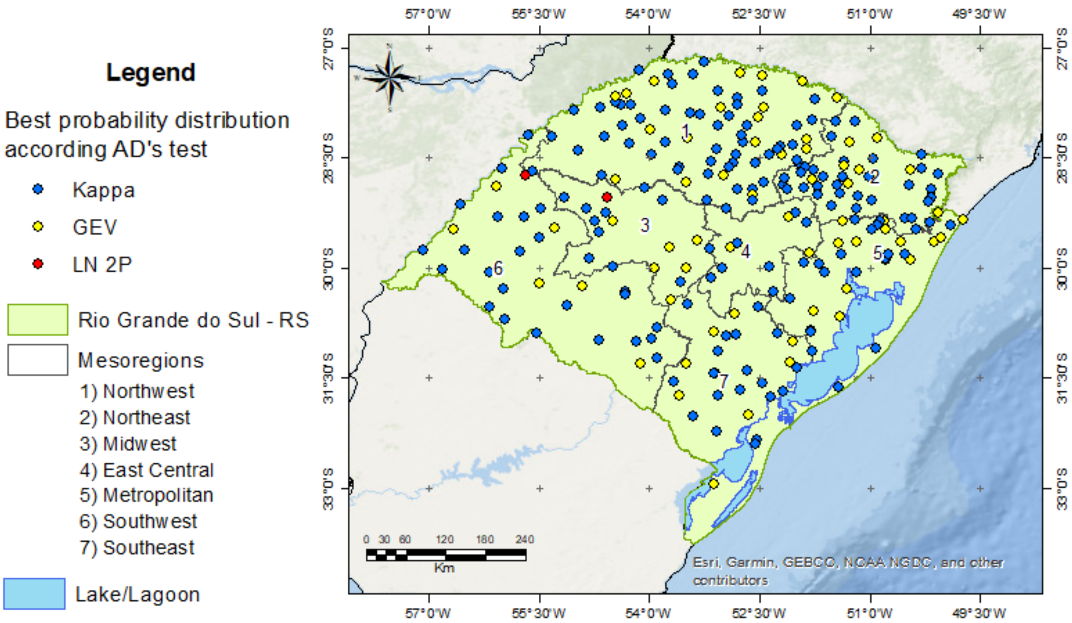

Figure 3. Best fit probability distribution for each AMDR series at a 5% significance level, according to the Anderson-Darling's test.

Table 2 shows the number of AMDR series whose adjustments were suitable or not for each probability distribution tested, according to the Anderson-Darling test results.

Table 2. Number of AMDR series adjusted by the probability distributions tested, according to the Anderson-Darling test.

Probability distribution	2P-LN		3P-LN		Gumbel		GEV		Kappa	
	Nº of series	%	Nº of series	%	Nº of series	%	Nº of series	%	Nº of series	%
Suitable	234	94.74	216	87.45	237	95.95	245	99.2	247	100
Unsuitable	13	5.26	31	12.55	10	4.1	2	0.8	0	0

As shown in Table 2, the 2P-LN and 3P-LN distributions were suitable for 234 and 216 AMDR series, respectively. For the others, the AMDR modeling with 2P-LN and 3P-LN distributions could not be carried out, possibly due to the asymmetry coefficient of these series being negative. According to Naghettini (2017), the LN variable is positive and has an asymmetry coefficient greater than zero, going against what occurred in these series. This was also reported by Caldeira *et al.* (2015), who used 2P-LN, 3P-LN, and Gumbel distributions for their AMDR probabilistic modeling study, in RS.

Gumbel distribution was suitable for 237 AMDR series and unsuitable for 10 (about 4%); however, this distribution did not show the best fit for any AMDR series. Even so, according to Naghettini (2017), Gumbel distribution is the most used in hydrological modeling and, therefore, many studies assume that the model will best represent the analyzed data, without checking the suitability of other probability distributions (Back *et al.*, 2011). In Brazil, Gumbel distribution is often used in the study of AMDR and the adjustment of IDF equation coefficients (Cotta *et al.*, 2016; Souza *et al.*, 2016; Oliveira, 2019; Back *et al.*, 2020).

Among the probability distributions adjusted to the AMDR series, only the Kappa distribution was suitable for all, according to the Anderson-Darling test, and the GEV distribution for 245 of the series, which corresponds to 99.2% (Table 2). The model was not suitable for AMDR modeling in only two series (rain gauges 2954030 and 3052009), and only the Kappa distribution was suitable for these two series. Since no characteristic that could explain this fact was observed in these series, the adjustment of the distribution to these series was verified with other goodness-of-fit tests which are less rigorous than AD’s one (such as Kolmogorov-Smirnov (KS) and Chi-Square (CS) (Naghettini, 2017)). According to them, the 5 probability distributions tested proved to be adequate at the significance levels of 5% and 10%.

According to Das (2021), multiparametric models in hydrological frequency analysis have gained popularity in recent years, as they provide better estimating on the frequency of the data, but also the shape, scale and position (Hosking, 1994). Also, many authors are modeling the extremes (Caldeira *et al.*, 2015, Cassalho *et al.*, 2018, Patel *et al.*, 2020). As pointed out by Beskow *et al.* (2015), when adjusting multiparametric probability distributions to the AMDR series in RS, the adjustment of various theoretical probability models is important, from the most simplified to the multiparameter, in order to get the one that best represents the frequency distribution of the sample data.

Another reason for the better performance of multi parametric models might be the choice of the method for adjusting those models, as concluded by Abreu *et al.* (2018). According to Fawad *et al.* (2019), some methods like the L-moments are more reliable as they are less sensitive to outliers and more suitable for smaller sample sizes.

Considering, as an example, the series from rain gauge 2954030, for which the Kappa distribution is the best fit according to the AD’s test, the AMDR depth associated with a 100-

year RP was about 186.25 mm; on the other hand, the estimation by 2P-LN distribution for this same series was of 177.76 mm, underestimating the AMDR by approximately 6%. Of course, it is not the main objective of this study to analyze the potential of different adherence tests to discern which is the best probability distribution for the AMDR series. However, it highlights the importance of choosing the best probabilistic model to avoid cumulative errors until the stage of estimating the IDF equation coefficients.

According to Naghettini (2017), the ability of the KS and CS tests to distinguish the veracity from the null hypothesis is reduced in the distribution tails, mainly due to the sample size and estimation errors, which are generally greater in these locations. Therefore, as the extreme values located on the tails can strongly impact the quality of the suitability of the models to the series, it is advantageous to use the AD's test to verify the suitability in asymptotic series instead of other less rigorous tests (Heo *et al.*, 2013).

Regarding the adjustment of the IDF equation coefficients for each location, the C_{NS} values varied between 0.994 and 0.999. According to the classification by Motovilov *et al.* (1999), the adjustment of the coefficients is considered good ($C_{NS} > 0.75$) and a value close to 1 reflects a nearly perfect adjustment. Figure 4 shows the spatial variation of the IDF equation coefficients, adjusted for the 247 locations.

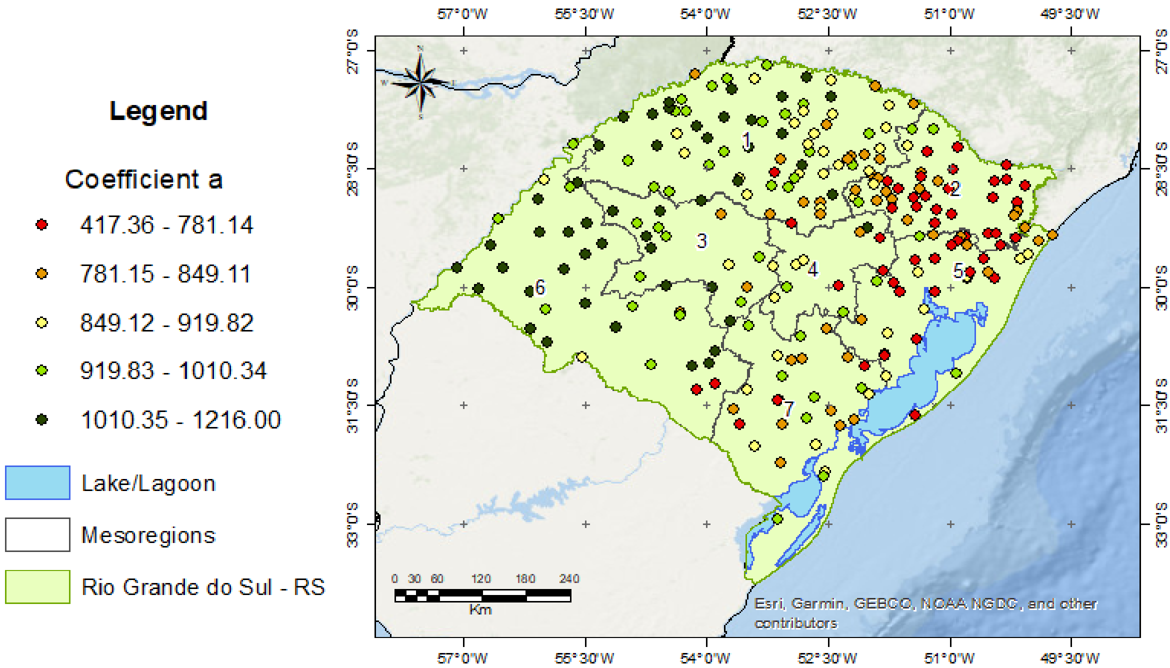

Figure 4. Spatial distribution of the values of the IDF equation coefficients adjusted for the 247 locations in RS.

The coefficient *a* ranged from 417.36 to 1216.00, with an average of 895.42 and a Coefficient of Variation of 15.17%. The lowest value of the coefficient *a* was found at Rain Gauge 2951021, located in Mesoregion 5, while the highest value of coefficient *a* was estimated at Rain Gauge 2955006, in Mesoregion 6 of RS.

In general, the lowest values of coefficient *a* of the IDF equation occur in the Northeast (2) and Metropolitan (5) mesoregions, in which the lowest mean AMDR values (Figure 2A) were also observed. The highest values were found in the Northwest (1) and Southwest (6) mesoregions, which are also the locations with the highest mean AMDR (Figure 2B).

To investigate if there was a correlation between the two variables, Spearman's Rho non-parametric correlation test (r_s) was employed. This test is widely used in the analysis of linear and non-linear correlations between continuous variables, mainly because it does not require that they be described by some specific probability distribution, in addition to being more robust regarding gaps in rainfall series (Schober *et al.*, 2018). The value of r_s was 0.84 ($p_{value} < 0.05$) indicating a strong positive and significant correlation between the mean AMDR and the *a* coefficient of the IDF equation.

Regarding coefficient *b* of the IDF equation, the spatial distribution of its values is shown in Figure 5. The values of coefficient *b* varied between 0.060 and 0.355, with an average of 0.158 and Coefficient of Variation of 31.72%. The lowest value of coefficient *b* was found in Rain Gauge 2752009, at the West portion of Mesoregion 1. The maximum value of coefficient *b* was found in Mesoregion 5, in Rain Gauge 2951021.

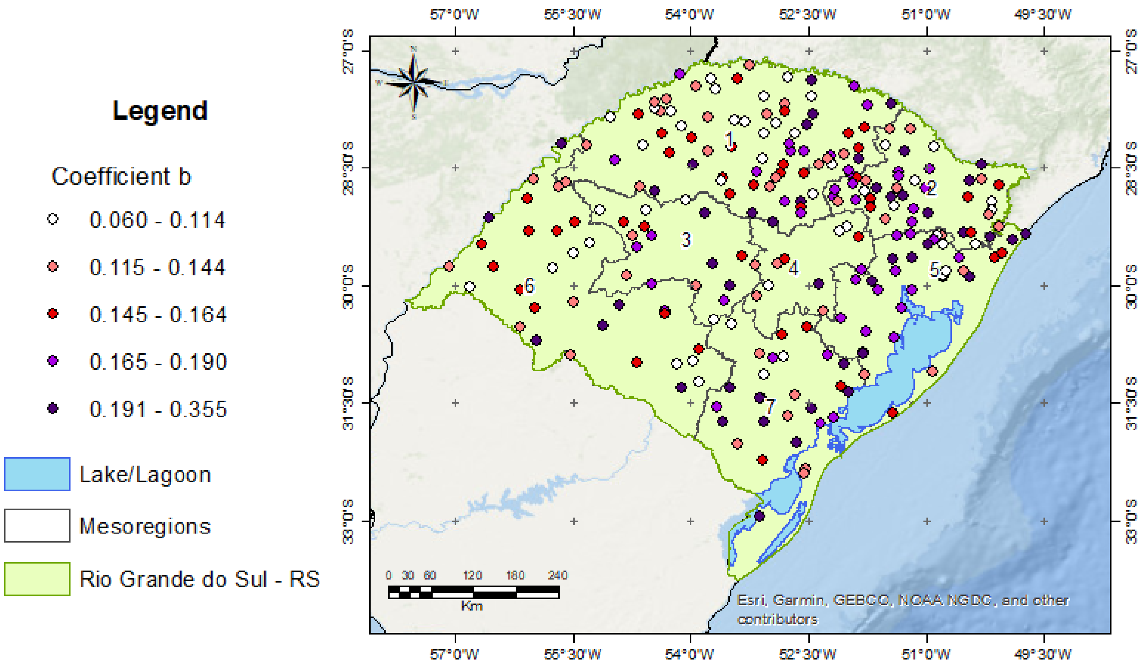

Figure 5. Spatial distribution of the values of coefficient *b* of the IDF equation, adjusted for the 247 locations in RS.

Different from what happened with coefficient *a*, the lowest values were more frequent in the western region of the state, while the highest ones were concentrated mainly in the eastern region of RS (Figure 5). Also, the coefficient *b* of the IDF equation has greater spatial variability and its distribution in the state is not similar to that of the mean AMDR (Figure 2b), as is the case with coefficient *a*.

To verify whether there is a correlation between the values of coefficient *b* and the AMDR, Spearman's Rho (r_s) correlation test was used considering, in this case, the maximum and mean AMDR of each station. The r_s were equal to 0.61 ($p_{value} < 0.05$), and approximately 0 ($p_{value} >$ 0.05) respectively. The r_s between the maximum AMDR and the coefficient *b* of the IDF equation suggests a moderate (Schober *et al.*, 2018) significant positive correlation between the variables.

The spatial variability of coefficients *a* and *b* was also evidenced by other studies in different regions of Brazil and the world, for example, Campus *et al.* (2014) in Piauí, Campos *et al.* (2015) in Maranhão, Souza *et al.* (2016) in Rondônia, Campos *et al.* (2017) in Paraíba, Braga *et al.* (2018) in Rio de Janeiro, Silva and Oliveira (2017) in the Northeast of Brazil, among others. Besides, the range of coefficients *a* (Figure 4) and *b* (Figure 5) is similar to those found by Oliveira (2019), who adjusted IDF equations for come locations in the state of RS based on pluviometric and pluviographic data.

The adjustment of the IDF equation resulted in coefficients *c* and *d* of 9.791 and 0.724, respectively. The values were practically constant in all locations, varying only from the 5th decimal place, which was also found in other studies (Aragão *et al.*, 2013; Souza *et al.*, 2016 and Campos *et al.*, 2017). As mentioned by Aragão *et al.* (2013), it is believed that the *c* and *d* values are nearly constant due to the DRD technique, since the studies that used it obtained constant values or with very low variability; when the IDF equations are adjusted based on pluviographic data, this does not occur.

Given the large number of possible combinations between the durations and RP used here, the duration of 30 minutes and the RPs of 5, 25, 50 and 100 years were chosen to demonstrate the estimate of the maximum intensity of rainfall in each location (Figure 6) based on their respective IDF equations.

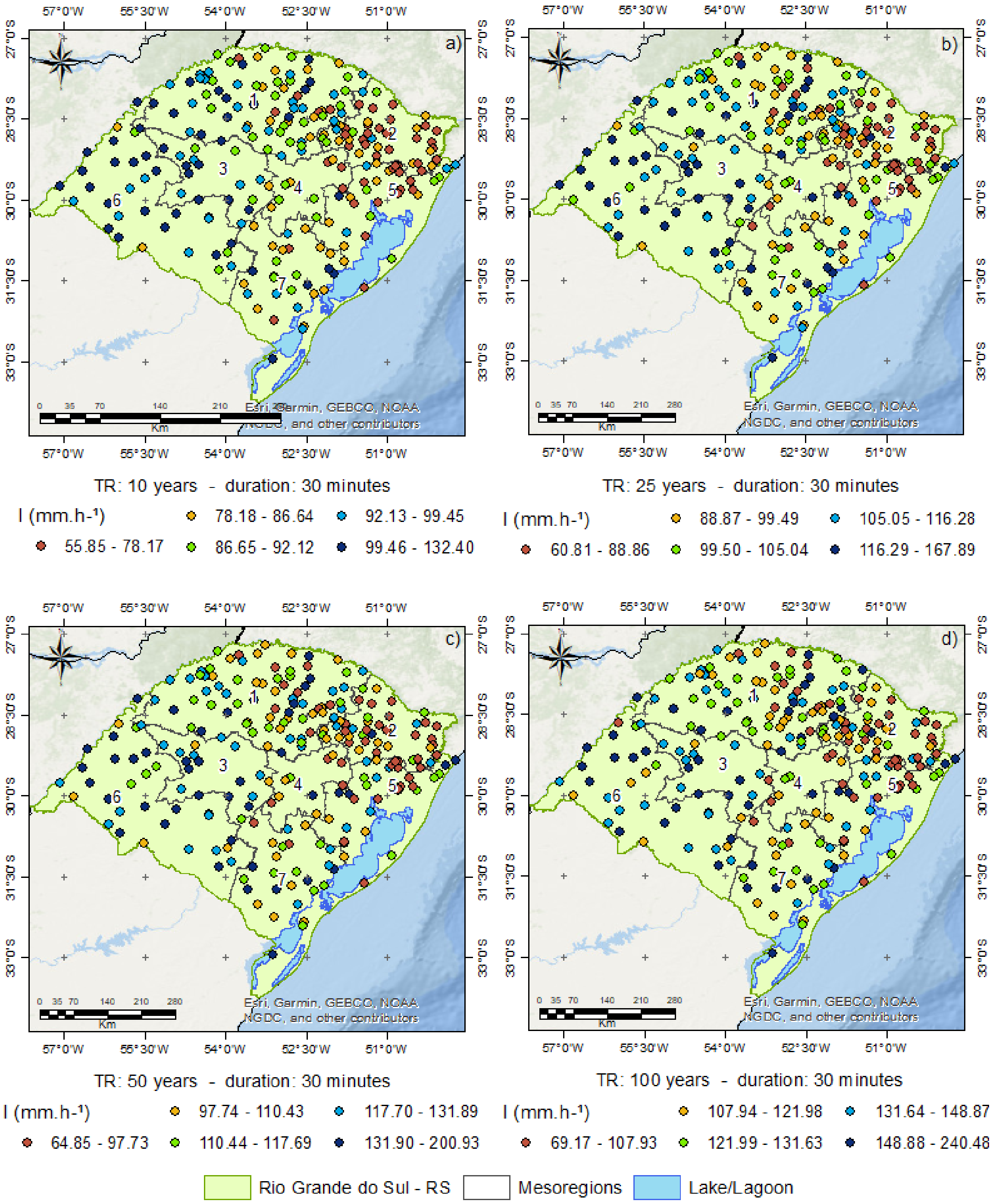

Figure 6. Spatial distribution of maximum rainfall intensity values for 30 minutes duration and: RP = 10 years (A); RP = 25 (B); RP = 50 years (C) and RP = 100 years (D).

For the same duration, as the RP increases, the estimated rainfall intensity values increase. The highest Intensity values occur, mainly, in the Northwest (1), Midwest (3), and Southwest (6) mesoregions, while the lowest values are concentrated in the eastern region of the state (Figure 6). Considering this, it should be noted that in the western portion of the state, where the highest rainfall intensities are more recurrent, there are also soils that due to their cultivational and textural characteristics are more susceptible to water erosion. In addition, there is the presence of the *Pampa* biome, which, according to SEPLAG (Rio Grande do Sul, 2019), is in an advanced state of fragmentation concerning its initial vegetation cover.

Also, it may be interesting for these regions to review and update the IDF equations used in environmental studies and water resources engineering projects, considering that this is one of the measures aimed at the execution or correction of projects for water structures, which normally interact with the elements that participate in the formation of flooding in small to medium watersheds, such as intense rainfall events.

Figure 7 shows the spatial distribution of the RMSE, which reflects the error between the rainfall intensity given by the IDF equations and the observed ones based on DRD. The RMSE values ranged from 0.84 mm h^{-1} to 5.57 mm h^{-1}, with the lowest values occurring in Mesoregions 2, 4, 5 and 7, while the highest values were found in the western portion of RS.

Another observation is that the spatial distribution of the RMSE resembles the pattern of spatial variability of the estimated rainfall intensity (Figure 6) and the average AMDR (Figure 2A).

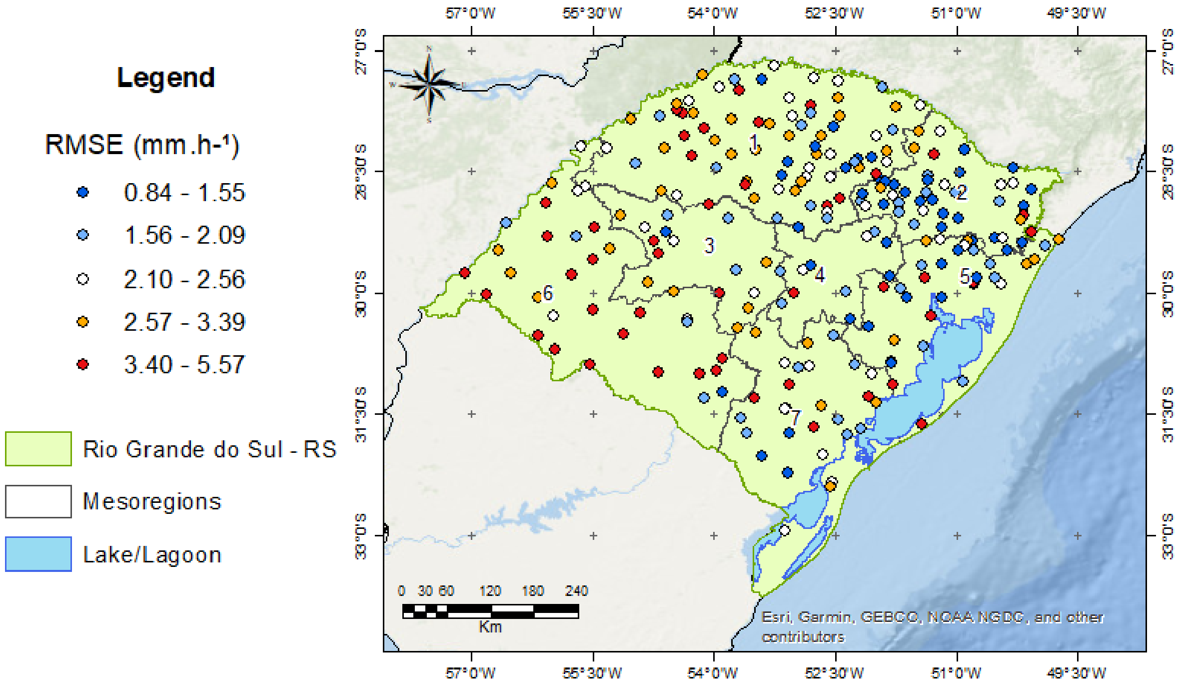

Figure 7. Spatial distribution of the RMSE values in the locations under study.

In fact, the temporal variability of the AMDR is more accentuated in western RS and the variability of the natural phenomenon under study influences its probabilistic modeling (Naghettini, 2017). Therefore, it is believed that this characteristic in association with some uncertainties inherent to the process of estimating IDF equation coefficients promotes greater differences between the intensity of the observed rainfall and that given by the IDF equation, resulting in higher values of RMSE. Besides, the RMSE results found by Oliveira (2019) are also very low, indicating that the adjusted IDF equations can be used in the locations under study.

4. CONCLUSIONS

From the results of this study, the following conclusions were reached:

i) Multiparametric probability distributions are more suitable for modeling the AMDR in Rio Grande do Sul and the importance of using the AD's adherence test to choose the probabilistic model that best fits this variable was brought to light.

ii) The IDF equation coefficients could be adjusted for all 247 locations in RS, with coefficients *a* and *b* varying across the territory and maintaining a correlation with the AMDR, while coefficients c and *d* were practically constant.

iii) It was possible to estimate the maximum intensities of rainfall associated with durations as short as 5 minutes and RP of up to 100 years, besides identifying which regions of RS are subject to intense events of rainfall and will need attention, mainly regarding the environmental and socioeconomic consequences triggered by these events and how to mitigate them.

iv) The use of AMDR series without significant temporal trend, of multiparametric probability distributions, of the set of disaggregation constants most suitable to the state's AMDR, and of a robust method for adjusting the coefficients of the IDF equations, provided excellent suitability, as well as low errors in estimating the coefficients of the IDF equations that were obtained.

Finally, despite the fact that the IDF equations obtained in this study represent a great gain of information about rainfall intensity in RS, as a large part of the state is not yet covered by this information and given the evidence that coefficients *a* and *b* are correlated with its geographical position, it is expected that future studies must confirm this hypothesis and allow the spatialization of IDF coefficients for the entire territory of Rio Grande do Sul.

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