Vol.65: e22220185, 2022
https://doi.org/10.1590/1678-4324-2022220185
ISSN 1678-4324 Online Edition

Article - Human and Animal Health

The Effects of Vaccination on Covid-19 Dynamics in Brazil: A Fuzzy Approach

Elenice Weber Stiegelmeier[1]
https://orcid.org/0000-0002-8834-4937

Glaucia Maria Bressan[1*]
https://orcid.org/0000-0001-6996-3129

André Luis Machado Martinez[1]
https://orcid.org/ 0000-0003-1888-648X

[1]Universidade Tecnológica Federal do Paraná, Departamento de Matemática, Cornélio Procópio, Paraná, Brasil.

Editor-in-Chief: Paulo Vitor Farago
Associate Editor: Paulo Vitor Farago

Received: 13-Mar-2022; Accepted: 12-Jun-2022.

*****Correspondence: glauciabressan@utfpr.edu.br (G.M.B.).

HIGHLIGHTS

- A fuzzy approach represents the COVID-19 dynamics under the effect of vaccination in Brazil.
- The infected population and the vaccination rate are considered as input variables.
- The effect of population-wide vaccination is modeled and analyzed.
- The level of infestation tends to decrease as the number of people vaccinated increases.

Abstract: The COVID-19 pandemic has spread widely through the world, since 2019. Then, the search for vaccines has become fundamental against the disease and has generated great expectations on population. In Brazil, the first case of COVID-19 was registered in February 2020 and the process of vaccination started in early 2021. In this context, considering the advance of vaccination and the behavior of epidemics, we present a study on the effect of population-wide vaccination on the evolution of the COVID-19 epidemic. To this end, we propose a new fuzzy system in order to model the COVID-19 epidemic evolution under the effect of vaccination in Brazil. This proposed model consists of a p-fuzzy system, where the input variables are the infected population and the vaccination rate. The output variable is the level of infection. Results of the proposed fuzzy system show that the level of infestation tends to decrease as the number of people vaccinated increases. Therefore, the population-wide vaccination strategy clearly contributes to suppressing the growth of the disease and to mitigating its impacts on society, including saving lives.

Keywords: fuzzy model; epidemic dynamic; COVID-19; effect of vaccination.

INTRODUCTION

The COVID-19 pandemic has spread widely through the world, since 2019 [1,2]. The spreading of SARS-CoV-2 in Brazil, based on official data available, is studied since March 22, 2020 [3]. Since then, the search for vaccines has become fundamental against the disease and has generated great expectations on population. In Brazil, the first case of COVID-19 was registered in February 2020 and the vaccination process of Brazilian population from priority groups started in January 2021. The Brazil came to occupy the third place in number of infected and dead in the world during the beginning of the year 2021 [4]. With the advance of vaccination in the country, the scenario has improved.

The World Health Organization (WHO) [2] recommends that intervention measures, such as personal protective measures, environmental measures and physical distancing measures, are still necessary, even with the vaccine. Relaxing social distancing restrictions to the pre-pandemic level would lead to a new COVID-19 outbreak [5]. Vaccination combined with physical distancing can stop the contamination, whereas a gradual vaccination process alone cannot contain resurgences [6].

Several mathematical models have been proposed in order to describe the dynamics of COVID-19 pandemic. In [7], a mathematical model that includes social isolation of susceptible individuals is proposed, when no vaccine was available. Results show that beginning social isolation, and with 70% of the population in isolation, seems to be promising.

According to [8], a legitimate question that one may ask is what is the best vaccination strategy that healthcare policymakers should implement and how this vaccination strategy would impact the evolution of the COVID-19 pandemic.

Considering the vaccination process, an optimal control strategy for vaccine administration in COVID-19 pandemic treatment is determined in [9], using Differential Evolution. The results provide information from which an optimal strategy for vaccine administration can be defined.

Biomathematical models, in particular population dynamics and epidemiology models, are based on physical-chemical hypotheses, in which the reaction between state variables is modeled by the product of their concentration [10]. In the models that deal with uncertainty, the use of linguistic terms to express quantitatively the variables is well applied and various types of uncertainty arising from biomathematical phenomena may generate distinct models. Fuzzy models [11] have been applied to describe biological phenomena, since this approach considers linguistic variables, uncertainty, and a rule base that make the inference process more understandable. In the literature, fuzzy approaches describing the behavior of the COVID-19 pandemic can be found [1,12,13]. The paper [1] presents a study about how intervention measures such as lockdown, partial lockdown, and no-lockdown help to impede the extent of the severe outbreak of COVID-19.

In this context, considering the advance of vaccination and the behavior of epidemics, the main objective of this work is to present a fuzzy approach [11] to describe the effects of the vaccination process on COVID-19 spread in Brazil. We further investigate the dynamics of the SARS-CoV-2 epidemic under the effect of vaccination within the Brazilian territory from January to November 2021. To achieve this task, we resort to a fuzzy model based on rule bases that allows us to describe the epidemic evolution.

This paper is organized as follows: Material and Methods section presents the Fuzzy Dynamical Systems theory and the formulation of the proposed fuzzy model to describe the effects of the vaccination process on COVID-19 spread in Brazil. Next, the results of the proposed fuzzy system are presented and discussed, which show that the level of infestation tends to decrease as the number of people vaccinated increases.

MATERIAL AND METHODS

This section initially provides a basic dynamic framework for the dynamic population based on p-fuzzy model and then shows the formulations of the fuzzy model to modelled the COVID-19 epidemic evolution considering the effect of vaccination in Brazil.

Fuzzy Dynamical Systems

Fuzzy dynamical systems illustrate the power of the fuzzy logic for studying evolutionary systems and two cases are distinguished: continuous and discrete. Continuous dynamic systems consist of a system defined by derivatives, since they represent rates of continuous variation. Discrete dynamic systems evolve over time through an iterative process [10]. We can describe the *discrete fuzzy dynamical systems* as a generalization of the classical ones.

In fuzzy systems theory, which consists of a system based on fuzzy rules, there are four fundamental components: an input processor (fuzzification), a collection of linguistic rules called a *rule base*, a fuzzy inference method (Mamdani or Sugeno) and an output processor (defuzzification). One of the most common rules based fuzzy systems is known as fuzzy controllers [11]. The *membership degree* refers to the extent to which it is possible for an element to belong to a set. Processing of fuzzy information is usually done through operations that aggregate, combine and compare the membership degree of their elements.

The *partially fuzzy systems,* or p-fuzzy systems, combine dynamical systems with fuzzy logic and use a rules based fuzzy systems in order to relate variables and their variations. The directions field, partially known a priori, it is obtained through methodologies of fuzzy controllers [10].

Definition 1 [10]. *A p-fuzzy initial value problem is given by*

$$\begin{cases} \frac{du}{dt} = F\big(t, u(t)\big) \\ u(a) = u_0 \end{cases} \tag{1}$$

where $F: [a\ b] \times \mathcal{F}(\mathbb{R}) \to \mathcal{F}(\mathbb{R})$, $u_0 \in \mathcal{F}(\mathbb{R})$ *and* $\frac{du}{dt}$ *represents the variation rate of function u depending on t. In addition,* F *is partly known and described by a fuzzy rule base.*

Discrete fuzzy modeling comes from fuzzy controllers. Therefore, a p-fuzzy discrete system is given by:

$$\begin{cases} x_{t+1} = F(x_t) \\ x_0 = x(t_0) \end{cases} \tag{2}$$

where $F(x) = x + \Delta x$ such as $F: R^n \to R^n$ and Δx is the defuzzificated output provided by fuzzy controller. Thus, it is possible to generalize a discrete p-fuzzy system as differences equation given by:

$$x_{t+1} - x_t = f(x_t) \tag{3}$$

where the variation function $f(x_t) = \Delta x$ is the output of the fuzzy controller. Therefore, a p-fuzzy system structure consists of the following elements: the input x_t, the fuzzy controller, the mathematical model (3) and the output Δx.

The controller output field f may present properties that guarantee the existence or uniqueness of the Initial Value Problems solution. However, depending on the complexity of f it is necessary to use numerical methods to obtain estimates for the Initial Value Problems solution.

According to [10], the fuzzy rule base satisfies some properties, in such a way that facilitates the search for possible equilibrium points of the system.

(i) The universes must be bounded interval sets of real numbers.
(ii) The fuzzy sets of the rule base must be fuzzy numbers.
(iii) The rule base must cover its associated universe, in the sense that each element of the universe has a non-zero degree of belonging to at least one of the fuzzy numbers of the rule base.
(iv) At most two rules can be activated at once.
(v) Any element whose membership value is 1 can belong to only one fuzzy set.
(vi) The rule base must be monotonically ordered (increasing or decreasing).

In the classical biomathematical models, in particular population dynamics and epidemiology models, the reaction between two state variables is modeled by the product of their concentration - the law of mass action. The same law is used in the Lotka–Volterra models of interactions between two species. When the state variables are modeled by fuzzy set theory, we have demographic fuzziness, and when only the parameters are fuzzy we have environmental fuzziness. The state variables and their variations are considered linguistic. This way, the state variables are correlated to their variations by means of fuzzy rules whose main characteristic is having the *state variables as input, while the variations are outputs.*

The Verhulst logistic population model is presented to describe the population variation at time *t*, as well as a p-fuzzy approach for this model. Verhulst (1838) then presented the population growth equation as a differential equation, assuming that the population grows to a sustainable maximum limit, tending to stabilize and due to the limited resources. Let *a* and *b* be positive constants, the separable differential equation is given by (4):

$$\frac{dp}{dt} = (a - bp)p \tag{4}$$

Constant functions $p(t) = 0$ e $p(t) = a/b \equiv k$ are equilibrium solutions of (1), since it refers to the case when there is no variation of *p* when *t* grows. Then, any equilibrium solution of (1) can be obtained by finding the roots of $f(p) = 0$ and the zeros of $f(p)$ are the critical points. For $p_0 \neq 0$ e $p_0 \neq \frac{a}{b}$, the solution is given by (5).

$$p(t) = \frac{ap_0}{bp_0+(a-bp_0)e^{-at}} \tag{5}$$

If $t \to \infty$, then $p(t) \to k,$ this value is called the limit population and it is the asymptotic value of the population, for any initial population $p_0 > 0$. If $p_0 > k,$ the population $p(t)$ decreases exponentially tending to *k*. If $0 < p_0 < k,$ the population increases tending to *k*. The tipping point is at $p(t) = \frac{a}{2b},$ that is, until reaching the value *k/2*, the population grows with a positive derivative and from there the growth is gives more slowly.

Verhulst model can be considered as a p-fuzzy model, considering some input variable as fuzzy. It is necessary to present rules with semantic opposition, that is, alternation of signs in the consequent variations.

In this way, using numerical data provided by the Ministry of Health (Brazil), from January, 03 to November 20, 2021, a fuzzy approach to describe the behavior of new daily cases of COVID-19 in Brazil under the effect of vaccination, using dynamic p- fuzzy model is presented.

Formulation of Fuzzy Model

Biological, medical and epidemic systems present several types of inherent uncertainties to its processes. Many of these uncertainties have been treated in an efficient way with statistical and Bayesian models. Though, these areas still lack of mathematical structures that make possible the treatment of the non-statistical uncertainties typical of some of these systems. Besides, the use of linguistic terms to express quantitatively the variables is very common in these areas. So, due to its features, the fuzzy logic comes as an appropriate theory to treat some of these problems.

Therefore, an attempt to reproduce the strategy of a human controller in his or her task execution is made by a Fuzzy Controller which is a typical case of a Fuzzy Rule-Based System (FRBS), that is, a system that makes use of the fuzzy logic to produce outputs for each fuzzy input.

This work provides an assessment of the epidemiological trends of the COVID-19 pandemic in Brazil under the influence of vaccination. The objectives are to model and to analyze the effect of population-wide vaccination on the evolution of covid-19 epidemic in Brazil.

In this paper, the dynamic p-fuzzy model is considered, in which the dynamic population is given by a fuzzy model. The proposed model is based on a system of fuzzy controls using the Mamdani inference method [2], in order to describe how the vaccination and the infected population can affect the COVID-19 dynamics. The input variables are: infected population and vaccination rate. The output variable is the level of infection. The structure of the proposed model is given in Figure 1.

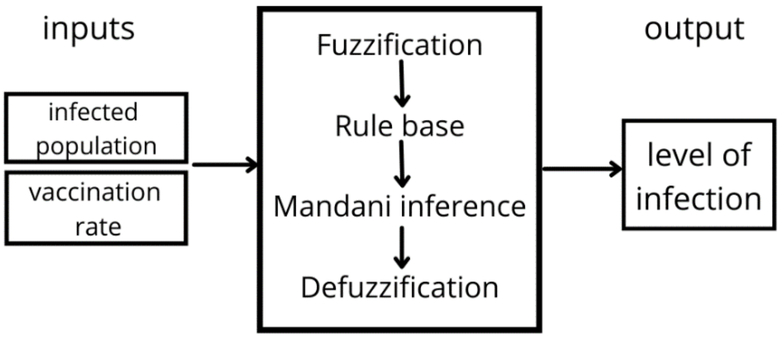


Figure 1. Architecture of the fuzzy model of COVID-19 evolution under vaccination in Brazil

We assume that the population, in this fuzzy model, is given by the infected individuals and the effectively vaccinated sub-population is represented by the accumulated daily-vaccination rate. The results show that the daily vaccination rates used in the proposed epidemic model represent the daily vaccination rate of fully vaccinated individuals (i.e., individuals that received the two doses of the vaccine). The outputs represent the variation of infected individuals.

So, let p_k be the population infected, v_k the accumulated daily-vaccination rate and $\Delta(p, v)$ the variation rate, in the iteration k. The proposed p-fuzzy system is formulated as:

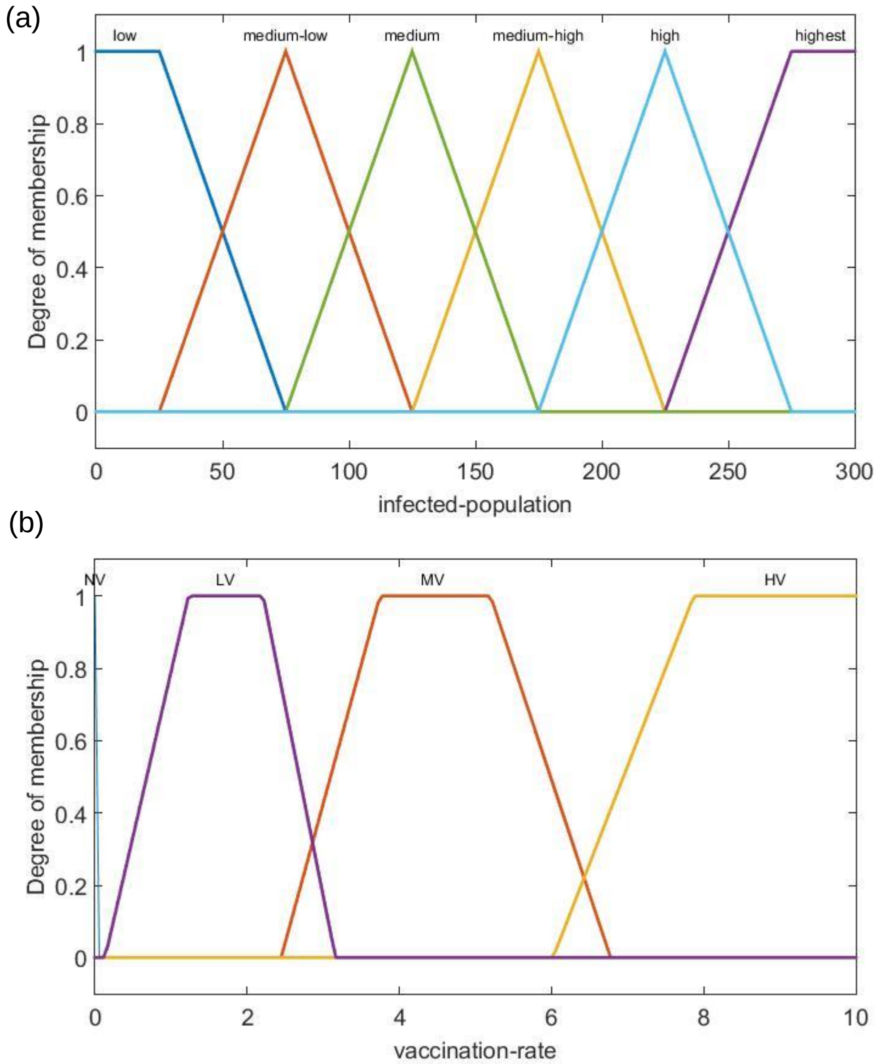
$$\begin{cases} p_{k+1} = p_k + \Delta(p_k, v_k) \\ \quad p_0 \in \mathbb{R} \end{cases}, \tag{6}$$

where $\Delta(p_k, v_k)$ is partly known and described by a fuzzy rule base.

The inputs and output variables are discretized into linguistic classes and are described in Table 1. Note that the infected population is discretized into 6 classes; the vaccination rate into 4 classes and the output, level of infection, into 6 classes. The two input variables and the output are performed using triangular or trapezoidal membership functions, as shown in Figure 2a–c.

Table 1. Classifications used in the fuzzy model of the evolution of COVID-19.

Infected population	Vaccination rate	Level of infection
low (L)	no vaccination (NV)	low-negative (L-)
medium-low (ML)	low vaccination (LV)	medium-negative (M-)
medium (M)	medium vaccination (MV)	high-negative (H-)
medium-high (MH)	high vaccination (HV)	low-positive (L+)
high (H)		medium-positive (M+)
highest (HH)		high-positive (H+)



(c)

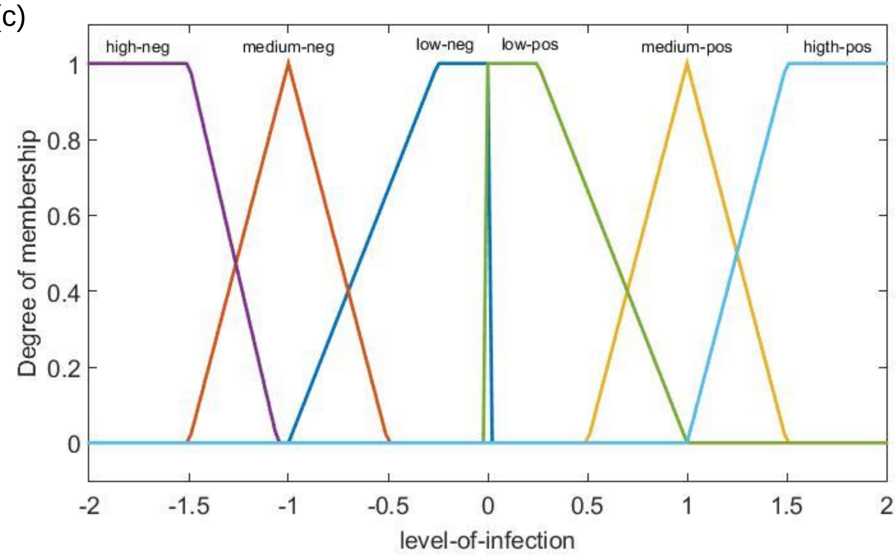


Figure 2. Membership functions: the inputs (a) infected population and (b) vaccination rate; the output (c) level of infected population.

Analyzing the data set with the inputs and output variables, it is possible to establish a knowledge base with linguistic rules relating them, to estimate the value of the level of infected population, where this estimation is made by the defuzzification process using the center of gravity.

The combination of input classes generates a linguistic IF-THEN rules base composed of 24 rules, as shown in Table 2. For example:

R1 - IF *(infected population)* is low AND vaccination rate is NV, THEN the variation $\Delta(p, v)$ will be M+.
R2 - IF *(infected population)* is low AND vaccination rate is LV, THEN the variation $\Delta(p, v)$ will be L+.

Table 2. Rules base for the p-fuzzy system

VACCINATION RATE / INFECTED POPULATION	NV	VL	MV	HV
LOW	M+	L+	L-	M-
MEDIUM LOW	H+	M+	L-	M-
MEDIUM	H+	H+	M-	M-
MEDIUM HIGH	M+	H+	M-	H-
HIGH	L+	L+	H-	H-
HIGHEST	L+	L-	H-	H-

Table 2 presents the rule base used in the proposed fuzzy model. Note that, if there is a favorable condition for the virus, that is, if no-vaccination is activated in the vaccination rate set, then the population growth rate increases. In other words, if the environment is always favorable, so the level of infection is always positive, regardless of the population size, even so, if there is an unfavorable environment condition, that is, if the control strategy is adopted and the vaccination rate set is activated, so the population variation rate makes negative (mortality higher than birth). Therefore, the variation of infected populations $\Delta(p, v)$ will change due to the control strategy adopted.

RESULTS

In this section, the dynamical behavior of the COVID-19 is numerically investigated. In the p-fuzzy model approach given by (6), p_0 represents the populations in generation k and $\Delta(p, v)$ the absolute populational variation in generation k up to $k + 1$.

We define a scenario for the simulation where the initial condition of infected population and the period of time are fixed, $p_0 = 800$ (daily cases), $k = 300$ days, respectively. The objective is to demonstrate how the wide-population vaccination rate can contribute to contain the spread of the virus.

The vaccination started in Brazil in January 18, 2021 for the population-wide and in November, 2021 we had about 60% of the population vaccinated [15].

Figure 3 illustrates the evaluation of the new-daily case of COVID-19 in Brazil versus the percentage of fully vaccinated population. These official data are available in Ministry of Health [14] and Fiocruz [15]. During the period from January to November 2021. We can notice that there is a significant reduction in the new daily cases of COVID-19 in Brazil during the period of time analyzed.

Considering the vaccination rate of the population-wide as a control variable, the solution of the fuzzy model can be compared with the official data provided by the Ministry of Health [14]. Therefore, Figure 4 illustrates the comparison between official data points and the results from the p-fuzzy model of the infected population dynamic considering the vaccination strategy. In Figure 5, the official data are considered as a moving average every seven days. These mentioned figures compare the official data versus the results from the proposed fuzzy model. Note that, with 10% of population vaccinated, there is a significant reduction in the number of new daily cases of COVID-19, around 150 days after the start of the vaccination. In November, with 60% of the population vaccinated, there is a reduction of 87% in the number of new daily cases compared to the peak of daily cases in Brazil. We went from 80,000 daily cases to around 10,000 at the end of the analyzed period.

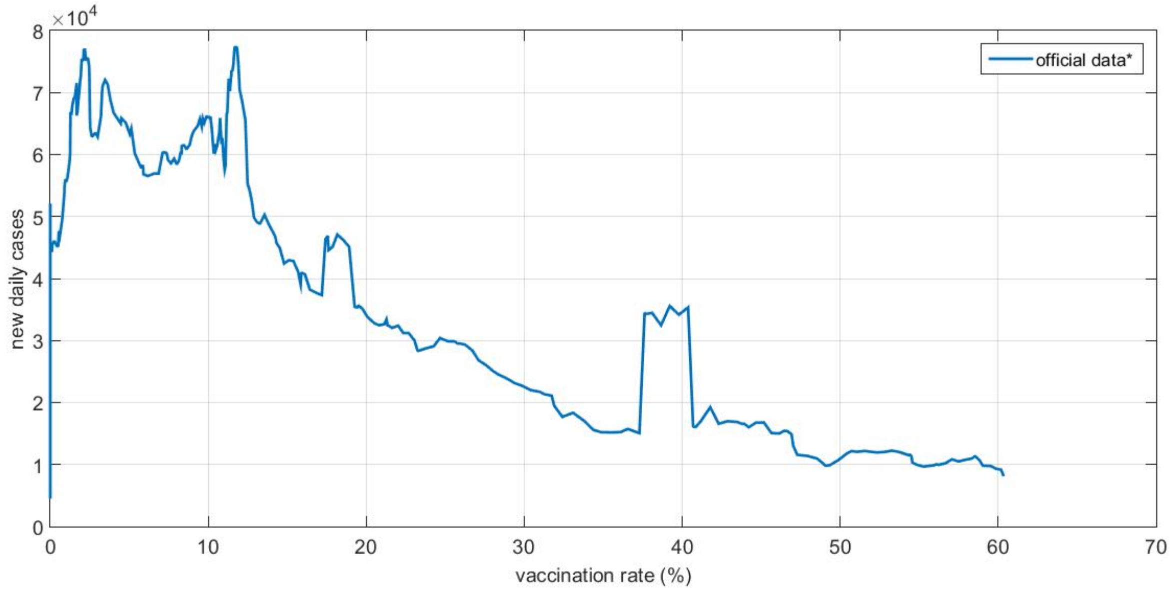


Figure 3. The new daily cases of COVID-19 in Brazil versus the percentage of fully vaccinated population. *The official data are from Ministry of Health [14] and Fiocruz [15].

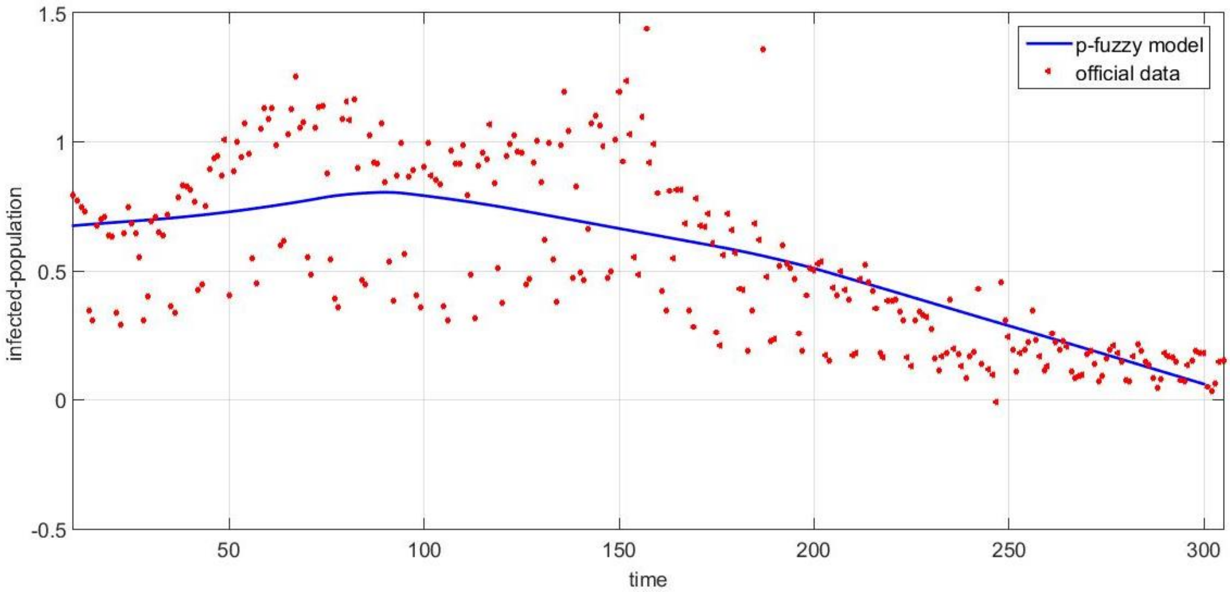


Figure 4. Number of new daily cases normalized. Official data is represented by points * and results from the p-fuzzy model are denoted by the solid line.

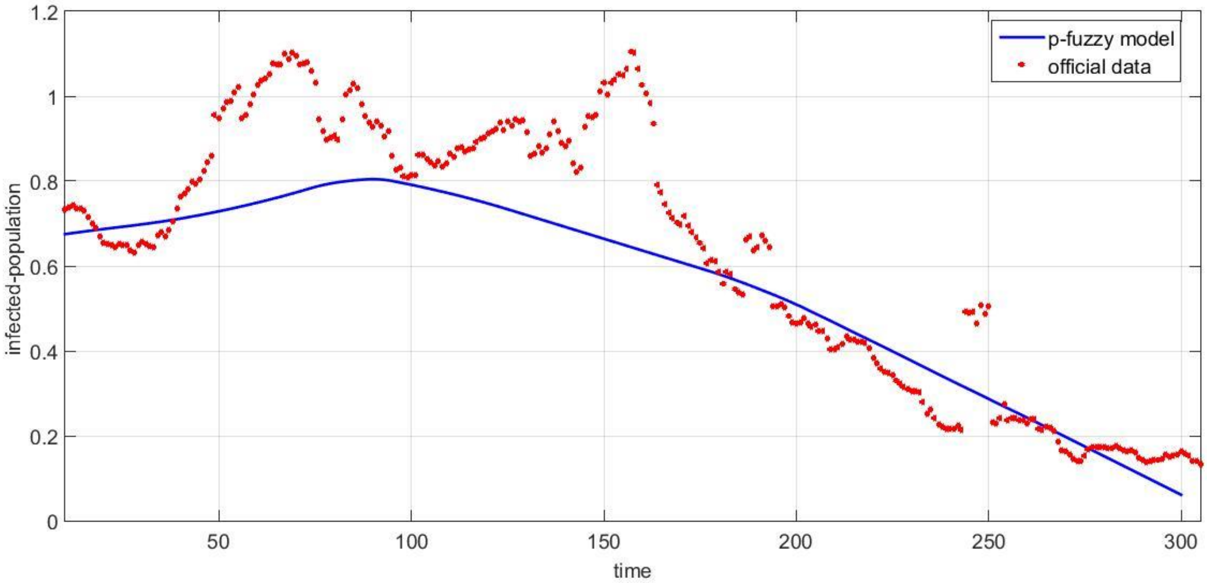


Figure 5. Number of new daily cases normalized considering the official data as a moving average every seven days.

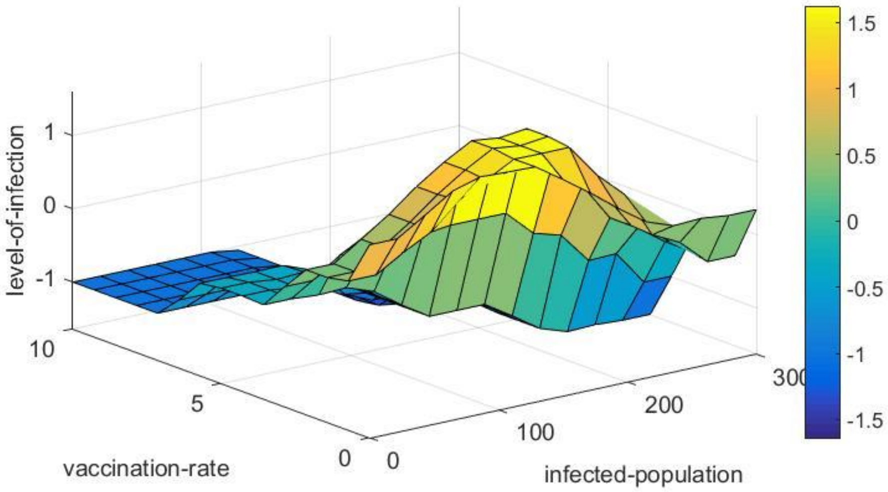


Figure 6. The output of the p-fuzzy system illustrated by a surface.

In Figure 6, the output of the p-fuzzy system is illustrated as a surface, using the variables of the problem.

Results of the proposed fuzzy system show that the level of infestation tends to decrease as the number of individuals vaccinated increases (see Figures 3-5). This fact is illustrated by the behavior of new daily cases of COVID-19 in Brazil, using numerical data provided by the Ministry of Health [14], from January to November, 2021. We note that the data dispersion occurs due to lack of regularity in data collection and testing. The spikes can be related to social behavior, around parties, holidays, etc.

The simulations were performed using MATLAB (www.mathworks.com) and the Fuzzy Logic Toolbox was used to implement the fuzzy rule base. All experiments were run on a 2.4 GHz Intel core i7 processor 8Gb of RAM memory and Windows 10 operating system. The average processing time is considered fast.

DISCUSSION

In this study, we investigated the association between new daily cases of COVID-19 and vaccination coverage during the year 2021. According to the results obtained by solving the proposed fuzzy model, the vaccination strategy adopted provides a significant reduction of the COVID-19 incidence in Brazil. In addition, the vaccination impact should capture the indirect protection provided by vaccination. Since the vaccine protects against infection, the number of infectious cases would decrease as more people are vaccinated. The lower number of infectious cases in the population would lead to a reduced probability for susceptible individuals to get in contact with infectious cases, thus leading to a reduction in incidence among all people, including non-vaccinated people.

This paper has suggested a fuzzy modelling to describe the evolution of infected population, focusing on the vaccination and their effect in the increase of new cases of COVID-19. According to official data from Ministry of Health [14], the new daily cases of coronavirus disease has a significance reduction since the vaccination started. The results presented in this paper demonstrate this fact, as can be seen in Figures 4 and 5. We can point out as a limitation of this research, the use of secondary data, from the health public system.

We also observed that the infection rate is tightly dependent on the safety measures implemented such as quarantine, lock-downs, face covering, hygiene, social distancing, etc. Therefore, the aforementioned results could be much worse if these measures are to be weakened even in the presence of the vaccine. This fact can be clearly seen when we analyze the official data from Brazil, in the period of the year 2022 during the presence of the new Omicron variant [14]. However, at the beginning of January 2022, the wide-population vaccination rate was around 67% (second doses), 75% (primary dose) and 12% (booster dose), which can be considered a good immunization coverage of the population. This fact contributes to the lower rate of the individual's ICU-hospitalized, complications from the COVID-19 when compared with the Firth and second wave in Brazil [15].

We can also highlight that the vaccination strategies influenced directly the increase wide-population protection against infections, severe diseases and deaths from COVID-19. Our results, combined with the available data about the COVID-19 epidemic in Brazil, suggest that mass vaccination is urgent, necessary and cost-effective to control the spread of the virus.

According to [5], [6] and [9], the COVID-19 vaccination is the efficient strategy to contain the coronavirus spread. In addition, [9] indicate that the efficient control strategy can help to stop the spread of COVID-19. In this sense, these results converge with the results presented by the fuzzy model proposed in this paper.

CONCLUSION

In this paper, a novel approach of dynamic of COVID-19 which considers the population-wide vaccination rate to control the spread of the COVID-19 and mitigate its effects on society is established. The model simulations were performed and compared with real data from COVID-19 of Brazil. We noticed that the fuzzy approach presents a good correspondence between the fuzzy model simulated and the official real data. However, it should be mentioned that in order to obtain a good correspondence, the membership functions should be adjusted for each situation under analysis, including new variables, such as the vaccine efficacy, measure intervention, among others.

The mathematical model used in the present work stands out for search into parameter uncertainty whereas the classic model does not, however, these were sufficient for a significant analysis of the COVID-19 dynamics, including a disease control strategy. The modeling can be refined further by incorporating the delay between the time the vaccine was administered and the time where the vaccine will provide the expected protection and the announced efficacy. However, any additional variable will make the model more complexity, also with the insertion of more parametric uncertainties.

Finally, this analysis confirms the role of vaccination strategy, as an effective way to reduce COVID-19 incidence.

Funding: This research received no external funding.
Conflicts of Interest: The authors declare no conflict of interest

REFERENCES

1. Stiegelmeier EW, Bressan GM. A fuzzy approach in the study of covid-19 pandemic in Brazil. Res. Biomed. Eng. 2021; 37(2): 263-71.
2. WHO. World health organization (2020, november 4). Considerations for implementing and adjusting public health and social measures in the context of covid-19, Interim guidance. 2020; 1-13. www.who.int/news-room/q-a-detail/q-acoronaviruses
3. Dos Santos IFF, Almeida GMA, De Moura F. Adaptive SIR model for propagation of SARS-CoV-2 in Brazil. Phys. A: Stat. Mech. Appl. 2021;569:125773.
4. Dong E, Du H, Gardner L. An interactive web-based dashboard to track covid-19 in real time. Lancet infect. Dis. 2020; 20(5):533–4.
5. Shen M, Zu J, Fairley CK, Pagán JA, An L, Du Z, et al. Projected covid-19 epidemic in the united states in the context of the effectiveness of a potential vaccine and implications for social distancing and face mask use. Vaccine. 2021;39(16):2295-302.

6. Huang B, Wang J, Cai J, Yao S, Chan PKS, Tam TH-W, et al. Integrated vaccination and physical distancing interventions to prevent future covid-19 waves in chinese cities. Nat. Hum. Behav. 2021;5(6):695-705.
7. Molter A, Quadros RS, Rafikov M, Buske D, Gonçalves G. A. Mathematical Modeling to Perform an Analysis of Social Isolation Periods in the Dissemination of COVID-19. Trends Comput. Appl. Math. 2021;22:595-608.
8. Berkane S, Harizi I, Tayebi A. Modeling the effect of population-wide vaccination on the evolution of COVID-19 epidemic in Canada. medRxiv pre-print, 2021.
9. Libotte GB, Lobato FS, Platt GM, Neto AJS. Determination of an optimal control strategy for vaccine administration in COVID-19 pandemic treatment. Comput. Methods Programs Biomed. 2020;196:105664.
10. Barros LCD, Bassanezi RC, Lodwick WA. A first course in fuzzy logic, fuzzy dynamical systems, and biomathematics: theory and applications. Vol. 347, Studies in fuzziness and Soft computing: Springer, 2017. 304p.
11. Pedrycz W, Gomide F. An introduction to fuzzy sets: analysis and design. Cambridge: MIT press, 1998. 465p.
12. Dhiman N, Sharma M. Fuzzy logic inference system for identification and prevention of coronavirus (covid-19). IJITEE. 2020;9(6),2278-3075.
13. Sharma M, Dhiman N, Mishra VN, Vandana. Mediative fuzzy logic mathematical model: A contradictory management prediction in covid-19 pandemic. Appl. Soft Comput. 2021;105:107285.
14. Brazil. Ministry of Health 2022 Brazil https://covid.saude.gov.br
15. Fiocruz. Instituto de Comunicação e Informação Científica e Tecnológica em Saúde (ICICT). MonitoraCovid-19. Rio de Janeiro, 2020. https://bigdata-covid19.icict.fiocruz.br