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# A Mathematical Model for the Removal of Organic Mater in Stabilization Ponds

**Waldir Medri*  and Vandir Medri**
*Universidade Estadual de Londrina, Departamento de Matemática Aplicada, Campus Universitário, CEP 86510-990, Londrina – PR, Brazil*

## ABSTRACT

*This work presents an application to systemize the construction of ponds systems for treatment of domestic sewage. It consisted of two anaerobic ponds operated in parallel during May/97 to April/99. These were connected in series with a chicaned facultative pond. The treatment system was controlled with samples collected from the crude sewage (compound sample), in the affluents and effluents of the ponds and along the flux of the anaerobic and facultative ponds. The following parameters were analyzed: pH, Biochemical Oxygen Demand, Chemical Oxygen Demand, Total Solids, Sedimentable Solids, Total Coliforms, Oxygen Consumed in Acid Medium (OCAM) and temperature.*

**Key words**: Stabilization ponds, mathematical model, optimization

## INTRODUCTION

The stabilization ponds are modeled to keep wastewater until the wished effluent is obtained by the activity of microorganisms present in the system. The treatment process is realized by the capacity of microorganisms to break complex organic molecules into more simple inorganic substances during the cellular synthesis processes (Dorego & Leduc, 1996).

Stabilization ponds systems are quite old techniques for waste treatment. These systems have some advantages over the conventional treatments such as: low capital, operational and maintenance costs and simplified operation. The disadvantage is that it needs a big area. This article presents models of capital costs (costs with land area occupied by the ponds system and its construction) and maintenance costs, in order to optimize the system, objecting to minimize of the total cost with adequate final effluent in terms of organic mater.

## METHODS AND MATERIALS

The wastewater treatment system belongs to Serviço Autônomo Municipal de Água e Esgoto– SAMAE. This system is situated in the south zone of Ibiporã city (state of Paraná) and consists of a grating box, a parshall gutter pipe, two anaerobic ponds (AP1) and (AP2), and a chicaned facultative pond (FP). The two anaerobic ponds are connected in parallel and these are connected in series with the facultative pond. Figure 1 shows a scheme of this system and Table 1 presents the physical and operational characteristics in the period of the experiment.

* Author for correspondence

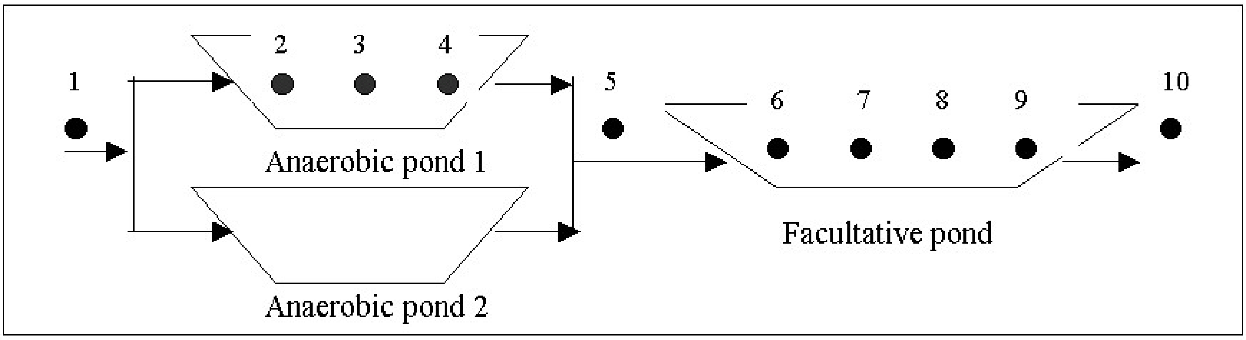


**Figure 1 -** Wastewater treatment system of Ibiporã/Pr South Zone

**Table 1 -** Physical and operational characteristics of the ponds

| DIMENSIONS | | PONDS | | |
|---|---|---|---|---|
| | | **Anaerobic pond 1** | **Anaerobic pond 2** | **Facultative pond** |
| Surface length | (m) | 154 | 154 | 210 |
| Bottom length | (m) | 149 | 147 | 209 |
| Surface width | (m) | 22 | 22 | 51 |
| Bottom width | (m) | 19 | 19 | 48 |
| Average surface | ($m^2$) | 3,106 | 3,085 | 10,370 |
| Depth | (m) | 2 | 1.9 | 1.5 |
| Volume | ($m^3$) | 6,211 | 5,862 | 15,555 |
| Discharge | ($m^3$/d) | 743 | 743 | 1486 |
| Detention time | (d) | 8.3 | 7.9 | 10.5 |

### Control and Physico – Chemical Analysis

The crude sewage control was realized every hour from the 10th to 17th of June/1996 in an amount of 168 collections as the samples were *compound* type (point 1), while for the anaerobic ponds effluents AP1 and AP2 (point 5) and the facultative pond FP (point 10), the samples were collected every 15 days between May/1997 and April/1999. For the samples collected from ponds AP1 (2, 3, 4) and FP (6, 7, 8, 9) the control was realized between May/1998 and April/1999. Figure 1 presents the points of sample collection. Following parameters were analyzed: pH, Biochemical Oxygen Demand (BOD), Chemical Oxygen Demand (COD), Total Solids (TS), Sedimentable Solids (SS), Total Coliforms (TC-24h), Total Coliforms (TC-48h), Oxygen Consumed in Acid Medium (OCAM) and temperature**,** according to Standard Methods for Examination of Water and Wastewater, 1992).

## RESULTS AND DISCUSSIONS

The average, minimum and maximum values obtained from the domestic sewage treatment system of SAMAE between May/1997 and April/1999 are presented in Table 2.

### Operational Datas

It was observed that the average results obtained along the flux of the anaerobic pond characterized a system closer to the *Complete Mixture* than to the *Piston Flux*, because it had not a high concentration of organic mater (BOD) near the pond entrance, where the rate of removal and the sequential degradation along it would be higher, and enabled a dispersion of dejects all over the pond so that the concentration of the wastewater when just entered was soon equaled to the effluent concentration in spite of the geometric configuration of the ponds related to length/width among them. Nevertheless, von Sperling (1996) argued that the hydraulic regime of the pond had a great influence over the treatment efficiency. Values obtained along the facultative pond were evidence of a system closer to the *Piston Flux*. It was important to emphasize that the facultative pond was chicaned. Yet, the *Complete Mixture* were the ponds efficiency models used in the system optimization.

During the control of the anaerobic ponds AP1 and AP2 and the facultative pond FP of SAMAE, the total $BOD_5$ was reduced from 661.8 mg/l (crude sewage) to 61.4mg/l in the effluent of FP pond (Table2), which corresponded to a total removal of 91%. Was a good removal efficiency, being according to the standards established by *Secretaria de Estado do Meio Ambiente – Instituto Ambiental do Paraná (IAP)*, which is 60mg/l. The $BOD_5$ was greatly removed in the anaerobic ponds AP1 and AP2, about 85%, for both ponds with detention time of approximately 8 days, while the removal in the FP pond was about 38%, with detention time of 10.5 days (Table 2).

**Table 2 -** Results from the treatment system, based on the control of samples from the *affluents* and *effluents* of each pond (total detention time = 18.6 days)

| Average, Minimum and Maximum of | affluents and effluents of the ponds | |
|---|---|---|
| | **AP1 - AP2** | **FP** |
| $BOD_5$ (mg/l) | 661.8 - 98.8<br>661.8 - 32.0<br>661.8 - 180.0 | 98.8 - 61.4<br>32.0 - 20.0<br>180.0 - 124.0 |
| COD (mg/l) | 1,289.4 - 231.9<br>1,289.4 - 47.0<br>1,289.4 - 538.0 | 231.9 - 149.2<br>47.0 - 37.0<br>538.0 - 393.0 |
| TS (mg/l) | 1,088.6 - 335.9<br>1,088.6 - 120.0<br>1,088.6 - 730.0 | 335.9 - 267.2<br>120.0 - 80.0<br>730.0 - 720.0 |
| SS (mg/l) | 10.6 - 0.3<br>10.6 - 0.01<br>10.6 - 1.3 | 0.3 - 0.1<br>0.01 - 0.01<br>1.3 - 0.5 |
| OD (mg/l) | - | A B C<br>0.1 1.0 1.9<br>0 0 0<br>2.4 7.6 10.4 |
| OCAM (mg/l) | 103.6 - 44.6<br>103.6 - 23.0<br>103.6 - 80.0 | 44.6 - 34.1<br>23.0 - 11.0<br>80.0 - 83.0 |
| pH | 6.98 - 7.2<br>6.98 - 7.0<br>6.98 - 7.6 | 7.2 - 7.7<br>7.0 - 7.3<br>7.6 - 8.2 |
| TC – 24 hours (nmp/100 ml) | 1.75E8 - 6.14E6<br>1.75E8 - 0.03E6<br>1.75E8 - 35.00E6 | 6.14E6 - 4.26E6<br>0.03E6 - 0.04E6<br>35.00E6 - 92.00E6 |
| TC – 48 hours (nmp/100 ml) | 2.85E8 - 16.77E6<br>2.85E8 - 0.33E6<br>2.85E8 - 92.00E6 | 16.77E6 - 5.58E6<br>0.33E6 - 0.09E6<br>92.00E6 - 92.00E6 |
| Temperature ($^0$ C) | 22.1<br>16.5<br>27.0 | 22.4<br>17.0<br>28.0 |

The anaerobic ponds affluents were assemblage samples, and the effluents were a common point (facultative pond affluent). Average environmental temperature during the control period was 20.4 $^oC$, with minimum of 13.5$^o$C and maximum of 26.5$^o$C.

## Pond Efficiency

Considering the average concentration values of the affluents and effluents in terms of $BOD_5$ of each pond, as well as their detention time, the efficiency of mathematical models for them can be presented by the equation:

$$E_i = \frac{k_i . t_i}{1 + k_i . t_i} \quad (1)$$

where: $E_i$ was the removal efficiency of pond i, and $t_i$ were the detention time, in days.

Although the BOD removal kinetic was the same for the anaerobic and facultative ponds (first order kinetic), the higher was the medium concentration the higher was the BOD removal rate. The adjusted efficiency curves of the anaerobic ponds (AP) and the facultative one (FP) are presented in Figures 2 and 3.

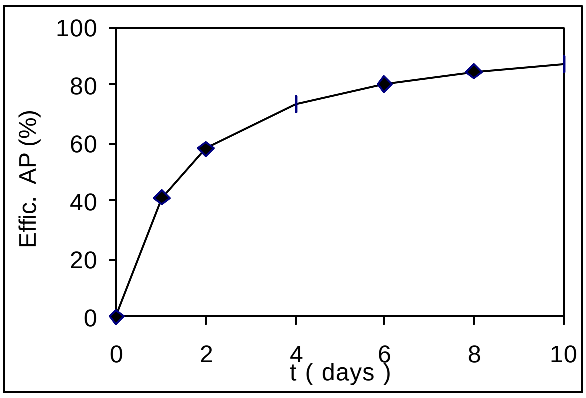


**Figure 2 -** Relation between the BOD efficiency and detention time in the pond AP

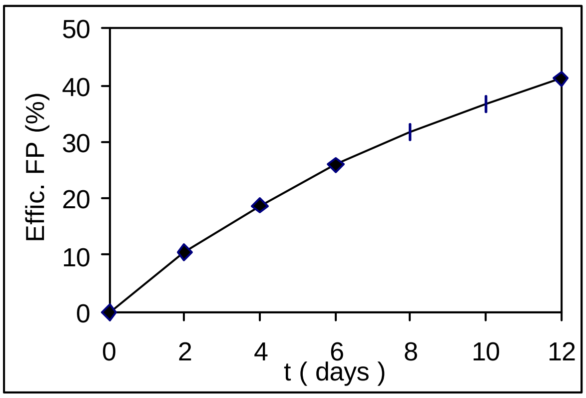


**Figure 3 -** Relation between the BOD efficiency and detention time in the pond FP

## Detention Time

The detention time of each stabilization pond is expressed by:

$$t_i = \frac{V_i}{Q} \quad (2)$$

where: $V_i$ is the volume the pond i, in $m^3$, and Q is the system capacity, in $m^3$/day.

## Costs Models

In the economical analysis of the wastewater treatment system, the total cost includes costs with the land occupied by the system, construction and maintenance. Therefore it is necessary to obtain the costs models, so that:

$$C_T = C_l + C_c + C_m \quad (3)$$

where: $C_T$ is the total cost of the system; $C_l$ is the land cost; $C_c$ is the construction cost and $C_m$ is the maintenance cost.

The initial investment includes the following costs: a) acquisition of the land occupied by the system plus 50% for the traffic of vehicles and/or people; b) ponds construction including the land cleanse, mechanical excavation, transport of the exceeding land and compaction.

## Land Cost

The land cost is associated not only to the area occupied by the ponds system, but also to its adjacent area. Thus, the mathematical model that better represents this cost is expressed by the equation:

$$C_{li} = 1.5\ \gamma_i\ Pl\ V_i \quad (4)$$

where: $C_{li}$ is the cost of the land occupied by pond i and its adjacent area, in US$; Pl is the land price, in US$/$m^2$; $\gamma_i$ is the relation between the surface and the volume of pond i, in $m^2/m^3$.

## Construction Cost

The adjusted mathematical model that better describes the cost of the earth transport is given by (Medri, 1997):

$$C_{ci} = 5.914\ V_i^{0.95} \quad (5)$$

where: $C_{ci}$ is the construction cost of pond i, in US$.

## Maintenance Cost

The maintenance cost results from a series of monthly expenses, along a certain time. Yang et al (1997), working with ponds for the treatment of swine dejects, recommended a period of 10 years. Hess (1980), studying domestic sewage ponds suggested a period of 20 years. In this case, the maintenance of the ponds system was characterized by the sum of the area reserved for the traffic of vehicles and/or people, which represented 50% of the ponds area. Thus, the month costs would be calculated in the date of the present investment by the equation (Medri, 1997):

$$C_m = f \;\; 0.164(\sum_{i=1}^{n} g_i \; V_i)^{0.83} \qquad (6)$$

where,

$$\Phi = \frac{(1 + r)^n - 1}{r\,(1 + r)^n} \qquad (7)$$

and $C_m$ is the total cost of the ponds system maintenance, in US$; $\phi$ is the factor of the present value; r is the annual interest rate and n is the life time of the ponds, in years.

From equation (4) to (6), it is possible to calculate the ponds cost:

$$C_i = 1.5\,\gamma_i\,Pl\,V_i + 5.914\,V_i^{0.95} + C_{mi} \qquad (8)$$

From equation (1) to (2), it is possible to have the ponds volume:

$$V_i = Q\,E_i\,[k_i\,(1 - E_i)]^{-1} \qquad (9)$$

Substituting equation (9) in equation (8), it gives the cost ponds:

$$C_i = 1.5\,\gamma_i\,Pl\,Q\,E_i\,[k_i\,(1 - E_i)]^{-1} + 5.914\{Q\,E_i\,[k_i\,(1 - E_i)]^{-1}\}^{0.95} + C_{mi} \qquad (10)$$

where $C_i$ is the cost the pond i, in US$.

## System Optimization

The optimization in this case consists of minimizing the total costs of the treatment system, making it as frugal as possible and with adequate effluent in terms of organic matter (Meisheng et al., 1992; Kezhao, 1994). Thus, the objective function in the minimization of the total costs is characterized by the sum of the total costs with land, construction, maintenance, and the system restriction condition is the wished final quality of the effluent, so that:

$$Min\ C_T = \sum_{i=1}^{n} C_i$$

$$s.to.:\ E_O \geq E_w \qquad (11)$$

$$0 \leq E_i \leq 1$$

where: $C_T$ is the system total cost, in US$; $E_o$ is the obtained efficiency and $E_w$ is the wished efficiency.

Thus, considering two anaerobic ponds in parallel followed by a facultative one, as the studied system, the problem can be formulated as followed:

$$\begin{aligned} Min C_T = {} & 0.5\,x\,1.5\,g_1\ Pl\ Q\ E_1\,[k_1\,(1-E_1)]^{-1} + 5.914\,\{0.5\,Q\,E_1\,[k_1\,(1-E_1)]^{-1}\}^{0.95} + \\ & 0.5\,x\,1.5\,g_2\ Pl\ Q\ E_2\,[k_2\,(1-E_2)]^{-1} + 5.914\,\{0.5\,Q\,E_2\,[k_2\,(1-E_2)]^{-1}\}^{0.95} + \\ & 1.5\,g_3\ Pl\ Q\ E_3\,[k_3\,(1-E_3)]^{-1} + 5.914\,\{Q\,E_3\,[k_3\,(1-E_3)]^{-1}\}^{0.95} + \\ & f.\,0.164\,\{0.5[0.5\,Q(g_1\,E_1(k_1\,(1-E_1))^{-1} + g_2\,E_2(k_2(1-E_2))^{-1}) + Q\,g_3\,E_3(k_3(1-E_3))^{-1}]\}^{0.83} \\ & 0 \leq E_1 \leq 1\,;\ 0 \leq E_2 \leq 1\,;\ 0 \leq E_3 \leq 1 \end{aligned}$$

$$s.to.:\ 1 - [1 - 0.5\,(E_1 + E_2)](1 - E_3) \geq E_w$$

## Practical Application

Making a study for 15 years and admitting interest rate of 10% per year, the factor of the present $\phi$ value given by equation (7) would be alike 95. Considering the average concentrations in the entrace and exit of the anaerobic (AP1) and facultative (FP) ponds and their detention time, the degradation constants k(BOD) were: 0.703 $d^{-1}$ for ponds AP1 and 0.058 $d^{-1}$ for the FP. These values were determined with average temperature around 22°C.

Land price in north Paraná region is approximately R$ 2,500.00/ha. Considering two years (1998-1999), the devaluation of Real comparing to American dollar approximately 70%, and relation between surface area and volume of each pond as $\gamma_1$=0.55; $\gamma_2$ = 0.58 e $\gamma_3$ = 0.69 $m^2/m^3$, for ponds AP1, AP2 and FP respectively, and admitting only the reduction of the organic matter (BOD), the mathematical model of model minimization is:

$$
\begin{aligned}
Min C_T =\; & 0.5x1.5x0.55x0.250 Q\, X_1[0.703(1-X_1)]^{-1} + 0.7x5.914[0.5 Q\, X_1\,(0.703(1-X_1))^{-1}]^{0.95} + \\
& 0.5x1.5x0.58x0.250 Q\, X_2[0.703(1-X_2)]^{-1} + 0.7x5.914[0.5\, Q\, X_2\,(0.703(1-X_2))^{-1}]^{0.95} + \\
& 1.5\,x\,0.69x0.250 Q\, X_3[0.058(1-X_3)]^{-1} + 0.7x5.914[Q\, X_3(0.058(1-X_3))^{-1}]^{0.95} + \\
& 0.7x95x0.164[0.5(0.5\, Q(0.55 X_1\,(0.703(1-X_1))^{-1} + 0.58 X_2\,(0.703(1-X_2))^{-1}) + \\
& Q\, 0.69\, X_3\,(0.058(1-X_3))^{-1})]^{0.83}\}
\end{aligned}
$$

$$
s.\ to.: \quad 1-[1-0.5\,(X_1+X_2)](1-X_3) \ge 0.85
$$

$$
0 \le X_1,\ X_2,\ X_3 \le 1
$$

**Table 3 -** presents the physical characteristics of ponds and costs with land, construction and system maintenance, supposing discharge of 1,500 $m^3$/day and system efficiency of 85%.

| Pond efficiency | Detention time (days) | Pond volume ($m^3$) | Pond area ($m^2$) |
|---|---|---|---|
| $E_1$ = 0.851 | $t_1$ = 8.1 | $V_1$ = 6,070 | $A_1$ = 3,338 |
| $E_2$ = 0.849 | $T_2$ = 8.0 | $V_2$ = 6,021 | $A_2$ = 3,492 |
| $E_3$ = 0 | $T_3$ = 0 | $V_3$ = 0 | $A_3$ = 0 |
| Land cost (US$) | Construction cost (US$) | Maintenance cost (US$) | |
| $C_{t1}$ = 1,251.90 | $C_{mt1}$ = 16,255.41 | $C_{ma1}$ = 4,565.52 | |
| $C_{t2}$ = 1,309.65 | $C_{mt2}$ = 16,132.04 | $C_{ma2}$ = 4,776.10 | |
| $C_{t3}$ = 0 | $C_{mt3}$ = 0 | $C_{ma3}$ = 0 | |
| Total: 2,561.55 | 32,387.45 | 9,341.62 | |

System total cost: R$ 44,290.62

As expected, the model excluded the secondary facultative pond, because it presented a low performance in the removal of the organic matter (BOD).

## CONCLUSIONS

The results obtained from the stabilization system, consisting of two anaerobic ponds and a chicaned facultative pond, treating the domestic sewage enabled to conclude that:

- the removal efficiency of the carbonaceous pollution (BOD and COD) was realized specially in the anaerobic ponds, with removal of 85% of BOD and 82% of COD with detention time of 8.1 days, while the facultative pond had 10.5 days of detention time and removed only 38% of BOD and 36% of COD;
- the facultative pond presented a low performance in the removal of the organic matter. Targetting a

BOD around 85%-90%, removal efficiency this pond could be excluded, because its inclusion only increased the costs of the process;
- the removal efficiency of the ponds system (AP1, AP2 and FP) was around 91% for BOD; 88% for COD; 75% for TS; 99% for SS; 98% for TC-24h and TC-48h, with total detention time of 18.6 days;
- in series stabilization ponds system for the treatment of domestic sewage, the degradation constant value of BOD of the anaerobic pond was highest than the facultative pond, because it was biodegraded easily, and the remained organic matter was more resistant to biodegradation.

The total cost (cost with land, construction and maintenance) of the ponds system: two anaerobic ponds in parallel (AP1 and AP2), followed by a facultative (FP) in series was US$ 42,290.62 for an efficiency of 85% of BOD, discharge of 1,500 $m^3$/day, and admitting interest rate off 10% per year during 15 years.


## RESUMO

Este trabalho apresenta uma aplicação para sistematizar a construção de sistemas de lagoas para tratamento de esgoto doméstico. O sistema consiste de duas lagoas anaeróbias operando em paralelo durante o período de maio/97 até abril/99. Estas lagoas eram conectadas em série com uma lagoa facultativa chicaneada. O sistema de tratamento foi monitorado com amostra coletadas no esgoto bruto (amostra composta), nos afluentes e efluentes das lagoas e ao longo dos fluxos das lagoas anaeróbia e facultativa. Os seguintes parâmetros foram analisados: pH, Demanda Bioquímica de Oxigênio (DBO), Demanda Química de Oxigênio (DQO), Sólidos Totais (ST), Sólidos Sedimentáveis (SS), Coliformes Totais (CT), Oxigênio Consumido em Meio Ácido (OCMA) e Temperatura.


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