ARTICLE

We are Living on the Edge: Managing Extreme-Severity Claims Using Extreme Value Theory

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ABSTRACT

Claims with high severity and low probability constitute a high risk for the insurance market. One tool to deal with this kind of event is the Extreme Value Theory (EVT). The main goal of this article is to apply the EVT to Actuarial Science using a different estimator for parameters, allowing the calculation of pure reinsurance premiums and the choice for the retention limit for insurance companies. The execution was split into two parts: (i) comparing the estimators through simulations, and; (ii) using data from 5 SUSEP lines of business with different natures, intending to estimate some extreme value statistics. In simulation studies, the Pickands estimator was very promising, but the limited amount of data resulted in a great variance when applied to real cases. Lastly, we concluded that the EVT is a powerful tool for insurance, capturing the behavior of extreme clams amount better than traditional models.

KEYWORDS

Insurance, Extreme values, Expected shortfall, High quantiles estimation

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Received: 03/07/2022.
Revised: 08/19/2022.
Accepted: 11/16/2022.
Published: 10/24/2023.
DOI: https://doi.org/10.15728/bbr.2022.1245.en

We are living on the edge: gerenciando sinistros de extrema severidade usando a Teoria de Valores Extremos

RESUMO

Ocorrências de altíssimo valor monetário e baixíssima probabilidade constituem grandes riscos no mercado segurador. Uma ferramenta para análise de eventos dessa natureza é a Teoria de Valores Extremos (TVE). O objetivo deste trabalho é aproximar a TVE das Ciências Atuariais, utilizando diferentes estimadores de parâmetros, possibilitando o cálculo de prêmios de resseguro e a escolha do limite de retenção ideal para seguradoras. A execução foi em duas etapas: (i) comparando os estimadores em simulações, e; (ii) usando microdados reais oriundos de 5 ramos SUSEP de naturezas distintas, com o intuito de estimar estatísticas de valores extremos. Em geral, o estimador de Momentos foi o mais consistente de todos. O estimador de Pickands nas simulações apresentou resultados promissores, mas a quantidade restrita de dados e sua grande variância o tornam inconstante na aplicação aos dados reais. Finalmente, observou-se que a TVE é um poderoso ferramental para a área de seguros, melhor capturando o comportamento de sinistros extremos do que métodos tradicionais.

PALAVRAS-CHAVE

Seguros, Valores extremos, *Expected shortfall*, Estimação em altos quantis

1. INTRODUCTION

Risks that cannot be avoided should be transferred or retained (Vaughan & Vaughan, 2013). Usually, the transferred kind of risks have low probability and high severity. For individuals, the process is simple: they buy insurance, and the insurance company takes on the responsibility to pay for eventual losses. However, insurance companies are also subject to catastrophic events, making it necessary to buy reinsurance. From an individual perspective, transferring risks is just an instrument to deal with losses, but for insurance companies it is the core business. If the designing process is not done precisely, the company may be under or over protected (Dempster et al., 2003; Dietz & Walker, 2019).

Low probability and high severity events are inherent to the insurance process, and typically happens when several unities with high financial value are damaged at the same time or have some dependency structure between them (Chen & Yuan, 2017; Gatto, 2020; Tanaka & Carvalho, 2019). In such cases, the company may face negative income and generate future solvency problems (Carvalho & Cardoso, 2021; Damasceno & Carvalho, 2021; Euphasio Junior & Carvalho, 2022; Ramsden & Papaioannou, 2019; Wüthrich, 2015).

To illustrate the dimension of such a problem, two catastrophic situations may be taken as examples. The first one is regarding the Mariana dam tragedy which occurred in 2015: a study from Terra Brasis (2017) showed that the disaster had an estimated impact of R$26.2 billions in damages, but only R$2.25 billions were insured. This kind of event, if not properly transferred, could lead the company Vale to bankruptcy. In a more general sense, according to SUSEP, in the same year, about R$95 billions were issued as insurance premiums (both Non-Life and Life). This means that just one event represented 2.1% of the total premium value for every line of business combined, demonstrating the importance of estimation of catastrophic losses. As an international example. according to a study made by Aon (2018), the damage caused by natural disasters has high variance throughout the years. In some years, climate and anthropogenic events can be more prone to generating insurance payments (Mendes-Da-Silva et al., 2021). Therefore, estimating the frequency and severity of such occurrences is crucial to the insurance activity, so that companies may plan how to better handle transference and make adequate their risk management policies (Cummins & Weiss, 2014; Kasumo et al., 2018).

One of the available tools to deal with high-risk and low-frequency events is the Extreme Value Theory (EVT). It is a methodology originated from the natural sciences and was adopted by several areas of finance by the early 1990s. It consists in a set of techniques that aim to model the distribution of maximum from collections of random variables, to estimate tails of distributions, and to measure the probability of occurrences in high quantiles. Applied to the Actuarial Science, it allows one to model the minimum solvency capital, calculate reinsurance premiums, and is also important to evaluating the estimation bias of parameters, dependence between observations, and to choose the ideal retention limit for risks by an insurer (McNeil, 1997). Although the development of EVT is not recent, and its spectrum of application is quite wide, in Brazil there are few articles that use this technique for Actuarial Science (Melo, 2006).

The main goal of this study is to obtain the best adjustments of claim's distribution tails using different parameter estimators provided by EVT. First, the efficiency of the methodology is evaluated through simulations in controlled scenarios. Then, the tested models will be used to estimate the tail of claims amount from 5 lines of business (LoB) with different characteristics, from light to heavy tails, in order to estimate the Expected Shortfall (ES), a summary statistic of extreme values. The results will be discussed according to each line of business nature, which EVT model best fit the data, and the consistency and reliability of the methods used. A comparison between the EVT and traditional models used in claims adjustment will also be made.

2. THEORETICAL BACKGROUND

One of the EVT's main approaches consists of splitting the distribution function into two disjointed domains, fitting a model for each subdomain. Many authors are more concerned with high quantiles than common occurrences because it is in the tail that events with greater impact can materialize and demand more careful risk management strategies. For example, Haan (1990) applied EVT to model sea levels in order to make decisions regarding the optimal size of dams in the Netherlands, a country where approximately 40% of the territory is below sea level.

Along with the development of heavy-tailed distributions in probability theory, some statistics were proposed to infer how frequent the extreme values of a distribution are. The estimator proposed by Hill (1975) for the Pareto distribution is commonly applied due to its simplicity and its useful statistical properties, such as being asymptotically Normal (Haeusler & Teugels, 1985) and converging to the smallest possible variance (Hall & Welsh, 1984). Brazauskas and Serfling (2000) suggested an estimator based on the sample median of Hill estimators, and compared its efficiency with other widely applied estimators, evaluating the impact that sample outliers may have.

Another historically important approach to study distribution tails was given by Pickands (1975) for the Generalized Pareto and it was based in order statistics. Estimators of this kind are generally less susceptible to bias than Hill's, but their variance is usually higher (Yun, 2000). To compensate for this disadvantage, Taylor and Falk (1994) considered a convex combination between two Pickands estimators, while Drees (1995) generalized this process to an arbitrary number of combinations. Yun (2000), on the other hand, approached the Pickands estimator using other order statistics, resulting in variance up to 10% lower.

In finance, EVT models are mainly used in the analysis of VaR (Value at Risk), which is a measure of investment portfolios exposed to severe losses, being constantly compared to results obtained by time series models. Daníelsson and Morimoto (2000) compared the application of GARCH models with EVT in Japanese market indices in the calculation of VaR, and concluded that, in addition to being more stable, the EVT methods exhibited better precision and lower variance. When modelling exchange rates, Iglesias (2012) proposed an estimator and made a comparison with Hill by using the Hausman test. He concluded that Hill is better in most cases in relation to his new estimator. Stupfler and Yang (2018), while seeking for an approach that captures higher levels of risk aversion in post-crisis situations, used the EVT for the pricing of CAT Bonds, securities that back catastrophic risks issued by insurance companies.

One of the main criticisms of this application of the EVT is the usual existence of correlation within the data, which could lead to a greater bias in estimates. Furthermore, there is no efficient method to determine the optimal threshold in which EVT is valid. For this reason, Haan et al. (2016) proposed an estimator that seeks to solve the previously mentioned problems, and their results point to greater efficiency when applied to the *Dow Jones* index.

Only recently, researchers in Actuarial Sciences started using EVT for insurance. In some cases, it may be necessary to model claims whose development has not yet completed, thus having a portion of their value estimated. Beirlant et al. (2018) adjusted several EVT estimators for a sample of car claims, in which part of the claim is still unknown. The authors concluded that the proposed estimator has reduced the bias. In life insurance, Huang et al. (2020) used EVT methods to complete actuarial tables at advanced ages. As few people reach ages beyond 100 years, these cases are commonly approximated or even excluded in conventional tables, which makes it difficult to estimate large losses in the case of retirement plans.

McNeil (1997) modeled Danish fire insurance data using the maximum likelihood estimator (MLE) along with complementary graphical methods. The author concluded that EVT should be more explored by actuaries because it is a useful tool in analyzing risks at higher quantiles, but the parameters may have high uncertainty because of the choice of the retention limit. This study, due to its impact as one of the first applications of EVT at Actuarial Science, was revisited by Resnick (2014) with new techniques developed during this period. In it, the author concluded that the assumptions of independence between observations made by McNeil (and, in general, in insurance modeling) are accurate, in addition to the estimators pointing to similar models in both articles. However, the sensitivity regarding the choice of the tail's threshold remains a problem to be explored.

In the Brazilian case, Melo (2006) used the EVT for tail modeling. The author's approach aimed for reinsurance in the comprehensive condominium and business insurance against fire, lightning, and explosions. The objective was to compare distributions traditionally used to fit claims value, such as Gamma and Log Normal, to others that could be more adequate (EVT) in modeling tails. The author concluded that the EVT models produced better results when estimating high quantiles, which agrees with the existing literature. Thus, based on the conclusion about EVT models and the wide existence of estimators, our intention is to expand the author's results when considering different methods of adjustment for the EVT models, especially the Generalized Pareto Distribution.

3. METHODOLOGICAL PROCEDURES

In this section, we will define the problem to be studied in a mathematical way. Then a small review of the EVT will be presented, a summary of the main results that gives us a basis for discussing the different estimators. Finally, we will give an overview about the simulation process.

3.1. *Peaks over Threshold*

Let $Y_1, \ldots, Y_n$ be a collection of random variables from a common distribution $F_Y(.)$, that represents claims in a given period of time. Consider U as a retention limit, y_f the supreme of this random variable support and let $Y_{n:i}$ be the order statistics as the i-th greatest term. The tail of a distribution is defined as each Borel set to the right from U, i.e., $X=(Y-U)_+>0$. It is important to make clear the difference between Y and X. The first represents the total amount of claim, and the latter is the exceeding value given a retention limit U, the theoretical reinsurance liability.

The *Peaks Over Threshold* (POT) method is a tool to model the Y distribution. It consists in dividing $F_Y(.)$ into two subdomains and modeling only the portion to the right-hand side of a given value, the portion pertaining to extreme values, as described in Equation 1. The method's advantage is that the data is often influenced by different phenomena, which could make a single distribution to have bias and unable to capture the entirety of the event. Using just a single model, it may fit the central part of the distribution better, but the extremes are more prone to errors. In this study, we focused into estimating $F_X(.)$.

$$F_Y(y) = \{\hat{F}_Y(y), \hat{F}_Y(U) + \left[1 - \widehat{F_Y}(U)\right] \times F_X(y-U), \frac{0 \le y \le U}{y > U} \quad (1)$$

3.2. The Extreme Value Distribution

The first approach to extreme values is through the distribution of random maximum. Consider $m < n$ and define $M_m = Max(Y_1, \ldots, Y_m)$. Knowing $F_Y(.)$, it is possible to model M_m as equation 2:

$$P\left(M_m = Max\{Y_1, \ldots, Y_m\} < y\right) = P(Y_1 < y \cap \ldots \cap Y_m < y) = P^m(Y < y) = F_Y^{\,m}(y) \quad (2)$$

Taking the limit when $m \to \infty$:

$$\lim_{m \to \infty} F_Y^m(y) = \begin{cases} 1, y \ge y_f \\ 0, y < y_f \end{cases} \quad (3)$$

In other words, the maximum distribution converges to a degenerate distribution and does not bring any useful information regarding the model. Thus, to obtain some relevant estimate for high values of the distribution we may consider transformations to rescale the random variable. According to Gnedenko (1943), if exists sequences $\{a_m\}$, $\{b_m\}$ such that $b_m>0$ for every m satisfying:

$$\mathbb{P}\left(\frac{M_m - a_m}{b_m} < y\right) = F_Y^m(y \times b_m + a_m) \overset{m \to \infty}{\to} H(y) \quad (4)$$

and if $H(.)$ is not degenerate then it will be:

$$H(y) = \begin{cases} exp\left\{-\left[1-(1+\gamma y)^{-1/\gamma}\right]\right\}, \gamma \neq 0 \\ exp\left\{-\left[1-e^{-y}\right]\right\}, \gamma = 0 \end{cases} \quad (5)$$

The H(.) function is called *Extreme Value Distribution* (EVD), and its format only depends on the shape parameter γ, called the extreme value index. The function H(.) falls into 3 different domains: if $\gamma<0$, H is a *Weibull* distribution; when $\gamma=0$ is the occurrence of a *Gumbel*; and, finally, with $\gamma>0$ it becomes a *Fréchet*. Each family is related to one of the cases cited above, in which it is said to belong to a certain domain of attraction of H(.), with γ less than, equal to or greater than 0. If there are two distinct pairs of sequences $\{a_m\},\{b_m\}$ e $\{a_m*\},\{b_m*\}$ for which H(.) is non-degenerate, then the final γ leads to the same domain of attraction (Resnick, 1971).

3.3. The Generalized Pareto Distribution

Another approach to study rare occurrences is to evaluate tail convergence properties. The objective is to find a limiting distribution that approximates the tail $F_X(.)$. An important result was obtained by Pickands (1975). Let $0<U<y_f$ be a retention limit. Then:

$$supr\left|\frac{F_Y(x+U)-F_Y(U)}{1-F_Y(U)}-G(x)\right|=supr\left|F_X(x)-G(x)\right|=0 \quad (6)$$

with

$$G(x)=\begin{cases}1-\left(1+\frac{|\gamma|x}{\sigma}\right)^{\frac{-1}{|\gamma|}},\gamma\neq 0\\ 1-e^{-\frac{x}{\sigma}},\gamma=0\end{cases} \quad (7)$$

In Equation 7, σ is a scale parameter and the G(.) distribution is called *Generalized Pareto Distribution* (GPD), and, just like the H(.) function, it unifies three families of distributions with different properties. When $\gamma<0$, G(.) corresponds to a *Beta*, used to model tailless or light tail random variables, having limited support ($y_f<\infty$) and all moments defined. Distributions contained in this case are *Uniform* and *Weibull*. For $\gamma=0$, the limiting distribution is an *Exponential*, having no heavy tail, finite moments, and infinite support. This family encompasses the largest number of functions, with Normal, Log-Normal and Gamma being the main representatives. Lastly, when $\gamma>0$, we have the *Pareto* distribution, which encompasses heavy-tailed distributions, with occasional undefined moments and power-law decay of the tail. Among its representatives are *Log-Gama*, *Cauchy*, and *Burr*. The Pareto family is usually chosen for modeling tails in insurance because it is more conservative to estimate high quantiles.

Pickands (1975) also defined a parametric estimator for location and scale of this distribution based in order statistics. Consider k integer such that $0<4k<n$. The estimators are:

$$\gamma_k^{\ P}=ln(2)^{-1}ln\left[\frac{X_{n:k}-X_{n:2k}}{X_{n:2k}-X_{n:4k}}\right] \quad (8)$$

$$\sigma_k^{\ P}=(X_{n:2k}-X_{n:4k})/\int_0^{ln2}e^{\gamma_k^{P}u}du \quad (9)$$

The same Greek letter γ was used in both the H(.) and G(.) distributions for the *extreme value index* because this indicator denotes the same domain of attraction (Pickands, 1975). We also calculated the maximum likelihood estimator of the GPD by maximizing the differentiation of equation (7) from the sample's perspective.

3.3.1. The Pareto Distribution

The case γ>0 (Pareto) is the most important. In general, distributions that converge to this domain of attraction can be represented by *L(x)*, a slowly varying function:

$$F_X(x) = 1 - x^{-\alpha} \times L(x) \quad (10)$$

where α is related to GPD through the ratio α=*1/*γ. When *L(x)=c*, a real-value constant, the MLE can be computed according to Hill's (1975) likelihood maximization procedure. The difference between Hill's procedure and the one cited in section 3.3 is that Hill considers only the γ parameter and has an analytical solution, while the one in section 3.3 considers γ and σ simultaneously. The mathematical formulation of the Hill index is given by:

$$\gamma_k^{\,H} = 1/\alpha_k^{\,H} = \frac{\sum_{i=1}^{k-1} ln\left(\frac{X_{n:i}}{X_{n:k}}\right)}{k} \quad (11)$$

A generalization of the Hill estimator proposed by Dekkers, Einmahl and Haan (1989) will also be used in this article. The Moments estimator has convergence properties similar to Hill, but with the advantage of being more reliable in non-heavy tail scenarios (Resnick, 2014). Its functional form is quite similar to Equation 11:

$$M_k^j = \frac{\sum_{i=1}^{k-1} ln\left(\frac{X_{n:i}}{X_{n:k}}\right)^j}{k} \quad (12)$$

$$\gamma_k^{\,M} = 1/\alpha_k^{\,M} = M_k^1 + 1 - \frac{1}{2}\left[1 - \frac{\left(M_k^1\right)^2}{M_k^2}\right]^{-1} \quad (13)$$

Estimators from this family have good statistical properties. However, they are subject to the *variance vs. bias tradeoff*: i.e., the more observations are used in the estimation, the greater the bias is and the smaller is the variance of the estimator, a fact that makes choosing the beginning of the tail a sensitive process (Caeiro et al., 2020).

3.4. Simulation Method

The execution will be split into two parts. Initially, the algorithms will be tested in controlled scenarios by simulations to observe the differences between the methods, and results obtained. Afterwards, the techniques will be applied to real anonymized data from an insurance company.

In the simulations, the data will be generated from a GPD with different magnitudes of the γ parameter and sample sizes. The main goal is to assess whether the estimators capture the correct domain of attraction and how close they get to the fixed γ. The algorithm process is described as:

1. Generate a random sample of n observations from claim amounts Y with distribution $F_Y(y)$;
2. From the sample, obtain *Horror Plots* (HP) to assess the estimators' behavior of convergence as a function of the number of observations;
3. Plot the histogram of each estimator and summarize some statistics in tables;
4. Determine which method came closer to the true value;

3.5. Other Analysis Tools

The HP of a sample consists of values sorted in descending order, and, starting from the i-th element, calculate the respective estimator for the subset between the first and the i-th element. The objective is to evaluate the convergence of estimates for tails, depending on the number of available observations used to calculate.

The *Expected Shortfall (ES)* is defined as the expected loss beyond a threshold u, and its behavior is determined by the domain of attraction of the Generalized Pareto. For positive (negative) γ, it has an increasing (decreasing) behavior as a function of u, i.e., the expected loss increases (decreases) according to the threshold. When $\gamma=0$, the Generalized Pareto becomes an Exponential, which *ES* assumes a constant value due to the memoryless property. Thus, graphs of ordered pairs described by Equation 14 can be used to search for regions with homogeneous tail behavior and to define the retention limit.

$$\left\{ \left(Y_{n:k}, \frac{\sum_{i=1}^{k-1} \left(Y_{n:i} - Y_{n:k} \right)}{k-1} \right), 2 \leq k \leq n \right\} \quad (14)$$

It is important to notice that multiplying the *ES* by the probability $1\text{-}F_Y(u)$ of exceeding the threshold constitutes the actuarial pure excess-damage reinsurance premium. Because an actuarial pure premium is the amount to be paid for a (re)insurance policy equivalent to the mathematical expectation of the future claims present values to which an economic agent is subject (Bowers et al., 1997; Landes, 2015).

Sample *QQ-Plots* against an exponential distribution with the same mean, representing the domain of attraction in which $\gamma=0$, will also be used. The objective is to indicate whether the data has a heavier (lighter) tail than an exponential, which would indicate a positive (negative) γ.

4. SIMULATION RESULTS

Figure 1 has the HP for the three tested estimators of a sample with 2,500 observations generated from a GPD with $\gamma=1.5$ and $\sigma=100$. It is observed that the Hill estimator requires a low number of data points in its estimation, otherwise the presence of bias becomes more intense. The Moments estimator follows the same trend as Hill, but its bias is lower. The Pickands estimator has the greatest variance of them; but it does not have bias, thus the need to use all available data for its calculation. The MLE does not need to be presented, due to its nature of being directly calculated, not being subject to bias due to the fact we know the density function.

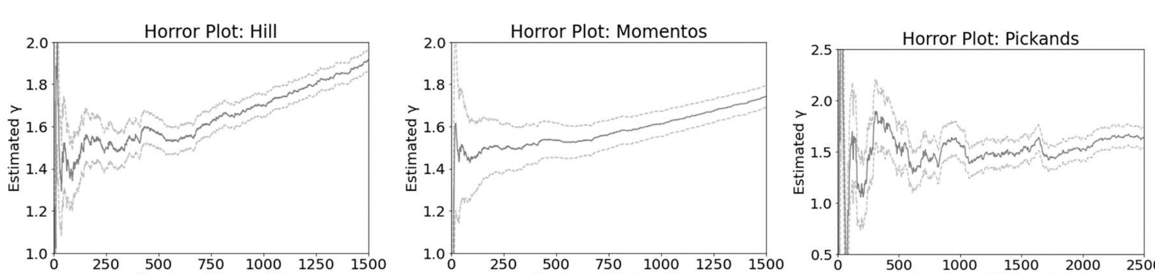

Figure 1. Resulting HP for different estimators simulated from a GPD with γ=1.5; σ=100; 2,500 observations.
Source: own elaboration.

In a second approach, 5,000 random samples of size 2,500 were generated and the 4 estimators fit for each, given a number of order statistics as a parameter. The histograms of the results with parameters γ=*1.5* and σ=*100* can be found in Figure 2, and in Table 1 we show the descriptive statistics. From Figure 2, the distribution of each estimator is close to a Gaussian. Table 1 shows that there is some difference between the actual value of the parameters and those obtained by the Hill and Moments estimators, highlighting the problem of variance vs. bias. The larger the sample is, the greater the bias present in its estimation and the smaller the variance of the estimator. However, the Moments estimator still achieves better results than Hill, both in terms of bias and standard deviation, when comparing Moments with n=300 and Hill with n=125.

Hill with n=500 is a scenario in which the variance and mean were very similar to the Moments estimator with n=700, and also the greatest bias, around 0.05. In this scenario, the estimator obtained would be the most conservative, as they indicate a heavier tail than the real one, in a scenario in which the variance of losses is not well defined (because γ>*1*). In contrast, because the sample contains a lot of observations, the Pickands estimator achieves a low amount of standard deviation, slightly smaller than Moments with 300 and Hill with 125 observations, while maintaining zero bias. As for the MLE, because the data was generated by its own function, it was the best scenario both in terms of bias and standard deviation, as expected. However, caution

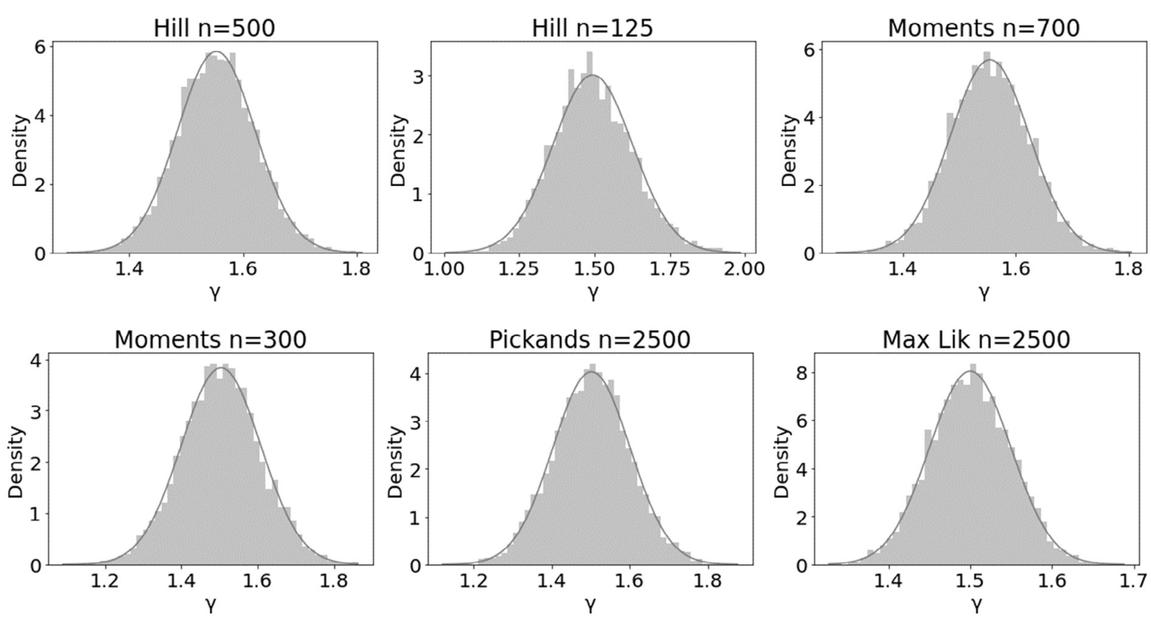

Figure 2. Resulting Histograms for different estimators simulated from a GPD with γ=1,5; σ=100; 2500 observations.
Source: own elaboration.

Table 1
Comparison between the estimators for γ from a GPD with parameters γ=1,5; σ=100; 2,500 observations

Estimator	Hill	Hill	Moments	Moments	Pickands	MLE
Used Data (n)	500	125	700	300	2,500	2,500
Mean γ	1.5529	1.4969	1.5530	1.5039	1.4975	1.4993
Standard Deviation	0.0687	0.1334	0.0707	0.1054	0.0987	0.0499
Bias	0.0529	0.0031	0.0603	0.0039	-0.0025	0.0007

Source: own elaboration.

is needed when considering this estimator as most adequate, because it was being tested on its own distribution. In practice, the true tail distribution is unknown.

The same exercise was replicated for samples of size 2,500 obtained from Generalized Pareto with parameters $\gamma=0.1$; $\sigma=10$. The results of an illustrative replication are presented in Figure 3. In this case, Hill had a very intense bias, even when almost no data was being used in its estimation due to its heavy tail estimation nature. The same comment applies to the Moments estimator, as its bias grows much faster than the previous scenario. Pickands is extremely volatile at the beginning but manages to stabilize at the end.

In the experiment of successive extractions of samples, due to space limitations the histograms were omitted because the formats were not very different from the previous case and the Normal approximation is still valid. Table 2 brings the descriptive statistics of this experiment. The Hill estimator bias is evident, because even while using only n=12 or 25 (amount of data in which the estimator still presents stability at some level), the difference between the estimated coefficient and the actual one was around 0.1. The Pickands estimator was again more efficient than the Moments estimator, since that to obtain the same bias, the standard deviation of the Moments estimator needed to be higher. If the objective is the same variance, however, then there will be a bias of 0.04. The MLE had once more results similar to Pickands in terms of mean, but its standard deviation was smaller.

It is worth noticing that, besides MLE, whose good results were already expected as it is a direct estimation method, Pickands in every controlled experiment was always the closest to the true γ coefficient value. In the unbiased cases, the Pickands variance was similar or smaller than Hill and Moments. However, when some bias is tolerated, it was possible to obtain estimates of lower variance as in the case $\gamma=1.5$ using Hill and Moments. Another important fact is that Hill and Moments were better applicable for larger values of γ, otherwise the bias can be significant due to the bias vs. variance trade-off aforementioned. However, the deviation always occurred towards being more conservative, and such situation may be preferable than incurring a greater variance. Especially in situations in which risk overpricing (generating potential opportunity costs) is less harmful than exposure to company's ruin.

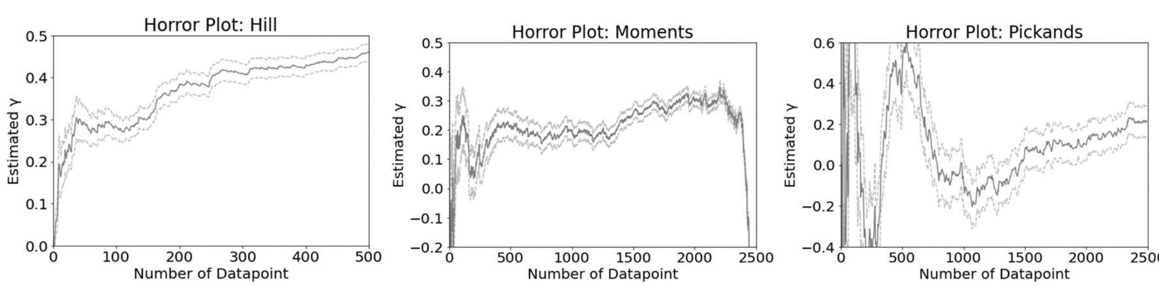

Figure 3. Resulting HP for different estimators simulated from a GPD with $\gamma=0.1$; $\sigma=10$; 2,500 observations.
Source: own elaboration.

Table 2
Comparison between the estimators for γ from a GPD with parameters γ=0.1; σ=10; 2,500 observations

Estimator	Hill	Hill	Moments	Moments	Pickands	MLE
Used Data (n)	12	25	62	225	2,500	2,500
Mean γ	0.1959	0.2270	0.1057	0.1397	0.0999	0.0993
Standard Deviation	0.0555	0.0422	0.1292	0.0700	0.0737	0.0217
Bias	0.0959	0.1270	0.0057	0.0397	-0.0001	-0.0007

Source: own elaboration.

5. APPLIED RESULTS

In this section, we will present the results obtained when applying the different methods of estimation to a database obtained from a multinational insurance company that operates in Brazil. The claims were incurred and settled between January/2006 and May/2012. Five lines of business (LoB) with different characteristics were used for comparison purposes: 0196 (Large and Operational Risks), 0167 (Engineering), 0351 (General Civil Liability), 0531 (Automobile) and 0993 (Group Life). All monetary values expressed in constant currency as May/2012, allowing direct comparison. Table 3 presents the descriptive statistics.

The *ES* and *QQ-Plot* for each LoB are presented at Figure 4. In LoB 0531, a tendency to light tails may be observed, as the *QQ-Plot* is below the Exponential distribution and *ES* suddenly drops. As for LoB 0993, its *ES* is increasing and its *QQ-Plot* is above the Exponential line, but there is a tendency towards stabilization at high quantiles. This may indicate a distribution without a heavy tail: i.e., for both cases it is not expected that there will be expressive gains from EVT adoption.

From Table 3, LoB 0196 has the greatest dispersion. This can be seen in Figure 4, as it is a LoB that traditionally presents high-magnitude claims and requires different model for extreme values. There is an important detachment of the *QQ-Plot* relative to the Exponential and the *ES* is increasing. However, an apparent stabilization in the tail of this LoB happens, as in 0993. This occurs because the last 3 observations were very close (R$36,827,195 and R$35,086,290). Lastly, in LoB 0351 and 0167, both *QQ-Plot* and *ES* indicate distributions with heavy tails.

As the determination of the "extreme values" threshold is discretionary for each case, a tail cutoff value was determined solely based on the *QQ-Plot* and *ES* behaviors. This cutoff can be understood as the retention limit at the reinsurance contract in the excess of loss modality, with the excess claim paid to the insurer by a reinsurer. We looked for region where there is greater stability in the most extreme observations in both elements (*QQ-Plot* and *ES*). The only exception was in LoB 0196, in which there is no well-defined behavior for both criteria, so HP[1] was used.

The comparison of the models was done by using the sum of squared errors (SSE), a measure defined as the difference between the observed and estimated accumulated function, as in Equation 16. For the plots and calculation of SSE of Pickands and MLE, the estimated scale parameter σ was calculated from their respective method described at section 3. Since Hill and Moments does not describe a way to obtain σ, the one that minimizes the SSE was used (numeric approach). A Gamma distribution was also fitted as a baseline model, to allow comparison between EVT and more traditionally used distributions.

Table 3
Descriptive statistics for each LoB

LoB	0531	0993	0196	0351	0167
Number Observations	38,783	1,375	414	441	34
Mean(R$)	10,165	18,186	577,989	29,570	221,195
Standard-Deviation(R$)	14,878	27,733	3,504,142	176,032	800,037
Maximum(R$)	126,601	344,748	39,261,086	3,576,987	4,649,752
Minimum(R$)	1,000	1,004	1,015	1,009	1,250

Source: own elaboration.

[1] Every HP was plotted, but because of space limitation, only LoB 0196 was shown due its importance. If the reader is interested, the authors can share the HP of the others LoB upon request.

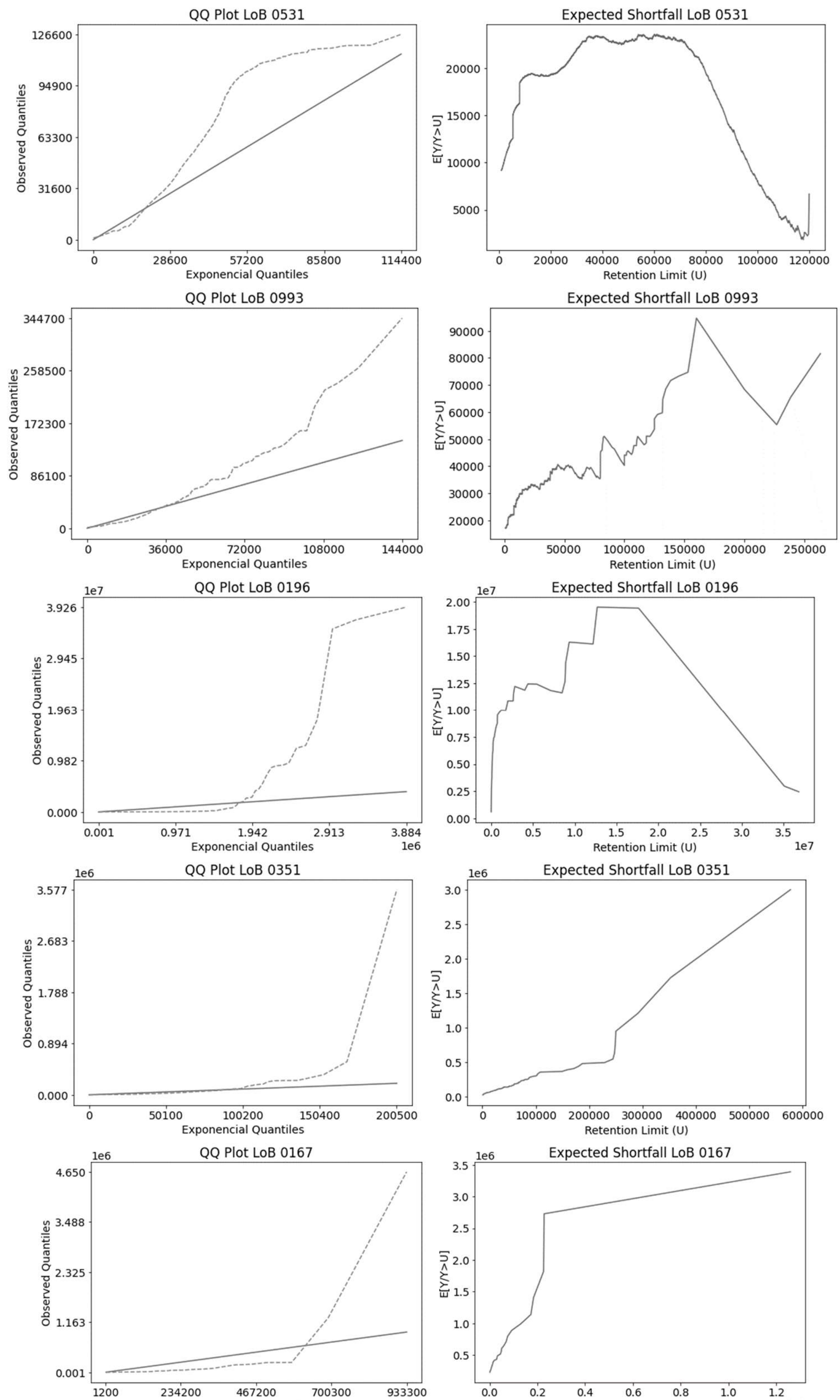

Figure 4. *QQ-Plots* and *ES* plot for each LoB.
Source: own elaboration.

$$SSE = \sum_{i=1}^{k} \left[\hat{F}\left(X_{n:i}\right) - F\left(X_{n:i}, \hat{\gamma}\right) \right]^2 \quad (15)$$

The results obtained for each LoB were compiled at Table 4, and the graphs comparing the empirical and theoretical cumulative functions are shown in Figure 5.

In LoB 0531, the threshold *U=100,000* was chosen, making 190 observations be above this limit. It is possible to infer that the tail in this case is light because the Moments, Pickands and MLE indicate a negative $\hat{\gamma}$. Among all models, the one that came closest was Moments (SSE=0.0570), followed by Pickands (SSE=0.1686) and Maximum Likelihood (SSE=0.2497) and Gamma (SSE=0.4083). On the other hand, Hill's estimator did not capture the light tail effect, being the only estimator to present a positive $\hat{\gamma}$.

For LoB 0993, *U=30,000* with 244 observations was chosen. This threshold was defined based on the *ES* graph, as the behavior is approximately linear within this subdomain. In this case, basically just two behaviors were observed: Moments and MLE with a low $\hat{\gamma}$ and a better SSE

Table 4
Fit results for each LoB

LoB	Exceeding Data	Retention Limit	Statistic	Hill	Moments	Pickands	ML	Gamma*
0531	190	100,000	$\hat{\gamma}$	0.0755	-0.6169	-0.9408	-0.3804	α=1.4429; β=5502
			Var	0.0055	0.1013	0.2567	0.045	–
			SSE	1.1982	0.057	0.1686	0.2497	0.4083
			ES	8313	7831	7757	7726	7939
0993	244	30,000	$\hat{\gamma}$	0.6108	0.2904	0.5923	0.1744	α=0.8784; β=38053
			Var.	0.0391	0.0667	0.2538	0.0752	–
			SSE	0.2441	0.1423	0.2459	0.1734	0.2251
			ES	54020	35645	56729	33490	33428
0196	137	15,000	$\hat{\gamma}$	2.0375	2.0228	1.8271	2.2245	α=0.2152; β=7999620
			Var.	0.1741	0.1928	0.4539	0.2755	–
			SSE	0.0424	0.0443	0.1013	0.0275	2.7369
			ES	∞	∞	∞	∞	1721734
0351	105	20,000	$\hat{\gamma}$	0.9694	0.8117	0.3358	0.6669	α=0.4800; β=178,814
			Var.	0.0946	0.1257	0.3708	0.1627	–
			SSE	0.0783	0.0449	0.0798	0.0310	1.0036
			ES	749910	130749	48794	79757	85827
0167	34	0	$\hat{\gamma}$	3.057	2.1584	1,0908	1.2289	α=0.3217
			Var.	0.5247	0.4080	0.7814	0.3823	β=687500
			SSE	0.3608	0.1269	0,1112	0.0318	0.6772
			ES	∞	∞	∞	∞	221195

Note: *Maximum likelihood; **For Gamma, instead $\hat{\gamma}$ and its variance, the table contains it estimated parameters.
Source: own elaboration.

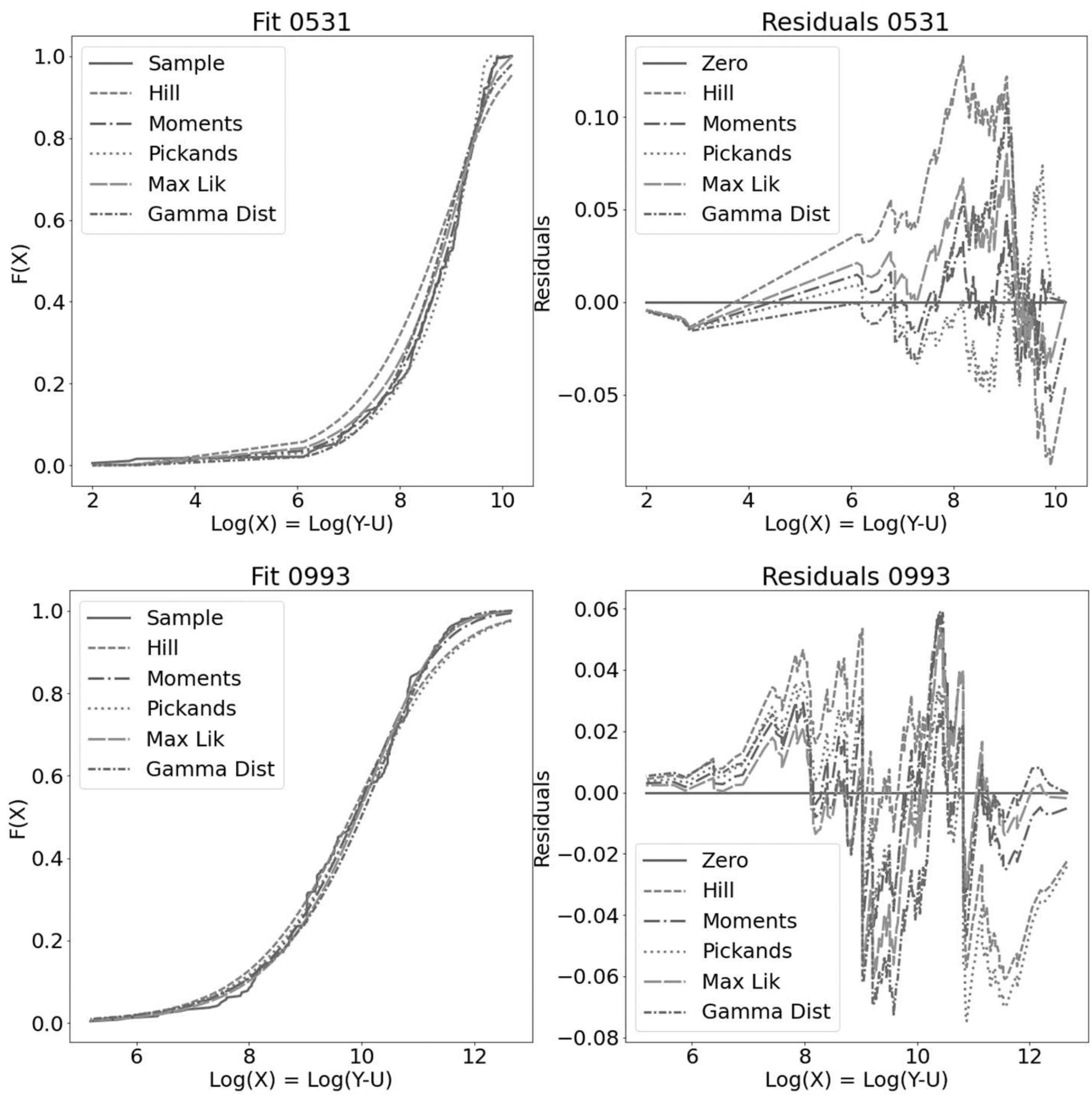

Figure 5. The fit (left) and the residuals (right) for each LoB and method
Source: own elaboration.

statistic (0.15); Hill and Pickands with a higher $\hat{\gamma}$ and lower SSE (0.24), a result similar to the Gamma distribution. In the second case, as discussed in section 4, Hill has the characteristic of greater bias and overestimation of the tail for low values of $\hat{\gamma}$.

In LoB 0196, the limit chosen was *U=15,000*, with 137 observations remaining, based on the HP from Figure 6. In this case, neither the *QQ-Plot* nor the *ES* exhibited well-defined behaviors. At the HP for Hill and Moments, a tendency for stabilization between 100 and 150 observations can be seen, hence the threshold *U* definition. From the results compiled at Table 4, the best estimate was given by the MLE (SSE=0.0275), followed by Moments and Hill with similar SSE (around 0.043), and lastly Pickands (SSE=0.1013). The coefficient $\hat{\gamma} \approx 2$ implies a heavy tail distribution without defined moments, being a potentially risk for the insurance activity given that its mean is unstable. The Gamma distribution had much worse fit than every Generalized Pareto estimator, indicating that traditional models fail to capture heavy tails. This is more evident when evaluating the residuals, which show a strong positive and then negative trend.

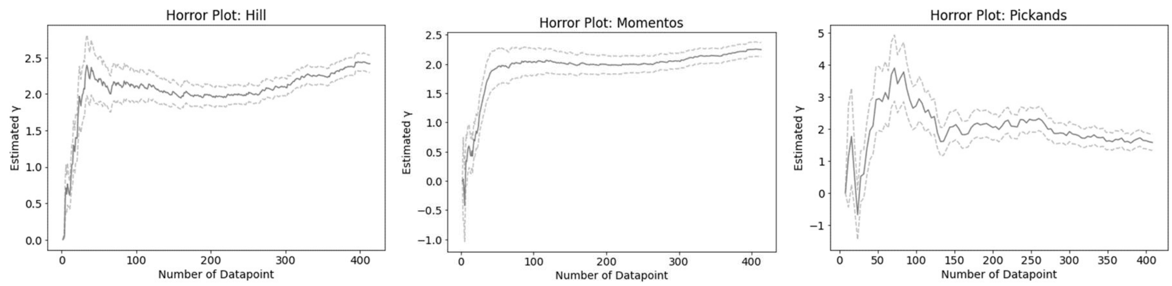

Figure 6. HP for LoB 0196– Named and Operational Risks.
Source: own elaboration.

In LoB 0351, the threshold *U=20,000* and *n=105* was chosen for the same reason as the previous LoB 0993: the *ES* is approximately linear, and some low values residuals are eliminated. Once more, the MLE and Moments achieve best results (SSE=0.0310 and 0.0449 respectively), while Hill (SSE=0.0783) overestimates and Pickands (SSE=0.0798) underestimates the tail. However, regardless of the chosen estimator, it is possible to infer that the tail of this distribution is slightly heavy with a defined mean, given that the coefficient never exceeded 1. In this LoB, the Gamma distribution was the worst fit again, having the highest SSE.

For LoB 0167, no retention limit was imposed due to limited amount of data. The estimators' variances in this case was quite high, and even Hill oscillated around 0.5. If another threshold was used, it would increase the variance even more, making the method unreliable. Once more, from Table 4, MLE was the most accurate (SSE=0.0318), followed by Pickands (SSE=0.1112), Moments (SSE=0.1269) and Hill (SSE=0.3608). At the fit graphs, it can be seen that Moments and Hill tended to overestimate the quantiles for the higher values of the distribution but was also more appropriate for lower quantiles. On the other hand, Pickands and MLE were the opposite: they underestimated the beginning of the distribution to better capture higher quantiles. The Gamma distribution had the worst performance again.

The theoretical *ES* were also calculated in Table 4 using each method. The *ES* of the sample and the Gamma distribution are the same because the α and β parameters were obtained through the moments generating function. In general, LoB with high $\hat{\gamma}$ made the *ES* were not well defined: the LoB 0196 and 0167 have 4 infinite *ES*, which reinforces the heavy nature of these tails. On the other hand, LoB 0351 had a high variance in its $\hat{\gamma}$, which is also reflected at the high dispersion of the *ES*. In this case, the highest estimate (Hill) is more than 15 times higher than the lowest (Pickands). In LoB with light or slightly heavy tails (0531 and 0993), there were no large variations between the theoretical *ES* estimates.

5.1. Adherence of fits by different estimators

In order to verify the adherence of the fits, the SSE obtained by varying the retention limit for each LoB were also computed. The results are shown in Figure 7.

In LoB 0531, the Pickands and Moments estimators have the best SSE throughout the tested interval. The MLE and the Gamma distribution at first shown a high SSE. However, for *U>100000*, the SSE approaches Moments and Pickands. Hill is always higher.

For LoB 0993, the Gamma distribution presented similar results to MLE and Moments when *U>30000*. On the other hand, Hill and Pickands have great variation at the tested range, although they are not totally inadequate for the interval *30000<U<35000*.

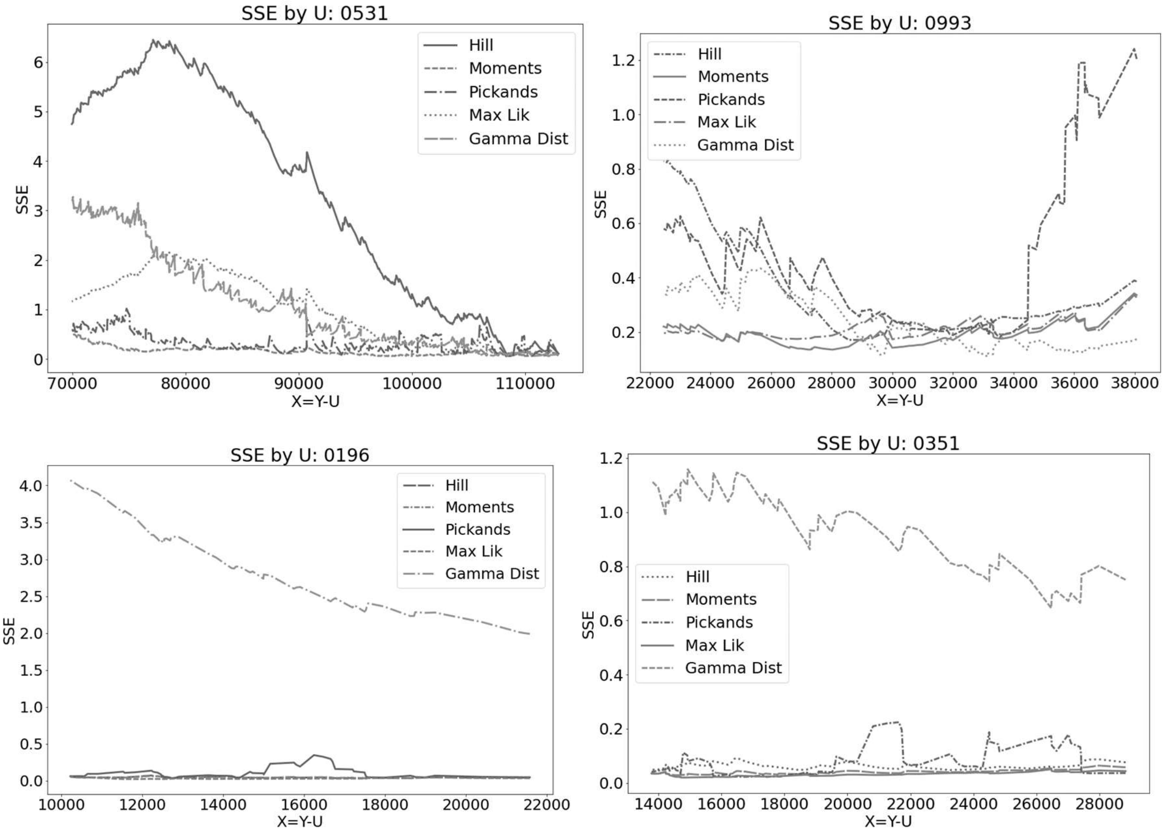

Figure 7. The SSE given U for each LoB
Source: own elaboration.

As for LoB 0196 and 0351, in both cases the Gamma distribution always had a high SSE, for the most of tested threshold interval. These LoB have a heavy tail nature and this distribution cannot capture this effect, making it inappropriate to use.

At last, the LoB 0167 does not have enough data for this analysis.

6. ESTIMATION OVER TIME

As a final way of analyzing the data, the time dimension was considered. To assess the estimates' sensitivity to arrival of new information, the data was arranged longitudinally. The objective is to identify the impact that claims of high severity, especially those that exceed the current maximum, have on the estimates derived from the different methods. Due to space limitations, LoB 0531 will be disregarded, because its tail is already limited (due to the imposition of a maximum indemnity limit), and also 0167, due to the insufficient amount of data. At the plots, vertical bars indicate claim values that exceed the current maximum. The results are shown in Figure 8.

The results show similar behavior between the estimators, both in increase and in decrease. In LoB 0351, there is a tendency for estimates to grow whenever a new maximum is reached in the sample, followed by a stabilization at the new level. MLE is the most affected of them, followed by Moments, while Hill has barely changed, maintaining the level since 2007.

As for LoB 0196, there is a rapid rise in peaks, reaching the final level of the sample in 2008. This occurs because one of the biggest accidents in the sample was observed in the very beginning. However, as in LoB 0351, the coefficients did not stabilize at the same speed. In fact, there is a

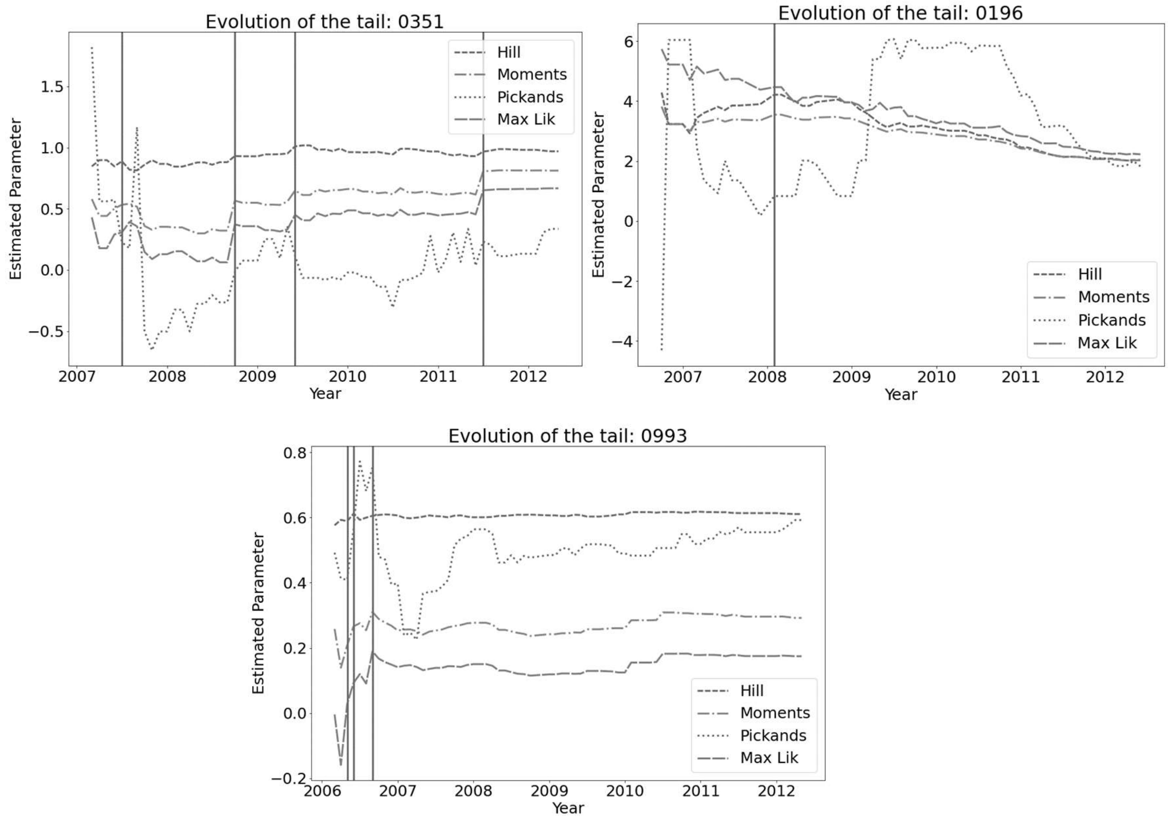

Figure 8. The evolution of the estimators for each LoB
Source: own elaboration.

downward trend over time, and the extreme values at the beginning of the period may have been considered sufficiently rare, so as not to severely affect the tail indices. But it could also signal a change in the company's underwriting process, better selecting risks based on past extreme losses.

Lastly, LoB 0993 in the same way as LoB 0196, shows rapid maximum maturation. However, there is no downward trend in the estimators, but a convergent behavior similar to LoB 0351. This could indicate maintenance of the underwriting process and risk management, or that there were no sudden changes in the portfolio profile of policyholders.

The purpose of Figure 9 is to assess the evolution of ES over time. It is important to remember that the *ES* is the actuarial pure reinsurance premium, and it is the cost that the insurer incurs to protect itself financially against the occurrence of extreme severity claims. Therefore, it is essential to evaluate the sensitivity of this measure when a new maximum amount appears and also when the same maximum lasts for a long time. Vertical bars indicate claim values that exceed the current maximum.

In general, all LoB had very similar behavior to what has already been verified in Table 4. One LoB that stands out in Figure 9 is 0196. Because estimates of γ are always greater than 1 (approaching 2), the *ES* for this LoB is infinite, and it appears as discontinuities. The immediate implication is to have reinsurance premiums so expensive in a way to render the risks non-reinsurable. As reinsurance premiums should be contained in insurance premiums, the consequence would be to make events uninsurable, reducing the supply capacity of insurers, since entities would consider the risks involved in these operations to be extremely high and unpredictable.

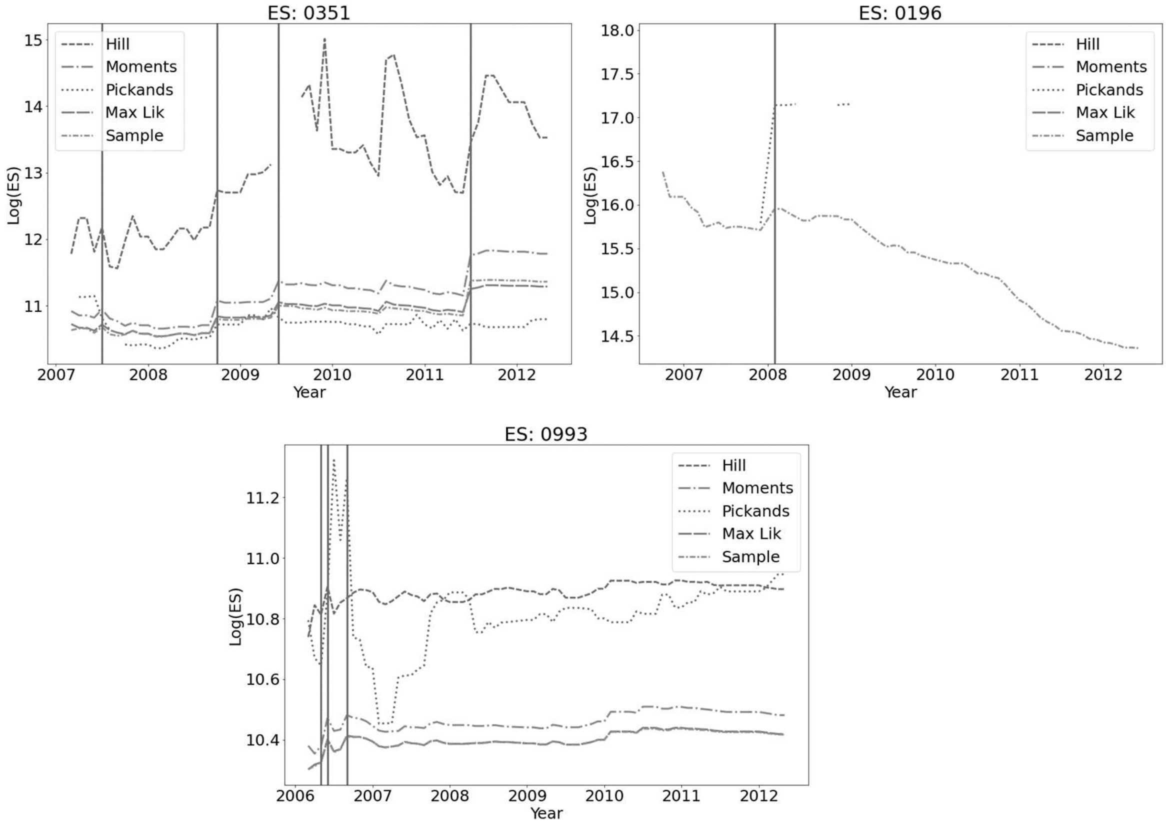

Figure 9. The evolution of the *Expected Shortfall* for each LoB
Source: own elaboration.

For LoB 0993, and Hill and Pickands estimators, the *ES* maintains a value close to R$50,000 throughout the period while MLE, Moments and the sample do not deviate from the values in Table 4. The *ES* of the MLE is close to the sample because the γ estimated in Figure 8 were close to 0. Lastly, in LoB 0351, with the exception of the Hill estimator, the *ES* also remained approximately constant, with small positive oscillations when a new maximum claim is materialized. In all cases, however, actuarial pure reinsurance premiums are positively sensitive to breaches of the old threshold, signaling that the value should be automatically updated because of a new maximum occurrence. Furthermore, the premiums tend to remain constant (or with a slight reduction) as the old maximum continues to be in force, denoting a certain persistence of the old severities influencing new contracts.

7. CONCLUSIONS

In the insurance business, where the goal is to guarantee a company's solvency in the long run, it is important not only to have tools to estimate the greatest potential risks to which the company is exposed, but also have good criteria for selecting the best estimators. After all, if materialized, claims of extreme severity can lead an insurer to present intense variations in the result and, ultimately, lead the entity to ruin. One of the tools to deal with this uncertainty is the reinsurance contract, especially the excess of loss. Given this scenario, this study aimed to evaluate different estimation methods for high quantiles of claims distributions, considering only the values that exceed a given retention limit. EVT was chosen to be our theoretical basis due to the promising results arising from techniques derived from this area of Statistics.

Regarding the tested estimation methods, in general, the Moments estimator seemed to be the most appropriate of all. Its bias in the simulations was a little intense when a large proportion of the sample was used for estimation, but always in the sense of overestimating the exposed risks and being more conservative. On the other hand, its variance is not very large, being applicable also for any γ, making it versatile. As for the Hill estimator, although it has the lowest variance among them, it has a very intense bias when used for tails that are not extremely heavy, arising from distributions without defined mean ($\gamma > 1$). In general, it had very good results when used in the LoB 0196, but even so, it did not manage to be better than Moments. It is important to note that Resnick (2014) has already mentioned the fact that Moments is more suitable for lower values of γ than Hill.

The Pickands estimator in the simulations obtained very promising results, but the restricted amount of data and its large variance make it unstable when applied to real data. In this way, unlike Hill or Moments, it is not possible to determine whether risks are being overestimated or underestimated, as any deviation is simply random. Lastly, the MLE also obtained good results, in addition to having a small variance when compared to the other models. Its bias, like Pickands, is null, thus being a choice that does not tend to overestimate or underestimate the occurrences of extreme values. Its only shortcoming, however, is when $\gamma < 0$. In this situation, it ended up being the estimator with the highest SSE. As for the traditional Gamma distribution, it obtained good results in light or slightly heavy tails, such as LoB 0531 and 0993, although it is not suitable for heavy tails (LoB 0351 and 0196).

In light of all the results obtained in this study, both in the simulated experiments and in the real application, it was observed that the tail estimation methods constitute a powerful tool for the insurance area and should be looked at more carefully. Although the results do not imply that there is a perfect and unique estimator that can be efficiently applied in all cases, the combination of all methods is vital to obtain good estimates. Understanding the nature and characteristics of each estimator is essential for the correct application of the model. A possible extension of this study would be to carry out a similar evaluation for other estimators, given their broad spectrum, and compare them among their families and methodologies.

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AUTHOR'S CONTRIBUTION

JC: conceptualization (leadership); data curation (equal); formal analysis (equal); investigation (equal); methodology (equal); project administration (leadership); software (equal); supervision (leadership); validation(equal); view(equal); writing - original draft (same); writing - proofreading and editing (equal); LO: data curation (equal); formal analysis (equal); investigation (equal); software (equal); validation(equal); view(equal); writing - original draft (same); writing - proofreading and editing (equal).

CONFLICTS OF INTEREST

The authors declare that they have no conflicts of interest.