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Influence of network configuration and stochastic model on the determination of the minimum detectable displacements (MDD) through sensitivity analysis and significance test

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Abstract:

This study investigates the influence of geodetic network configuration, stochastic model, and the approach local or global on the determination of minimum detectable displacements (MDD) using sensitivity analyses and significance tests. The proposed approach integrates sensitivity characteristics to establish confidence regions based on MDD. In addition, we examine the equality between the critical value of a significance test and the non-centrality parameter derived from a chi-square distribution to compute concentric ellipsoids representing sensitivity and accuracy. The analyses were focused on evaluate how variations in network configuration, stochastic model, and the type of analysis (if global or local) affect the relationship between sensitivity and accuracy. Our results showed the importance of considering these factors, providing valuable insights for robust network design and analysis in practical applications.

Keywords: Statistical tests; deformation analysis; minimal detectable displacements; geodetic monitoring.

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1. Introduction

In the pre-analysis of the geodetic monitoring networks, the sensitivity analysis provides valuable insights related to the capacity of the geodetic network to detect deformations. The sensitivity analysis is applied for specific probability levels based on the minimum detectable displacement (MDD) of monitoring points (Even-Tzur, 2010). Here, some aspects such as the network configuration, stochastic model, and the sensitivity analyses type, namely, global (for the entire network), and local (for specific points) play a key role. Thus, if the computed MDD exceeds the desired threshold based on the specified probability levels and the number of points tested simultaneously, it indicates a need for design improvement. This can be achieved by adding new observations and points, reducing their standard deviation, changing the probability levels or the approach (global or local). The computation of MDD is essential in geodetic monitoring as it accounts for both Type I errors (false positive) and Type II errors (false negative). These error types define a false alarm, namely, a deformation incorrectly detected and an undetected deformation. The last one is a critical condition for geodetic monitoring (Carvajal et al., 2022).

In addition to the sensitivity analysis, the network accuracy is also analyzed in the pre-analysis or design stage. Here, the thresholds for confidence and significance tests are defined to provide the best network design according to requirements (Prószyński & Łapiński, 2021). As in the sensitivity analysis, in accuracy analysis, the configuration of the network, the stochastic model, and the probability levels are aspects that can influence the results. In this context according to Prószyński & Łapiński (2021), the analysis of accuracy and sensitivity are traditionally separately applied due to the lack of a theoretical basis to consider a unique analysis. Therefore, the same authors provide a theoretical basis to consider the confidence region and significance test in sensitivity analysis in a unified approach based on MDD determination.

The method called *variance factor (I)* supports the confidence region in a network sensitivity characteristic. For this, the method provides equality between the critical value of the significance test of displacements $\Phi_{h,\alpha}$ (associated with confidence region) and the non-centrality parameter $\lambda_{h,\alpha,\beta}$ based on χ^2-distribution for a specific value for the power of test γ_0 determined by a Type II error probability β_0 $\left(\gamma_0 = 1 - \beta_0\right)$, coordinated with the stipulated Type I error probability or level of significance α and the h-dimensional displacement vector such that $\Phi_{h,\alpha_0} = \lambda_{h,\alpha_0,\beta_0}$. After the equality determination, the MDDs are computed and represented as concentric ellipsoids where the MDD corresponds to the semi-major axis (Aydin, 2014). Here the relation between accuracy and sensitivity depends only on probabilistic concepts and does not consider aspects such as the network configuration and the stochastic model.

In this study several experiments were carried out to analyze the relationship between accuracy and sensitivity. Thus, initially, the sensitivity analysis characteristics were included in the deformation detection analysis through the global congruence test. Furthermore, we presented aspects related to the network configuration, stochastic model, and simultaneous displacements (multivariate and univariate approaches). Here, our results showed that the configuration network, stochastic model, and the type of analysis, namely, global or local influence the deformation analysis (MDD value) for both approaches. Finally, we presented experiments to analyze the influence of the network configuration, and the stochastic model in the relation between sensitivity and significance analysis presented by Prószyński & Łapiński (2021).

2. Theoretical basis

2.1 Deformation analysis: Global Congruence Test (GCT)

The theoretical approach initially considers the deformation test. Here a displacement vector is given by:

$$\hat{d} = x_2 - x_1 \tag{1}$$

Where x_1 and x_2 are the least squares solution for the monitoring network parameters (point coordinates) in the first and second epoch respectively. The covariance matrix is computed as:

$$C_d = \sigma_0^2 Q_d \tag{2}$$

Here Q_d is the cofactor matrix of the displacement vector, and $\;_0^2$ is the a-priori variance factor. Based on hypothesis testing, two hypotheses are formulated:

$$H_0 : E\left(\hat{d}\right) = 0 \; and \; H_A : E\left(\hat{d}\right) \neq 0 \tag{3}$$

While the test statistic is given by:

$$\Phi = \hat{d}^T C_d^+ \hat{d} \tag{4}$$

Which follows (central) X^2-distribution with h degrees of freedom in H_0, is compared with the theoretical value of $\chi^2_{(\alpha,h)}$ corresponding to the α -significance level. If the test statistic Φ is smaller than the threshold value, the null hypothesis H_0 is not rejected with the confidence level of $1\text{-}\alpha$, and it is concluded that there is no deformation between the two epochs. Otherwise, it is decided that displacement has occurred with the probability risk of a false positive given by α. This congruence test may be performed by considering the estimated variance factor. In this case, the test follows F (Fisher)-distribution. Nonetheless, this case is outside the scope of this paper. More information can be found in Aydin (2014).

2.2 Sensitivity analysis

The capacity of the network to detect displacements can be quantified by sensitivity analysis (Aydin, 2014). Based on the deformation test, a theoretical vector of expected displacement, denoted by Δ, is related to the alternative hypothesis defined as:

$$H_A : E\left(\hat{d}\right) \neq 0 = \Delta \tag{5}$$

Here the non-rejection of H_A implies that the expected displacement values of the vector Δ can be detected by the monitoring network. The theoretical relationship of the non-centrality parameter (λ) and the test statistic (T) is given by:

$$T = \lambda = \Delta^T C_d^+ \Delta \tag{6}$$

The condition to define if the network is sensitive to displacements is given by:

$$\lambda \geq \lambda_0 \tag{7}$$

Where λ_0 is the so-called lower bound of the non-centrality parameter which fulfills the given power of the test γ_0 being the complement of the type-II error probability β_0 : $\gamma_0 = 1 - \beta_0$. Here λ_0 is obtained from Aydin & Demirel (2004).

2.3 Minimal detectable displacements (MDD)

To evaluate the MDD, firstly a vector with the expected displacements is defined from a vector of directions g and a scale factor value denoted by b (Aydin, 2014). Thereby, the condition $\Delta = bg$ is fulfilled. If $b = b_{min}$ $(\Delta_{min} = b_{min}\ g)$, then the determination of b_{min} is given by:

$$b_{min} = \sqrt{\lambda_0 \lambda_{max}} \tag{8}$$

Where λ_{max} is the maximum eigenvalue of the covariance matrix of C_d. According to Küreç & Konak (2014) b_{min} is the best sensitivity level of the network. On the other hand, the worst sensitivity level of the network can be computed as:

$$b_{max} = \sqrt{\lambda_0 \lambda_{min}} \tag{9}$$

Where λ_{min} is the minimum eigenvalue of C_d. According to Hsu & Hsiao (2002) the average between b_{max} and b_{min} can be interpreted as the global sensitivity for the entire network.

To obtain the vector of directions of displacements, Aydin (2014) computed the (unity) eigenvectors corresponding to the maximum eigenvalue (λ_{max}) and the minimum eigenvalue ($\lambda_{\min}$) of the C_d matrix (Λ_{max} and Λ_{min} respectively). Hence: $\Lambda_{min} = b_{min}\Lambda_{max}$ and $\Lambda_{max} = b_{max}\Lambda_{min}$. The MDD in each i^{th} element of the vector $\hat{d}$ is given by the respective i^{th} element of Δ_{min} or Δ_{max}. For details, we suggest Aydin, (2014).

2.4 Global and Local sensitivity analysis

The sensitivity analysis can be carried out under a global or local approach. Here an important analysis related to simultaneous and unitary displacements arises. If the global analysis is applied the MDD represents simultaneous displacements of all the points that make up the monitoring network. Conversely, if the local sensitivity is applied the MDD is computed for a specific point. These conditions imply that the h-dimensional vector of displacements changes and therefore the non-centrality parameter also. Here, for 2D and 3D networks, the local sensitivity can be computed under multivariate (simultaneous displacements) or univariate approaches. For example, a multivariate analysis for a 3D point with $\alpha_0 = 5\%$ and $\gamma_0 = 80\%$ implies $\lambda_{(\gamma_0=80\%, \alpha_0=5\%, h=3)} = 10.9$. On another hand, if the univariate approach is used, the non-centrality parameter is computed as $\lambda_{(\gamma_0=80\%, \alpha_0=5\%, h=1)} = 7.85$. These conditions are fulfilled for the significance test also ($\chi^2_{(\alpha_0=5\%, h=3)} = 7.81$ and $\chi^2_{(\alpha_0=5\%, h=1)} = 3.84$) (Bandeira et al., 2021).

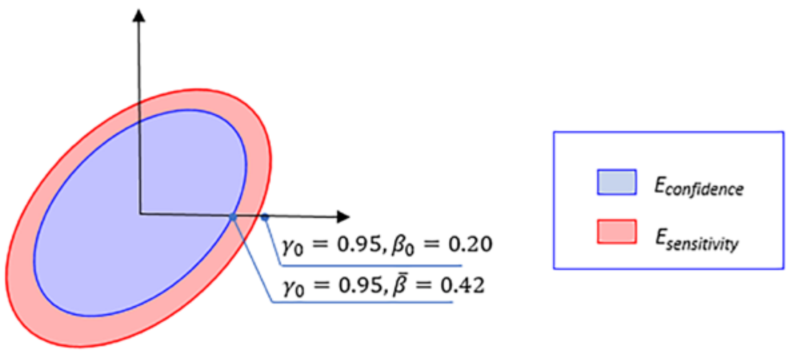
Therefore, the multivariate and univariate approaches are related to aspects of detectability, where the multivariate approach has more difficulty to detect deformation due to critical values being calculated from stochastic models without covariance (e.g., GNSS). For the univariate approach, the neglect of covariance can be generated false positives and false negatives in addition to providing only a displacement magnitude and not their directions. Based on this, the multivariate approach is recommended Bandeira et al. (2021).

2.5 Confidence region determination supported by network sensitivity characteristics

The theoretical basis for the confidence region determination supported by network sensitivity characteristics is developed for the scenarios where σ_0^2 is used. Here, a specific value for the power of test γ_0 is determined through a value of β_0 coordinated with the stipulated level of significance α_0 and the h-dimensional displacement vector. This approach provides equality between the critical value for the significance test and the non-centrality parameter of the sensitivity test: $\Phi_{(h,\alpha_0)} = \lambda_{(h,\alpha_0,\beta_0)}$ (Prószyński & Łapiński, 2021).

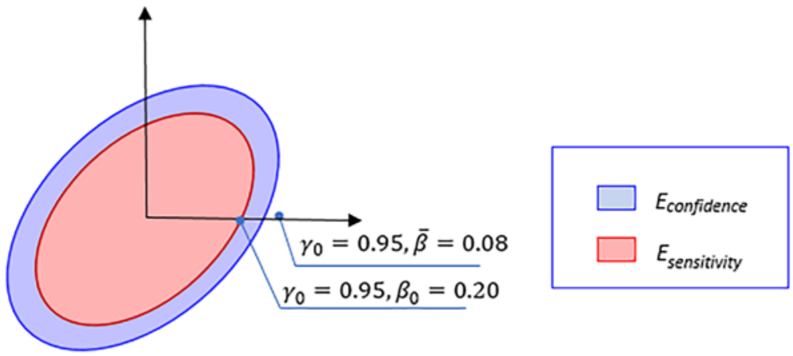
Initially for $h=1$ the relation $\lambda_{h,\alpha_0,\beta_0} > \Phi_{h,\alpha_0}$ is fulfilled. If the h value increases, both values $\lambda_{h,\alpha_0,\beta_0}$ and Φ_{h,α_0} also increase. However, for a specific value of h (namely h^*), the relation $\Phi_{h,\alpha_0} = \lambda_{h,\alpha_0,\beta_0}$ is achieved (Prószyński & Łapiński, 2021). The results presented by the authors show that for $h^* = 7.3$ the above equality holds to $\alpha_0 = 0.05$ and $\beta_0 = 0.20$. In addition, for values greater than $h^* = 7.3$, the relation $\Phi_{h,\alpha_0} > \lambda_{h,\alpha_0,\beta_0}$ is fulfilled. Note that for different values for α_0 and β_0, the value for h^* also changes.

The approach proposed by Prószyński & Łapiński (2021) is based on the size comparison of three concentric ellipsoids; sensitivity ellipsoid, confidence ellipsoid, and significance ellipsoid. This comparison is carried out by global sensitivity. For the significance ellipsoid, the relation $\hat{u}^T \ {}_{d}^{+} \ \hat{} = \Phi_{h,\alpha_0}$ is fulfilled for a α_0 significance level; for the confidence ellipsoid, $\hat{d}^T C_d^+ \hat{d} = \Phi_{h,CL}$ is fulfilled for a given confidence level (CL) and for the sensitivity ellipsoid, $\hat{d}^T C_d^+ \hat{d} = \Phi_{h,\alpha_0,\beta_0}$. Considering $CL = 1-\alpha_0$, the significance ellipsoid turns into the confidence ellipsoid and the analysis focuses on the determination of $\Phi_{h,\alpha_0} = \lambda_{h,\alpha_0,\beta_0}$. In this case, two scenarios were defined: $h > h^*$ and $h < h^*$, where h^* is the value for h that satisfies the equality $\Phi_{h,\alpha_0} = \lambda_{h,\alpha_0,\beta_0}$. Then the confidence and sensitivity ellipsoids were determined, for $h < h^*$ the confidence ellipsoid is smaller than the sensitivity ellipsoid (Figure 1) while for $h > h^*$ the sensitivity ellipsoid is smaller than the confidence ellipsoid (Figure 2).



Source: Adapted from Prószyński & Łapiński (2021).

Figure 1: Ellipsoids of confidence and sensitivity for $h < h^*$.



Source: Adapted from Prószyński & Łapiński (2021).

Figure 2: Ellipsoids of confidence and sensitivity for $h > h^*$.

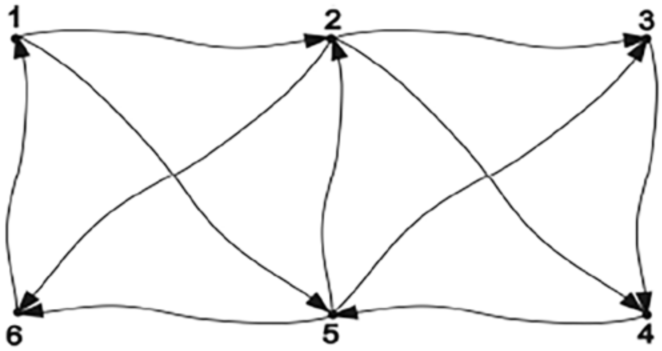
3. Experiments and analyses

In this section, we conducted several experiments to evaluate the behavior of the MDD under different scenarios, such as network configuration, number of observations, local or global MDD, as well as different stochastic models. For this, we utilized two leveling networks: Network A, which comprises 6 points as depicted in Figure 3, and Network B, which consists of 9 points as shown in Figure 5. The number of observations for each network is denoted by n, while h represents the dimensionality of the displacement vector. Table 1 provides a description of each experiment.

Table 1: Summary of experiments.

Experiment	Description
Experiment 1	Local MDD *(h = 1)* for network A with *n = 11*
Experiment 2	Local MD *(h = 1)* for network A improved redundancy *(n = 15)*
Experiment 3	Test of local MDD values for *n = 15* on network A with *n = 11*
Experiment 4	MDD for three simultaneous points *(h = 3)* on network A with *n = 15*
Experiment 5	Global MDD *(h = 6)* on network A with *n = 11*
Experiment 6	Global MDD *(h = 9)* on network B with *n = 12*
Experiment 7	Global MDD for network A with *n = 15* and network B with *n = 20*
Experiment 8	Global MDD for network A with *n = 15* and network B with and gradients of different precisions *n = 20*

For the first experiment, Figure 3 shows a leveling monitoring network with 11 height differences and a standard deviation of 1 mm (Nowel, 2018). To evaluate the MDD, a trial and error methodology presented by Bandeira et al., (2021) was applied for each point of the network (local sensitivity approach). In this step, in addition to the application of the significance test, which considers only false positives in H_0; the sensitivity analysis was applied also, namely, the occurrences of false positives in H_0 and false negatives in H_A were considered, in both cases $Q_d = 2 \cdot Q_x$ (Q_x, cofactor matrix of the unknowns). In the last case, the critical value is determined by the non-centrality parameter $\lambda_{(\gamma_0, \alpha_0, h)}$ instead of $\chi^2_{(\alpha_0, h)}$.



Source: Adapted from Nowel (2018).

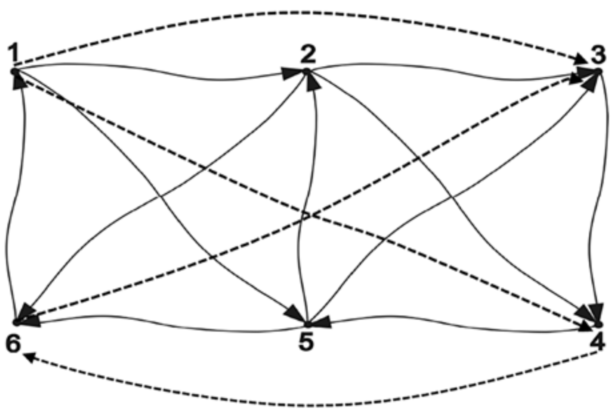
Figure 3: leveling network.

The results of the first experiment show that the MDD values are lower for the significance approach (Table 2). These results are equivalent to the theoretical basis presented in Prószyński & Łapiński, (2021), since $h < h^{*}$. Another relevant aspect of this experiment is related to points 2 and 5. These points have a small magnitude for MDD in comparison with the points 1,3,4,6. The main difference between these two groups is the number of observation connections which are 5 and 3 respectively.

Table 2: Local MDD for significance and sensitivity approach based on the GCT.

Point	$d_{(i,1)}$ (mm) (Significance analysis)	$d_{(i,1)}$ (mm) (Sensitivity analysis)
1	1.5	2.1
2	1.1	1.5
3	1.5	2.1
4	1.5	2.1
5	1.1	1.5
6	1.5	2.1

To evaluate the role of the network configuration, observations were added between points 1-3,1-4,3-6,4-6 with 1 mm of standard deviation. Figure 4 shows the new network configuration.



Source: Author’s.

Figure 4: leveling network.

From the new configuration, all the points have the same local MDD. Thus, the local MDD is 1.1 mm and 1.5 mm for the significance and sensitivity respectively. Therefore, here the configuration of the network influences the MDD values. To evaluate this condition, the new local MDD found was tested in the first configuration (11 leveling differences). The results are presented in Table 3.

Table 3: Results for tested of MDD obtained in the second configuration inserted in the first configuration.

Deformation considered for significance analysis				Deformation considered for sensitivity analysis			
Point	$d_{(i,1)}$ **(mm)**	$d^T{}_{(i,1)} \cdot C^+_{d(i,i)} \cdot d_{(i,1)}$	$\chi^2_{(\alpha_0=5\%,d=1)}$	**Point**	$d_{(i,1)}$ **(mm)**	$d^T{}_{(i,1)} \cdot C^+_{d(i,i)} \cdot d_{(i,1)}$	$\chi^2_{(\alpha_0=5\%,d=1)}$
1	1.1	2.2926	3.8415	1	1.5	4.2632	7.8488
2	1.1	4.3560	3.8415	2	1.5	8.1	7.8488
3	1.1	2.2926	3.8415	3	1.5	4.2632	7.8488
4	1.1	2.2926	3.8415	4	1.5	4.2632	7.8488
5	1.1	4.3560	3.8415	5	1.5	8.1000	7.8488
6	1.1	2.2926	3.8415	6	1.5	4.2632	7.8488

From Table 3, the detection was successful for points 2 and 5 for both tests. Note that for these points the local of MDD was the same as in the first experiment. For the rest of the points, the test under the significance and sensitivity could not identify the deformations. These results showed the importance of the network configuration in the design stage.

Subsequently, simultaneous displacements were tested. For this, three displacements in the network with 15 leveling differences and 1 mm of standard deviation were defined. In this case, the critical value for significance and sensitivity analysis becomes $\chi^2_{(\alpha_0=5\%,h=3)} = 7.81$ and $\chi_{(\alpha\ =5\%,d=3)} = 10.9$, for both cases $\gamma_0 = 80\%$. Here two MDDs were computed, in the direction of the largest and in the direction of the smallest variances. The results are presented in Tables 4 and 5 for significance and sensitivity analysis respectively.

Table 4: MDDs for simultaneous displacement for significance analysis.

Point	**MDDs (largest variance direction)**			**Deformation considered for sensitivity analysis**		
	$d_{(i,j,k)}$ **(mm)**	$d^T \cdot C^+_{d(i,i)} \cdot d$	$\chi^2_{(\alpha_0=5\%,d=3)}$	$d_{(i,j,k)}$ **(mm)**	$d^T \cdot C^+_{d(i,i)} \cdot d$	$\chi^2_{(\alpha_0=5\%,d=3)}$
1,3,5	1.4,0.2,0.2	9.4800	7.8147	0.6,-0.1,1.3	9.4800	7.8147
2,4,6	1.4,0.2,0.2	9.4800	7.8147	0.7,-0.2,1.2	9.4800	7.8147

Table 5: MDDs for simultaneous displacement for sensitivity analysis.

Point	**MDDs (largest variance direction)**			**Deformation considered for sensitivity analysis**		
	$d_{(i,j,k)}$ **(mm)**	$d^T \cdot C^+_{d(i,i)} \cdot d$	$\chi^2_{(\gamma_0=80\%,\alpha_0=5\%,h=3)}$	$d_{(i,j,k)}$ **(mm)**	$d^T \cdot C^+_{d(i,i)} \cdot d$	$\chi^2_{(\gamma_0=80\%,\alpha_0=5\%,h=3)}$
1,3,5	1.5,0.3,0.3	11.8200	10.903	0.7,-0.2,1.4	11.8200	10.903
2,4,6	1.5,0.3,0.3	11.8200	10.903	0.8,-0.1,1.3	11.8200	10.903

From Tables 4 and 5 note that the local MDD from Table 3 can be less or greater than the global MDD (univariate or multivariate analysis). These results show the importance of the type of analysis (if local or global) to compute the MDD values in both significance and sensitivity approaches.

The next experiments are focused on evaluating Prószyński & Łapiński, (2021) method. Therefore, we used the leveling network from Figure 3 with a standard deviation of 1 mm for each height difference (11 observations). The inner-constrained approach is applied in the adjustment (see Ogundare, (2018)). The covariance matrix for the deformation vector $\hat{d}$ was obtained by $Q_d = 2Q_x$ In this case $h = rank\ (Qd) = 5$. The values for the non-centrality parameter were obtained from Aydin & Demirel, (2004). Table 6 shows the results.

Table 6: non-centrality parameter for sensitivity and confidence for leveling network of Figure 3.

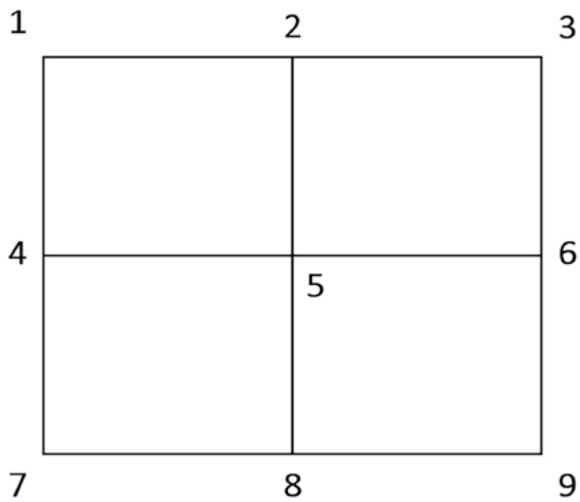
Ellipsoid	α_0	β	$\lambda_{h,\alpha,b}$
Sensitivity	0.05	0.20	12.828
Confidence	0.05	0.27	11.070

For the computation of the global sensitivity values (see Eqs. 4 and 5), we used the maximum and minimum eigenvalues of C_d and the non-centrality parameters of Table 6. The results are presented in Table 7:

Table 7: Global sensitivity values for the third experiment.

Ellipsoid	$b_{\min}$ **(mm)**	$b_{\max}$ **(mm)**	**Average (mm)**
Sensitivity	2.06	3.58	2.82
Confidence	1.92	3.32	2.62

By analyzing Table 7, we note that the size of the sensitivity ellipsoid is higher than the confidence ellipsoid as expected, once that $h = 5 < h^* = 7.3$ (Figure 1). After, a new experiment was developed with a leveling network with 9 points and 12 leveling differences with a standard deviation of 1 mm (Figure 5).



Source: Author’s

Figure 5: Leveling network with 9 points.

The adjustment procedure, the determination of non-centrality parameter, and global sensitivity values were carried out according to the previous experiment. The results are presented in Table 8.

Table 8: non-centrality parameter for sensitivity and confidence ellipsoids for leveling network with 9 points and 12 height differences.

Ellipsoid	α_0	β	$\lambda_{h,\alpha,b}$
Sensitivity	0.05	0.20	15.022
Confidence	0.05	0.18	15.507

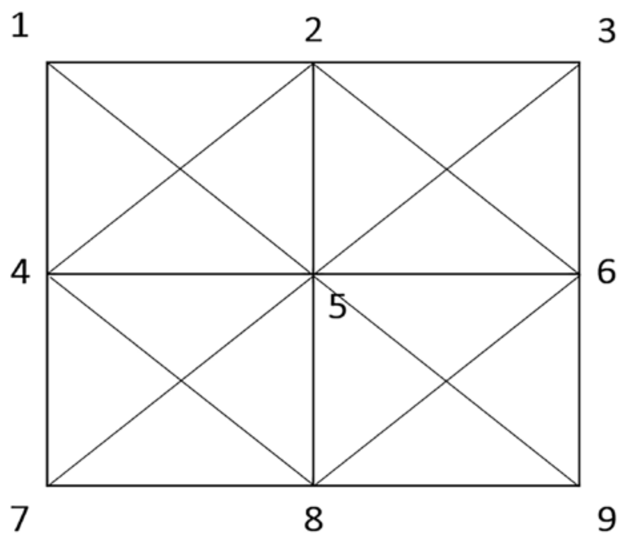
The global sensitivity values are presented in Table 9.

Table 9: Global sensitivity values for leveling network with 9 point and 12 height differences.

Ellipsoid	$b_{\min}$ **(mm)**	$b_{\max}$ **(mm)**	**Average (mm)**
Sensitivity	2.24	5.48	3.86
Confidence	2.27	5.67	3.97

The results in this experiment show that the non-centrality parameter (and thus the respective ellipsoid) obtained for the confidence approach was higher than the sensitivity approach, according to $h = 8 > h^* = 7.3$. The difference between these experiments and those presented in Prószyński & Łapiński (2021) is that here we addressed geodetic network applications rather than a theoretical analysis without displacement values.

To evaluate the network configuration influence, observations were included in both networks analyzed. Figures 4 and 6 showed the new configuration for each network.



Source: Author's.

Figure 6: 1D network adapted.

In both cases, the new observations have a standard deviation of 1 mm. Note that the non-centrality parameters are the same as in Tables 6 and 8 respectively since we do not change the number of network points (and thus the value of h). The new global sensitivity values are presented in Tables 10 and 11.

Table 10: Global sensitivity values for the first network (6 network points) with new observations.

Ellipsoid	$b_{\min}$ (mm)	$b_{\max}$ (mm)	Average (mm)
Sensitivity	2.07	2.07	2.07
Confidence	1.92	1.92	1.92

Table 11: Global sensitivity values for the second network with new observations.

Ellipsoid	$b_{\min}$ (mm)	$b_{\max}$ (mm)	Average (mm)
Sensitivity	1.83	3.64	2.73
Confidence	1.86	3.69	2.77

By Analyzing Tables 7 and 10 or Tables 9 and 11, we note that the global sensitivity values decrease for both networks adding new observations, especially the b_{max} values for the first network. We can also note that $b_{min} = b_{max}$ for the network when all points are tied with each other as in Figure 4 and all observations have the same precision (the same is not true for the second network as shown in Figure 6). Furthermore, the differences between the global sensitivity values of the sensitivity and the confidence approach decrease for both networks.

This kind of analysis provides interesting tools for the pre-analysis or design of deformation networks, being not covered before in the theoretical experiments of Prószyński & Łapiński (2021). To evaluate the role of the stochastic model, the new observations in Figure 4 were now defined with a standard deviation of 2 mm. The results are presented in Table 12.

Table 12: Global sensitivity values for network of Figure 4 with new observations with a standard deviation of 2 mm.

Ellipsoid	$b_{\min}$ (mm)	$b_{\max}$ (mm)	Average (mm)
Sensitivity	2.07	2.93	2.50
Confidence	1.92	2.72	2.32

The same case was considered with the new observations in Figure 6. Each new observation has now a standard deviation of 2 mm and the results are presented in Table 13.

Table 13: Global sensitivity values for the second network with new observations with a standard deviation of 2 mm.

Ellipsoid	$b_{\min}$ (mm)	$b_{\max}$ (mm)	Average (mm)
Sensitivity	2.18	4.76	3.47
Confidence	2.21	4.83	3.52

By Analyzing Tables 10 and 12 or Tables 11 and 13, we note that the global sensitivity values increase if the standard deviation of the new observations increases from 1 mm to 2 mm as expected. However, the global sensitivity values decrease in relation to the original case without new observations for both networks. Besides that, the differences between the global sensitivity values of the sensitivity and the confidence approach decrease again for both networks (see Tables 7 and 12 or Tables 8 and 13). Consequently, these experiments also show the role of the stochastic model in this kind of analysis, especially when designing deformation networks.

Therefore, the addition of new observations reduces the MDD values, even if these observations are of poorer precision than the previous ones. Furthermore, increasing the network's redundancy reduces the discrepancies between the results of significance and sensitivity analysis.

4. Conclusions

In this work, we have studied the relationship between significance and sensitivity in MDDs computation. First, under the GCT approach, we compare the detectability of significance and sensitivity analysis. Here we found that, the network configuration, stochastic model, and the type of analysis, i.e., if global or local influences on the MDD values.

In addition, we analyzed the Prószyński & Łapiński, (2021) method under the same conditions. Thus, the influence of network configuration and stochastic model on the variance factor method *(I)*, which jointly analyzes aspects of sensitivity and accuracy in the pre-analysis of geodetic networks showed that If the network and stochastic model improvement, namely, the addition of more observations and better standard deviations for the observations, provides on average better values for MDD and reduces the magnitude between the semi-major axis of the sensitivity and significance ellipsoids. These results provides key information for the optimization of geodetic network design.

Hence, the geodesist must be aware of the following issue: only the occurrence of false positives will be considered (significance analysis) or also the occurrence of false negatives (sensitivity analysis). It should be noted that in the case of geodetic monitoring, the occurrence of false negatives (undetected deformations) is generally more critical than the occurrence of false positives ("false alarm"). For future studies, we recommend analyze some properties of the MDD directions for the smallest and largest directions for simultaneous displacements (Table 4 and Table 5) and the design or pre-analysis of a real monitoring geodetic network considering all the aspects addressed here.

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AUTHOR´S CONTRIBUTION

Author1: Conceptualization, methodology, drafting (writing – original and review), data collection, data analysis, editing, final approval; Author2: Conceptualization, methodology, drafting (writing – original and review),

data collection, data analysis, editing, final approval; Author3: drafting (writing – original and review), data curation, final approval; Author4: Conceptualization, methodology, drafting (writing – original and review), data collection, data analysis, editing, final approval.

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